Midcourse Guidance Strategies For Exoatmospheric Intercept

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Abstract

This paper considers the interception of an exoatmospheric target with a ship-based interceptor which employs both midcourse and terminal guidance. Before and during the midcourse phase of flight the target is tracked with a ship-based radar. With target state estimates derived from the radar measurements, the interceptor is launched at the expected intercept point. The inevitable intercept point prediction errors are reduced during the interceptor's flight with midcourse guidance updates from the ship. When the interceptor's seeker acquires the target, noise free terminal guidance information is assumed for guiding on the actual target. The purpose of this paper is to briefly investigate various midcourse guidance strategies which will influence the missile's terminal performance.

Introduction and Overview

In this paper, a hypothetical surface launched exoatmospheric anti-tactical ballistic missile interceptor is considered. The interceptor consists of three propulsion stages and a fourth stage with a kinetic kill vehicle KKV. The first two stages speed up the interceptor in the atmosphere taking it from a vertical launch and then pitching it over to a commanded injection reference vector (i.e., constant flight path angle). This guidance philosophy enables the interceptor to nearly be on a collision course with the predicted intercept point at the end of the second stage. In other words, if the intercept time is known precisely and the flight path angle is correctly chosen the missile would intercept the target without any further steering corrections.

As the interceptor ascends and the dynamic pressure drops, the missile can no longer maintain aerodynamic control and the exoatmospheric midcourse stage separates. This third stage is designed to support long range exoatmospheric missions and to further correct any errors resulting from the first and second stages. In this paper the third stage motor is assumed to consist of two pulses. The first is fired early to boost the average speed to intercept while correcting for known heading or intercept point prediction errors. The second pulse is then ignited after some interpulse delay time to allow other errors to be corrected. The steering correction on this stage is referred to as a midcourse divert because it is conducted without the use of an on-board seeker.

Finally, a kinetic kill vehicle is deployed which acquires the target using a terminal seeker. This vehicle must carry sufficient fuel to allow for a final steering corrections. Since fuel, weight and cost are intimately related the interceptor's lateral divert and acceleration capability is limited. The homing time of the KKV and the divert sets the amount intercept point prediction error for which it may correct. This sets the handover basket to which the midcourse stage (i.e., third stage) must deliver the vehicle.

During a mission, each of the stages must steer out the residual errors from the previous stage. Errors will result because of inaccuracies in the predicted intercept point (i.e., estimation errors, lack of knowledge of the target, etc.), saturation effects and choosing improper reference angles.

The interceptor is command guided for the first three stages based upon information derived from ship based radar measurements of the target. The paper will illustrate, via examples, that the radar update rate is an important factor in determining system performance. Two different methods for implementing second stage guidance will be explored and the impact on overall system performance will be investigated. The importance of the acceleration limits in the homing phase and the length of homing time will be demonstrated. Finally, it will be shown how the loss of radar data impacts system performance.

Generic Interceptor

Table 1 presents the thrust weight properties of the generic interceptor considered in this paper. We can see that the first stage burns for 9 sec at a high thrust level while the second stage burns for 30 sec at
a lower thrust level. There is a delay of 1.5 sec before the third stage begins to burn. As mentioned before, the third stage consists of two pulses. The first pulse burns for 10 sec and then after an interpulse delay (not shown in Table 1) the second pulse also burns for another 10 sec. The thrust levels for each pulse of the third stage are the same.

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>Weight (lbs)</th>
<th>Thrust (lbs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2806</td>
<td>24612</td>
</tr>
<tr>
<td>9-</td>
<td>2000</td>
<td>24612</td>
</tr>
<tr>
<td>9+</td>
<td>1888</td>
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<td>274</td>
<td>4044</td>
</tr>
<tr>
<td>60.5+</td>
<td>67</td>
<td>0</td>
</tr>
</tbody>
</table>

*There can be a delay between the 2nd and 1st pulse

Table 1. Generic interceptor's thrust-weight profile

It is assumed that the generic interceptor has a zero lift drag coefficient of .3 and a reference area of .994 ft². The nominal interpulse delay of the third stage is considered to be 4 sec.

