Experimental and Theoretical Studies of the Seismo-Acoustic Noise Field at Very Low Frequencies

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While the first ambient-noise spectra were published in 1948, it was the growing interest in passive sonar systems in the 1960's that stimulated research into this aspect of underwater acoustics. The first published results relating to low-frequency ambient noise appeared in a paper by Wenz, describing data in the range 50 Hz to 1000 Hz. The noise sources reported included turbulence, wave motion, earthquakes, biological activity, and shipping. Other contributions appearing in that period dealt with wind and wave generated noise, underwater volcanism, and relevant to this report are given in the Abstract.
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While the first ambient-noise spectra were published in 1948[1] it was the growing interest in passive sonar systems in the 1960's that stimulated research into this aspect of underwater acoustics. The first published results relating to low-frequency ambient noise appeared in a paper by Wenz[2], describing data in the range 50 Hz to 10 kHz. The noise sources reported included turbulence, wave motion, earthquakes, biological activity and shipping. Other contributions appearing in that period deal with wind and wave-generated noise[3]-[5], underwater volcanism[6],[7] and relevant to this report a large rise in noise levels at frequencies below 4 Hz [8],[9].

Improved low-frequency data came with the 1970's[10]-[14]. Perrone's results in particular identified several significant spectral characteristics, among which were a slope of -20 dB/octave in the region between 2 Hz and 5 Hz and a predicted maximum between 1 Hz and 2 Hz. The 1980's saw an upsurge of interest in VLF acoustics in general and the natural physical sources responsible for second generation at the ocean surface in particular. Several contributions established the general form of the spectrum in the range 0.1 Hz to 4 Hz[16]-[21] more clearly and Kibblewhite and Ewans[18] were able to demonstrate convincingly that nonlinear wave-wave interaction was the mechanism responsible for the rise in levels below 4 Hz. The spectral levels observed, the spectral slope of -18 dB/octave between 0.3 and 1 Hz, the wind-dependence and the 2:1 ratio of the noise and sea-state spectra, all corresponded closely to the predictions of the theory.
describing the wave-interaction process.

This mechanism was first studied by Longuet-Higgins\textsuperscript{[22],[23]} in connection with the phenomenon of microseisms. He demonstrated that the nonlinear interaction of ocean waves produces pressure fluctuations, which are not attenuated with depth and therefore capable of generating microseisms at the ocean seafloor. In the years to follow Hasselmann\textsuperscript{[24]} and others were also to make significant contributions to the subject. The first complementary analysis in the context of underwater acoustics came from Brekhovskikh\textsuperscript{[26]}. His results express the wave-induced acoustic intensity spectrum, $S(2f)$, in terms of a surface-wave spectrum, $H(f)$, the properties of which were unknown at that time, but $S$ is predicted to be proportional to $f^2H^2$. Later Hughes\textsuperscript{[27]} and Lloyd\textsuperscript{[28]} used the relationships $H \propto f^{-4.5}$ and $H \propto F^{-4}$ to establish spectral slopes above the peak of -21 dB/octave and -15 dB/octave respectively. Kibblewhite and Ewans\textsuperscript{[18]} interpreted their experimental data using the relation $H \propto F^{-4.5}$, actually established in parallel measurements of the ocean-wavefield, to obtain the spectral slope of -18 dB/octave mentioned above. This result, coupled with the other spectral characteristics they identified, led them to conclude that the dependence of the noise field on seastate was consistent with the predictions of the wave-interaction theory as formulated by Longuet-Higgins rather than turbulence\textsuperscript{[25]}. The results of Adair et al\textsuperscript{[19]} and Cotaras et al\textsuperscript{[21]} also supported this interpretation.

From a theoretical point of view, the ultimate understanding of the phenomenon requires the solution of the nonlinear fluid dynamic equation under the constraints of non-stationarity, nonlinear surface conditions and quasi-linear, range-dependent bottom properties. However in all analyses to date the non-stationarity of the seastate and the range-dependence of the sea-bed structure are ignored and the problem, though still difficult, simplifies to the solution of a nonlinear equation under nonlinear surface conditions and linear bottom conditions.

