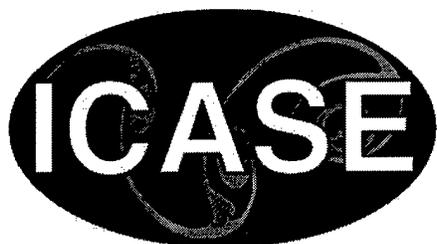


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# SOME REMARKS CONCERNING RECENT WORK ON ROTATING TURBULENCE\*

YE ZHOU<sup>†</sup>, CHARLES G. SPEZIALE<sup>‡</sup>, AND ROBERT RUBINSTEIN<sup>§</sup>

**Abstract.** A recent paper on rotating turbulence by Canuto and Dubovikov is examined from both an historical and scientific perspective. It is first shown that their claim of finding a new energy spectrum scaling is inaccurate; such a scaling law has been published in the literature by other authors using the same physical assumptions. Canuto and Dubovikov actually only offered a different estimate for the constant. Finally, it is demonstrated that the alternative model for the dissipation rate transport equation proposed by Canuto and Dubovikov does not have the desired physical features in rotating isotropic turbulence. It is physically inconsistent in both the weak and strong rotation limits.

**Key words.** turbulence modeling, rotating turbulence, rotation modified energy spectrum

**Subject classification.** Fluid Mechanics

**1. Introduction.** Recently, Canuto and Dubovikov [1-6] published six papers under the general title “A dynamical model of turbulence”. An application to rotating turbulence by Canuto and Dubovikov [5] (hereafter, denoted as CD) was offered by the authors as an example that the extension of their “dynamical model” is capable of “providing a coherent explanation of a wide variety of apparently disjoint qualitative and quantitative LES results”.

We feel that it is appropriate to address the three major issues in CD:

(1) It is rather regrettable that CD made somewhat inaccurate statements on what is new and what has been published in the literature by other researchers in the field.

(2) The abstract of CD stated that for rotating turbulence, “the spectrum exhibits a new form  $E(k) \sim (\epsilon\Omega)^{1/2}k^{-2}$ ”.

(3) In the introduction, the authors further stated that “we find that the spectrum exhibits a new form  $E(k) \sim (\epsilon\Omega)^{1/2}k^{-2}$ ”.

These statements are rather misleading and give the readers the erroneous impression that the  $E(k) \sim k^{-2}$  scaling of the energy spectrum of rotating turbulence was first found by CD. Hidden in the middle of the paper, the authors admitted that “In Ref. [22] a spectrum of the [same] type ... was obtained.”

Indeed, the rotation modified energy spectrum has been published by Zhou [7] in the *Phys. Fluids*. Ref. [22] of CD is based two fundamental assumptions: first, the energy transfer is local, and second, the time scale of the triple velocity correlation in the energy flux function is of the the order of  $1/\Omega$ . Here  $\Omega$  is the rotation rate of the fluid. These two assumptions lead directly to the  $k^{-2}$  scaling of the rotation modified energy spectrum<sup>7</sup>

$$(1) \quad E(k) = C_{\Omega}(\Omega\epsilon)^{1/2}k^{-2},$$

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where  $\epsilon$  is the dissipation rate. The constant  $C_\Omega$  is estimated as  $C_\Omega = 1/A = 1.22 - 1.87$  for the typical range of the Kolmogorov constant<sup>7</sup>. These results are supported by recent mathematical analysis [8] and direct numerical simulations [9].

**2. Analysis.** Now we turn our attention to CD, and review the procedure by which they derived the  $E(k) \sim k^{-2}$  spectrum for strongly rotating flows. It turns out that the main physical assumption in their “dynamical model” is “*that the transfer of energy is mainly local in character*”. Another assumption is that “*in the presence of rapid rotation, the triple correlation decays on a time scale of the order of  $\Omega^{-1}$* ”. Hence, these two basic assumptions in CD are exactly the same as in previously published results<sup>7</sup>. As we have shown in Ref. [7], these two assumptions are all that are needed for deducing an  $E(k) \sim k^{-2}$  energy spectrum for strongly rotating turbulence.

