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A dynamical model of behavioural specifications

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Tomasz Nowicki

Abstract

We give a description of the *behavioral specification* in terms of infinite sequences. This description allows the use of the well developped theory of *subshifts of finite type* and the formulation of some invariance properties. Our aim is to introduce a new language in this area.

0 Introduction

In this report we introduce a mathematical model for the *system behaviour paradigm.* This behavioral specification is explained in [3], where the motivation, general introduction and specific examples are given. We assume that this paper is known to the reader.

First we give the outline of the idea. We deal with a system being a (finite) set of states. We observe a state, it may change into another one by some prescribed rules, this is the evolution of the system.

We assume the following : The system behaviour is described by the sequences of its states. Not all the sequences are possible. The restrictions are given by the rules, represented by tasks, which are transformations of the set of states into itself.

In fact this transformations are parameterized by the same set of states (or : a task is a map from the carthesian product of two copies of the set of states into one copy). We understand this in a dynamical way, given an initial state (paprameter) a rule transforms a given (source) state into a new (target) one. We say that a task starts at one state, ends at the second one and the state evolued to the result of the task.

Possible sequences are those for which next element is an image of the previous one by a task started some time before. We understand that the set of states and the set of tasks are given and finite (or countable).

Our aim is the understanding and classification of asymptotical behaviour of the system, i.e. of such sequences. The first step is to separate possible behaviors of the system (possible sequences) into two classes, in one the behaviour is eventually constant (which may be called a dead-lock), in the other is not. Of course this is very rough, as for example periodic behaviour is mixed up with an aperiodic one, but the method we use can be developed in order to deal with more involved problems

Now we sketch the model. In the next section (1) we give precise definitions. In section 2 we construct some interesting invariant sets in the most general situation. Section 3 follows with some more specific models, next (section4) we describe our model in terms of the theory of subshifts of finite type and we show he links with a thermodynamical formalism.

- In order to settle the notion of the system we introduce the set of possible states called a world. The behaviour of the system is a sequence of states, so we assume that the changes occur discretely (i.e. not continuously). We shall say earlier or later (i.e. introducing time) in the sense of the precedence in such a sequence. The time is discrete and is modeled by natural (or entire) numbers.

- Actions are transformations of the world into itself. So we may say that an action changes a state, meaning that the next element of the sequence is an image of the previous one under the action.
- In fact we want the action to be a rule how to change the state when the action is terminated given a state when it is initiated. Therefore we may speak about tasks. A task is a function from world into itself parameterized by the world.
- The set of parameters such that the task is not identity on the world is called *a preguard* of the task.
- The set of states which are changed by some parameters is called *a postguard* of the task.
- The notions of *guards* appearing in [3] are defined in a different way, but essentially describe the same properties.
- In what follows we think of an action to be a task with a fixed parameter. The preguard and the postguard of an action are those of corresponding task.
- The world and some set of tasks is called a project. Possible sequences realized by a project are such that may be submitted to some additional restrictions.
- One of such restrictions could be described by the notion of the agency (see section 3).
  - Given an abstract (finite) set $\mathcal{A}$ we assign to each moment a collection of card $\mathcal{A}$ actions.
  - The change in state is due to one of the actions (called the active action in contrast to other pending actions).
  - The change in assignment is due to two rules. First we skip the active action and maybe some actions for which the current state is not in the postguard. Then we add actions such that the new state is in their preguards.
- We may impose some restrictions on the distribution of the actions. For example we may want that an action persists until it is terminated i.e. until there was an attempt to implement it in the system. That means that for a given agent an action assigned to it at some moment (initiated) does not change in following moments until it becomes active (at some moment the system is changed by this action or if not in the postguard the action is abandonned).
- We may also want that any initiated action terminates (becomes active or is skipped - 'tried to be active but did not meet a postguard condition').
- Often a carthesian model is used. Then the world is a carthesian product of some sets and the actions consist in changing some coordinates of the state by the function of same coordinates of the parameters.
- The carthesian model is idempotent, by this we mean that using twice one action consecutively gives the same result as using it just once.

- There is a natural dynamic in the set of sequences, which is given by a shift (see section 4). This corresponds to the evolution in time. If we say that some evolution is (time-)invariant we mean that some subset of such a collection is invariant under the shift.
- In the set of sequences we may introduce distance and probability based on distance and probability in the world. Thus we may speak of convergence, limits and attractors in the topological and measure–theoretical senses.
- For a finite world the theory of such sequences is called the theory of subshifts of finite type. In this theory one may describe such physical notions as energy, pressure or entropy.

1 Definitions

In this section we introduce the notions which will be used in this report. We shall use the following notation :

- We denote by $\mathcal{F}(\mathcal{A},\mathcal{B})$ the set $\mathcal{B}^{\mathcal{A}}$ of all maps from $\mathcal{A}$ to $\mathcal{B}$.
- For $A, B \subset \mathcal{W}$ and $\mathcal{Z} \subset \mathcal{F}(\mathcal{W}\times\mathcal{W},\mathcal{W})$ we denote by $\mathcal{Z}(A,B)$ the set $\{w \in \mathcal{W} : \exists_{Z\in\mathcal{Z},a\in A,b\in B}\ w = Z(a,b)\}$.

Now we construct the system.

