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BASIC CONCEPTS OF CYBERNETICS

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BASIC CONCEPTS OF CYBERNETICS

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As in the case of other disciplines it is hard to fix the exact time when cybernetics was born. The fact of the matter is that certain formulations of problems and a number of ideas relating to the field appeared long before our time. It can be said, however, that the shaping of cybernetics into a scientific discipline began in the middle of the twentieth century. This process was fostered by a series of problems presented by practice. Among them we must include the need for intricate computing machines, the automation of production, the automation of certain thinking actions and the study of the mechanisms involved in heredity, evolution and nervous activity. The first attempt to present a unified exposition of cybernetics was made by N. Wiener in 1948. Since, however, Wiener's Cybernetics [1] dealt more with the ideal side of the question, controversies arose among a broad segment of scientists. Some of them, while recognizing cybernetics, demanded a clearer definition of the subject and formulations of its fundamental problems; others, while finding nothing unscientific in cybernetics, said that it was at best a mechanical combination of a number of questions or that it was a part of automatics; still others, not have fully grasped the facts, regarded cybernetics as an attempt to create a new "science of sciences" and therefore called it a "scientific fraud."

In this paper we offer an exposition of the basic concepts of cybernetics, with the intention of filling the gaps referred to above. The conception on which the paper is based arose in 1955 and the writer has used it in different variations in reports on cybernetics.

Here, in the introduction, we will operate with concepts which are not, from the mathematical viewpoint, exact. As far as a strictly mathematical grounding is concerned, it is presented in Chapter I. Inasmuch as the only way of explaining the concepts is via the substance of the matter, we will begin our examination by considering a group of examples.

In nature, in production, in engineering, in science, and elsewhere, we are constantly dealing with so-called control systems. To set off the informality yet concreteness of these objects we will henceforth call them physical control systems. Examples of such systems are:

a) nerve tissue exerting some effect on organs;
b) automatic lines of machine tools;
c) digital computers;
d) molecules having certain properties;
e) algorithms for solving mathematical problems, etc.

The question naturally arises what marks a particular object as a physical control system? To clear up this matter let us examine an example a little more closely.

Figure 1 shows a physical control system for switching on and off light on a staircase with the aid of switches located on each floor.

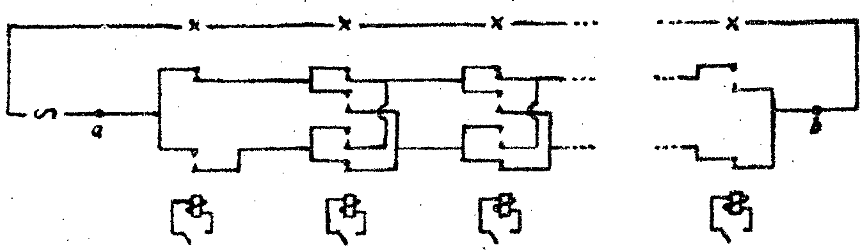

Fig. 1

The system we are interested in is shown in the drawing between points a and b. It includes circuit-closing and circuit-opening contacts and a group of relay windings. Thus we have several "elementary" devices--closing and opening contacts--which are connected together so as to form a circuit or scheme. If we use $-x_i-$, $-\bar{x}_i-$ respectively to denote the closing and opening contacts of relay i, the scheme can be pictured as in Figure 2. On the other hand, the system produces a given effect, or, otherwise, performs a certain function.

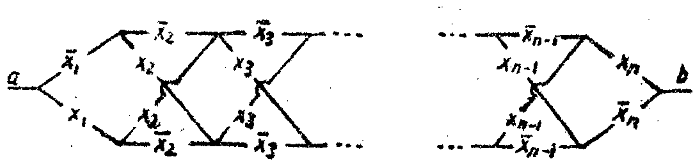

Fig. 2

Indeed, let the relay windings with numbers $1, 2, \ldots, n$ and the scheme, be represented by the Boolean variables $x_1, x_2, \ldots, x_n$ and the Boolean function $f(x_1, x_2, \ldots, x_n)$ so that the variable $x_i (i = 1, 2, \ldots, n)$ has a value of 1 or 0 depending on whether or not the winding of relay i is excited, and the function $f(x_1, x_2, \ldots, x_n)$ has a value of 1 or 0 where $x_1 = a_1, x_2 = a_2, \ldots, x_n = a_n$ according to whether or not there is conductance between points a and b when the state of relay windings with numbers $1, 2, \ldots, n$ corresponds to values $a_1, a_2, \ldots, a_n$. It is evident that the function $f(x_1, x_2, \ldots, x_n)$ characterizes the scheme's action where

$$f(x_1, x_2, \ldots, x_n) = x_1 + x_2 + \ldots + x_n \pmod 2.$$

The above example leads us to the idea that an arbitrary physical control system should be taken to mean the unity of two things: scheme and function.

We must emphasize here that in our understanding both the concept of scheme and the concept of function have an extremely broad meaning. Schemes may be mathematical formulas, the arrangement of pieces on a chessboard, the structure of connections between centers in computers, the configuration of atoms in a chemical molecule, the structure of neuron synapses in nerve tissue, and so on. Functions may be functions corresponding to mathematical formulas, the set of possible moves in a particular chess position, the set of elementary operations performed by a computer, the properties of a molecule, the relationship between the states of a particular group of neurons and the presence or absence of stimuli acting in a given fashion on nerve tissue, and so on.

The study of physical control systems is carried on by different disciplines related to the most varied fields of learning. For instance, physical control systems bound up with mathematical formulas are studied in mathematics: molecules are examined in both chemistry and physics; nerve tissue is studied in physiology. It is characteristic of these disciplines that they deal with specific, individual physical control systems. Not infrequently they examine only certain aspects of control systems. Algebra, for example, studies identical transmutations of algebraic formulas (schemes), the theory of functions studies different properties of functions, and so forth. Since these disciplines study specific physical control systems or separate aspects of such systems, they do not require a general definition of the physical control system.

The confrontation of different physical control systems leads us to an important concept, the concept of the control system. To explain the essence of the matter we will turn to some examples.

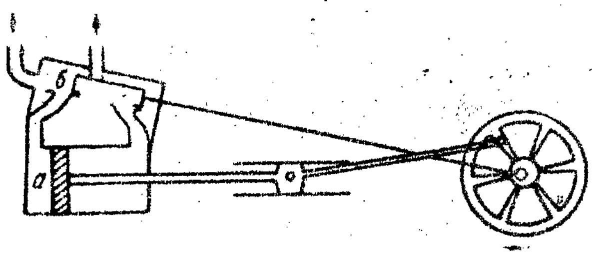

Fig. 3

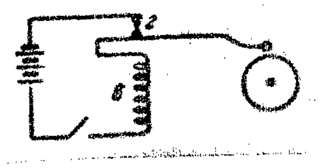

Fig. 4

In Figures 3 and 4 we see the rough sketches of two physical control systems--a steam device and an electric bell.

From the standpoint of use, the essential fact is that the energy of the steam in the steam device is transformed into the forward-and-back motion of the piston and that the electrical energy in the bell is transformed into the oscillatory motion of the hammer. This enables us to treat the examples in a somewhat idealized form. Notably, we will in all places substitute continuous changes with discrete ones.

The basic elements making up the steam device and the electric bell are, respectively, the piston a, the slide valve b, the relay winding c and the circuit-closing contact d. With respect to source of energy, these elements are connected as shown in Figures 5 and 6. These drawings are not schemes, since we see from them only the direct effect of the valve on the piston and of the contact on the winding. They are, however, easily changed into the schemes of corresponding physical control systems once the feedback element is pointed out. Feedback between the valve and piston is accomplished by means of the flywheel, and feedback between the contact and winding is produced by the electromagnetic field of the relay coil.

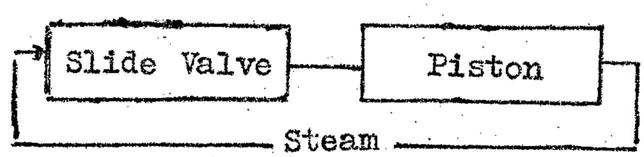

Fig. 5

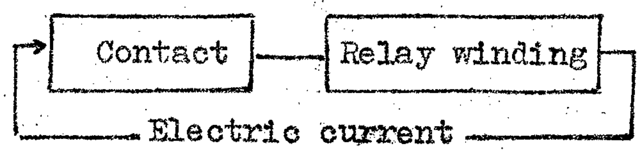

Fig. 6

Now, let us say, that the piston and slide valve (Fig. 3) can only be in two states--left (0) or right (1)--and that the relay winding is either dead (0) or live (1) and the contact either open (0) or closed (1). To find the functional working characteristics of the systems we will first construct tables showing the direct connection and feedback between the elements' states. In constructing the direct connection tables (Tables land 2) we must mentally dispose of the feedback.

Table 1

State of valve at moment t	State of piston at moment $t+\Delta t_1$
0	0
1	1

Table 2

State of contact at moment t	State of winding at moment t
0	0
1	1

Here we must draw attention to the fact that the change in state of the piston does not occur immediately upon the change in state of the slide valve but with a certain delay Δt_1. On the other hand, the change in state of the relay winding occurs almost momentarily upon the change in state of the contacts. In constructing the feedback tables (Tables 3 and 4) we must analyze the feedback processes.

Table 3

State of piston at moment t	State of valve at moment t
0	1
1	0

Table 4

State of winding at moment t	State of contact at moment $t+\Delta t_2$
0	1
1	0

We see from the tables that the change in state of the valve caused by the change in state of the piston due to the feedback (via a lever system) occurs almost momentarily, while the change in state of the contact occurs with a delay Δt_2 after the change in state of the relay winding due to the inertia of the contact's spring.

Since the state of each system is completely determined by the state of one of its elements, it is sufficient to set the functioning of that element. Taking the piston as that element in the steam device and the relay winding in the electric bell, we get the desired functional description (Tables 5 and 6).

Table 5

Time	t	$t+\Delta t_1$	$t+2\Delta t_1$	...
State of piston	0	1	0	...

Table 6

Time	t	$t+\Delta t_2$	$t+2\Delta t_2$	...
State of winding	0	1	0	...

A comparison of schemes and functional characteristics for the two physical control systems shows that what we have here are essentially identical systems. Their alikeness becomes entirely clear when we use special designations. Let X and x be the piston (relay winding) and its state, and $\bar{x}$ — the slide valve (circuit-opening contact) and its state. We can then draw a scheme as in Figure 7, which also shows the feedback between elements $\bar{x}$ and $\bar{X}$. The functioning of the system can be set by means of the formula $x(t+\Delta t)=\bar{x}(t)$, where $^{-}$ signifies logical negation.

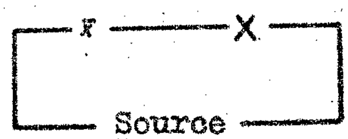

Fig. 7

Thus we have seen directly that the physical control systems being examined have identical (more precisely, isomorphic) schemes and functions. Comparison of physical control systems leads to the conclusion that the multiplicity of all physical control systems breaks down into classes. Each class consists of systems having in some sense identical schemes and functions. By definition, each class sets the control system. Or, otherwise, a control system is a certain mathematical object characterizing the common element contained in identical physical control systems. In this sense the concept of control system has a kinship with other mathematical abstractions, e. g. the concepts of figure, number, etc. A rigorous mathematical definition of a control system is given in Chapter I.

We can now go on to the question of what cybernetics should be taken to mean. In our understanding cybernetics is a mathematical discipline studying control systems. This definition cannot, however, be regarded as complete since it refers to the subject matter of cybernetics and does not indicate either the basic problems solved by cybernetics or its methods. This latter can only be done, and even then with some approximation, on the basis of a precise definition of the control system. Nevertheless, the present definition makes it possible to draw a rough distinction between cybernetics and other disciplines. It is characteristic of cybernetics that it deals with abstract control systems. Other fields, generally speaking, deal with concrete physical control systems. A somewhat special place is held by mathematical logic. It does indeed study special classes of control systems but the essential feature in its work is that the solution of problems relies on the connection between the formulas (schemes) and their realization in some model (functions). This is the basis, for instance, of the solution of such problems as the totality, consistency and independence of systems of axioms. In the case of cybernetics, another sphere of problems is typical, as we will see below.