It is clear that the statements in CD and those cited above in (2) and (3) are inaccurate. CD should not have made the claim that “*we find that the spectrum exhibits a new form  $E(k) \sim (\epsilon\Omega)^{1/2}k^{-2}$* ”. Instead, CD only estimated a proportionality constant with a different value ( $\sqrt{45/8}$  to be specific.).

CD also presented a critique of the modeled dissipation rate transport equation

$$(2) \quad \dot{\epsilon} = -C_{\epsilon 2}^\Omega |\Omega| \epsilon$$

in Rubinstein and Zhou [10]. The energy equation for isotropic turbulence

$$(3) \quad \dot{K} = -\epsilon$$

and Eq. (2) implies that in a rotating isotropic turbulence, the turbulent kinetic energy  $K$  approaches a constant and the turbulent dissipation rate  $\epsilon$  approaches zero:

$$(4) \quad \epsilon(t) = \epsilon(0)e^{-\Omega t}$$

$$(5) \quad K_0(t) = K_0(0) - \frac{\epsilon(0)}{\Omega} [1 - e^{-\Omega t}].$$

CD argued that since in a  $k^{-2}$  inertial range cut off at a scale  $k_0$ , the relationship

$$(6) \quad K = C_\Omega \sqrt{\epsilon\Omega} k_0^{-1},$$

holds and the integral scale  $k_0^{-1}$  must increase exponentially. We shall now address the issue raised by CD.

We stress that in rotating turbulence, both a  $k^{-2}$  and a Kolmogorov  $k^{-5/3}$  scaling region can and do co-exist. The continuous transition between these regions was treated heuristically by Zhou<sup>7</sup>. At sufficiently low Rossby numbers (strong rotation), the energy spectrum  $E(k)$  is theoretically expected to follow the  $E(k) \sim k^{-2}$  scaling law for  $k \geq k_0(t)$ . Here  $k_0(t)$  denotes the lower edge of the inertial range. The  $E(k)$  profile shape for  $k \leq k_0(t)$  is, in general, problem dependent. The function  $k_0(t)$  is a property of the solution. To simplify our discussion, we use the “split spectrum” model

$$(7) \quad E(k) = \begin{cases} Ck^5 & \text{if } k \leq k_0 \\ C_\Omega \sqrt{\Omega\epsilon} k^{-2} & \text{if } k_0 \leq k \leq k_\Omega \\ C_K \epsilon^{2/3} k^{-5/3} & \text{if } k \geq k_\Omega. \end{cases}$$

Here, the parameter  $k_\Omega = (\Omega^3/\epsilon)^{1/2}$  appeared in Refs. [11] and [7] but was used in Ref. [5] without reference. The parameter  $k_\Omega$  separates the inertial range modified by rotation ( $k \leq k_\Omega$ ) from that of the traditional Kolmogorov inertial scales ( $k > k_\Omega$ ). In decaying turbulence,  $\epsilon \rightarrow 0$ , so that  $k_\Omega \rightarrow \infty$ . We therefore consider the case when the Kolmogorov inertial range does not appear in Eq. (7). The existence of the range of

scales  $k \leq k_0$  suggests an alternative to the spectral dynamics proposed in CD's argument against the model Eq. (2). Namely, suppose that in the limit of asymptotically long time,  $k_0$  approaches a constant. Then the  $k^{-2}$  range will disappear, since  $\epsilon$  approaches zero. Unlike the scenario proposed by CD, the turbulence kinetic energy can nevertheless approach a constant as required by Eq. (5), because energy can be trapped in the far infrared region of scales  $k \leq k_0$ , where it will undergo purely viscous decay. The single-point model proposed in Eq. (2) therefore does not entail indefinite exponential growth of the integral scale. Since single-point modeling entails a drastic loss of information, it is not surprising that a given single-point model can be consistent with several quite distinct models of the underlying spectral dynamics.