Since the wave-induced pressure field is the result of second-order effects in the fluid motion, Longuet-Higgins introduced the perturbation procedure to establish solutions up to the second order. Further, because two interacting gravity-wave trains must be travelling in nearly opposite directions to generate a propagating acoustic wave, he adopted
a "standing-wave" approximation in his calculation of the induced pressure field. Hasselmann's paper[24], providing a statistical description of the process and a spectral form of the source pressure field, is also based on this approximation.

Longuet-Higgins' perturbation analysis and Hasselmann's spectral description of the wave-induced pressure field have provided the basis for most of the theoretical studies which were to follow[18],[19],[27],[37],[39],[40]. In the past this treatment has proved adequate for most purposes, but more recently the proper interpretation of new experimental data, now becoming available from field programs based on vastly improved instrumentation, has called for the resolution of a number of key theoretical issues and extensions to the theory in its original form. Our activities under this research grant have been aimed at providing these extensions. Essentially all elements outlined in our proposal of 19/3/1990 have been addressed successfully.

The main issues involved have been concerned with:

(1) The role of the inhomogeneous component of the pressure field - a generalised expression for the wave-induced noise field in deep water.

The inhomogeneous part of the wave-induced pressure field comprises those components for which the horizontal wave number, \( k \), is greater than the acoustic wave number in water, \( \omega/\alpha_1 \). Though this part of the pressure field was recognised by Longuet-Higgins it has generally been ignored in the past, all calculations of pressure and seismic spectra being based on the homogeneous wave component alone, for which \( 0 < k < \omega/\alpha_1 \). This approximation has been justified by the fact that the inhomogeneous component decays exponentially with depth and is accordingly negligible in deep water. In shallow-water environments and in the upper levels of the deep ocean, however, the implications of this simplification need to be examined to establish the relative importance of the two components as a function of depth below the seafloor. In shallow-water many seabeds are characterised by sedimentary layers of low rigidity, in which the shear-wave velocities (and compressional) can be less, even much less, than in seawater. As Frisk[37] was the first to demonstrate the inhomogeneous component of the pressure field can be of real relevance to the seismo-acoustic response of such structures. In the case of the wave-induced pressure field the quantitative description of the inhomogeneous component involves an
extension of the "standing-wave" approximation through a full-wave analysis of the interaction process, and the development of a "generalised" spectral expression, which provides a complete description of the pressure field as a function of wave-number, \( k \), frequency \( f \), and observation depth.

This analysis was provided in our contribution\cite{36} - A.C. Kibblewhite and C.Y. Wu, "The theoretical description of wave-wave interactions as a noise source in the ocean", J.Acoust.Soc.Am. 89, 2241–2252 (1991). (See also reference \cite{38}.)

The main features of this analysis are:

(i) A geometric description of the wave-interaction process is developed, which shows that the horizontal-wave number of the induced pressure field cannot become infinite but is limited to a value \( \omega^2/g \) uniquely determined by frequency. This restriction, not previously recognised, leads to a generalised expression for the wave-number frequency spectrum of the pressure field in deep water, which can be integrated over the \( k \)-plane to obtain a total frequency spectrum embracing both the homogeneous and inhomogeneous components.

(ii) Some limited calculations of pressure spectra are made, based on the generalised expression, to demonstrate the relative importance of the homogeneous and inhomogeneous components as a function of wind speed and observation depth, in a deep-water environment.

(2) A theoretical description of the ULF noise spectrum in deep water.


Based on the generalised deep-water formalism described in \cite{36}, this analysis provided a theoretical basis for the form of the whole ULF spectrum. The relative importance of the homogeneous and inhomogeneous components as a function of wind speed and observation depth was shown to play an important role in establishing the "noise notch".

(3) The inhomogeneous component and acoustic source levels.
We also used the generalised expression for the pressure field \(^{(36)}\) in an examination of the influence of the inhomogeneous component on the source levels associated with the nonlinear interaction process. This analysis\(^{(39)}\), "Acoustic source levels associated with nonlinear interactions of ocean waves", J.Acoust.Soc.Am. 94, 3358–3378 (1993), confirmed the effect reported by Schmidt and Kuperman\(^{(32)}\) but demonstrated that the increased seismic response observed in shallow water was a direct consequence of the influence of the inhomogeneous component of the pressure field and not "bottom-magnification" as they suggested. The analysis showed further that the inhomogeneous component drops quickly as the observation point is moved outside the active wave region, and emphasised the relevance of this behaviour to the onshore observation of wave-induced microseisms.