Nevertheless, logical problems like those of totality, consistency and the like, also arise in cybernetic investigations [2]. Thus cybernetics can be reduced neither to theory of regulation, to automatics, to programming, nor to other disciplines. It would furthermore be mistaken to say that cybernetics embraces the aforesaid disciplines, that it is some kind of "science of sciences." It is entirely obvious that although cybernetics does study the general rules governing control systems, the peculiar features of specific physical control systems lie outside its sphere. For instance, in a cybernetic study of steam devices one is not concerned with the machine's efficiency or operating conditions. Similarly, in the logical computation of electrical schemes one does not treat the scheme's operating stability, its possible length of service, etc. It follows from this that cybernetics does not embrace disciplines studying specific physical control systems. At the same time there is a close tie between cybernetics and the disciplines related to it. And this demands close contact between specialists in different fields in order that, in the course of their joint labors, mathematicians will gain knowledge in related disciplines while specialists in other fields will learn to apply the devices of mathematics in concrete situations.

In expounding the subject of cybernetics it is natural to deal separately with the following questions:

1. The content of the subject (basic concepts and their analysis);
2. Examples of cybernetic objects;
3. Basic tasks of cybernetics;
4. Applications of cybernetics;
5. Philosophical problems of cybernetics (mainly the limits of its applicability).

In this article we examine the first question (Chapter I). At some future date we propose to publish a paper on the two next questions, which will be connected in part with the applications of cybernetics. Philosophical problems require special grounding and we will not, therefore, deal with them.

Chapter I

THE CONTENT OF THE SUBJECT OF CYBERNETICS

Since the central idea of this chapter is the concept of control systems and since this concept requires in turn the defining of many other notions, it is natural to begin the discussion by examining the structure of these relations (Fig. 8).

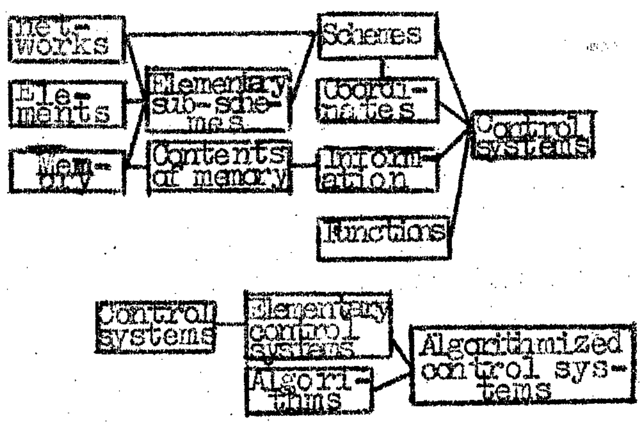

Fig. 8

To make the exposition clearer we will treat these notions along the same pattern. First we will give a definition; then we will comment on the substantive aspect of the notion and its peculiar features. After that we will take up the question of how the identity of the objects being defined is to be understood, i. e. the question of their isomorphism. Lastly, we will offer a rough classification.

In the second part we examine the basic tasks of cybernetics, having in mind of course only those tasks and problems which have come to stand out fairly clearly at the present time.

1. Networks

Let $\mathfrak{M}=\{a_\nu\}$ — be a multiplicity of different objects a_ν, the multiplicity having a capacity m. Also, let E_0 and $E_i (i \geq 1)$ represent sets of objects (a) from the multiplicity $\mathfrak{M}$, the sets having a capacity allowing for repetitions of objects e_0 and e_i respectively. Let us assume that subscript i runs through the segment of transfinite numbers of capacity h, with different subscripts being able to answer to the same sets.

Definition. Multiplicity $\mathfrak{M}$ with a distinct aggregate of sets $E_0, E_1, \ldots$ is called a network and denotes by $\mathfrak{M}(E_0, E_1, \ldots)$, if $|E_0| \subset \bigcup_{1 \leq i \leq h} |E_i|$, where the symbol $|E|$ here and subsequently represents the multiplicity of all objects from set E. The objects making up multiplicity $\mathfrak{M}$, are called the apexes of the network and the objects from set E_0 — are the poles of the network.

It is easily seen that the concept of the network contains within itself the concept of the graph [3]. It would be incorrect, however, to think that every network is a graph with distinct apexes (poles).

A fundamental role in the study of physical control systems is played by networks in which the numbers m, h and $e_i (i=0, 1, \ldots, h)$ are natural numbers. We will call such networks finite. Since we will henceforth be concerned principally with finite networks, we will comment on them at somewhat greater length.

Let $\mathfrak{M}(E_0, E_1, \ldots, E_h)$ — be a finite network and $E_i = (a_1^i, \ldots, a_{e_i}^i)$, where $a_j^i \in \mathfrak{M}\ (i=0, 1, \ldots, h)$. Let us represent each set E_i in three-dimensional space by a small circle, and objects $a_1^i, \ldots, a_{e_i}^i$ from set E_i — by rays extending out from this circle (Fig. 9).

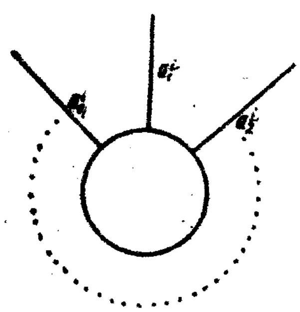

Fig. 9

We will represent E_0 in three-dimensional space as points having one ray each with each of the latter corresponding to one of the objects $a_1^0, a_2^0, \ldots, a_{e_0}^0$. We assume that all rays corresponding to the same objects of multiplicity $\mathfrak{M}$, are connected with each other. The resultant bundles of rays, answering to the same objects a_i, can be made to correspond with the apexes $a_i (i=1, 2, \ldots, m)$. The figure gotten as a result of the constructions is called the geometric realization of the network if the following two conditions are fulfilled:

a) no pair of circles making up the figure may have points in common;

b) the bundles of rays answering to different apexes a_i and a_j have no points in common.

<u>Example</u>. Let $\mathfrak{M} = \{1, 2, 3, 4, 5, 6, 7\}$. We will examine a network $\mathfrak{M}(E_0, E_1, E_2, E_3, E_4, E_5)$, in which $E_0 = (1, 2, 6)$, $E_1 = (1, 3, 3, 4, 5)$, $E_2 = (4, 4, 4, 5, 6)$, $E_3 = E_4 = (2, 4)$ and $E_5 = (2, 5, 6, 7)$. The geometric realization of this network is shown in Figure 10.

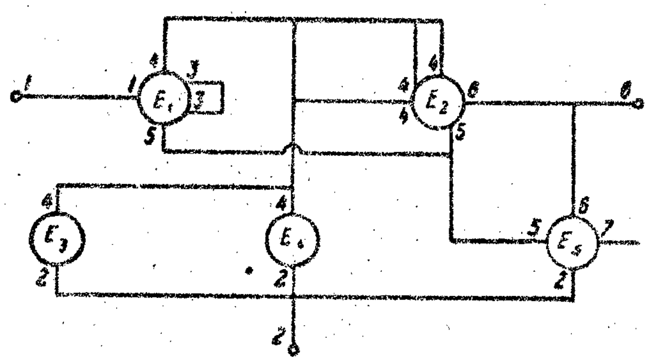

Fig. 10

From topology we know [4] that every finite network may be realized in three-dimensional space. This fact can easily be generalized to the case of calculating networks.

<u>Definition</u>. Networks $\mathfrak{M}_1(E_0^{(1)}, E_1^{(1)}, \ldots)$ and $\mathfrak{M}_2(E_0^{(2)}, E_1^{(2)}, \ldots)$ are called <u>isomorphic</u> if between the objects from $\mathfrak{M}_1$ and $\mathfrak{M}_2$, and also between $\{E_j^{(1)}\}$ and $\{E_j^{(2)}\}$ it is possible to establish an equivalent correspondence such that a) $E_0^{(1)}$ and $E_0^{(2)}$ correspond to each other and b) the corresponding sets $E_j^{(1)}$ and $E_j^{(2)}$ consist of corresponding elements.

We will not henceforth distinguish between isomorphic networks. For example, every finite network is isomorphic in its geometric realization and can therefore be identified with it. This fact, indeed, permits us to regard the concept of network as a topological concept.

In studying control systems we must often avoid considering networks of arbitrary form, restricting ourselves to networks of some special type. The matter of a classification of networks arises in this connection. Below we give a rough classification of networks based on the following characteristics:

1) Capacity m of multiplicity $\mathfrak{M}$. In terms of capacity m networks are divided into:

a) networks with a finite number of apexes;

b) networks that can have an infinite number of apexes.

2) Capacity h. Depending on the magnitude of h we have:

a) networks with a finite number of sets E_i ;

b) networks admitting an infinite number of sets E_i.

3) Capacity e_0 of set E_0. Here we distinguish three classes:

a) networks having an assigned number of poles;

b) networks with a finite number of poles;

c) networks in which an infinite number of poles is possible.

4) Capacities e_i of sets E_i. In this case we have:

a) networks in which capacities e_i are limited in the aggregate;

b) networks in which capacities e_i are finite;

c) networks which can have infinite capacities e_i.

An important particular case in 1 - 4 is the class of finite networks, i. e. networks in which the capacities m, h, e_0 and e_i are finite. This class contains in turn a sub-class of so-called π-networks (parallel-series networks) (b).

5) Connectedness of the network (c). Here we form the following two classes:

a) connected networks;

b) networks admitting, possible, breakdown into unconnected components.

6) Presence of offshoots (d). We have the following cases:

a) networks without offshoots;

b) networks in which offshoots are possible.

7) Presence of cycles (e). In this case we distinguish:

a) networks not containing cycles (trees);

b) networks in which there can be cycles.

8) Ability to be put into a plane. Here we have:

a) plane networks (f);

b) networks which may not be plane.

It is entirely possible that this classification can be even further refined if some other characteristics were taken into consideration.

2. Memory, elements and elementary sub-schemes.

Definition. The multiplicity $\mathfrak{X}=\{X_\alpha\}$ of different objects X_α is called the memory; the objects X_α are called cells.

Substantively memory is taken to mean the repository for storing and recalling information.

Definition. Memories $\mathfrak{X}^1$ and $\mathfrak{X}^2$ are called isomorphic if multiplicities $\mathfrak{X}^1$ and $\mathfrak{X}^2$ have equal capacity.

Since the memory is determined by the capacity of multiplicity $\mathfrak{X}$, a natural classification of memory suggests itself. We indicate here the following most important cases:

a) empty memory;
b) finite memory;
c) calculating memory;
d) non-calculating memory.

Further study of memory is bound up with the definition of the content of memory, given in section 4.

Definition. The symbol $S_\alpha(\ ,\ ,\ )$, having three empty spaces and a certain number of poles (g), is called an element if we are told:

a) the number s_α of poles which the symbol S_α has (h);

b) the cardinal numbers $u_\alpha, v_\alpha, w_\alpha$, corresponding to the first, second and third empty spaces.

We should note here that the numbers $u_\alpha, v_\alpha, w_\alpha$ may be zero. Let us now clarify somewhat the meaning of the numbers $s_\alpha, u_\alpha, v_\alpha$ and w_α. In that which follows the elements will be linked up with the networks and with the memory, namely the poles of element S_α will be brought into an equivalent correspondence with the objects of set E_i; this is possible only on the condition that $s_\alpha = e_i$. With respect to the empty places in the symbol $S_\alpha(\ ,\ ,\ )$, they will be filled with sets $X^\alpha, Y^\alpha, Z^\alpha$ of the cells of some memory $\mathfrak{X}$; ; here an indispensible requirement must be observed--the capacities of sets X^α, Y^α и Z^α are to equal to u_α, v_α and w_α respectively.