First Stage Guidance

The purpose of the interceptor's first stage is to first enable the missile to clear the ship structures by going straight up (90 deg flight path angle) and then after a certain altitude is reached to gradually pitch over to and hold a flight path angle $\gamma_{DES}$ (70 deg in this paper) which will both help the interceptor go in the general direction of the predicted intercept point, avoid excessive loadings and efficiently glide through the atmosphere. In this case the missile uses thrust vector control to attain the desired flight path angle where the missile burn angle $\delta$ is the control variable. The appropriate forces and angles for the first stage portion of the flight are displayed in Fig. 1. In this diagram the missile body and velocity vectors are assumed to be aligned which means the angle of attack is considered to be zero.

![Figure 1. Forces and angles for second stage portion of flight](image)

If the desired flight path angle is $\gamma_{DES}$ then a simple control law to hold the missile on course is given by

$$\delta = \frac{\gamma_{DES} - \gamma}{\tau}$$

where $\tau$ is a time constant chosen by the designer (.25 sec in this paper) and $\gamma$ is the actual flight path angle. We can see that with this type of control law we are trying to drive an error signal to zero which will make $\gamma$ and $\gamma_{DES}$ the same. From Fig. 1 we can see that the accelerations along and perpendicular to the velocity vector (i.e., we are assuming small angle of attack and therefore missile body and velocity vector are aligned) are given by

$$a_M = n_A \cos \gamma - n_L \sin \gamma$$

$$a_{M2} = n_A \sin \gamma + n_L \cos \gamma - g_M$$

As was mentioned previously, initially the missile goes straight up and then pitches over to 70 deg during the first stage of this generic interceptor. Since no target or predicted intercept point information is required during first stage guidance, the missile is flying open loop. The first stage enables the interceptor to fly efficiently in the general direction of the predicted intercept point and sets up conditions for handover to the second stage. In
practice, this goal is often accomplished by attaining the desired flight path angle at a specified altitude and downrange.

The target considered in the paper is drag free and is impulsively launched at 10,000 ft/sec on an initial flight path angle of 45 deg (towards the missile). The target is initially located 1000 km downrange of the missile launch point. In the paper’s simplified analysis only gravity acts on the target. The initial target launch angle is chosen so that the target will hit the missile launch point (i.e., ship). Therefore in this example the interceptor serves as a self defense weapon. The generic interceptor with the thrust-weight properties of Table 1 has tremendous open loop reach capability (i.e., in excess of 1700 km) and can attain a velocity of approximately 4 km/sec. Figure 2 shows the trajectory and velocity profile of the missile for the first 40 sec of flight. Except for the very beginning, the trajectory is nearly a straight line and we can see that the missile velocity reaches 4 km/sec and then gradually diminishes.

![Figure 2. Missile speed reaches 4 km/sec](image)

The first stage guidance is successful from the point of view that the flight path angle has been reduced from 90 deg to 70 deg as depicted in Fig. 3. We can also see that after the flight path angle has been reduced to 70 deg, the flight path angle remains constant until the end of the first stage burn. After that gravity takes over and the flight path angle gradually diminishes.

![Figure 3. First stage guidance brings flight path angle to 70 deg](image)

The burn angle required to reduce the flight path angle to 70 deg and to hold it at that level is displayed in Fig. 4. We can see that initially the burn angle saturates at 25 deg (the limit chosen for this paper) and then diminishes. The burn angle then becomes zero after the first stage guidance is completed and aerodynamic control is used next.