- We shall call the set $\mathcal{W}$ **a world.** Essentially we think that $\mathcal{W}$ is finite but large.
- We call an element from $\mathcal{F}(\mathcal{W}\times\mathcal{W},\mathcal{W})$ **a task.**
- Given $p \in \mathcal{W}$ and a task Z we call $Z_p \in \mathcal{F}(\mathcal{W},\mathcal{W})$ defined by $Z_p(w) = Z(p,w)$ **an action.**
- Let $\mathcal{Z} \subset \mathcal{F}(\mathcal{W}\times\mathcal{W},\mathcal{W})$. (We think of it to be finite). **A project** is a pair $(\mathcal{W},\mathcal{Z})$.
- **A path** of the project is a sequence $(w_n)_{n=0}^{\infty} \in \mathcal{W}^{\mathcal{N}}$ such that for any $i > 1$ there exists a $Z \in \mathcal{Z}$ and $j < i$ such that $w_{i+1} = Z(w_j, w_i)$.
- **Remark** We may say also that for any path in a project there exists a sequence of natural numbers j_i and a sequence of tasks Z_i such that $w_{i+1} = Z_i(w_{j_i}, w_i)$, and $j_i < i$ for $i > 0$. The sequence $j = j_i$ might not be defined in an unambiguous manner. E.g. if $w_0 = w_1$ and $w_3 = Z(w_0, w_2)$ then j_2 may be either 0 or 1.
- Given a project we call **a preguard** of the task $Z \in \mathcal{Z}$ the set
$$P_Z = \{p \in \mathcal{W} : \exists_{w\in\mathcal{W}}\, Z(p,w) \neq w\} \subset \mathcal{W} .$$
We call **a postguard** of Z the set
$$Q_Z = \{q \in \mathcal{W} : \exists_{w\in\mathcal{W}}\, Z(w,q) \neq q\} \subset \mathcal{W} .$$
- Remark The notions of *guards* may be introduced in a more general way. For any task $Z \in \mathcal{Z}$ sets P and Q called pre-and post guards such that $Z(p,w) = w$ for each $p \in \mathcal{W} \setminus P$ and each $w \in \mathcal{W}$ and $Z(w,q) = q$ for each $q \in \mathcal{W} \setminus Q$ and each $w \in \mathcal{W}$. Clearly the above defined sets P_Z and Q_Z are minimal with this properties as allways $P_Z \subset P$ and $Q_Z \subset Q$. For some technical reasons it may be sometimes usefull to have less restrictive definitions. This have no importance in following considerations and we shall not discuss this any further.

- The preguard and the postguard of the project are the unions

$$\mathcal{P} = \bigcup_{Z \in \mathcal{Z}} P_Z \qquad \mathcal{Q} = \bigcup_{Z \in \mathcal{Z}} Q_Z \, .$$

2 Invariance

Now we shall now define inductively some subsets of $\mathcal{W}$ and $\mathcal{Z}$.

Definition 2.1 *First we set $\mathcal{W}_0 = \mathcal{W}$. Suppose $\mathcal{W}_n$ is defined for some $n \geq 0$ then we put*

$$\begin{aligned}
\mathcal{P}_n &= \{p \in \mathcal{W}_n : \exists_{w \in \mathcal{W}_n, Z \in \mathcal{Z}} \, Z(p,w) \neq w\} \\
\mathcal{Q}_n &= \{q \in \mathcal{W}_n : \exists_{w \in \mathcal{W}_n, Z \in \mathcal{Z}} \, Z(w,q) \neq q\} \\
\mathcal{Z}_n &= \{Z \in \mathcal{Z} : \exists_{p \in \mathcal{P}_n, q \in \mathcal{Q}_n} \, Z(p,q) \neq q\} \\
\mathcal{W}_{n+1} &= \mathcal{Z}_n(\mathcal{P}_n, \mathcal{Q}_n) = \{w \in \mathcal{W}_n : \exists_{p \in \mathcal{P}_n, q \in \mathcal{Q}_n, Z \in \mathcal{Z}} \, w = Z(p,q)\}
\end{aligned}$$

Lemma 2.1 *We have $\mathcal{W}_{n+1} \subset \mathcal{W}_n$, $\mathcal{P}_{n+1} \subset \mathcal{P}_n$, $\mathcal{Q}_{n+1} \subset \mathcal{Q}_n$ and $\mathcal{Z}_{n+1} \subset \mathcal{Z}_n$.*

Proof :

1. Because $\mathcal{Z}_0 \subset \mathcal{Z} \subset \mathcal{F}(\mathcal{W} \times \mathcal{W}, \mathcal{W})$, $\mathcal{P}_0 \subset \mathcal{W}$ and $\mathcal{Q}_0 \subset \mathcal{W}$ we have $W_1 = \mathcal{Z}_0(\mathcal{P}_0, \mathcal{Q}_0) \subset \mathcal{Z}(\mathcal{W}, \mathcal{W}) \subset \mathcal{W} = \mathcal{W}_0$.

2. Suppose that $\mathcal{W}_{i+1} \subset \mathcal{W}_i$ for $i < n$ with $n > 0$.

3. Let $p \in \mathcal{P}_{i+1}$ then there exists $w \in \mathcal{W}_{i+1} \subset \mathcal{W}_i$ such that $Z(p,w) \neq w$ for some $Z \in \mathcal{Z}$ therefore $p \in \mathcal{P}_i$.

4. Similarly if $q \in \mathcal{Q}_{i+1}$ then there exists a Z and $w \in \mathcal{W}_{i+1} \subset \mathcal{W}_i$ such that $Z(w,q) \neq q$ therefore $q \in \mathcal{Q}_i$.

5. If $Z \in Z_{i+1}$ then there are $p \in \mathcal{P}_{i+1} \subset \mathcal{P}_i$ and $q \in \mathcal{Q}_{i+1} \subset \mathcal{Q}_i$ such that $Z(p,q) \neq q$ therefore $Z \in \mathcal{Z}_i$.

6. Now we can prove that $\mathcal{W}_{n+1} \subset \mathcal{W}_n$. We have

$$\mathcal{W}_{n+1} = \mathcal{Z}_n(\mathcal{P}_n, \mathcal{Q}_n) \subset \mathcal{Z}_{n-1}(\mathcal{P}_{n-1}, \mathcal{Q}_{n-1}) = \mathcal{W}_n \, .$$

These sets have following properties.

- Three conditions are equivalent $\mathcal{P}_n \neq \emptyset \Leftrightarrow \mathcal{Q}_n \neq \emptyset \Leftrightarrow \mathcal{Z}_n \neq \emptyset$

  Proof : Each of this conditions says that there exist $Z \in \mathcal{Z}$, $p \in \mathcal{W}_n$ and $q \in \mathcal{W}_n$ such that $Z(p,q) \neq q$.

- For any $A \subset \mathcal{W}_n \setminus \mathcal{P}_n$ and any $B \subset \mathcal{W}_n$ we have $\mathcal{Z}_n(A,B) \subset B$.