Let us suppose that we have two multiplicities of elements $S' = \{S_{\alpha'}(\ ,\ ,\ )\}$ and $S'' = \{S''_{\alpha''}(\ ,\ ,\ )\}$. In each of these multiplicities elements having different designations may a priori be considered identical. To characterize the identicalness of elements we will introduce predicates of equality $R'(S_{\alpha'}, S_{\beta'})$ and $R''(S''_{\alpha''}, S''_{\beta''})$, determined for multiplicities S' and S'' respectively and having the value 1 or 0 according to whether or not the elements in pairs $(S_{\alpha'}, S_{\beta'})$ and $(S''_{\alpha''}, S''_{\beta''})$ are a priori identical. It is also clear that we must further postulate for the identical elements S_α and S_β

$$s_\alpha = s_\beta, \quad u_\alpha = u_\beta, \quad v_\alpha = v_\beta \text{ и } w_\alpha = w_\beta.$$

Definition. The multiplicities of elements S' and S'' are called isomorphic if between the elements of these multiplicities we can establish an equivalent correspondence

$$S'_{\alpha'} \longleftrightarrow S''_{\alpha''},$$

such that

a) the numbers $s'_{\alpha'}, u'_{\alpha'}, v'_{\alpha'}, w'_{\alpha'}$ and $s''_{\alpha''}, u''_{\alpha''}, v''_{\alpha''}, w''_{\alpha''}$, ascribed to elements $S'_{\alpha'}$ and $S''_{\alpha''}$ are respectively equal, namely:

$$s'_{\alpha'} = s''_{\alpha''}, \quad u'_{\alpha'} = u''_{\alpha''}, \quad v'_{\alpha'} = v''_{\alpha''} \text{ и } w'_{\alpha'} = w''_{\alpha''}.$$

b) if $S'_{\alpha'} \longleftrightarrow S''_{\alpha''}$ and $S'_{\beta'} \longleftrightarrow S''_{\beta''}$, then

$$R'(S'_{\alpha'}, S'_{\beta'}) = R''(S''_{\alpha''}, S''_{\beta''}).$$

It can easily be seen that the isomorphic multiplicities S' and S'' are distinguished from one another only by the designations of elements. It follows from the definitions that a reasonable way to classify the elements is in terms of:

a) the difference in symbols S_α, as determined by the predicate R;

b) the number s_α of poles of element S_α;

c) the capacities $u_\alpha, v_\alpha, w_\alpha$.

Remark. Sometimes the elements display two non-intersecting sub-multiplicities of poles, called input and output. In this case we must refine the concept of the indenticalness of elements. In the definition of isomorphism, similarly, we must further demand that the appropriate elements have not only the same number of poles but also the same number of input and output poles. In the classification of elements we must break down the elements in accordance with the number of input and output poles.

The concept of the elementary sub-scheme is built up on the basis of the concepts of memory and elements.

Let $\mathfrak{X}$ - be some memory, E^α — a set of objects from multiplicity $\mathfrak{M}$, and $S_\alpha(\ ,\ ,\ )$ - an arbitrary element having s_α poles and whose empty spaces are represented by cardinal numbers $u_\alpha, v_\alpha, w_\alpha$.

Definition. The symbol $S_\alpha^{E^\alpha}(X^\alpha, Y^\alpha, Z^\alpha)$ is called an elementary sub-scheme over memory $\mathfrak{X}$, if the poles of element $S_\alpha(\ ,\ ,\ )$ are represented respectively by objects of set E^α, having a capacity s_α, and if sets $X^\alpha, Y^\alpha, Z^\alpha$ of memory $\mathfrak{X}$, having capacities $u_\alpha, v_\alpha, w_\alpha$, respectively fulfill the condition

$$(|X^\alpha| \cup |Y^\alpha|) \cap |Z^\alpha| = \Lambda.$$

We will now briefly explain the meaning of the sets referred to in the definition of the elementary sub-scheme. The set Z^α determines the cells of memory $\mathfrak{X}$, which are rigidly connected (i) with the given elementary sub-scheme, these cells containing both the information necessary for the work of the elementary sub-scheme and the information arising as a result of its work. The set X^a singles out the cells of memory $\mathfrak{Z}\setminus|Z^a|$, which contain the information needed for the work of the elementary sub-scheme $S_a^{E}(X^a, Y^a, Z^a)$. Lastly, set Y^a fixes the cells of memory $\mathfrak{Z}\setminus|Z^a|$, which receive information that has appeared owing to the work of the elementary sub-scheme $S_a^{E}(X^a, Y^a, Z^a)$. On the basis of these clarifications it is not difficult to see that the following definitions follow naturally.

<u>Definition.</u> Let $S_a^{E}(X^a, Y^a, Z^a)$ be the elementary sub-scheme over memory $\mathfrak{Z}$. We will call the multiplicities of cells $|Z^a|$ and $\mathfrak{Z}\setminus|Z^a|$ the <u>internal</u> and <u>external</u> memories, respectively, of elementary sub-scheme $S_a^{E}(X^a, Y^a, Z^a)$.

We should point out here that the breakdown of memory into internal and external depends on the choice of the elementary sub-scheme. The introduction of memory breakdown enables us to place the meaning as defined completely within the condition $(|X^a|\cup|Y^a|)\cap|Z^a| = \Lambda$, namely, the multiplicities of cells $|X^a|$ and $|Y^a|$ should belong to the external memory.

<u>Definition.</u> We say that an elementary sub-scheme has <u>feedback</u> if $|X^a|\cap|Y^a| \neq \Lambda$.

Where there is no feedback, the information arising in the work of the elementary sub-scheme does not destroy the initial information. This is also possible, generally speaking, where $|X^a|\cap|Y^a| \neq \Lambda$, namely, when the state of cells from multiplicity $|X^a|\cap|Y^a|$ does not change during the work of the given sub-scheme. We will not deal with this situation here, however, because we have not yet introduced the precise concepts of the state of cells and the work of the elementary sub-scheme.

Thus we have defined the elementary sub-scheme as an element having a determined link with the memory. Consequently, unlike the element, concretized in the elementary sub-schemes are the multiplicities of cells providing the initial information and receiving the results. In that which follows, the schemes will be composed of elementary sub-schemes. For greater convenience we will picture the elementary sub-schemes $S_a^{E}(X^a, Y^a, Z^a)$ as a circle with s_a numbered (as objects of set E^a) rays extending from it and with the symbol $S_a^{E^a}(X^a, Y^a, Z^a)$ in the center (Fig. 11).

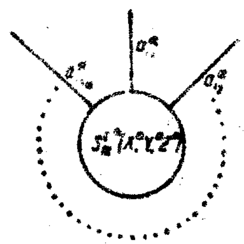

Fig. 11

Remark. In practice frequent use is made of a variety of signs with the appropriate number of poles to designate elementary sub-schemes in depictions of schemes. In this case we need not mark a sign with the symbol $S_a^{E^a}$, but simply write next to it the sets X^a, Y^a, Z^a.

We will use $\mathfrak{S}_0'$ and $\mathfrak{S}_0''$ to designate the multiplicities of the elementary sub-schemes

$$\{S'^{E'^{a'}}_{a'}(X'^{a'}, Y'^{a'}, Z'^{a'})\} \text{ и } \{S''^{E''^{a''}}_{a''}(X''^{a''}, Y''^{a''}, Z''^{a''})\}$$

over memories $\mathfrak{X}'$ and $\mathfrak{X}''$. Let $S'(S'')$ be the multiplicity of all elements belonging to $\mathfrak{S}_0'$ (or $\mathfrak{S}_0''$ respectively).

Definition. The multiplicities of the elementary sub-schemes $\mathfrak{S}_0'$ and $\mathfrak{S}_0''$ are called isomorphic if there is an isomorphism between the multiplicities of elements S' and S'', and between memories $\mathfrak{X}'$ and $\mathfrak{X}''$, such that an equivalent correspondence results between multiplicities $\mathfrak{S}_0'$ and $\mathfrak{S}_0''$, in which the corresponding elementary sub-schemes consist of the corresponding elements and corresponding sets.

Since the concept of elementary sub-scheme is derivative from the concept of element and memory, the classification of elementary sub-schemes amounts to the same thing as the classification of e lements and memories.

Remark. Where the elements have input and output poles, the definition of isomorphism and the particular classification must accordingly be supplemented (see first remark in this section).

3. Schemes. Coordinates.

Definition. Let $\mathfrak{S}_0$ be the multiplicity of elementary sub-schemes over memory $\mathfrak{X}$ and $\mathfrak{M}(E_0, E_1, \ldots)$ a network. The symbol

$$\mathfrak{M}(E_0, S^{E_1}_{a_1}(X^{a_1}, Y^{a_1}, Z^{a_1}), S^{E_2}_{a_2}(X^{a_2}, Y^{a_2}, Z^{a}), \ldots)$$

is called a scheme (over the given multiplicity $\mathfrak{S}_0$ of elementary sub-schemes), if it is gotten as a result of substituting into the network $\mathfrak{M}(E_0, E_1, \ldots)$ in the place of sets $E_1, E_2,$ of the elementary sub-schemes $S_{a_1}^{E_1}(X^{a_1}, Y^{a_1}, Z^{a_1})$, $S_{a_2}^{E_2}(X^{a_2}, Y^{a_2}, Z^{a_2})$, with $e_i = s_{a_i} (i = 1, 2, \ldots)$ and the poles of elementary sub-schemes $S_{a_i}^{E_i}(X^{a_i}, Y^{a_i}, Z^{a_i})$ being placed in a determined fashion (see first six definitions in preceding section) in correspondence with the apexes of set $E_i (i = 1, 2, \ldots)$.

In particular, for each elementary sub-scheme $S_a^{E^a}(X^a, Y^a, Z^a)$ the symbol $\mathfrak{M}(E_0, S_a^{E^a}(X^a, Y^a, Z^a))$, in which $E_0 = E_1 = E^a$, is a scheme. It is natural to identify this scheme with the initial elementary sub-scheme. This may be written as the identity

$$\mathfrak{M}(E_0, S_a^{E^a}(X^a, Y^a, Z^a)) = S_a^{E^a}(X^a, Y^a, Z^a).$$

In the case where input and output poles are isolated in the elementary sub-schemes we can define the concept of such poles as follows: pole $a_v \in E_0$ is called an input (output) if in all sets in which it is found it corresponds to the input (output) poles of the substituted elementary sub-schemes.

In the case of schemes, as in the case of elementary sub-schemes, we may introduce the concept of an internal and external memory and also formulate what we mean by feedback in schemes.

Definition. Multiplicities $Z = \bigcup |Z^{a_i}|$ and $\mathfrak{Z} \setminus Z$ are called, respectively, the internal and external memory of the scheme

$$\mathfrak{M}(E_0, S_{a_1}^{E_1}(X^{a_1}, Y^{a_1}, Z^{a_1}), \ldots).$$

Definition. We say that the scheme $\mathfrak{M}(E_0, S_{a_1}^{E_1}(X^{a_1}, Y^{a_1}, Z^{a_1}), \ldots)$ has feedback if $(|\bigcup X^{a_i}|) \cap (\bigcup |Y^{a_i}|) \neq \Lambda$.

We may make the same remarks about these definitions as about the analogous definitions for elementary sub-schemes. We will point out in addition that the scheme may have feedback even where this is not present in each of the elementary sub-schemes making up the scheme.

Schemes constructed over finite networks are clearly represented geometrically. For this purpose we need only, in the geometrical picture of the network, inscribe in the circle depicting set E_i, the representation of the elementary sub-scheme $S_{a_i}^{E_i}(X^{a_i}, Y^{a_i}, Z^{a_i})$, identifying the corresponding apexes (Fig. 12).

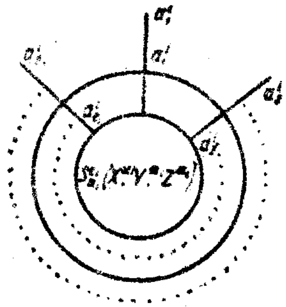

Fig. 12

<u>Definition</u>. Schemes $\mathfrak{M}'(E_0', S_{\alpha_i'}^{E_i'}(X'^{\alpha_i'}, Y'^{\alpha_i'}, Z'^{\alpha_i'}), \ldots)$ and $\mathfrak{M}''(E_0'', S_{\alpha_i''}^{E_i''}(X''^{\alpha_i''}, Y''^{\alpha_i''}, Z''^{\alpha_i''}), \ldots)$ over the multiplicities of elementary sub-schemes $\mathfrak{S}_0'$ and $\mathfrak{S}_0''$ respectively, are called <u>isomorphic</u> if there is:

a) isomorphism of networks $\mathfrak{M}'(E_0', E_1', \ldots)$ and $\mathfrak{M}''(E_0'', E_1'', \ldots)$

b) isomorphism of multiplicities $\mathfrak{S}_0'$ and $\mathfrak{S}_0''$ of elementary sub-schemes with an equivalent correspondence of poles in corresponding elementary sub-schemes such that corresponding sets are replaced by corresponding elementary sub-schemes.

It follows from the definition of schemes that their classification comes to the same as the classification of networks and elementary sub-schemes.

In that which follows we will deal not only with schemes in themselves but also, roughly speaking, with their arrangement in space. We will assume that Ξ is a multiplicity of different objects ξ_ν.

We will look upon these objects as points (coordinates) in some space Ξ.