(4) Removal of the deep-water restriction.

As was mentioned above the traditional description of the wave-induced pressure field has been based on the deep-water assumption and the use of the approximate dispersion relation, \(\omega^2 = gk\). The analyses in (1) to (3) above were also based on this approximation.

In recent years a renewed interest in continental shelf environments and the use of the wave-induced pressure field as a source in inversion studies, designed to determine the value of key geoacoustical parameters of ocean sediments\(^{(41),(42),(43)}\), has called for a description of the pressure field which is applicable to environments of any water depth. Such an analysis must be based on the generalised dispersion relation governing the ocean-wave field, include both the primary- and double-frequency components of the induced pressure field and take the elasticity of the seabed into account.

An analysis which provides these developments has been completed. We presented a preliminary account of these extensions to theory to the Lake Arrowhead Conference on Sea Surface Sound '94\(^{(40)}\), the proceedings of which are in press. A more detailed presentation has been prepared and along with other studies not yet reported will be submitted for publication in the near future\(^{(46)}\).

(5) The "difference-frequency" component.

In all previous analyses the calculation of pressure and seismic spectra has involved only the "sum-frequency" component of the wave-interaction process, for which the fre-
quency of the second-order pressure wave, $\omega$, is the sum of the frequencies of the two interacting components of the surface-wave field; that is $\omega = \sigma_1 + \sigma_2$ with $\sigma_1$ and $\sigma_2$ being both positive or both negative. While the reality of the difference-frequency interactions, in which $\sigma_1$ and $\sigma_2$ can be of opposite sign, is acknowledged, no quantitative calculation of this contribution has yet been made. A complete analysis of the process should incorporate the difference-frequency interactions and include an assessment of their relevance to the VLF and ULF (ultra-low-frequency) spectra.

We have now completed an analysis which incorporates both the sum and difference frequency components of the interaction process in the analytical description of the wave-induced pressure field\cite{46}. Calculations of wind- and depth-dependent spectra based on this formalism, show that the difference-frequency interactions can be a significant component of the ULF spectrum at frequencies below the main spectral peak (i.e. below $f \approx 0.04$ Hz).

(6) Bottom-reflection loss investigations.

Because of the computational limitations of the time, the early analyses of the wave-induced seismo-acoustic fields were of necessity restricted to simple geoacoustic models in which the seabed was regarded as either a liquid or an homogeneous elastic halfspace. With the modern facilities now available it is possible to consider the influence of more realistic representations of the seabed on the seismo-acoustic response. The significance of the inhomogeneous component of the wave-induced pressure field, demonstrated in the analyses described above, make such developments essential for shallow-water environments.

Our examination of these issues has involved two aspects of the problem. Given the significance of the inhomogeneous component of the wave-induced pressure field, we first embarked on a study of the reflection loss of a multilayered visco-elastic seabed at very low frequencies, for both the homogeneous and inhomogeneous components of the wave-induced noise field. In an extension of these studies we have also examined the effect of introducing porosity as a sediment parameter.

The results of these investigations have revealed aspects of the reflection-loss process, not previously well recognised, which are of significance to all bottom-interaction acous-

The transfer functions used to calculate the wind-dependent pressure and seismic spectra resulting from the wave-induced source field, and described in (1) to (6) above, were evaluated using reflection coefficients based on these studies.

(7) A point-source solution to the acoustic-gravity equation.

The first- and second-order equations of the perturbation analysis are the acoustic-gravity wave equations, \( L\Phi^{(1)} = 0 \) and \( L\Phi^{(2)} = F(\bar{R}, t) \) where \( L \equiv c^2 \nabla^2 - g \partial / \partial z - \partial^2 / \partial t^2 \) and the corresponding boundary conditions apply. Traditionally, the first-order field, \( \Phi^{(1)} \), has been established by introducing a plane-wave solution and determining the complex amplitude from the boundary conditions. Solution of the second-order field, \( \Phi^{(2)} \), has been based either on a plane-wave solution (with the active wave region regarded as infinite in size) or use of the Green's function and simpler acoustical wave equation, and its associated boundary conditions, where the operator is \( L \equiv c^2 \nabla^2 - \partial^2 / \partial t^2 \). To maintain theoretical consistency and remove the restriction of water depth and frequency range, the Green's functions must be derived directly for the operator \( L \). Since this operator is not self-adjoint certain complications are involved in the derivation. In a recent paper, Guo[49] claimed to provide a form of the Green's function for an ocean of infinite depth but gave no derivation. To facilitate general application, Green's functions are required, with appropriate analytical detail, for both the first- and second-order fields and any specified environment.