  Proof : If $A = \emptyset$ then $\mathcal{Z}_n(A,B) = \emptyset$. Otherwise let $a \in A$ then for any $w \in \mathcal{W}_n$ and any $Z \in \mathcal{Z}$ we have $Z(a,w) = w$. Therefore for any $Z \in \mathcal{Z}_n$ we have $Z(a,B) = B$ and $\mathcal{Z}_n(A,B) = B$.

- For any $B \subset \mathcal{W}_n \setminus \mathcal{Q}_n$ and any $A \subset \mathcal{W}_n$ we have $\mathcal{Z}_n(A, B) \subset B$.

  Proof : If $B = \emptyset$ or $A = \emptyset$ then $\mathcal{Z}_n(A, B) = \emptyset$. Otherwise let $b \in B$ then for any $w \in \mathcal{W}_n$ and any $Z \in \mathcal{Z}$ we have $Z(w, b) = b$. Therefore for any $Z \in \mathcal{Z}_n$ we have $Z(w, B) = B$ and $\mathcal{Z}_n(A, B) = B$.

Theorem 2.1 *For any n the pair $(\mathcal{W}_n, \mathcal{Z}_n)$ is a project.*

That means that $\mathcal{Z}_n \subset \mathcal{F}(\mathcal{W}_n \times \mathcal{W}_n, \mathcal{W}_n)$ or in other words that $\mathcal{Z}_n(\mathcal{W}_n, \mathcal{W}_n) \subset \mathcal{W}_n$.
Proof : We have

$$\begin{aligned} \mathcal{Z}_n(\mathcal{W}_n, \mathcal{W}_n) &= \mathcal{Z}_n(\mathcal{W}_n \setminus \mathcal{P}_n, \mathcal{W}_n) \cup \mathcal{Z}_n(\mathcal{P}_n, \mathcal{W}_n \setminus \mathcal{Q}_n) \cup \mathcal{Z}_n(\mathcal{P}_n, \mathcal{Q}_n) \\ &\subset \mathcal{W}_n \cup (\mathcal{W}_n \setminus \mathcal{Q}_n) \cup \mathcal{W}_{n+1} \subset \mathcal{W}_n \ . \end{aligned}$$

□

Corrolary 2.1

1. *For any n the pair $(\mathcal{W}_n, \mathcal{Z})$ is a project.*
2. *We have the following invariance $\mathcal{Z}(\mathcal{W}_n, \mathcal{W}_{n+1}) \subset \mathcal{W}_{n+1}$.*

Proof :

1. Again we have to prove $\mathcal{Z}(\mathcal{W}_n, \mathcal{W}_n) \subset \mathcal{W}_n$. Consider $w = Z(p, q)$, for $p, q \in \mathcal{W}_n$. If $w = q$ then $w \in \mathcal{W}_n$, if $w \neq q$ then $Z \in \mathcal{Z}_n$, $p \in \mathcal{P}_n$ and $q \in \mathcal{Q}_n$. Hence $w \in \mathcal{W}_{n+1} \subset \mathcal{W}_n$.
2. Again we use the fact that if for $p, q \in \mathcal{W}_n$ and $Z(p, q) \neq q$ then $p \in \mathcal{P}_n, q \in \mathcal{Q}_n$ and $Z \in \mathcal{Z}_n$ and $Z(p, q) \in \mathcal{W}_{n+1}$. Let $Z \in \mathcal{Z}$, $p \in \mathcal{W}_n$ and $q \in \mathcal{W}_{n+1} \subset \mathcal{W}_n$. If $w = Z(p, q) = q$ then $w \in \mathcal{W}_{n+1}$. Otherwise $w \in \mathcal{Z}_n(\mathcal{P}_n, \mathcal{Q}_n) = \mathcal{W}_{n+1}$.

□

We remind that for a path (w_i) of a project we may define for any i the number $j_i < i$ is such that there exists $Z \in \mathcal{Z}$ such that $w_{i+1} = Z(w_{j_i}, w_i))$. This sequence is thought to be fixed for a given path, its properties are however valid for any such sequence.

- Define $\nu(i) = \max\{n : w_i \in \mathcal{W}_n\} \leq \infty$, where ∞ is assumed of $w_i \in \mathcal{W}_n$ for all n. We have

  $$\nu(i+1) \geq \min(\nu(j_i) + 1, \nu(i)) \ .$$

  Proof : Let $\nu' = \min(\nu(j_i), \nu(i))$, therefore $w_{j_i}, w_i \in W_{\nu'}$. We have either $w_{i+1} = w_i$ and then $\nu(i+1) = \nu(i)$ or $w_{j_i} \in \mathcal{P}_{\nu'}$ and $w_i \in \mathcal{Q}_{\nu'}$ and $Z \in \mathcal{Z}_{\nu'}$. Therefore $w_{i+1} = Z(w_{j_i}, w_i) \in \mathcal{W}_{\nu'+1}$, and $\nu(i+1) \geq \nu' + 1$. Hence if $\nu(j_i) \geq \nu(i)$ then $\nu(i+1) \geq \nu(i)$ and if $\nu(j_i) < \nu(i)$ then $\nu(i+1) \geq \nu(j_i) + 1$.

- Define $\tilde{\nu}(i) = \min\{\nu(k) : k \geq i\} = \max\{n : \forall_{m \geq i} w_m \in \mathcal{W}_n\}$. Then $\tilde{\nu}(i)$ is not decreasing.

  Proof : We have $\tilde{\nu}(i) = \min(\tilde{\nu}(i+1), \nu(i)) \leq \tilde{\nu}(i+1)$.

Theorem 2.2 *If a path of a project satisfies $j(i) \to \infty$ for $i \to \infty$ for some sequence j_i then either $w_m = w_M$ for some M and all $m \geq M$ or for each n there exists an M such that $w_m \in \mathcal{W}_n$ for all $m \geq M$.*

Proof : It goes through following steps.

1. If the path is not eventually constant then the number of changes in the path $m(i) = \text{card}\{0 < k < i : w_{k+1} \neq w_k\}$ increases with i to ∞.

2. In this case we shall prove inductively that for any $n < \infty$ there exists an M_n such that $\tilde{\nu}(M_n) \geq n$.