<u>Definition</u>. We say that scheme $\Sigma = \mathfrak{M}(E_0, S_{\alpha_i}^{E_i}(X^{\alpha_i}, Y^{\alpha_i}, Z^{\alpha_i}), \ldots)$ is <u>arranged in space</u> Ξ, if there is a sub-multiplicity of coordinates $\Xi^0 \subset \Xi$ of capacity Λ (see the definition of a network) such that between the objects of multiplicity Ξ^0 and the elementary sub-schemes forming part of the given scheme there is an equivalent correspondence.

Thus, in the scheme arranged in space, a coordinate is ascribed to every elementary sub-scheme. It should be emphasized that, not only do different elementary sub-schemes of the given scheme answer to different coordinates, but if some sub-scheme is encountered in a scheme several times, its different entries answer to different coordinates. It follows from the definition that multiplicity Ξ^0 determines the position of the scheme in the space. Indicating this or that object from Ξ^0, we can isolate any elementary sub-scheme of the particular scheme. This fact permits us, below, to discuss each elementary sub-scheme of the initial scheme.

Definition. The multiplicities of schemes $\{\Sigma'_i\}$ and $\{\Sigma''_i\}$, determined over multiplicities $\mathfrak{S}'_i$ and $\mathfrak{S}''_i$ of elementary sub-schemes and arranged in spaces Ξ' and Ξ'', are called isomorphic if there is:

a) an equivalent correspondence between the objects of spaces Ξ' and Ξ''.

b) isomorphism of multiplicities $\mathfrak{S}'_i$ and $\mathfrak{S}''_i$.

c) an equivalent correspondence between schemes from multiplicies $\{\Sigma'_i\}$ and $\{\Sigma''_i\}$ such that the corresponding schemes Σ'_i and Σ''_i are isomorphic for the given isomorphism of the multiplicities of elementary sub-schemes, with the sub-multiplicities Ξ'_i and Ξ''_i of the coordinates consisting of corresponding objects.

From this, among other things, there directly follows the definition of the isomorphism of two schemes arranged in spaces Ξ' and Ξ''.

Below we will examine the multiplicity of schemes $\{\Sigma_i\}$ over a given multiplicity of elementary sub-schemes $\mathfrak{S}_i$ and arranged in the same space Ξ.

The elaboration of the concepts of scheme and coordinates completes the first phase in defining the control system. The second phase consists in elucidating the concepts which make it possible to characterize the functioning of the control system. With this aim we first introduce the functional characteristics of memory enabling us to describe memory states and information. Then we will define the functional characteristics determining the behavior of control systems.

4. Memory state. Information. Functions.

Let us suppose that we are given a memory $\mathfrak{X} = \{X_\alpha\}$. Each cell X_α will be represented as a sub-multiplicity of objects which are described by the values of a variable x_α.

Definition. The values of variable x_α are called the states of cell X_α. The assignment of its state for each cell determines one of the possible states I of memory $\mathfrak{X}$.

It is evident that, with respect to the amount of possible states, cell X_α is characterized by the number ρ_α, which represents the capacity of the multiplicity of values of variable x_α (i).

We will introduce into the discussion the variable t, which takes its value from a sub-multiplicity T of actual numbers. This variable can be interpreted as the time, while the multiplicity of all actual numbers can be regarded as the time scale. We suppose that sub-multiplicity T

does not have more than calculating capacity. This means that the variable t can change only discretely. The variable t, sometimes has a value t_{min} such that $t \geqslant t_{min}$; in this case the variable changes as of a moment t_{min}, called the initial moment.

Henceforth we will study memory in time, i. e. we will consider that $\mathfrak{X} = \mathfrak{X}(t)$. Let us suppose that the following postulate is fulfilled for the states of the memory's cells.

At each moment of time $t \in T$ the cell X_α is exactly in one (but in an arbitrary) state $x_\alpha = x_\alpha(t)$.

Definition. The state $I = I(t)$ of memory $\mathfrak{X}$, corresponding to the state of the cells at time t, is called the memory state at time t.

The memory state can change with time. This change can occur:

a) owing to the effect of the control system on the memory;

b) spontaneously.

The times and the character of the changes in memory state bound up with the functioning of the control system can be clarified on the basis of a complete definition of the control system. As to spontaneous changes reflecting the process of forgetting, to describe them we must introduce the parameter $\tau_\alpha(t) > 0$ (degree of recall) (k). This parameter is dependent on the number of the cell α and the time t and shows how long this or that cell state can last if during this period the cell has not been subjected to the action of the control system. Consequently, where

$$t < t' \leqslant t + \tau_\alpha(t)$$

$$x_\alpha(t') = x_\alpha(t).$$

Thus we see that the memory has two functions. On the one hand it acts as a repository or intermediary. This is mainly typical of the external memory and determines the link between the control system and the outside world. On the other hand, the memory has the ability to recall, owing to which it is the most fully equipped part of the control system to take previous experience into consideration.

Let us now try to refine the concept of the isomorphism of memories.

Definition. Memories $\mathfrak{X}'(t')$ and $\mathfrak{X}''(t'')$, in which the variables t' and t'' take their values from sub-multiplicities T' and T'', respectively, are called isomorphic memories if there is an equivalent correspondence $X'_{\alpha'} \longleftrightarrow X''_{\alpha''}$ between the cells of memories $\mathfrak{X}'$ and $\mathfrak{X}''$ and a mutually continuous (homeomorphic) similar correspondence of time scales such that:

a) corresponding cells $X'_{\alpha'}$ and $X''_{\alpha''}$ have the same number of states, i. e. $p'_{\alpha'} = p''_{\alpha''}$;

b) sub-multiplicities T' and T'' consist of corresponding elements;

c) if $\tau'_{\alpha'}(t')$ and $\tau''_{\alpha''}(t'')$ are the degrees of recall of the corresponding cells $X'_{\alpha'}$ and $X''_{\alpha''}$ and the time t' corresponds to the time t'', then the times $t' + \tau'_{\alpha'}(t')$ and $t'' + \tau''_{\alpha''}(t'')$ correspond to each other.

It is easily seen that the classification of memories given in section 2 can be naturally extended in the following directions.

1. Depending on the number p_α of states of cells, we find:

a) a memory in which the cells have in total a limited number of states, i. e. $p_\alpha < c$;

b) a memory in which each cell has a finite number of states;

c) a memory in which the cells can have an infinite number of states.

2. Bearing in mind the types of states of cells, we find:

a) a homogeneous memory, i. e. a memory in which $p_\alpha \equiv$ const;

b) a non-homogeneous memory, i. e. a memory in which there are cells with differing numbers of states.

3. Depending on the magnitude of the degree of recall, we get:

a) a memory with limited magnitudes of degrees of recall, i. e. $\tau_\alpha(t) < C$;

b) a memory with finite magnitudes of degrees of recall;

c) a memory in which there can be infinite degrees of recall.

4. Bearing in mind the character of the degrees of recall, we find:

a) a memory with identical degrees of recall $\tau_\alpha(t) \equiv \text{const}$ (in particular, where $\tau_\alpha(t) = \infty$ we get an unforgetting memory);

b) a memory having differing degrees of recall.

The memory state $I(t)$ represents the code of some message needed for the functioning of the control systems. This code is not identical with the message but is only its symbolic notation. It does, however, contain information about the message and we will therefore interpret $I(t)$ in that which follows as information and will think of memory as the repository for storing the information.

In view of the ability of the memory to recall information we are able to define more clearly the difference between the external and internal memories for the given scheme $\Sigma = \mathfrak{M}(E_s, S^{E_i}_{s_i}(X^{s_i}, Y^{s_i}, Z^{s_i}), \ldots)$ over memory $\mathfrak{X}$. Namely, through the external memory the tie with the outside world is established, and through it comes the original data, the initial information. The internal memory is characterized by the fact that for all schemes over memory $\mathfrak{X}$ with the given multiplicity of elementary sub-schemes at the starting time t_{min} the state of each cell is either always the same or fortuitous. Consequently, at the starting time it is impossible, in the internal memory, to set the arbitrary states at will. But we note here that with the passage of time, with the "evolution" of the control system, the breakdown of memory into external and internal can change.

<u>Definition</u>. We will say that two multiplicities of information $I' = \{I'_{i'}\}$ and $I'' = \{I''_{i''}\}$, connected with memories $\mathfrak{X}'$ and $\mathfrak{X}''$, are <u>isomorphic</u> if there is an equivalent correspondence of informations $I'_{i'} \longleftrightarrow I''_{i''}$, brought about by the isomorphism of memories $\mathfrak{X}'$ and $\mathfrak{X}''$ and by the equivalent correspondence of states of corresponding cells.

We will not present a classification of information since it is difficult to make a natural differentiation of information into different types.

Let $\Phi = \{\Phi_i\}$ — be a multiplicity of objects Φ_i.

<u>Definition</u>. Objects $\Phi_i \in \Phi$ are called functions.

The meaning of the functions will be revealed below when we examine control systems.

<u>Definition</u>. The multiplicities of functions Φ' and Φ'' are called <u>isomorphic</u> if they are equal in capacity.

It is obvious that multiplicities of functions can be divided in terms of capacity into:

a) multiplicities with a finite number of functions;

b) multiplicities with an infinite number of functions.

Further specification of multiplicities Φ requires the disclosure of the substance of the functions and will therefore be given later.

<u>5. Control systems. Thesis regarding control systems.</u>

We will now turn to defining the concept of the control system. This can be done in two ways: the naive multiplicity theory way and constructively. In the former case, roughly speaking, the function of the control system is not bound up with the control system's structure but arises out of experimental considerations. Here the appointment of a function for each control system is an individual problem. In the second approach it is held that the functions are known for the so-called elementary control systems but that for other control systems they are found with the aid of some algorithm. These two aspects are connected with each other in such a way that the constructive definition is possible only on the basis of the multiplicity theory definition and represents a refinement of the latter. These approaches recall, essentially, two ways of determing truths in mathematical logic: the naive method and in the form of a realization according to Kleene [5].

In this section we give a definition of the control system proceeding from the naive multiplicity theory point of view. The logic of this definition follows from the preliminary study of physical control systems in which one experimentally demonstrates the structure of the scheme, function, etc.

Let us regard as fixed the memory $\mathfrak{X}$, the multiplicity of elementary sub-schemes $\mathfrak{S}_0$, the space of coordinates Ξ, the sub-multiplicity T of time moments, and the multiplicity Φ of functions. We will consider the schemes $\Sigma_{i_\beta} = \mathfrak{M}_{i_\beta}(E_0, S_{\alpha}^{\Xi_1}(X^{*_1}, Y^{*_1}, Z^{*_1}), \ldots)$ over memory $\mathfrak{X}^{i_\beta} \subset \mathfrak{X}$ and over the multiplicity of elementary sub-schemes $\mathfrak{S}_0$. We understand ${}^{\mathfrak{X}_{i_\beta}}\Xi$ to be, as always, a multiplicity of coordinates corresponding to scheme Σ_{i_β}, and we think of I_{k_β} as information determined by the memory state $\mathfrak{x}^{i_\beta}$. We suppose, finally, that schemes Σ_{i_β}, coordinates Ξ_{j_β}, information I_{k_β} and function Φ_{i_β} are considered in time.

The last condition will sometimes be written

$$\Sigma_{i_\beta}(t_\beta), \Xi_{j_\beta}(t_\beta), I_{k_\beta}(t_\beta) \text{ и } \Phi_{i_\beta}(t_\beta), \text{ where } t_\beta \in T.$$

We will denote as $\mathfrak{U}$ the multiplicity of symbols

$$U_\beta = \{\Sigma_{i_\beta}(t_\beta), \Xi_{j_\beta}(t_\beta), I_{k_\beta}(t_\beta), \Phi_{i_\beta}(t_\beta)\}.$$

Definition. The symbols $U_\beta = \{\Sigma_{i_\beta}(t_\beta), \Xi_{j_\beta}(t_\beta), I_{k_\beta}(t_\beta), \Phi_{i_\beta}(t_\beta)\}$ from multiplicity $\mathfrak{U}$ are called control systems (over $\mathfrak{X}, \mathfrak{S}_0, \Xi, T$ and Φ) if each symbol $U_\beta \in \mathfrak{U}$ represents a submultiplicity $\mathfrak{U}_\beta = \{U_\gamma\} \subset \mathfrak{U}$ of symbols U_γ with $t_\gamma > t_\beta$ ($\mathfrak{U}_\beta$ may be empty) and a distribution of probabilities where $p(U_\beta, U_\gamma)$, and $U_\gamma \in \mathfrak{U}_\beta$ и $\sum_{U_\gamma \in \mathfrak{U}_\beta} p(U_\beta, U_\gamma) = 1$.