![Figure 4. Burn angle profile required to maintain 70 deg flight path angle](image)

### Second Stage Guidance

In this paper an approximate second stage scheme is used which attempts to guide the interceptor towards the predicted intercept point by trying to hold a constant flight path angle. In reality the desired flight path angle could have chosen from flight tables (based on extensive simulation analysis) but for simplicity was chosen by a simple iterative scheme. Therefore the simplified scheme used in this paper might be a representation of the complex logic which is used in practice. The logic for the simplified scheme is as follows:

Given a launch time, an intercept time somewhat greater than the launch time is arbitrarily chosen and the location of the target at the intercept time is determined based on a perfect model of the target (i.e., integrating target equations of motion forward until desired intercept time). The target location at intercept is known as the predicted intercept point. The predicted intercept point will normally be different from the actual intercept point because the true intercept time cannot be known exactly in advance. In addition, imperfect knowledge of the current target states (i.e., estimation errors due to filtering radar data) will also cause the predicted intercept point to be in error. Assuming a constant speed interceptor, the missile velocity required to reach the predicted intercept point at the intercept time is calculated. If the required missile velocity is larger than the true average missile velocity (an input based on the interceptor’s velocity profile), a slightly larger intercept time is then chosen and the process is repeated. If the computed missile velocity always exceeds the average missile velocity, the target is not considered to be reachable. When the computed missile velocity does not exceed the average missile
velocity the desired flight path angle is calculated according to,

\[
\gamma_{DES2} = \tan^{-1} \left( \frac{R_{T2F} - R_{M2}}{R_{T1F} - R_{M1}} \right)
\]

where \( R_{T1F} \) and \( R_{T2F} \) are the downrange and altitude components of the predicted intercept point (i.e., location of the target at the intercept time) and \( R_{M1} \) and \( R_{M2} \) are the components of the missile downrange and altitude at the end of the first stage guidance. The lateral missile acceleration required to maintain the desired flight path angle is given by

\[
\alpha = \frac{V_M (\gamma_{DES2} - \gamma)}{\sqrt{L}}
\]

where \( V_M \) is the missile velocity. This type of control law will attempt to make the actual and desired flight path angles the same. Therefore the total inertial downrange and altitude accelerations (along 1 and 2 axis) acting on the interceptor during the second stage are determined by thrust, drag, the lateral acceleration and gravity and are given by

\[
a_{M1} = \left( \frac{T}{W} \right) \cos \gamma \cdot n \sin \gamma
\]

\[
a_{M2} = \left( \frac{T}{W} \right) \sin \gamma \cdot n \cos \gamma \cdot g
\]

As implemented for this paper, the second stage guidance is very simple and will not exactly hit the predicted intercept point if no other guidance is available. However it will generally come to within a few miles of the predicted intercept point which is considered good enough for purposes of the paper since the third and fourth stage guidance systems (i.e., midcourse and terminal respectively) require very little fuel to remove those additional intercept point prediction errors (since the time to go until intercept is very large).

As an illustration of the general accuracy of the simplified second stage guidance of this paper, a case is selected in which the interceptor is located at the intended impact point of the 1000 km target whose characteristics were previously described. For this example the interceptor is launched 300 sec after target liftoff and the engagement geometry of Fig. 5 results.

The flight path angle chosen for the second stage guidance is based on an average velocity of 2.5 km/sec. Although this velocity is considerably smaller than the real average missile velocity it was the correct value to use to place the interceptor on a collision triangle (i.e., get a good flight path angle). In practice, look-up tables, based on extensive high-fidelity simulation work, would be used to find the best flight path angle for each possible scenario. We can see that for this example the approximate second stage guidance comes to within 5663 ft of the target at intercept. For this example this is the error that will have to be taken out during the midcourse and terminal phases of flight.

Figure 6 shows how the flight path angle varies with each stage. As was mentioned previously, the first stage guidance reduces the flight path angle from 90 deg to 70 deg. During second stage guidance a flight path angle \( \gamma_{DES2} \) of approximately 40 deg is desired and attained. The angle was determined using 2.5 km/sec of average missile velocity and a launch time of 300 sec to keep the interceptor on a collision triangle with the predicted intercept point. After the end of second stage guidance the interceptor simply falls due to gravity and the flight path angle decreases.