3. For $n = 0$ we have $M_0 = 0$ as $w_i \in \mathcal{W} = \mathcal{W}_0$ for all i.

4. Let now $n \geq 0$ and suppose that we found $M = M_n$. We have by definition of M_n that $w_m \in \mathcal{W}_n$ for all $m \geq M$.

5. Let $k \geq M$ be minimal such that $j_i \geq M$ for all $i \geq k$. This is possible as $j_i \to \infty$. Let $N \geq k$ be minimal with $w_{N+1} \neq w_N$. Such N exists as the sequence is not eventually constant.

6. We prove inductively that $w_m \in \mathcal{W}_{n+1}$ for $m > N$.
   - For $m = N$ we have by definition of N that $w_N \in \mathcal{Q}_n, w_{j_N} \in \mathcal{P}_n$ and $w_{N+1} \in \mathcal{Z}_n(\mathcal{P}_n, \mathcal{Q}_n) = \mathcal{W}_{n+1}$.
   - Inductively for $m > N$ we have $w_m \in \mathcal{W}_{n+1}$ and $w_{j_m} \in \mathcal{W}_n$ and $w_{m+1} \in \mathcal{Z}(\mathcal{W}_n, \mathcal{W}_{n+1}) \subset \mathcal{W}_{n+1}$.
   - We have therefore $\tilde{\nu}(N+1) \geq n+1$.

7. We set $M(n+1) := N+1$, because of the last remark the proof is done.

□

Corrolary 2.2 *If for some w and for all $Z \in \mathcal{Z}$ we have $Z(w,w) \neq w$ and $j_i \to \infty$ then there is no eventually constant sequences terminating by $w_i = w$ for large i.*

Proof : Clearly for i large enough $w_{i+1} = Z(w_{j_i}, w_i) = Z(w,w) \neq w$. □

3 Agencies and Managements

In this section we introduce a specific model which realizes the sequences from last section. First we say what are the actions which can change the world.

- A project $(\mathcal{W}, \mathcal{Z})$ defines the set of **available actions** $\mathcal{V} \subset \mathcal{F}(\mathcal{W}, \mathcal{W})$ given by

$$\mathcal{V} = \{\phi \in \mathcal{F}(\mathcal{W}, \mathcal{W}) : \exists_{Z \in \mathcal{Z}, p \in P_Z} \forall_{w \in \mathcal{W}} \, \phi(w) = Z_p(w) = Z(p, w)\} .$$

- An action Z_p is called **idle**, denoted by $Z_p = \text{id}$, if $Z(p_w) = w$ for any $w \in \mathcal{W}$. A task Z is called idle if $Z_p = \text{id}$ for any $p \in \mathcal{W}$ (for a generalized notion of a preguard we demand only $Z_p = \text{id}$ for p in the preguard of Z).

- We say that the project is **complete** if either $\mathcal{P} = \bigcup_Z P_Z = \mathcal{W}$ or there is an idle task in $\mathcal{Z}$.

- If a project is not complete we complete it by adding an idle task to $\mathcal{Z}$.

Now we are ready to describe tha way the world is changed.

Definition 3.1 *Given a (finite) set $\mathcal{A}$ (called* **the agency** *or the set of agents) we define* **a management** *of a (completed) project $\mathcal{P} = (\mathcal{W}, \mathcal{Z})$ to be a set of functions $\mathcal{M} = \mathcal{F}(\mathcal{A}, \mathcal{V})$ valued in the set of available functions $\mathcal{V}$.*

Each elements of this set may be thought of as a decision how to distribute among the agents $A \in \mathcal{A}$ available actions from $\mathcal{V}$. The terms *larger* or *smaller* agency will refer to the cardinality of the set $\mathcal{A}$.

An enterprise $\mathcal{E}$ of an agency $\mathcal{A}$ realizing (or managing) a (completed) project $\mathcal{P}$ is a (maximal) subset of $\mathcal{E} \subset (\mathcal{W} \times \mathcal{M})^{\mathcal{N}}$ of sequences which fulfills the condition stated below. First we explain the structure of an enterprise.

If $e \in \mathcal{E}$, then $e = (w_j, \alpha_j)_{j=0}^{\infty}$. For any $j \in \mathcal{N}$ we have $\alpha_j \in \mathcal{M}$, therefore for any $A \in \mathcal{A}$ the function $\alpha_j(A) \in \mathcal{V}$ is an available map of the form Z_p for some $i \in I$, $Z \in \mathcal{Z}_i \subset \mathcal{P}$ and $p \in P_i$. Hence an agent may act on the world $\mathcal{W} \to \mathcal{W}$ by $w \mapsto Z_p(w) = Z(p, w)$, which we may write $w \mapsto \alpha_j(A)(w)$.

The condition reads : For any j there exists an $A \in \mathcal{A}$ such that

- $w_{j+1} = \alpha_j(A)(w_j)$ and
- $\alpha_{j+1}(A) = Z_{w_{j+1}}$ for some $Z \in \mathcal{P}$. (It means implicitly that w_{j+1} fulfills the preguard of $Z \in \mathcal{Z} \subset \mathcal{P}$, this is always possible for some $\mathcal{Z}$, as the project was completed).
- Moreover if $\alpha_{j+1}(A) \neq \alpha_j(A)$ then $\alpha_{j+1}(A) = Z_{w_{j+1}}$ for some Z, i.e. new available actions assigned to the agents at this moment must be anchored at w_{j+1}.

We say that an enterprise is **quiet** if the distribution of non-idle activities of agents α_{j+1} does not differ to much from the distribution α_j. By this we mean that if $w_{j+1} = \alpha_j(A_0)(w_j)$ and $A \neq A_0$ then if $\alpha_j(A) \neq \mathrm{id}$ then $\alpha_{j+1}(A) = \alpha_j(A)$.

A short way to say it is that at the moment j the change in the distribution of available actions may occur only at idle actions and the action which just took place. The management takes as little decision as possible, summoning only agents which are not active and the agent which just finished its job.

We say that the enterprise is **active** if for any agent with an idle activity $\alpha_j(A) = \mathrm{id}$ we have $\alpha_{j+1}(A)$ is not idle if possible. By this we mean that if w_{j+1} belongs to a preguard P_i, whose task is not idle then $\alpha_{j+1}(A) \neq \mathrm{id}$ for $A \in \mathcal{A}$.

