A Fracturing of Polygonal Objects

by Andrew M. Neiderer, Mark A. Thomas, and Richard Pearson

APRIL 1998

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A Fracturing of Polygonal Objects

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Abstract

This report presents an approach for the fracture of polygonal objects. The technique was developed by the U.S. Army Research Laboratory (ARL) for real-time display of munitions penetrating urban structures. A weapons effects model and exterior polygon clipper were designed to accomplish this. The algorithm has been successfully implemented as a component of a combat simulator.
Acknowledgments

The authors of this report would like to thank Timothy Gion, a student at The University of Delaware, for his invaluable programming support. He was responsible for implementing this work on a Silicon Graphics, Inc., ONYX Reality Engine using Performer for real-time display.
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1. Introduction

In an effort to better prepare soldiers in a battlefield environment, the U.S. Army Research Laboratory (ARL) is developing a comprehensive combat simulator [1]. Many components, such as dynamic terrain and battlefield smoke, have already been included. Recently, a capability for the approximation of a munition penetrating an urban structure was added. In order to maintain real-time display of this phenomenon, weapons effects had to be simplified.

Weapons effects can be divided into blast and kinetic energy (KE) effects. A blast weapon detonates its high explosive (HE) warhead at the target surface or inside the structure. This usually results in extensive damage; large HE warheads can even cause structural collapse. However, this paper only addresses KE effects. A solid KE penetrator delivers energy to a localized region of the target, and typically results in a small perforation with respect to the size of the impacted object.

The following sections present a technique for geometrical change and attribute update of a polygonal object that undergoes fracture. The algorithm can handle penetrations in any part of a polygonal object. This includes the ideal situation that results in a circular hole, as well as the more general case of elliptical hole formation. A simple empirical solution for more realistic perforations is then described. Finally, ARL’s implementation of the method is discussed, and an example is given.

2. Fracture of a Polygonal Object

The penetrating projectile $p$ is assumed to have a circular, cross-sectional area of radius $r_p$; the target $t$ is a polygonal object, where each surface is defined by coplanar vertices. These two restrictions allow for the application of polygon clipping.

Polygon clipping is a procedure for determining which portions of a subject polygon lie inside and/or outside a clipping region. Typically, the algorithm starts by clipping a polygon against a single, infinite clip edge (2D) or plane (3D). If there is no intersection, the polygon is either trivially accepted or rejected. Otherwise, the points where polygon edges intersect the clipping region are computed. These intersection points are vertices of a new intermediate polygon that is then clipped against the next clipping region and so on. Note that polygons must remain closed (i.e., both input and output to a polygon clipper are polygons).

The next section first presents an algorithm for the generation of a circular hole in a target, then a solution for the more general case, where the normal direction vector of the target $\hat{u}_t$ does not necessarily coincide velocity vector of the projectile $\vec{v}_p$, is described. Both involve geometrical manipulation of objects in the 3D scene and a subsequent application of an exterior polygon clipper.

2.1. Circular Hole Formation

The following algorithm could be used for a normal impact, where the direction of the projectile velocity vector aligns with the unit normal of the target. The geometry in this case is simpler than that for an oblique impact and allows one to focus on the process of hole formation.
2.1.1. Algorithm

Every object in the 3D scene is described by one or more planar polygons. Each of these polygons is positively oriented (i.e., the order of vertices is counterclockwise as you face the polygon). Because polygon vertices are ordered, an edge can be treated as a vector and therefore the normal vector \( \vec{n} \) to the plane of a polygon is computed by taking the cross product of adjacent edge vectors; the unit normal \( \hat{n} \) is \( \vec{n} / |\vec{n}| \).

Once the point of impact \((x_i, y_i, z_i)\) is known, each polygon of the affected object is geometrically transformed so that \((x_i, y_i, z_i)\) is at the origin, and the unit normal \(\hat{n}\) is aligned with the positive z-axis. This is accomplished by first rotating about the x-axis through an angle \(\alpha\), and then about the y-axis by \(\beta\) degrees [2]. An exterior polygon clip is then performed about a region containing \((x_i, y_i)\), where \(z_i = 0\) for each rotation \(\delta\) about the z-axis. Thus, the complete procedure is to

1. translate the object so that \((x_i, y_i, z_i)\) is at the origin,
2. rotate about the x-axis till \(\hat{n}\) lies in the xz plane,
3. rotate about the y-axis to make \(\hat{n}\) coincident with the z-axis,
4. for each rotation \(\delta\) about the z-axis perform an exterior polygon clip,
5. apply the inverse of (3),
6. apply the inverse of (2), and
7. apply the inverse of (1).