Thus we define, in fact, not one control system but a whole multiplicity $\mathfrak{U}$ of control systems. Each system is defined as the unity of a scheme, coordinates, information and a function. To set it we must indicate the submultiplicity of control systems $\mathfrak{U}_\beta$. This sub-multiplicity determines the direct transitions

$$U_\beta \to U_\gamma$$

of the given control system U_β to the control systems U_γ from $\mathfrak{U}_\beta$ (from time t_β to time t_γ). It is clear that $\mathfrak{U}_\beta$

may be empty. For example, if there is $t_{max} = \max_{t \in T} \{t\}$, then in a control system U_β, where $t_\beta = t_{max}$, the sub-multiplicity $\mathfrak{U}_\beta$ is empty. In these cases the control system does not admit direct transitions and is therefore termed final. We will call the control system initial if there is no control system U_β such that $U_\gamma \in \mathfrak{U}_\beta$. It is clear that if there is $t_{min} = \min_{t \in T} \{t\}$, then each control system of the type U_γ, where $t_\gamma = t_{min}$, is initial. An indication of the possible transitions of a given control system does not in itself define the completeness of its transmutations. Therefore, where $\mathfrak{U}_\beta \neq \Lambda$ we must set a distribution of probabilities $p(U_\beta, U_\gamma)$ for transitions $U_\beta \to U_\gamma$

We must postulate separately for the case $\mathfrak{U}_\beta \neq \Lambda$, where for control system U_β exactly one transmutation $U_\beta \to U_\gamma$, is realized at the moment of time under consideration. This precludes for the same control system the possibility of several transmutations simultaneously.

Let us now clarify the meaning of the symbol for the function Φ_β. We will, notably, regard it as an operator which acts on the control system U_β, if $\mathfrak{U}_\beta \neq \Lambda$. For convenience we may use the notation

$$\Phi_\beta U_\beta = U_\gamma,$$

using this to signify that the result of applying the operator Φ_β to U_β, i. e. $\Phi_\beta U_\beta$ with probability $p(U_\beta, U_\gamma)$, is U_γ, where $U_\gamma \in \mathfrak{U}_\beta$.

Thus, the definition of the control system supposes that in each symbol $\{\Sigma_{i_\beta}(t_\beta), \Xi_{j_\beta}(t_\beta), I_{k_\beta}(t_\beta), \Phi_{l_\beta}(t_\beta)\}$ from $\mathfrak{U}$ the symbol of function $\Phi_{l_\beta}(t_\beta)$, is interpreted unambigously, or, in other words, that the control system is completely defined by its symbol notation. It can happen that in multiplicities $\mathfrak{U}$ and $\mathfrak{U}'$ we find control systems

$$U_\beta = \{\Sigma_{i_\beta}(t_\beta), \Xi_{j_\beta}(t_\beta), I_{k_\beta}(t_\beta), \Phi^l{}_\beta(t_\beta)\}$$

$$U'_\beta = \{\Sigma_{i_\beta}(t_\beta), \Xi_{j_\beta}(t_\beta), I_{k_\beta}(t_\beta), \Phi_{l_\beta}(t_\beta)\},$$

with the same symbolic notation but having, generally speaking, different interpretations, i.e.

$$(\mathfrak{U}_\beta(t_\beta), P(U_\beta, U_\gamma)) \neq (\mathfrak{U}'_\beta(t_\beta), P'(U'_\beta, U'_\gamma)).$$

In this case the simultaneous examination of multiplicities $\mathfrak{U}$ and $\mathfrak{U}'$ is not permissible.

It is easy to divide functions Φ_{l_β} where $\mathfrak{U}_\beta \neq \Lambda$ into two types: determined and random.

Definition. The function Φ_{i_β} is called determined if sub-multiplicity $\mathfrak{U}_\beta$ consists of one object; in the case where $\mathfrak{U}_\beta$ contains more than one object, the function Φ_{i_β} is called random.

It follows directly from the definition that a determined function is completely defined by the indication of sub-multiplicity $\mathfrak{U}_\beta$; to set the random function we must, together with sub-multiplicity $\mathfrak{U}_\beta$, set the distribution law $p(U_\beta, U_\gamma)$ for the direct transitions $U_\beta \to U_\gamma$.

Let us suppose that we have a control system U_β. In this case we obtain a whole series of direct transmutations characterized by the correlations:

if $\mathfrak{U}_\beta \neq \Lambda$, то $\Phi_{i_\beta} U_\beta = U_{\gamma_{i_1}}$, where $U_{\gamma_{i_1}} \in \mathfrak{U}_\beta$,

if $\mathfrak{U}_{\gamma_{i_1}} \neq \Lambda$, то $\Phi_{i_{\gamma_{i_1}}} U_{\gamma_{i_1}} = U_{\delta_{i_1 i_2}}$, where $U_{\delta_{i_1 i_2}} \in U_{\gamma_{i_1}}$,

if $\mathfrak{U}_{\delta_{i_1 i_2}} \neq \Lambda$, то $\Phi_{i_{\delta_{i_1 i_2}}} U_{\delta_{i_1 i_2}} = U_{\varepsilon_{i_1 i_2 i_3}}$, where $U_{\varepsilon_{i_1 i_2 i_3}} \in \mathfrak{U}_{\delta_{i_1 i_2}}$

etc.

For greater clarity we can show these transmutations graphically in the form of a "tree" (Fig. 13).

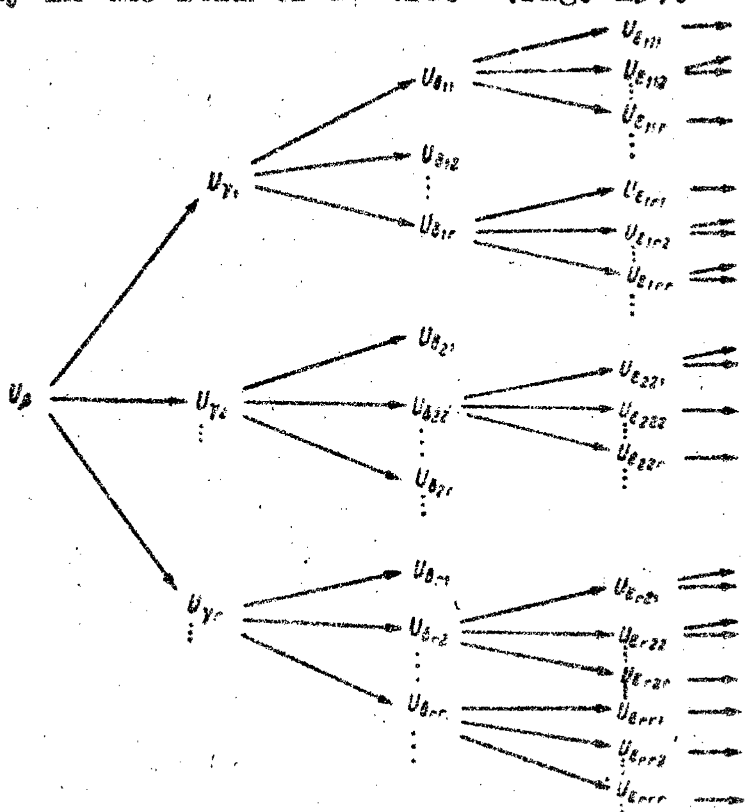

Fig. 13

Here $U_{\delta_{12}}, U_{\delta_{11}}, U_{\delta_{22}}, \dots, U_{\delta_{r_1}}$ are final control systems. It is obvious that the tree makes it possible to follow all the transmutations of control system U_β. The aggregate of all the transmutations of control system U_β will henceforth be called the <u>evolution</u> of the control system. Thus the evolution of a control system is characterized by a tree.

Following from our postulate we will actually always have (with one probability of another) only one individual branch of the tree, i. e.

$$U_\beta \to U_{\gamma_{i_1}} \to U_{\delta_{i_1 i_2}} \to \dots,$$

which either breaks off at the final step or contains an infinite number of objects.

In the case where all the functions are determined, we get instead of a tree, one branch

$$U_\beta \to U_\gamma \to U_\delta \to \dots$$

The correlation $\Phi_{l_\beta} U_\beta = U_\gamma$ can be written out in greater detail in the following way:

$$\Phi_{l_\beta}\{\Sigma_{i_\beta}(t_\beta), \Xi_{j_\beta}(t_\beta), I_{k_\beta}(t_\beta), \Phi_{l_\beta}(t_\beta)\} = \{\Sigma_{i_\gamma}(t_\gamma), \Xi_{j_\gamma}(t_\gamma), I_{k_\gamma}(t_\gamma), \Phi_{l_\gamma}(t_\gamma)\}.$$

It is clear from this that where the function Φ_{l_β} acts on control system U_β, there is, generally speaking, a simultaneous change of scheme, of coordinates, of information and of function. Thus a classification of functions can be made according to the following principle:

1) depending on the type of function Φ_{l_β} we distinguish:

a) determined functions;

b) random functions.

2) in terms of the character of action of function Φ_{l_β} on the control system we have:

a) functions altering the scheme and function not altering the scheme;

b) functions altering the coordinates and functions not altering the coordinates;

c) functions altering the information and functions not altering the information;

d) functions altering the functions and functions not altering the functions.

Earlier (in section 4) we spoke of the action of the control system on the memory. Here we can reformulate this in exact terms.

<u>Definition.</u> Cell X_α of memory $\mathfrak{X}$ is acted upon by control system U_β at time t_β, if function Φ_{l_β} alters the content of cell X_α, i. e. the content of cell X_α of control system U_β is different from the content of the

cell of control system $\Phi_{i_\beta}U_\beta$.

Knowing the evolution of the given control system U_β, we are able to determine for each cell all the moments of time at which it is acted upon by the control system in all branches of the evolution

$$U_\beta \to U_\gamma \to U_\delta \to \dots$$

Remark. In examining control systems we are often interested not in the entire evolution, i. e. not in the alteration of all the quantities $\Sigma_{i_\beta}, \Xi_{i_\beta}, I_{k_\beta}$ and Φ_{i_β}, but only in some of them, e. g. only the coordinates, only the information, and so on. We therefore use different functional characteristics of control system. Thus for determined control systems of the type

$$U_1 \to U_2 \to \dots$$

(here U_i denotes a control system arising at step i) we can consider the following functional characteristics:

a) the functional characteristic of the elementary change in information

$f_i(I_i) = I_{i+1}$ (for a control system with number i);

b) the functional characteriistic of the change in information in time

$f(I_1, t) = I_i$ where $t = t_i$ (for a control system with number i);

c) if the class of control systems is such that for some $k = k(I_1)$ we find that $I_k = I_{k+1} = \dots$, we can consider the functional characteristics of the change in information through $k(I_1)$ steps, i. e.

$$f(I_1) = I_{k(I_1)}.$$

It is clear that the above-mentioned characteristics are defined entirely in terms of Φ_{i_β} and U_β, i. e. in terms of the evolution. The opposite assertion does not, generally speaking, hold: the evolution (functions Φ_{i_β}) are not re-established in terms of these characteristics. In these cases the control systems themselves are merely means of realizing these arbitrary functions. However, in solving the problem of synthesizing control systems we are nevertheless obliged to resort to the initial concept of the function, i. e. to Φ_{i_β}. Consequently, control systems may be connected with different functional characteristics. Their choice influences the solution of many problems (the synthesis of control systems and so forth). The solution itself demands a knowledge of the functions in our first meaning.