The projection of an enterprise on $\mathcal{W}^{\mathcal{N}}$ is called **a development** $\mathcal{D}$

$$\mathcal{D} = \{(w_j)_{j=0}^{\infty} : \exists\, e \in \mathcal{E}\ e = (w_j, \alpha_j)_{j=0}^{\infty}\}$$

for some sequence of α_j. An element of a development (i.e. a sequence (w_j)) is called **a path.**

- Given the world $\mathcal{W}$ we say that the enterprise $\mathcal{E}$ is *wider* than $\mathcal{E}'$ if $\mathcal{D}' \subset D$.
- We say that an enterprise $\mathcal{E}$ is developing *quicker* than $\mathcal{E}'$ if for any sequence $(w') \in \mathcal{D}'$ and $j' > 0$ there exists a sequence $(w) \in \mathcal{D}$ and $j < j'$ such that for any $l \geq 0$ we have $w'_{j'+l} = w_{j+l}$. That means that any subsequence from $\mathcal{D}'$ appears sooner in $\mathcal{D}$.
- We use the word *eventually* in front of wider or quicker if there exists an N such that these relations are restricted for the all sequences after rejecting first N elements.

In order to compare two developments we have to consider the following equivalence relation

Definition 3.2 *We say that a sequence is* **simple** *if the equality $w_j = w_{j+1}$ for some j implies $w_i = w_j$ for all $i > j$.*

A simplification of a sequence is its maximal simple subsequence.

We may think of it as of a map $s : \mathcal{W}^{\mathcal{N}} \to \mathcal{W}^{\mathcal{N}}$ such $s(w_j) = w_{k_j}$ with $k_0 = 0$ and if k_j is defined then either $k_{j+1} = i$ is minimal $i > k_j$ such that $w_i \neq w_{k_j}$ or if $w_i = w_j$ for all $i > j$ then $k_{j+1} = k_j + 1$.

We say that two sequences are *simply the same* if they have same simplifications

Example 3.1 *Consider the world $\mathcal{W} = \{0,1\}^N$ for some $N \in \mathcal{N}$. Let $\mathcal{P} = \cup_{i=0}^{N+1}\{Z_i\}$ where for $i = 1,\ldots,N$ we have $P_i = \{w \in \mathcal{W} : w_1 = 0\}$, $Q_i = \{w \in \mathcal{W} : w_j = 1, j < i\}$ and $Z_i(p,w)_j = 1$ for $j \leq i$ and $Z_i(p,w)_j = 0$ for $j > i$. For $i = 0$ we set $P_0 = \{(1,\ldots,1)\}$, $Q_0 = \mathcal{W}$ and $Z_0(p,w) = (0,\ldots,0)$. For $i = N+1$ we set the preguard $P_{N+1} = \mathcal{W} \setminus \cup_{i=0}^{N} Z_i(\cdot,\mathcal{W})$, postguard $Q = \mathcal{W}$ and $Z(p,w) = (0,\ldots,0)$. We complete the project in a standard way.*

This project is ending in idle actions if the agency has cardinality smaller than N.

Example 3.2 *Let $\mathcal{W} = \{0,1,2,3\}$. For $i = 1,2$ let $\mathcal{Z}_i = \{Z_i\}$ with $Z_i(p,w) = 0$ for $w = 0,i$, $Z_i(p,3) = i$ and $Z_i(p,3-i) = i$. If $Q_i = \{0\}$ then the task $\mathcal{Z}_i$ is idle. If $0 \notin Q_i \neq \emptyset$ then the task is tight. If the project consists only of one of these tasks then it lands at 0 after at most three steps. If the project includes both of them then eventually periodic sequences as 312121212... appear in the development.*

Theorem 3.1 *Two agencies of the same cardinality realizing same project have the same development.*

Proof : We have to show that any sequence (w_j) from the development $\mathcal{D}$ realized by the management $\mathcal{M}$ of the agency $\mathcal{A}$ belongs to the development $\mathcal{D}'$ realized by the management $\mathcal{M}'$ of the agency $\mathcal{A}'$. If $(w_j) \in \mathcal{D}$ then there exist α_j and A_j, $j = 0,1,\ldots$, such that the sequence $(w_j,\alpha_j,A_j) \in \mathcal{M}$. In view of equal cardinalities there exists a bijection $\sigma : \mathcal{A} \to \mathcal{A}'$. We define a sequence (w_j,α'_j,A'_j) by $A'_j = \sigma(A_j)$ and $\alpha'_j(A') = \alpha_j(\sigma^{-1}(A'))$. This sequence lies in $\mathcal{M}'$. □

In fact we proved more : there is a bijection between two managements of two agencies of the same cardinality. We proved it by showing a conjugacy. The same art of proof is valid for $\operatorname{card}\mathcal{A} < \operatorname{card}\mathcal{A}'$ if there is no special conditions on the management.

Theorem 3.2 *Larger agencies have larger managements (and larger developments).*

Proof : Let $\mathcal{A}'' \subset \mathcal{A}'$ and $\operatorname{card}\mathcal{A}'' = \operatorname{card}\mathcal{A}$. If $(w_j,\alpha_j,A_j) \in \mathcal{M}$ then $(w_j,\alpha_j \circ \sigma^{-1},\sigma(A_j)) \in \mathcal{M}''$ and we may extend the definition of $\alpha' = \alpha \circ \sigma^{-1}$ on the set $\mathcal{A}' \setminus \mathcal{A}''$ by setting constant tasks. Clearly $(w_j,\alpha',\sigma(A_j)) \in \mathcal{M}'$. □

However if we want to have some additional conditions on the management we have to be more carefull.