If the point of impact is designated as the centroid of a 2D clipping square, then performing an exterior polygon clip for \(0 \leq \delta < 90\) at small increments results in a “circular” hole (see Figure 1). The details of an exterior polygon clip are now examined.

2.1.2. Clipping a Polygon Exterior to a Region

There are many published solutions for clipping polygons (convex or concave) interior to a region. We have modified Heckbert’s implementation of the Sutherland-Hodgman (SH) algorithm [3] for exterior polygon clipping. Heckbert uses an interpolation mask for the efficient update of polygon vertex attributes; relevant attributes are bilinearly interpolated across the face of the polygon. However, the code has two restrictions: (1) subject polygons must be convex, and (2) the clipping region is defined as a 3D rectangular parallelepiped.

A subject polygon from the appropriate object is passed through the modified SH polygon clipper. A trivial accept/reject is first determined by counting vertices outside/inside with respect to each of the four clip edges. If the subject polygon is completely outside each of the four edges, it does not have to be clipped. If it is entirely within each of the four edges it is discarded. Only if neither of these conditions applies (i.e., crosses an edge) does it have to be clipped.

The subject polygon, which is convex, is clipped against one of the four infinite clip edges. The resulting exterior polygon, if any, will also be convex. This remainder polygon is then clipped against each of the other three clip edges in succession.

The choice of the z-axis is arbitrary. The idea is to make \(\hat{n}\) coincide with one of the coordinate axes, which will then be the rotation axis.
Figure 1: A "circular" (5° increments for $0 \leq \delta < 90$) hole.

Figure 2 shows an example of a nontrivial exterior polygon clip. The planar polygon consists of 48 subject polygons (Figure 2a). As shown in Figure 2b, only polygons 19 and 20 need to be clipped against the clipping square (i.e. there are 46 trivial cases). Let us examine the exterior polygon clip of polygon 19 (Figure 2c). Clipping this polygon against the left, infinite clip edge results in an exterior polygon and an intermediate polygon (Figure 2d). In Figure 2e, the intermediate polygon is clipped to the right clip edge, which results in a second exterior polygon. The new intermediate polygon does not have to be clipped against the bottom clip edge since there is no intersection (Figure 2f). Finally, the intermediate polygon is clipped against the top clip edge (Figure 2g). The three resultant exterior polygons are saved and could be either output or used as input for a new orientation of the clipping square.
2.2. Elliptical Hole Formation

This section considers the geometry for a projectile penetrating at an oblique angle. A simple solution for realistic perforations of a target is presented.

2.2.1. A True Elliptical Hole

A penetrating projectile of radius $r_p$ results in a conic (minus the hyperbola) perforation of the target. The shape is a function of $\cos(\gamma)$, where $\gamma$ is the angle between $\vec{v}_p$ and $\vec{u}_t$ satisfying $0 \leq \gamma < 90$. The upper limit of $\gamma$ is the grazing angle. $\gamma = 0$ corresponds to normal impact and forms a circular perforation. Any other value of $\gamma$ results in an elliptical shape.

The true elliptical hole has semiminor axis length $b = r_p$ and semimajor axis length $a = r_p / \cos(\gamma)$. The first 3 steps of the algorithm presented in section 2.1.1 place the point of impact at the origin with the unit normal of the appropriate polygon aligned with the positive $s$-axis. The next step is to rotate about the $z$-axis by some angle $\eta$, aligning the semimajor axis with the $x$-axis at the plane of the target. The inverse of these four steps needs to be applied after exterior polygon clipping at this orientation.

An aligned, origin-centered ellipse can be mathematically described by the parametric functions

$$x(\theta) = a \cos(\theta), \text{ and } y(\theta) = b \sin(\theta)$$

for $0 \leq \theta < 2\pi$ radians. The 2D coordinates $[x(\theta) \ y(\theta)]$ of points along the perimeter of the ellipse in Euclidean space are computed by taking uniform angular increments of the parameter $\theta$. However, one can decrease computation time by restricting the range to $0 \leq \theta \leq \pi/2$, and then use four-way symmetry (4WS) to determine the rest of the points.
Figure 3: Hole formation for an origin-centered ellipse.