Definition. Let us suppose that we have two aggregates of control systems $\{U'_{\beta'}\}$ and $\{U''_{\beta''}\}$ over $\mathfrak{X}', \mathfrak{S}'_0, \mathfrak{Z}', T', \Phi'$ and $\mathfrak{X}'', \mathfrak{S}''_0, \mathfrak{Z}'', T'', \Phi''$, respectively. We will call these aggregates isomorphic if there is an equivalent correspondence between the control systems

$$U'_{\beta'} \longleftrightarrow U''_{\beta''},$$

giving rise to a) isomorphism $\Sigma'_{i'_{\beta'}} \longleftrightarrow \Sigma''_{i''_{\beta''}}, \mathfrak{Z}'_{j'_{\beta'}} \longleftrightarrow \mathfrak{Z}''_{j''_{\beta''}}$ of the multiplicities of schemes $\{\Sigma'_{i'_{\beta}}\}$ and $\{\Sigma''_{i''_{\beta''}}\}$, situated in spaces $\mathfrak{Z}'$ and $\mathfrak{Z}''$; b) isomorphism $I'_{k'_{\beta'}} \longleftrightarrow I''_{k''_{\beta''}}$ of the multiplicities of information $\{I'_{k'_{\beta'}}\}$ and $\{I''_{k''_{\beta''}}\}$; c) isomorphism $\Phi'_{l'_{\beta'}} \longleftrightarrow \Phi''_{l''_{\beta''}}$ of the multiplicities of functions $\{\Phi'_{l'_{\beta'}}\}$ and $\{\Phi''_{l''_{\beta''}}\}$, and such that

1) if $U'_{\beta'} \longleftrightarrow U''_{\beta''}$ and $U'_{\gamma'} \longleftrightarrow U''_{\gamma''}$, then it follows from $t'_{\beta'} < t'_{\gamma'}$ that $t''_{\beta''} < t''_{\gamma''}$;

2) sub-multiplicities $\mathfrak{U}'_{\beta'}$ and $\mathfrak{U}''_{\beta''}$, represented by control systems $U'_{\beta'}$ and $U''_{\beta''}$, consist of corresponding control systems;

3) $\rho(U'_{\beta'}, U'_{\gamma'}) = \rho(U''_{\beta''}, U''_{\gamma''})$, where $U'_{\gamma'} \in \mathfrak{U}'_{\beta'}$, $U''_{\gamma''} \in \mathfrak{U}''_{\beta''}$ and $U'_{\gamma'} \longleftrightarrow U''_{\gamma''}$.

It is clear that isomorphic control systems behave similarly and it is therefore sufficient to study one of them to be able to judge another.

The classification of control systems is connected with the classification of schemes, memory and functions. In addition it is natural to subdivide control systems, with respect to their role in the evolution process, into

a) initial control systems;
b) final control systems;
c) other control systems.

It is also important to note that the class of determined control systems having a finite number of functions contains a sub-class of control systems in which the evolution represents a periodic sequence.

In the introduction we spoke of physical control systems and of control systems. The latter were viewed as classes of isomorphic physical control systems. It goes without saying that all these concepts were drawn from the analysis of concrete examples with the aid of substantive examinations and that they therefore lack the strictness that is typical of mathematical concepts.

We have just constructed a mathematical object called a control system which for the sake of accuracy should perhaps be called a mathematical control system. There naturally arises the question what connection there is between this object and the control systems and physical control systems of which we spoke in the introduction. It is entirely understandable that this connection cannot be formulated in exact mathematical terms. It can only be expressed as a thesis, similar to the thesis of Church [5] in the theory of algorithms.

Fundamental thesis regarding control systems. Every physical control system may be adequately depicted by means of the mathematical control system we have defined.

The peculiar feature of this thesis consists in that it cannot be proved in the mathematical sense. Experiment can only confirm it.

Let us offer some clarifications. In formulating the thesis we use the expression "adequately depicted." This should be regarded as synonymous to: for every physical control system we can construct a mathematical control system whose schematic and functional characteristics clearly and without distortion depict the schematic and functional characteristics of the initial physical system. Thus, for instance, in the introduction we considered a control system for turning on and off light on a stairway (Fig. 1). Abstracting from this, however, we soon replaced this physical control system essentially by a mathematical control system (Fig. 2 and corresponding text). This substitution, being a natural one, might have escaped the notice of the reader. The thesis we have put forward relates specifically to the fact that such a substitution of a physical control system with a mathematical control system is always possible.

The thesis can also be understood in this way: every mathematical control system is a control system in our initial meaning, i. e. a physical control system. Its peculiar feature is its complete mathematical determination. We defined control systems as a class of physical control systems having identical (isomorphic) schemes and functions. It follows from the thesis that every such class contains a mathematical control system. Since each class is entirely characterized by any of its representatives, we can treat the control system as a mathematical control system. In this way we give a precise mathematical meaning to the term control system, identifying it with the term mathematical control system. Owing to this fact we have nowhere except in this discussion, and will not in that which follows, used the word "mathematical" in connection with the concept of control systems.

. Elementary control systems. Control systems over a given multiplicity of elementary control systems. Algorithmized control systems.

The naive multiplicity theory definition of control systems is convenient in a preliminary study of physical control systems. However, to be able to reach the higher stage in the study of physical control systems at which not only external observation of the behavior of control systems is possible, but also their modeling, we must narrow somewhat the class of control systems.

Definition. A multiplicity of control systems $\mathfrak{U}$ is called regular if the function of the control system is unambiguously determined by its scheme, coordinates and information, i. e. if

$$\Phi_{i_\beta}(t_\beta) = f(\Sigma_{i_\beta}(t_\beta),\ \Xi_{j_\beta}(t_\beta),\ I_{k_\beta}(t_\beta)),$$

where $U_\beta \in \mathfrak{U}$. It follows from the definition that in the case where multiplicity $\mathfrak{U}$ of control systems is regular, every control system $U_\beta \in \mathfrak{U}$ is completely determined by the setting of scheme, coordinate and information:

$$\{\Sigma_{i_\beta}(t_\beta),\ \Xi_{j_\beta}(t_\beta),\ I_{k_\beta}(t_\beta)\},$$

since function Φ_{i_β} is found from the correlation

$$\Phi_{i_\beta}(t_\beta) = f(\Sigma_{i_\beta}(t_\beta),\ \Xi_{j_\beta}(t_\beta),\ I_{k_\beta}(t_\beta)).$$

However, we must bear in mind that this correlation is not, generally speaking, effective, as it guarantees for every group of three $\{\Sigma_{i_\beta}(t_\beta), \Xi_{j_\beta}(t_\beta), I_{k_\beta}(t_\beta)\}$ the existence of a single function Φ_{i_β} and no more. Below we define the class of control systems for which the function Φ is determined from $\{\Sigma, \Xi, I^0\}$ effectively, i. e. f is a calculable function.

A very important special case of control systems is the elementary control system, i. e. a control system of the type

$$U^0 = \{S(t),\ \xi(t),\ I^0(t),\ \Phi^0(t)\},$$

where $S(t)$ is an elementary sub-scheme and $\xi(t)$ is its coordinate.

Definition. The multiplicity of control systems $\mathfrak{U}$ are called correct if, regardless of the elementary sub-scheme $S \in \mathfrak{S}_0$, its coordinate $\xi \in \Xi$, the information $I^0 \subset I$, expressing the state of the memory of the elementary sub-scheme S, and the time $t \in T$, such a $\Phi^0 \in \Phi$, can be found that the elementary control system

$$U^0 = \{S(t),\ \xi(t),\ I^0(t),\ \Phi^0(t)\}$$

belongs to multiplicity $\mathfrak{U}$.

We will denote by $\mathfrak{U}^0$ the sub-multiplicity of all elementary control systems from $\mathfrak{U}$.

Theorem. If the multiplicity $\mathfrak{U}$ of control systems is correct and regular, then with every control system $U_\beta \in \mathfrak{U}$ it is possible unambiguously to connect a set

$$\{U^0_{\alpha_1},\ U^0_{\alpha_2},\ \ldots\}$$

of elementary control systems from $\mathfrak{U}^0$ such that the control system U_β may be regarded as a control system over the given set of control systems.

In this connection let us consider a scheme $\Sigma_{i_\beta}(t_\beta)$, forming part of U_β. The scheme $\Sigma_{i_\beta}(t_\beta)$ is made up of a set of elementary sub-schemes

$$\{S_{i_{a_1}}(t_\beta),\ \ S_{i_{a_2}}(t_\beta),\ldots\}.$$

In control system U_β every elementary sub-schemes $S_{i_{a_r}}(t_\beta)$ unambiguously corresponds to a coordinate $\xi_{j_{a_r}}(t_\beta)$. Furthermore, information $I_{k_{a_r}}(t_\beta)$ determines the memory state (information) of the elementary sub-scheme $S_{i_{a_r}}(t_\beta)$. Thus we have $S_{i_{a_r}}(t_\beta)$, $\xi_{j_{a_r}}(t_\beta)$, $I_{k_{a_r}}(t_\beta)$. By virtue of the correctness and regularity of multiplicity $\mathfrak{U}$ there is a single elementary control system $U^0_{a_r}$ such that

$$U^0_{a_r}=\{S_{i_{a_r}}(t_\beta),\ \ \xi_{j_{a_r}}(t_\beta),\ \ I_{k_{a_r}}(t_\beta),\ \ f(S_{i_{a_r}}(t_\beta),\ \ \xi_{j_{a_r}}(t_\beta),\ \ I_{k_{a_r}}(t_\beta))\}$$

and $U^0_{a_r}\in\mathfrak{U}^0$. Thus we have constructed a set of elementary control systems $\{U^0_{a_1}, U^0_{a_2},\ldots\}$, connected with the given control system U_β. Now we can consider the control system U_β as a control system over a given multiplicity of elementary control systems understanding this to mean that U_β is constructed of elementary control systems $U^0_{a_1}, U^0_{a_2},\ldots,$ belong to sub-multiplicity $\mathfrak{U}^0$.

Since the elementary control systems play an important role, let us go into greater detail on the specific features of functions Φ^0. Every function Φ^0 represents an operator which can be viewed as the "composition" of a series of simple operators. The most important of these operators carries out the following transmutations.

1. Alteration of the connection between the elements and the external memory. Here, in the elementary sub-schemes whose coordinates are determined by the information $I_{k_{a_r}}(t_\beta)$, the sets of cells $(X^{i_{a_r}}, Y^{i_{a_r}})$, are replaced by the sets of cells $(X'^{i_{a_r}}, Y'^{i_{a_r}})$, where $(|X'^{i_{a_r}}|\bigcup|Y'^{i_{a_r}}|)\bigcap|Z^{i_{a_r}}|=\Lambda$. The character of this substitution is also determined by $I_{k_{a_r}}(t_\beta)$.

2. Alteration of the connection between the poles of elementary sub-schemes and the apexes of the sets of network $\mathfrak{M}_{i_\beta}$. The coordinates of these elementary sub-schemes and the character of the alterations are determined by the information $I_{k_{a_r}}(t_\beta)$.

3. Alteration of the individuality (type) a_ρ of elements S_ρ. The coordinates of the corresponding elementary sub-schemes and the character of the alterations are indicated in the information $I_{k_{a_r}}(t_\beta)$.

4. Alteration of the coordinates of elementary sub-schemes. The information necessary for this is determined by $I_{k_{a_r}}(t_\beta)$.

5. Alteration of the states of the external memory

$$\Phi^{q}_{l_{\alpha_r}} (X^{i_{\alpha_r}}, Z^{i_{\alpha_r}}) \rightarrow Y^{i_{\alpha_r}},$$

i. e. the information stored in the cells from sets $X^{i_{\alpha_r}}, Z^{i_{\alpha_r}}$, is reworked with the aid of function $\Phi^{q}_{l_{\alpha_r}}$ and registered in the cells from set $Y^{i_{\alpha_r}}$.

6. Alteration of the states of the internal memory

$$\Phi^{q}_{l_{\alpha_r}} (X^{i_{\alpha_r}}, Z^{i_{\alpha_r}}) \rightarrow Z^{i_{\alpha_r}},$$

i. e. the information stored in the cells from sets $X^{i_{\alpha_r}}, Z^{i_{\alpha_r}}$, is reworked with the aid of function $\Phi^{q}_{l_{\alpha_r}}$, and registered in the cells from set $Z^{i_{\alpha_r}}$.

7. Alteration of the topology of scheme

$$\mathfrak{M}_{i_\beta} \rightarrow \mathfrak{M}'_{i_\beta}.$$

The character of alterations is determined by the information $I_{k_\alpha}(t_\beta)$.

8. Auxiliary operations relating respectively to the work of the algorithm (see below). This category contains signals for the start and end of work, indices of the order of work - random choices, logical functions, brackets, and so forth. Control systems of this type provide a certain "auxiliary alphabet" required for the work of the algorithm [5].

In many cases the function $\Phi^{q}_{l_{\alpha_r}}$ in the presence of action on the control system U_β produces simultaneously several simple acts, and we therefore say that the operator represents a "composition" of simple operators.

Let $\mathfrak{U}$ - be an arbitrary multiplicity of control systems U_β.