Example 3.3 *Assume that any initiated action terminates and let us consider the world $\mathcal{W} = \{0..N-1\}$ and the task $Z(w) = w + 1(\mathrm{mod}\, N)$ with $P_0 = \{0\}$. If $\operatorname{card}\mathcal{A} = M$ then for $M < N$ all the paths starting from 0 land in M and stay there ($w_j = M$ for j large enough). For $M \geq N$ all of them are cyclic.*

Theorem 3.3 *In an agentural project (a project managed by an agency) when all the actions terminate for any sequence (w_i) we can pick the sequence $j_i \to \infty$.*

Proof : The chosen sequence is defined by the sequence of active actions. As each action terminates the function $j_i : \mathcal{N} \to \mathcal{N}$ is at most $\operatorname{card}\mathcal{A}$-to-1. □

4 Subshifts of finite type

If we suppose that the world $\mathcal{W}$ is finite and the project $\mathcal{P}$ is finite then we may use the modeling *via* the theory of subshifts of finite type. In this section we give an outline of this theory.

Let $\mathcal{K}$ be a finite set and $\mathcal{X} = \mathcal{K}^{\mathbf{N}}$ be the set of (one-sided) sequences valued in $\mathcal{K}$. The dynamical system $(\mathcal{X}, \sigma)$ is called a full shift, where the shift σ acts on the sequence (a_n) by skipping its first element, $\sigma((a_n)_{n=0}^{\infty}) = (a'_n)_{n=0}^{\infty}$ with $a'_n = a_{n+1}$.

Now let $T = (t_{m,n})$ be a $k \times k$ matrix, $k = \operatorname{card} \mathcal{K}$, and $t_{m,n} \in \{0,1\}$. We define a subspace $\mathcal{X}_T \subset \mathcal{X}$ by the condition $(a_n) \in \mathcal{X}_T$ iff for any n $t_{a_n, a_{n+1}} = 1$. We can describe the sequences from $\mathcal{X}_T$ as the sequences for which the possible successors $a' = a_{n+1} \in \mathcal{K}$ (in the sequence) of the element $a = a_n \in \mathcal{K}$ are described by the permission matrix T. The matrix is called irreducible if there is some m such that T^m has all entries strictly positive. It means that after m steps we may go from any state to any state. The entries of T^m, which are obviously natural numbers, represent the number of paths joining two states in m steps.

Example 4.1 *Let* $\mathcal{K} = \{1,2\}$ *and*

$$T = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix},$$

then $\mathcal{X}$ *is the set of all sequences of 1 and 2 and* $\mathcal{X}_T$ *are the sequences with no two consecutive 2's.*

If $\mathcal{K} = \{1,2,3\}$ *and*

$$T = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix},$$

then $\mathcal{X}_T$ *is the set of all sequences of 1,2 and 3 where 3 must be followed by 2 and preceded (if not staying at the beginning of the sequence) by 1.*

The dynamical system $(\mathcal{X}_T, \sigma)$ is called the subshift of finite type. For more information see [1]. In this system the following notions are naturally defined.

- **The metric.** Let $\tilde{d}$ be the discrete metric on the set $\mathcal{K}$. Then we can define the metric d on $\mathcal{X}$ (and on $\mathcal{X}_T$ as well) by $d((a_n),(b_n)) = \sum \tilde{d}(a_n, b_n)/2^n$. The balls consist of sequences with same values of in given finite number of initial coordinates. Two sequences are close if their sufficiently long initial parts are identical.

- **The topology** is the metric topology in the above sense. The base of neighbourhoods are the so called cylinders, i.e. the sets with fixed values on some finite coordinates. The cylinders are the unions of finite number of balls.

- **The measure.** Let $(p_i)_{i=1}^{k}$ be a probability distribution on the set $\mathcal{K}$. We may define the probability of the cylinder by the product of p_i's corresponding to the fixed values defining the cylinder. This definition expands (*via* Kolmogorov's theorem) to the measure on Borel sets in $\mathcal{X}$. The measure on $\mathcal{X}_T$ may be (with some care) derived by taking the conditional measure.

We want to describe the evolution of a system with acting agents as a sequence of states. One sequence represents one possible development of a system. The state is a configuration of the system including *the real world* and *the agents with their private worlds, flags of actual activities and preguard and postguard information.*

The idea borrowed from the *subshifts of finite time* is to reduce the space of all sequences of configurations (states) to the sequences for which only some successors of a state are possible. This corresponds on one hand to the conditions making *actions acceptable* and on the other to the permission matrix. We shall be more specific in the next section.

Let us now consider in detail an easy example. The goal of this example is to present a system small enough to make the full description readable but which has the features leading to more complicated applications as a system with a flip-flop and a counter controlled by several agents.

Example 4.2 (An agent copies the flip-flop into a register) *Consider the system S with a flip--flop x and a register r. The flip-flop changes the value from 0 to 1 and from 1 to 0 independently from the rest of the system. The aim of the acting agent A is to put in r the value of x in case when the actual state of the flip-flop is different from the one remembered by the agent. The agent has two registers : a working one w and an activity one a.* □

Let $\mathcal{K}$ be the set of following vectors $(S, A) = ((x, r), (w, a))$ with $x, r, w, a \in \{0, 1\}$. The cardinality of $\mathcal{K}$ is $2^4 = 16$. A 0-1 matrix $T = (t_{V,V'})$ describes the admissible followers $V' \in \mathcal{K}$ of a state V in the sequence of the evolution of the system (S, A).

Instead of writing the 16×16 matrix T we shall describe the permissions, the entry of the matrix is 1 iff the following conditions are fulfilled :

1. $((x, r), A)$ can follow and be followed by $((x', r), A)$ for any choice of x, x' and r, A, i.e. $t_{((x,r),A),((x',r),A)} = 1$. This is the independence of the flip-flop.

2. The value of w is changed into the value of x when some condition $G(S)$ (*a preguard* from [3]) is fulfilled, then also a changes from 0 to 1 (the agent becomes active). In other words under $G(S)$ for $V = ((x, r), (w, 0))$ and $V' = ((x', r), (x, 1))$ we have $t_{V,V'} = 1$, for any x, x', w and r. In our case G is fulfilled by any state S (there is no *preguard* condition).