As an illustration of the general accuracy of the simplified second stage guidance of this paper, a case is selected in which the interceptor is located at the intended impact point of the 1000 km target whose characteristics were previously described. For this example the interceptor is launched 300 sec after target liftoff and the engagement geometry of Fig. 5 results.
Third Stage Guidance

During the third stage burnout reference guidance is used to bring the missile very close to the predicted intercept point. Burnout reference guidance attempts to find the appropriate axial angle assuming there will not be any further axial accelerations after burnout of the third stage. This is accomplished by driving the missile velocity component perpendicular to the line-of-sight at third stage burnout to be equal to that of the target. This novel and elegant guidance technique was developed by Hughes Missile Systems Company. The method is based on common sense and the principles of feedback. For the interested reader, another more complex and less robust guidance approach for accomplishing the same goals can be derived from optimal control theory and is more fully described in Ref. 2. We can see from Fig. 7 that burnout reference guidance is indeed very effective. The 5663 ft miss from the end of second stage guidance has been reduced to 3 ft enabling us nearly hit the target without any terminal guidance at all! Of course, in this simplified analysis no other error sources have been considered (i.e., noise). In reality, the miss distance would be much larger and a terminal guidance system would be required to take out the remaining errors.

Figure 7. Burnout reference guidance is very effective

In this particular example there was a 4 sec delay between the first and second pulse of the third stage. We can see from Fig. 8 that approximately 10 deg of burn angle is required for the first pulse and less than 2 deg of burn angle for the second pulse of the third stage in order to accurately guide towards the target.

Figure 8. Very little burn angle is required to take out a few miles of intercept point prediction error

Fourth Stage Guidance

The last stage or KKV uses a terminal guidance scheme based on seeker measurements of the line of sight rate. With this stage the KKV is finally guiding on the actual target rather than the predicted intercept point. For purposes of this paper it is assumed that the method of guidance is proportional navigation, based on noise free seeker information, and that the acceleration limit on the guidance commands is as shown in Fig. 9. The acceleration limit \( a_1 \) is large at the beginning of terminal guidance (\( t = 0 \) in Fig. 9) until time \( \Delta t \) in order to take out the errors resulting from burnout reference guidance without causing saturation. The limit is then reduced to a smaller level \( a_2 \) to conserve fuel. Intercept is assumed to occur at time \( t_F \). The amount of lateral divert and divert distance covered are also indicated in the equations of Fig. 9. For the same amount of divert there are several possibilities for energy management. The amount of lateral divert available during terminal in this paper is considered to be 675 ft/sec. Therefore it is possible to have an acceleration limit of .7 g for 30 sec (i.e., \( a_1 = 7\times32.2, a_2 = 7\times32.2, \Delta t = 0, t_F = 30 \)) or an acceleration limit of 4.2 g for the first 2.5 sec and then a limit of .38 g for the remaining 27.5 sec (i.e., \( a_1 = 4.2\times32.2, a_2 = .38\times32.2, \Delta t = 2.5, t_F = 30 \)). Even though the amount of lateral divert is identical in both cases the former case will yield a divert distance of 10143 ft while the later will yield 14347 ft. The question is from a miss distance point of view does it make any difference?
Unsatisfactory
Mini Studies

In this section several mini studies are conducted for purposes of identifying key system issues. It is not the purpose of this section to reach universal conclusions but simply to identify trends which should be investigated in further detail.

An experiment was run in which the radar measurement noise was present (i.e., 2 mr of angle noise and 10 ft of range noise). The initial missile location was made a parameter for purposes of the mini study. Launch conditions were selected so that in the noise free case, second stage guidance would enable the interceptor to come within a few miles of the target without the need for further guidance. If midcourse and terminal guidance were turned on in the noise free case the miss in these cases would be near zero. The launch conditions for each of the cases are described in Table 2. It is important to note that the average missile velocity in Table 2 does not represent the interceptor’s real average missile velocity but is simply a fudge factor used in second stage guidance to minimize guidance errors (i.e., get correct flight path angle to follow).