Definition. The multiplicity $\mathfrak{U}$ of control systems is called algorithmized if there is an algorithm A, defined for $\mathfrak{U}$, such that:

1. For every control system

$$U_\beta = \{\Sigma_{i_\beta}(t_\beta),\ \Xi_{j_\beta}(t_\beta),\ I_{k_\beta}(t_\beta),\ \Phi_{l_\beta}(t_\beta)\}$$

from $\mathfrak{U}$ $AU_\beta = \Phi'_{l_\beta}(t_\beta)$, where $\Phi'_{l_\beta}(t_\beta)$ belongs to some multiplicity Φ'.

2. For every elementary control system

$$U^{0}_{\alpha} = \{S_{i_\alpha}(t_\alpha),\ \xi_{j_\alpha}(t_\alpha),\ I_{k_\alpha}(t_\alpha),\ \Phi^{0}_{l_\alpha}(t_\alpha)\} \text{ from } \mathfrak{U}^{0},$$

$$AU^{0}_{\alpha} = \Phi'_{l_\alpha}(t_\alpha) = \Phi^{0}_{l_\alpha}(t_\alpha), \text{ where } \Phi^{0}_{l_\alpha}(t_\alpha) \in \Phi'.$$

3. Symbols U'_β from multiplicity $\mathfrak{U}'$, where

$$U'_\beta = \{\Sigma_{i_\beta}(t_\beta),\ \Xi_{j_\beta}(t_\beta),\ I_{k_\beta}(t_\beta),\ \Phi'_{l_\beta}(t_\beta)\}$$

are control systems.

In particular, symbols $U'^{0}_{\alpha} = \{S_{i_\alpha}(t_\alpha),\ \xi_{j_\alpha}(t_\alpha),\ I_{k_\alpha}(t_\alpha),\ \Phi^{0}_{l_\alpha}(t_\alpha)\}$ and only they will be control systems from the multiplicity $\mathfrak{U}'^{0}$ - the sub-multiplicity of all control systems from $\mathfrak{U}'$.

4. The multiplicities of control systems $\mathfrak{U}$ and $\mathfrak{U}'$ satisfy the condition or coordination, i.e. if control

systems

$$U_\beta = \{\Sigma_{i_\beta}(t_\beta),\quad \Xi_{j_\beta}(t_\beta),\quad I_{k_\beta}(t_\beta),\quad \Phi_{i_\beta}(t_\beta)\}$$

and $U'_\beta = \{\Sigma_{i_\beta}(t_\beta),\ \Xi_{j_\beta}(t_\beta),\ I_{\beta}(t_\beta),\ \Phi_{i_\beta}(t_\beta)\}$ from $\mathfrak{U}$ and $\mathfrak{U}'$ respectively have the same symbolic notation, then

$$\mathfrak{U}_\beta(t_\beta) = \mathfrak{U}'_\beta(t_\beta) \text{ and } P(U_\beta, U_\gamma) = P(U'_\beta, U'_\gamma).$$

In other words, control systems U_β and U'_β are identical, i.e. $U_\beta = U'_\beta$.

Let us give some clarification of this definition. In the case where the multiplicity $\mathfrak{U}$ of the control systems are algorithmized, every control system

$$U_\beta = \{\Sigma_{i_\beta}(t_\beta),\quad \Xi_{j_\beta}(t_\beta),\quad I_{k_\beta}(t_\beta),\quad \Phi_{i_\beta}(t_\beta)\} \text{ from } \mathfrak{U}$$

is unambiguously, with the aid of algorithm A. , in correspondence with the control system

$$U'_\beta = \{\Sigma_{i_\beta}(t_\beta),\quad \Xi_{j_\beta}(t_\beta),\quad I_{k_\beta}(t_\beta),\quad \Phi'_{i_\beta}(t_\beta)\} \text{ from } \mathfrak{U}',$$

which, generally speaking, does not always coincide with the control system U_β . As regards the elementary control systems, they turn into themselves in this correspondence, i. e.

$$U'^{0}_{*} = U^{0}_{*}.$$

Furthermore, the multiplicities $\mathfrak{U}$ and $\mathfrak{U}'$ have identical sub-multiplicies of elementary control systems, i. e. $\mathfrak{U}^{0\prime} = \mathfrak{U}^{0\prime}$. Finally, the fact that the symbols from multiplicity $\mathfrak{U}'_\gamma$ are control systems signifies that with every symbol U'_β is connected a sub-multiplicity $\mathfrak{U}'_\beta = \{U'_\gamma\}$ of symbols U'_γ , where $t_\gamma > t_\beta$ ($\mathfrak{U}'_\beta$ may be empty) and the distribution of probabilities $p(U'_\beta, U'_\gamma)$, where $U'_\gamma \in \mathfrak{U}'_\beta$ and $\sum_{U'_\gamma \in \mathfrak{U}'_\beta} p(U'_\beta, U'_\gamma) = 1$.

The latter conditions determine the direct transitions $U'_\beta \to U'_\gamma$ of the control system U'_β to control system U'_γ from $\mathfrak{U}'_\beta$. These transitions are characterized by the operator $\Phi'_{i_\beta}(t_\beta)$, namely, with a probability of $p(U'_\beta, U'_\gamma)$ operator $\Phi'_{i_\beta}(t_\beta)$ carries control system U'_β to control system U'_γ.

The condition of coordination signifies that control systems U_β and U'_β from multiplicities $\mathfrak{U}$ and $\mathfrak{U}'$, respectively, having identical symbolic notation behave identically in the process of evolution and are consequently identical. In particular, the corresponding elementary control system from multiplicities $\mathfrak{U}$ and $\mathfrak{U}'$ are identical. Therefore

$$\mathfrak{U}^{0} = \mathfrak{U}'^{0}.$$

<u>Theorem. If an algorighmized multiplicity $\mathfrak{U}$ of the control systems is regular and correct, then $\mathfrak{U}'$ represents a multiplicity of control systems which is also regular and correct.</u>

It is entirely evident that in the terms of the theorem, control systems from $\mathfrak{U}'$ are determined respectively over the same sets of elementary control systems as control systems from $\mathfrak{U}$.

In certain cases an indispensable condition for the work of algorithm A is the existence of elementary control systems with auxiliary functions (see point 8 in the list of simple operations).

<u>Definition.</u> We will consider that the algorithmation of the multiplicity $\mathfrak{U}$ with the aid of algorithm A is <u>exact</u> for the given control system $U_\beta \in \mathfrak{U}$, if

$$AU_\beta = \Phi_{i_\beta}(t_\beta),$$

and the control system itself is called <u>exactly algorithmized.</u>

Thus with each algorithm A, effectuating the algorithmation of multiplicity $\mathfrak{U}$, is connected a sub-multiplicity $\mathfrak{U}_A$ of those control systems from $\mathfrak{U}$, for which the algorithmation is exact. Obviously

$$\mathfrak{U}^0 \subseteq \mathfrak{U}_A \subseteq \mathfrak{U}.$$

i. e. the class of exactly algorithmized control systems contains a sub-multiplicity $\mathfrak{U}^0$ of elementary control systems.

In the case where the algorithmation is exact for all control systems from $\mathfrak{U}$

$$\mathfrak{U}_A = \mathfrak{U}.$$

Algorithmized control systems from a regular and correct multiplicity $\mathfrak{U}$ of control systems are important in that they not only indicate the correspondence between $\Phi_{i_\beta}(t_\beta)$, $\Sigma_{i_\beta}(t_\beta)$, $\Xi_{j_\beta}(t_\beta)$ and $I_{k_\beta}(t_\beta)$, i. e.

$$\Phi_{i_\beta}(t_\beta) = f(\Sigma_{i_\beta}(t_\beta),\ \Xi_{j_\beta}(t_\beta),\ I_{k_\beta}(t_\beta)),$$

but also set the effective method of calculating the function f. In the same way, the mechanism is described for obtaining the function Φ_{i_β}, and consequently such a control system becomes similar in action to an automatic device.

7. Peculiar features of the control systems studied in cybernetics.

In the introduction we defined cybernetics as the methematical discipline studying control systems. However, since we had not yet clearly defined the control system we were unable to discuss the peculiar features of the control systems studied in cybernetics and their relationship with physical control systems. But now, with an exact definition, we shall return to this matter.

A characteristic feature of the control systems studied in cybernetics is the fact that they represent in essence objects of a discrete nature, namely, schemes, functions, coordinates, information and time. The discreteness of the schemes and functions is entirely understandable and is manifested in the fact that between all schemes and all functions there is no continuous transition. As regards the coordinates, information and time, they are frequently determined with the aid of parameters that may take on values from some segment of real numbers. Let us suppose that the chart (Fig. 14) shows the change in state x_α of cell X_α in time. We will assume that the only important thing for the functioning of the control system is whether x_α is or is not larger than x_α^0. In these conditions the functioning of the control system would not change if, other things being equal, the change in state of cell x_α were set by the chart (Fig. 15). Thus the functioning of the control system is completely determined by the behavior of the predicate $P(x_\alpha(t)>x_\alpha^0)$ which is shown graphically in Figure 16. We have then three graphs bound up with the identical functioning of a control system. The first two characterize a contiuous quantity while the third pictures a discrete quantity. It is also evident that each of them bears in some sense information on the state of cell X_α, the information in the first (and accordingly in the secong) being greater than the information contained in the third graph, since the third graph is unambiguously determined by either of the aforegoing. This shows that a necessary condition for the functioning of the control system is, in essence not all the information on the state of the cell, i. e. not complete knowledge of the change of state graph, but only that part characterized by the third graph. Thus the state of the cell emerges as a discrete quantity although its carrier may be a continuous quantity. A similar situation also arises in considering space-time characteristics. In a whole series of important cases there is far greater restriction - the aforesaid quantities are not only discrete but also take on only a finite number of values (though this number may be very lart, it is true). The discrete character of control systems also places its mark on the devices used in cybernetics. Here a large part is played by the multiplicity theory, probability theory, number theory, algebraic and particularly the logical methods.

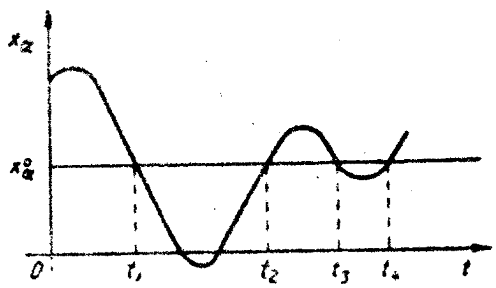

Fig. 14

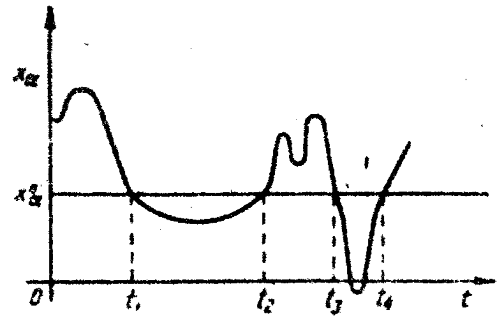

Fig. 15

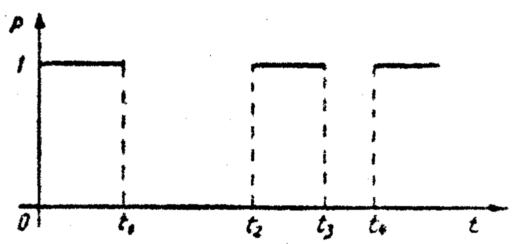

Fig. 16

The second feature of control systems requiring special cybernetic examination is their complexity. Namely, study is made of control systems consisting of a great number of elementary control systems, controls systems having intricate schemes, etc. The fact is that in order to study simple control systems such as the electric bell, $\sin x$ programs, etc., it is not necessary to use any device or theory - such problems are solved directly "in the head" by simple mental calculation. In the case of intricate control systems, even if they are connected with quantities that can take only a finite number of values, mental calculation is no longer possible. It is known, for instance, that the number of all chess possibilities is finite, or that the number of chemical molecules with a given number of atoms, say less than 20, is also finite. Despite this, in practice, even using modern computing machines, we are unable to sift through all the chess possibilities so as to select the best move or go through all the chemical molecules to find the one with the desired properties.

All this goes to demonstrate that the finiteness of a multiplicity of values is actually symbolic in nature, finite multiplicities emerging in these conditions as infinite ones. Thus complex control systems (even in the finite case) require special approaches for their study.

The third feature is connected with the fact that real objects can be viewed as control systems, generally speaking, in many ways. The fact is that a control system is not an absolute concept but a concept that depends on what is taken as the elementary control systems and on, what aspect of the object one intends to study. Thus one speaks of the relativity of control systems. In explanation we will mention the control systems bound up with computing machines.