3. When $G(S)$ is fulfilled and $a = 0$ then the only possible follower of V is V' with $a' = 1$ and $w' = x$.

4. a may change from 1 to 0 only if w changes to $w' = f(w)$ and y changes to $F(S, A)$. f is a predefined function depended of the aim of the agent, in our case $f(w) = w$, and $F(S, A) = r$ or $f(w)$ depending if some condition (*a postguard* from [3]) is fulfilled. In our case $F(S, A) = f(w)$ if $f(w) = w \neq x$ and y when $f(w) = w = x$. That means that $t_{V,V'} = 1$ for

   (a) $V = ((0, r), (1, 1))$ and $V' = ((x', 1), (1, 0))$ or

   (b) $V = ((0, r), (0, 1))$ and $V' = ((x', r), (1, 0))$ or

   (c) $V = ((1, r), (1, 1))$ and $V' = ((x', r), (1, 0))$ or

   (d) $V = ((1, r), (0, 1))$ and $V' = ((x', 0), (0, 0))$

5. We do not permit the register r to change in a different way than by converting it into the value of w as described above at (4).

6. We do no say what happens to w when a stays equal to 0 (this situation is in our case excluded by (2,3)), but when $a, a' = 1$ w cannot change. When $a = 1$ and $a' = 0$ there is no condition on w.

$V \setminus V'$	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
0									1	1						
1													1	1		
2											1	1				
3															1	1
4									1	1						
5													1	1		
6											1	1				
7															1	1
8	1	1			1	1			1	1						
9	1	1			1	1			1	1						
A			1	1			1	1			1	1				
B	1	1			1	1					1	1				
C			1	1			1	1					1	1		
D	1	1			1	1							1	1		
E			1	1			1	1							1	1
F			1	1			1	1							1	1

Table 1: The permission matrix for a system copying a flip-flop. Explanation in the text.

Here is the whole permission matrix T. We represent the state by the hex-digit equal to $x + 2r + 4w + 8a$. The double lines separates different activities ($a = 0$ and $a = 1$) and single lines different w. We mark only the entries equal to 1. Repeated double ones say that the flip-flop may change its state in an independent way.

The first quadrant ($a = 0, a' = 0$ of the matrix is empty because in our example the system must change from idle to active (condition 3) in particular because there is no restriction given by a *preguard*, as $G(S)$ is always fulfilled. The second quadrant ($a = 0, a' = 1$) shows the copying of x into w' while leaving r. The third one ($a = 1, a' = 0$) shows what happens when the agent terminated its job. There are two pairs of entries in each row due to the fact that the new value of w' is not determined. The register r changes only in rows B and D. The fourth quadrant shows that (up to the flip-flop changes) the system stays identical when the agent is busy.

Let us take now the set $\mathcal{X}$ of all (one-sided) sequences of symbols $0 \ldots F$, and $\mathcal{X}_T$ the subshift derived from $\mathcal{X}$ with the matrix T. The sequences from $\mathcal{X}_T$ represent all possible evolutions of the system which fulfill the rules of permission. We can now compare two evolutions, try to find an invariant measure then attractors (in both topological and metrical sense).

5 The thermodynamics

There is already a very well developed theory of subshift of finite type. In this section we point out how the notions of thermodynamics may be used in the investigations of dynamical systems consisting of sets of sequences as the phase space and a shift as the transformation. This is based on the books [1, 2].

Probability measures, introductory notions. Let $(X, \mathcal{B}, \mu)$ be a *probability space*, i.e. $\mathcal{B}$ is a σ-field of subsets of X (called *measurable* sets) and μ is a nonnegative *measure* on $\mathcal{B}$ with $\mu(X) = 1$. Usually we work with a fixed transformation of the space. We want this transformation to preserve the measure (or the measure to be invariant with respect to the transformation).

An *automorphism* is a measurable bijection $T : X \to X$ (i.e. $T^{-1}E \in \mathcal{B}$ iff $E \in \mathcal{B}$) for which $\mu(T^{-1}E) = \mu(E)$, $E \in \mathcal{B}$. An *endomorphism* of a probability space is a measurable transformation such that $\mu(T^{-1}E) = \mu(E)$, $E \in \mathcal{B}$.

In case when X is a compact space and T is a homeomorphism (or a continuous map) one usually sets $\mathcal{B}$ to be the family of Borel sets. The measure is called then a Borel probability measure. Let $M(X)$ be the set of Borel probability measures and $M_T(X)$ its subset of measures which are invariant with respect to T. We have by definition $\mu \in M_T(X)$ iff $\mu \circ T^{-1} = \mu$. For any $\mu \in M(X)$ we can define $T^*(\mu) = \mu \circ T^{-1}$.

Real-valued continuous functions $C(X)$ on the compact metric space X form a Banach space with the norm $||f|| = \sup_{x \in X} |f(x)|$. The weak $*$-topology on the space $C(X)^*$ (i.e. the space of continuous linear functionals $\alpha : C(X) \to \mathcal{R}$) is generated by the sets of the form $U(f, \epsilon, \alpha_0) = \{\alpha \in C(X)^* : |\alpha(f) - \alpha_0(f)| < \epsilon\}$ with $f \in C(X)$, $\epsilon > 0$, $\alpha_0 \in C(X)^*$.

Riesz Representation. For each $\mu \in M(X)$ define α_μ by $\alpha_m u(f) = \int f \, d\mu$. Then $\mu \to \alpha_\mu$ is a bijection between $M(X)$ and $\{\alpha \in C(X)^* : \alpha(1) = 1 \text{ and } \alpha(f) \geq 0 : f \geq 0\}$. We identify α_μ and μ. We call the weak topology on $M(X)$ the topology induced by this identification from the weak *–topology on $C(X)^*$.

We have following properties of the spaces $M(X)$ and $M_T(X)$.

- *$M(X)$ is a compact, convex, metrizable space.*
  This follows from the fact that the weak topology on $M(X)$ is equivalent with the topology induced by the metric $d(\mu, \nu) = \sum_{n=1}^{\infty} | \int f_n \, d\mu - \int f_n \, d\nu| \cdot ||f_n||^{-1}/2^n$, where (f_n) is a dense subset of $C(X)$.

- *$M_T(X)$ is a nonempty closed set of $M(X)$.*
  T^* is a homeomorphism of $M(X)$ and $M_T(X) = \{\mu \in M(X) : T^*(\mu) = \mu\}$. For $\mu \in M(X)$ let ν be an accumulation point from $\frac{1}{n} \sum_{k=0}^{n-1} (T^*)^k \mu$. Then ν is T invariant.