<table>
<thead>
<tr>
<th>MsLoc (Km)</th>
<th>Lnch (S)</th>
<th>AvVel (Km/S)</th>
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</thead>
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<tr>
<td>0</td>
<td>300</td>
<td>2.5</td>
</tr>
<tr>
<td>250</td>
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</tr>
<tr>
<td>750</td>
<td>100</td>
<td>2.9</td>
</tr>
</tbody>
</table>

Table 2. Launch conditions for mini study

Cases were first run with all the guidance turned on, but without noise, and all resulted in direct hits. Next, cases were run in which the radar measurement noise was present and the radar sampling time was 1 sec. We can see from the single flight results of Fig. 14 that direct hits are now achieved in all cases! A careful examination of these results indicates that the terminal homing time is now closer to the design goals. The 0 km, 250 km, and 500 km interceptor launches had homing times of 30 sec while the 750 km launch had a slightly reduced homing time of 25 sec. Since the homing time was adequate in all cases, acceleration saturation was avoided and near direct hits were achieved.
Figure 15. Miss is small when second stage guidance logic is improved

To demonstrate that the interceptor does not have to be launched as soon as possible the 750 km case of Fig. 15 was repeated for different launch time delays from the earliest possible launch (i.e., 50 s, 100 s and 125 s). We can see from Fig. 16 that in this case the launch can be delayed for up to 125 sec without losing interceptor performance. If the launch is further delayed the interceptor will not be able to reach the target. Thus we can say the engagement window is quite large for this interceptor.

Figure 16. Engagement window for 750 km launch is large

If there is only one target for the radar to track, small radar sampling times can be achieved. However, if the radar must track several targets, one is interested in seeing how well the system performs at the larger sampling times. The four earliest launch cases of Fig. 15 were rerun, except this time the sampling time was increased from 1 s to 2 s. Near direct hits were achieved for the 0 km, 250 km and 500 km missile launches (.1 ft, .14 ft and .06 ft respectively) but the miss was 1760 ft for the 750 km missile launch. Figure 17 displays the components of the homing guidance command in the earth or inertial coordinates frame (i.e., AGUIJD1 and AGUIJD2). We can see that since the acceleration components are constant, the missile is in acceleration saturation for the entire 25 sec of homing.

Figure 17. 750 km launch has large miss when sampling time is increased because of acceleration saturation

If we increase the acceleration limit from .7 g to 1 g during the terminal phase of flight we can see from Fig. 18 that the missile comes out of saturation after a while. This enables the missile to hit the target at the larger sampling time of 2 sec for the 750 km launch.

Figure 18. Increasing acceleration limit yields near zero miss for 750 km launch

To confirm that the solution of Fig. 18 always works, a Monte Carlo version of the program was written and the miss distances from 25 runs appear in Fig. 19. We can see from the figure that the miss distances are always large when the acceleration limit is .7 g.

Figure 19. Increasing sampling time can cause very large miss distances
Increasing the acceleration limit can reduce miss distance when there is a saturation problem. Figure 20 shows that the miss distances can be reduced to near zero when the acceleration limit is increased from .7 g to 1 g.

\[
\Delta V = (a_1 - a_2) \Delta t + a_2 \Delta F
\]

\[
\Delta V = (7 \times 32.2 - 7 \times 32.2) 0 + .7 \times 32.2 \times 22 = 496 \text{ ft/sec}
\]

Simulation results show that for this case, if the terminal guidance system was shut down entirely, the miss distance would be approximately 7600 ft. For a constant target acceleration of .7 g the maximum divert distance that could be covered would be

\[
\text{dist} = .5 \times .7 \times 32.2 \times 22 = 5455 \text{ ft}
\]

which is less than 7600 ft and is why we have a large miss distance in this case. If we increase the acceleration limit to 1 g we find that the divert distance covered is

\[
\text{dist} = .5 \times 32.2 \times 22 = 7792
\]

which is more than 7600 ft and is why we hit the target when the acceleration limit was increased from .7 g to 1 g. Of course, increasing the g limit also increased the lateral divert. For the same lateral divert, we could increase the divert distance by having a larger acceleration at the beginning of terminal and a smaller acceleration later on. From Fig. 9 we know that

\[
\text{dist} = \Delta t (a_1 - a_2) (t_1 - .5 \Delta t) + .5 a_2 \Delta F
\]