CD propose an alternative dissipation rate transport equation. It is also essentially that proposed by other authors and actually does not have the requisite physical properties. There is a problem at both weak and strong rotation rates as well as at both high and low turbulence Reynolds numbers. In the strong rotation limit of isotropic turbulence, where  $\Omega \rightarrow \infty$ , the dissipation rate transport model proposed by CD reduces to the form

$$(8) \quad \dot{\epsilon} = -C_{\epsilon 2} \frac{\epsilon^2}{K}$$

where the coefficient  $C_{\epsilon 2}$  is approximately equal to 2.84. This gives rise to a power law decay [12]

$$(9) \quad K \sim t^{-\alpha}$$

where the exponent  $\alpha$  is approximately equal to 0.54. The turbulence Reynolds number  $R_t \equiv K^2/\nu\epsilon$  actually undergoes a weak power law growth

$$(10) \quad R_t \sim t^\beta$$

where the exponent  $\beta$  is approximately equal to 0.46. It is now well established that the cascade is so disrupted by a rapid rotation that, at high turbulence Reynolds numbers, the turbulent kinetic energy remains approximately constant and  $\epsilon \rightarrow 0$  (the turbulence undergoes a linearly viscous decay since the phase coherence needed to cascade energy from the large to the small scales is scrambled; see Mansour, Cambon and Speziale [13]). Hence, there is no question that a power law decay for the turbulent kinetic energy and dissipation rate is physically incorrect in a rotating isotropic turbulence at high turbulence Reynolds numbers. Furthermore, it is even incorrect for low turbulence Reynolds numbers. It has been shown by DNS that, at low turbulence Reynolds numbers, the turbulence Reynolds number peaks, after undergoing a dramatic increase, and then decays – an effect that cannot be represented by the weak power law growth given in (10). This is a precursor to the high Reynolds number result of  $\epsilon \rightarrow 0$  and, thus,  $R_t \rightarrow \infty$ . It results from the imbalance between the production of dissipation by vortex stretching and the leading-order part of the destruction of dissipation caused by the rapid rotation which scrambles the turbulence [13]. A power law decay cannot properly describe this effect. More sophisticated models – that account for non-equilibrium vortex stretching – are needed [13].

As far as the weak rotation limit is concerned, the dissipation rate model of CD reduces to the asymptotic form

$$(11) \quad \dot{\epsilon} = -C_{\epsilon 2} \frac{\epsilon^2}{K} - C_{\epsilon 3} K \Omega^2$$

where  $C_{\epsilon 2}$  and  $C_{\epsilon 3}$  are constants that assume the values of 11/6 and 1/10, respectively. This equation can be integrated analytically yielding  $\epsilon/K$  going as the  $\tan ct$  to a power where  $c$  is a constant. Thus, the solution is oscillatory. DNS and physical experiments have *not* indicated that there are oscillations in the

turbulent kinetic energy and dissipation rate for this problem. So this model is, unquestionably, incorrect. Equation (11) – which actually forms the basis of the Hanjalić and Launder [14] model – has been integrated numerically by a Runge-Kutta numerical integration technique (see Speziale and Gatski [15]). It was found that – for an initial condition of  $\Omega K_0/\epsilon_0 = 0.496$  which is not that far removed from the region of small rotation rates – the turbulent kinetic energy grew during the first few eddy turnover times and the solution became unrealizable through the development of negative dissipation rates [15]. Realizability is violated by (11) since  $\epsilon/K = 0$  is not a fixed point in that equation unlike in the classical dissipation rate equation; this allows negative values of the turbulent dissipation rate to develop ( $\epsilon = 0$  is no longer an invariant plane in dynamical systems terms). Hence, it is clear that the asymptotic form (11) proposed by CD for weak rotation rates is fraught with problems.

Finally, it should be noted that the results of CD are only applicable to rotating isotropic turbulence where it still has problems as shown above. CD gave the erroneous impression that their results were more general, which is simply not accurate.

**3. Conclusions.** We have examined a recent paper on rotating turbulence by Canuto and Dubovikov [*Phys. Fluids* **9**, 2132 (1997)] from both an historical and scientific perspective. We have shown that their claim of finding a new energy spectrum scaling is inaccurate; such a scaling law has been published in the literature by other authors using the same physical assumptions. Canuto and Dubovikov actually only offered a different estimate for the constant. Finally, we have demonstrated that the alternative model for the dissipation rate transport equation proposed by Canuto and Dubovikov does not have the desired physical features in rotating isotropic turbulence. It is physically inconsistent in both the weak and strong rotation limits

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