- *$\mu \in M_T(X)$ iff $\int (f \circ T) \, d\mu = \int f \, d\mu$ for all $f \in C(X)$.*
  This is Riesz representation theorem applied to $T^*\mu = \mu$.

Suppose that $A = (a_{ij})$ is a $n \times n$ matrix of nonnegative integers. We may consider a (closed) subset Σ_A (resp. Σ_A^+) of Σ (resp. Σ^+) consisting of the sequences $\mathbf{x}$ such that $a_{x_k x_{k+1}} > 0$ for any k. We may assume that A is such that for any $k \in F$ there is an $\mathbf{x} \in \Sigma_A^{(+)}$ with $x_0 = k$, otherwise one can take an $m \times m$ matrix B with $m < n$ and $\Sigma_A^{(+)} = \Sigma_B^{(+)}$. These sets with a shift transformations are subshifts of finite type.

Let us state the following result. The shift τ is topologically mixing (i.e. for any U, V nonempty open subsets of $\Sigma_A^{(+)}$ there is an N such that $\tau^M U \cap V \neq \emptyset$ for $M > N$) iff $A^M > 0$ (i.e. all entries are strictly positive) for some M.

In the set $C(\Sigma_A^{(+)})$ there is a special of continuous real-valued functions on $\Sigma_A^{(+)}$ where is a special family $\mathcal{F}_A$ of functions with positive Hölder exponent with respect to the metric d_γ. $\phi \in \mathcal{F}_A$ iff $\mathrm{var}_k(\phi) := \sup\{|\phi(\mathbf{x}) - \phi(\mathbf{y})| : x_i = y_i : |i| \leq k\} \leq b\alpha^k$, for some $b > 0$ and $\alpha \in (0, 1)$.

Gibbs measures Suppose that a 'physical system' has possible states $1, \ldots, n$ and the energies of these states are $E_1, \ldots, E_n$. Suppose further that this system is not isolated but in permanent contact with a 'large heat source' which remains at the constant temperature T. Therefore the total energy of the system is not fixed and any state of the system may actually occur. There is a following 'physical' fact. The probability p_j that the system is at state j is given by *Gibbs distribution* $p_j = \exp(-\beta E_j)/\sum_i \exp(-\beta E_i)$, with $\beta = 1/kT$, k a 'physical' constant.

This is connected to the following 'mathematical' fact. Given real numbers $a_1, \ldots, a_n$ the function $F(p_1, \ldots, p_n) = \sum_i (a_i - \log p_i) p_i$ attains in the simplex $\sum_i p_i = 1$, $p_j \geq 0$ a maximum $\log \sum_i \exp(a_i)$ at the point $p_j = \exp(a_j)/\sum_i \exp(a_i)$. The quantity $h(p_1, \ldots, p_n) = \sum_i -p_i \log p_i$ is called the *entropy* of the distribution (p_i). We assume $0 \log 0 = 0$ for $x \log x$ to be continuous at $x = 0$.

If we put $a_i = -\beta E_i$ then we have $\sum a_i p_i = -\beta E$ with average energy E and the Gibbs distribution *maximizes* $S - \beta E$, where S stays for entropy. The *minimized* quantity $P = E - kTS$ is called *free energy*. Therefore the principle reads 'nature maximizes the entropy' when the energy is fixed but 'nature minimizes the free energy' when the energy is not fixed. One can generalize such a distribution to the system Σ_A.

Let $\phi : \Sigma_A \to \mathcal{R}$ be Hölder continuous. Then there is a unique invariant measure $\mu \in M_\tau(\Sigma_A)$ for which one can find constants c_1, c_2 and P such that $c_1 \leq \mu\{\mathbf{y} : y_j = x_j, j = 0, \ldots, m-1\}/exp(-Pm + \sum_{j=0}^{m-1} \phi \circ \tau^j(\mathbf{x})) \leq c_2$. One can call $P = P(\phi)$ a pressure of ϕ. The exact definition of the pressure is more complicated.

For such generalized 'Gibbs distribution' μ one has the *Variational Principle.* $h(\mu) + \int \phi \, d\mu = P(\phi)$.

The measure satisfying the above principle is called an *equilibrium state.* In one-dimensional lattice the equilibrium states for Hölder continuous functions are (unique) Gibbs distributions.

We can call $\phi \in C(X)$ observables, and $\mu \in M(X)$ states in the sense that $\mu(\phi) = \int \phi \, d\mu$ is the average value of ϕ in the space.

The configurations (i.e. the sequences $\mathbf{x} \in \Sigma_A$) can describe possible evolutions of the system. The matrix A describes *pre-* and *post-guards.* If we are interested in asymptotical behaviour of the limit sets we may define an observable ϕ and see what is a state μ which realizes the variational principle. This measure μ shows what configurations are 'important' from the point of view of the observable ϕ.

Now there is a couple of notions which needs to be interpreted in the setting of the behavioral specifications. We may want to know what are *energy, pressure, temperature* ? We may want to look for a 'good' potential energy, so that we see 'interesting' sequences with large probabilities. We may want to understand what are equilibrium states μ and observables ϕ ? Are we interested at all in the infinite setting ? Should we rather concentrate in the finite (but large) systems ?

The easiest case is when one considers as $\phi(\mathbf{x}) = -d_\gamma(\mathbf{x}, S)$, the distance of the configuration to a given set $S \subset \Sigma_A$. In the case of an *algorithm* — by this I mean that we are interested in an evolution of finite number of steps, i.e. finite iterations of the shift — this set S should be (forward) shift invariant or even a fixed point of the shift. The function ϕ cannot be split into two parts, describing the energy of the site 0 (due to x_0) and the potential energy of interactions between the site 0 and sites j for $j \in \mathcal{N}$. Nevertheless it describes in a sense how 'far' the actual configuration is from the desired form. Natural candidate equilibrium measures (states) should have supports on S (the integral part is then 0) and spread equally on S to maximize the entropy. If the set S is not forward invariant then the (invariant) measure cannot be supported only on S.

On the other hand the 'potential energy' may also describe the preferences of the observer, e.g. a weighted distance to some disjoint sets of acceptable 'final' configurations.

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