Therefore, assuming \(\Delta V=496 \text{ ft/sec}, t_f=22 \text{ sec}\) and that \(\Delta t=2 \text{ sec}\), we find that the acceleration limit at the beginning of terminal and for the first 2 sec should be 4.44 g (i.e., \(a_1=143 \text{ ft/sec}^2\)) and for the rest of the time should be .33 g (i.e., \(a_2=10.5 \text{ ft/sec}^2\)).

Figure 21 shows that the variable acceleration concept does not reduce the miss distance in this case. The reason for this is that proportional navigation did not require all the acceleration allotted by the divert formula (i.e., \(a_1\) too large) at the beginning but required more acceleration at the end (i.e., \(a_2\) too small). Reducing \(a_1\) and increasing \(a_2\) would not work because we want to preserve 496 ft/sec of divert and 8100 ft/sec of divert distance.

Figure 21. Having two acceleration limits did not reduce the miss in the challenging case

To further explain why performance degraded when the sampling time was increased from 1 s to 2 s in the 750 km launch case a plot of the filter's velocity errors was made. Figure 22 shows how the filter's error in the estimate of the altitude velocity for the two different sampling times varies with time from target launch. We can see that the longer the filter operates on the measurements the smaller the error will be. However, since interceptor launch occurs at 50 sec for this example the velocity error is 11 ft/sec for the 1 s sampling time and approximately 17 ft/sec for the 2 s sampling time. Since the intercept point prediction error is proportional to the velocity error there will be nearly twice the prediction error to remove - thus requiring more terminal acceleration.

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Often the question is asked concerning how much data can be lost without degrading system performance. The answer to the question can be found by performing a simple experiment. A case was run for the 0 km launch and the filter’s velocity errors are displayed in Fig. 23. We can see that the filter’s velocity error’s decrease with time. If data is lost the filter should be able to coast and the velocity errors can be quite small provided the filter has had enough time to settle. We can see from Fig. 23 that if the radar was tracking the target for 100 sec, the altitude velocity errors should be approximately 20 ft/sec whereas the downrange velocity error should be approximately 5 ft/sec. Theoretically, if all the data was lost after 100 sec the filter should coast intelligently since it has a perfect model of the target and all the filter transients should be gone.

Figure 24 confirms the preceding hypothesis - the filter should be able to coast intelligently when data is lost. We can see from Fig. 24 that if all the data is lost after 100 sec the downrange and altitude velocity errors only increase slightly. Simulation results indicate that in this case where the missile launch time was 274 s, a successful intercept resulted because there was adequate terminal acceleration to take out the intercept point prediction errors.

Of course the previous example assumed that the radar was tracking the target since target lift-off. This may not be realistic since the radar may not have sufficient detection range nor can it see the target at such a small elevation angle. However, the principal is still correct and if there is sufficient time for the filter to reduce the velocity errors, losing data afterwards should not seriously degrade system performance.

Summary

This paper has shown the results of some sample experiments conducted with a ship-based interceptor which employs both midcourse and terminal guidance. The paper illustrates that modeling the radar noise and filtering is important in determining system performance and that the radar update rate is also an important factor. Two different methods for implementing second stage guidance were explored. It was found that the method which controlled the fourth stage homing time was the preferred choice in getting near zero miss distance. It was shown that in cases in which the miss distance was very large, near zero miss distance could be obtained by slightly increasing the value of the acceleration limit in the terminal phase of flight. Variable acceleration limits which optimized the divert distance did not appear to help in reducing the miss distance. Finally, it was shown that if the filter has sufficient settling time, losing data afterwards - even for long periods of time - does not necessarily have to degrade system performance.

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References