We have completed this analysis. The results will be presented in [46]

(8) The coherence properties of the wave-induced pressure field.

The analysis in (7) allows the spatial coherence properties of the wave-induced pressure field to be examined. These calculations have demonstrated clearly the nature of the coherence function and its wind- and depth-dependence. A preliminary account of these investigations was given in [40]. A more complete account will be reported in [46].
(9) Propagation of the wave-generated microseism field.

In the early treatments of the seismic field only the response to the homogeneous component of the pressure field was considered. Furthermore calculations were restricted to the far field, for which the "residue" analysis was particularly suitable. Given the increasing sophistication of the subject and the new computational techniques now available, a more comprehensive analysis is required, which describes the propagation of the seismic response in multilayered environments from the near to the far field.

A preliminary account of an examination of this problem was presented in [39]. A study which extends consideration to range-dependent environments is partially completed.

(10) Comparison of the perturbation procedure and the acoustical analogue analysis.

In the historical development of the subject two different theoretical approaches have been used to describe the wave-induced acoustic field. The first is that developed by Longuet-Higgins in which the velocity potential is chosen as the field variable, the field and the boundary conditions are both linearized explicitly through standard perturbation procedures, and the pressure fields of different orders are derived from the corresponding potential. The other approach, developed by Lighthill [50], chooses the pressure as the field variable and rearranges the terms of the fluid dynamic equation to form a nonhomogeneous acoustic wave equation. The equation is linearized by assuming the nonhomogeneous terms on the right-hand side of the equation are known, while the equations governing the surface conditions are linearized by integration over the average plane, \( z = 0 \), instead of the random surface, \( z = \zeta(x,y) \). Apart from the choice of the main field variable the two approaches are therefore basically the same in the context of the present subject. This is still the case in the more recent applications of the Lighthill procedure [47],[48],[49] where the wave surface is described using the Heaviside function or the divergence theorem.

As no formal attempt has previously been made to reconcile the two procedures, we have attempted such an analysis. This study does reveal differences between the higher-order solutions obtained by the two analytical techniques. The results of this study will be presented in [46].

(11) An integrated account of the seismo-acoustic effects of wave-wave interactions.

While the pioneering works of Longuet-Higgins and Hasselmann remain as key bench
mark contributions in the development of the subject, the improved computational power now available has permitted some aspects of the subject to be revisited. Extensions to theory are now available which address many of the issues outlined above, at least in part, and remove restrictions imposed by necessity in the original treatments. These have not only clarified the nature of the physical processes involved but facilitated the exploitation of the wave-induced pressure field as a unique source of low-frequency energy in the exploration of seabed structures.

The accounts of many of these developments have, unfortunately, appeared in the literature in a somewhat disjointed manner, with the result that the evolution of the subject has not been easy to follow. A contribution is being prepared[46], which aims to present a more coherent account of those developments which have been published in recent years and relate them to others we have not yet reported. The latter, which are concerned primarily with the extensions to theory required for shallow-water environments, form the bulk of the contribution. Exact expressions are developed, which allow the wave-induced pressure and seismic fields to be calculated for any geophysical environment that can be approximated by a combination of parallel, porous and/or viscoelastic layers. Material will be included which is fundamental to the analyses described, but is too detailed for publication in a standard scientific journal. Examples of theoretical wind-dependent spectra based on this formalism are given for a number of typical environments. Comparisons with experiment are made although field data based on the new technologies is still not extensive at this time. However the results of major programmes are expected soon. It is hoped that publication of a coherent account of the theoretical developments made in recent years will facilitate the interpretation of these data. The data will of course provide the means of checking further the validity of the extensions to theory.

References


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