1. The machine. The scheme of this control system is the scheme of the machine. It involves the following elementary control systems: tubes, semiconductors, condensers, resistors, transformers, induction coils, cathode-ray tubes, and so forth. The internal memory is made up of tubes, rolls, triggers, hold-back lines and hold-backs in the scheme itself. The external memory is made up of tapes, racks, punched cards, etc. The functions of the machine are conditioned by the algorithm, the latter being determined by the laws of radio mechanics, and the functions represent select groups of "numbers," simple "number" operations, the recording of "numbers", changes in state of the machine, etc., and combinations of these.

2. The program controlling the work of the machine. Its scheme represents a sequence of elements (commands) each of which has its coordinate (address) - the number of the cell in which the given command is stored. The elementary control systems are the individual commands: arithmetic commands, readdressing and dispatching commands, control commands. The internal memory is bound up with the states of the machine during operation. Thus, in the "Strela" machine, the sign ω is produced when carrying out certain commands, this sign being found in the internal memory. The external memory is made up of cells of tapes, rolls, cathode-ray tubes, emitters of constants, registering devices, etc. The algorithm determining the program function is set by the machine itself.

3. The algorithm realized by the program. Its scheme expresses the connection of operators. These are elementary control systems and are broken down into arithmetic operators, operators for change in parameter and formation, control operators, and their combinations. The internal memory represents all the parameters and indices of the operators. The external memory is bound up with the storing of input quantities and results of information processing. The functioning of the given scheme is determined by the setting of information processing by each operator and by

the indication of a superalgorithm which computes the function of the entire control system in terms of the function of each operator and in terms of the scheme.

From these examples we see that the role of the various units in the machine depends on what kind of control system is involved.

Finally, the fourth trait is bound up with the fact that in algorighmizing a multiplicity of control systems we neglect certain secondary phenomena as a result of which the algorithmizing proves inaccurate for a number of control systems. In this way we replace one multiplicity of control systems with others, the latter representing a certain approximation.

Consequently, study of the multiplicity of control systems amounts to finding an approximation with one or another degree of exactness. This picture emerges in constructing transfer algorithms, in finding tactics, etc.

8. Ways of studying control systems and the basic problems of cybernetics.

As we have stated earlier, a control system is an object of a discrete nature which consists, generally speaking, of a large number of elementary control systems. This fact leads us to conclude that the control system is not only an object have a microstructure but also a macroscopic object. As a result, in studying control systems two approaches are possible: the micro approach and the macro approach.

The essence of the macro approach is determined by certain specific features of control system investigation. Notably, it arises when the physical system being studied does not permit direct and complete examination. The only things lending themselves to direct observation are the poles of the system, its external memory and its "behavior." For the rest, the structure of the control system is unknown. Such a situation arises, for instance, in studying inaccessible control systems (in games, etc.) or in examining control systems whose structure is incompletely investigated (in biology, etc.). Thus in such cases the control system can roughly be treated as a box in which poles and an external memory are set apart (Fig. 17). Examination of the poses and external memory with their specific features represent the preliminary analysis of the scheme of a control system. Further study is bound up with an infinite variety of experiments not involving entering the box. Each of these experiments leads to some reprocessing of information. This can be described in precise language and, as a rule, in more than one way. The simplicity of such descriptions is essentially a reflection of how the information is fed to the control system. In this connection there arises the problem of coding information, which involves the question of differentiating the states of the memory cells and establishing their number. Having properly selected the time step we can determine the functional characteristic describing the information processing. Finding them furthermore comprises the task of function analysis in the control system, or more precisely the task of analyzing the functions describing the external behavior of the control system. It turns out, further, that experiments may permit us to "peek" into the box. Notably, in studying a function it is possible to reveal the internal memory somewhat and to give an estimate of its volume from below. Thus we move on to the problem of secondary analysis of a scheme. It should be noted that the macro approach is limited since it does not enable the investigator to clarify the control system's structure completely. Thus it is quite evident that the macro approach gives almost no idea of the scheme's physical structure. As a rule it does not enable us to find the function of the control system, because it is impossible by external experiment to reveal the nature of the state changes in the internal memory or to detect transmutations of the scheme. Despite this, the macro approach is of great importance in studying control systems, particularly in the initial stage.

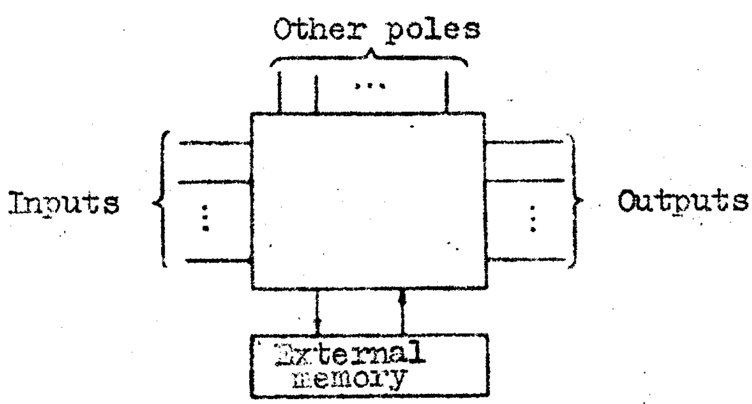

Fig. 17

The micro approach is characterized by the fact that it takes account of the specific qualities of the internal structure of the control system. One first elucidates the control system's scheme: its poles, memory and elements and their relation. This comprises the content of the problem of scheme analysis. After this one solves the problem of coding information bound up with the entire memory. Having properly established the time step and also the coordinates of the elementary sub-schemes, one finds the functional characteristics of the control system, taking into consideration not only the processing of information but also, generally speaking, the entire control system, including the reworking of the scheme, coordinates and functions. All this enters into the problem of function analysis.

The problems outlined above pertain to both approaches, they have the purpose of analyzing individual control system and comprise the first stage im the study. In essence this stage enables one to understand the given physical control system as a control system in the naive multiplicity theory sense. The following cycle of micro approach problems is bound up with the study of classes of control systems. We have in mind control systems over a fixed memory, a multiplicity of elementary sub-schemes, a space of coordinates, a sub-multiplicity of times and a multiplicity of functions. The problem arises here of the algorithmizing of control systems in a given class. The solution of the problem involves the isolation of the elementary control systems, their analysis and the construction of a sufficiently effective algorithm permitting calculation of the function of the control schemes. For algorithmized control systems it is possible to set the problem of function analysis with the aid of the algorithm. It should be especially emphasized that, in the case of algorithmed control systems, the function analysis can be done in two ways: experimentally and on the basis of the algorithm. The latter method is of great importance, particularly in theoretical analysis. The problem includes that of identical transmutations of control systems, i. e., the problem of finding on the basis of the algorithm the transmutations offering the possibility of switching over from one control system to any other possessing the same functional characteristics as the former (see remark in section 5). A very important problem is that of the synthesis of control systems. This problem is stated as follows: given the functional characteristics, construct a control system of a given type with these characteristics. Inasmuch as the problem usually has several solutions, it is required further that the desired solution should reach the extreme for some parameter. The synthesis of control systems is significant, incidentally, in that it enables us to model physical control systems. Among other problems we should point out that of monitoring control systems. In connection with the fact that physical control systems under external influences may undergo some change, the question arises of ascertaining these changes (localization) with the aid of various techniques. A similar problem arises in medicine in the diagnosis of diseases; in technology it arises when possible faults must be ascertained in operation of control systems.

The list of problems given above should not be considered final, since several other problems may be formulated. But it apparently includes the most important of these.

In conclusion we will comment briefly on the approaches outlined above. It is clear that the micro approach is more comprehensive than the macro approach. This is achieved, however, at the expense of great labor. To make the micro approach simpler the attempt is being made to examine whole classes of control systems on the basis of algorithmation. Another way of minimizing the difficulties is to use the macro approach at first and then, taking into account the resultant information on the control systems, to switch over to the micro approach.

FOOTNOTES

(a) We take "objects" to mean an unordered aggregate of objects in which their repetition is possible.

(b) The concept of a π-network may be defined by induction in terms of the number h. Where $h=1$ a network of the type $\mathfrak{M}(E_0, E_1)$, in which $\mathfrak{M}=E_0=E_1=\{a, b\}$ is called a π-network.

Let us assume that the concept of the π-network has been defined for all cases of $h \leqslant h_0$. We will call a two-pole network $\mathfrak{M}(E_0, E_1, \ldots, E_h)$, where $\mathfrak{M}=\bigcup_{i=1}^{h} \{E_i\}$ and $E_0=\{a, b\}$ a π-network if there is a breakdown of the set $\{E_1, \ldots, E_h\}$ into the direct sum of two sets (i. e. the multiplicity of subscripts $\{1, \ldots, h\}$ breaks down into a direct sum) $\{E'_1, \ldots, E'_{h_1}\}$ and $\{E''_1, \ldots, E''_{h_2}\}$, i. e. $\{E_1, \ldots, E_h\}=\{E'_1, \ldots E'_{h_1}\} \cup \{E''_1, \ldots, E''_{h_2}\}$, where $h_1, h_2 \leqslant h_0$, and such that either $\mathfrak{M}_1 \cap \mathfrak{M}_2=\{a, b\}$, where $\mathfrak{M}_1=\bigcup_{i=1}^{h_1} \{E'_i\}$, $\mathfrak{M}_2=\bigcup_{i=1}^{h_2} \{E''_i\}$, $\mathfrak{M}_1(E_0, E'_1, \ldots, E'_{h_1})$ and $\mathfrak{M}_2(E_0, E''_1, \ldots E''_{h_2})$ are π-networks;

or $\mathfrak{M}_1 \cap \mathfrak{M}_2=\{c\}$, $c \notin \{a, b\}$, but $a \in \mathfrak{M}_1$ and $b \in \mathfrak{M}_2$ and $\mathfrak{M}_1(E'_0, E'_1, \ldots, E'_{h_1})$, $\mathfrak{M}_2(E''_0, E''_1, \ldots, E''_{h_2})$, where $E'_0=\{a, c\}$ and $E''_0=\{c, b\}$ are π-networks. In the first case the breakdown is called parallel and in the second it is called sequential.

(c) We call network $\mathfrak{M}(E_0, E_1, \ldots)$ connected if there is no breakdown of the set $\{E_1, E_2, \ldots\}$ into the direct sum of the two sets $\{E'_1, E'_2, \ldots\}$ and $\{E''_1, E''_2, \ldots\}$ such that $\{\bigcup_i \{E'_i\}\} \cap \{\bigcup_i \{E''_i\}\}=\Lambda$ (Λ stands for an empty multiplicity).

(d) We say that network $\mathfrak{R}(E_0, E_1, \ldots)$ has an offshoot if the set $\{E_1, E_2, \ldots\}$ breaks down into the direct sum of the two sets $\{E'_1, E'_2, \ldots\}$ and $\{E''_1, E''_2, \ldots\}$ such that $|\bigcup_i |E'_i|| \cap |\bigcup_i |E''_i|| = \{a\} \subset \mathfrak{R}$, where either $\bigcup_i |E'_i|$ has no poles or the only pole in $\bigcup_i |E'_i|$ is the apex a.

(e) The sequence of sets $E_{i_1}, E_{i_2}, \ldots, E_{i_r}, E_{i_1}$ forms a cycle if $r > 1$ and there is a sequence of apexes $a_{j_1}, a_{j_2}, \ldots a_{j_r}$ such that

$$a_{j_1}, a_{j_2} \in |E_{i_1}|, \quad a_{j_2}, a_{j_3} \in |E_{i_2}|, \ldots, \quad a_{j_r}, a_{j_1} \in |E_{i_r}|.$$

(f) A network is called plane if its geometric realization on a plane is such that any two poles may be joined by a broken line the ends of which are the only common points contiguous with the network.

(g) It can be said formally that the symbol involves some multiplicity of objects called poles.

(h) For the sake of brevity we will henceforth write S_a. instead of $S_a(,)$

(i) This means that the cells of memory Z^a are realized in the elementary sub-scheme $S_a^{E^a}(X^a, Y^a, Z^a)$.

(j) If $\rho_a = 1$, the memory cell provides the constant.

(k) It is also possible to introduce a more complete characteristic which would take into account both the character of a spontaneous change in time and the probabilities of such changes. In application, however, it can usually be considered that there is either no forgetting $(\tau_a(t) = \infty)$, or that total forgetting occurs over the period $\tau_a(t)$. On the basis of this we have limited ourselves to examining only one characteristic - the degree of forgetting.

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