Figure 3 illustrates the previously mentioned technique. The points satisfying $0 < \theta < \pi/2$ at an angular increment $d\theta = \pi/12$ are shown in Figure 3a; the upper and lower limit, namely $\theta = 0$ and $\theta = \pi/2$, are not needed when determining clipping rectangles for elliptical hole formation. Symmetrical points and clipping rectangles are then determined (Figure 3b).

2.2.2. A Jagged Elliptical Hole

An infinite number of clipping rectangles $n$, where $n$ is a natural number, are needed to generate a true elliptical hole using this approach, and the number of computations increases as $n \rightarrow \infty$. Decreasing $n$ reduces computation time, but may result in unacceptable error. Consider Figure 4, where $n = 5$. An exterior polygon clip against any of these clipping rectangles could result in a vertex within the shaded region (see Figure 4a). This is not physically possible since the hole must be at least the size of the true ellipse. No matter how large of an $n$ is used, there will still be some associated error. Therefore, the exterior polygon clip needs to be done against circumscribed clipping rectangles (see Figure 4b). In this figure $\varepsilon$ corresponds to the distance for a minimum enclosing band. A uniform random number generator (RNG) is used to add noise to position vectors of polygon vertices that are within some $\Delta$ of the true ellipse. This technique provides for a more realistic-looking perforation.

An example of elliptical fracture using five clipping rectangles is given in Figure 5. The impact point for the penetrating projectile of velocity direction vector $[-0.577 0.577 0.577]$ is located at the center of the $z = 0$ planar polygon.
Figure 4: An ellipse described by five clipping rectangles.

Figure 5: Elliptical fracture of a polygon for \( \hat{\mathbf{p}} = [-0.577, 0.577, 0.577] \).
3.  Real-Time Implementation

The fracture algorithm has been integrated into a Distributed Interactive Simulation (DIS). The following discusses the implementation for ARL's Stealth DIS simulator. This involved the development of a modular library libhole; but before using this library, a determination is made as to whether penetration even occurs.

3.1.  Penetration Mechanics

The Detonation Protocol Data Unit (DPDU) and the Collision Protocol Data Unit (CPDU) contain properties of the projectile and target. The projectile has a circular, cross-sectional area of radius \( r_p \). It is also characterized by its mass \( m_p \) and a velocity vector \( \vec{v}_p \). The target is homogeneous and isotropic throughout its thickness \( t_t \). Its orientation is specified by the normal direction vector \( \vec{u}_t \). The penetration depth \( z \) is computed by using one of the following empirical relationships [4].

1. For a ceramic target,

\[
z_c = \frac{222 \cdot p_p \cdot d_p^{0.215} \cdot |\vec{v}_p|^{1.5}}{\sigma_t^{0.5}} + 0.5 \cdot d_p,
\]

where

- \( p_p \) is the sectional pressure of the projectile (lb/in\(^2\)),
- \( d_p \) is the diameter of the projectile (in),
- \( |\vec{v}_p| \) is the velocity of the projectile (ft/s x 1000), and
- \( \sigma_t \) is the compressional strength of the ceramic target (lb/in\(^2\)).

2. For a metal target,

\[
z_m = 0.21 \cdot w_p^{0.33} \cdot |\vec{v}_p|^{1.22} \quad \text{and} \quad 
z_m = z_0 \cdot \epsilon \left( 8.77 \cdot 10^{-6} \cdot (B_t^2 - B^2) \right) - 5.41 \cdot 10^{-3} \cdot (B_t - B),
\]

where

- \( w_p \) is the weight of the projectile (oz),
- \( |\vec{v}_p| \) is the velocity of the projectile (ft/s x 1000),
- \( B_t \) is the Brinell hardness of the metal target, and
- \( B \) is the Brinell hardness of common mild structural steel (i.e., \( B = 150 \)).

3. For a wood target,

\[
z_w = \frac{10041 \cdot |\vec{v}_p|^{0.41} \cdot w_p^{0.59}}{\rho_t \cdot h_t^{0.54}},
\]

where

- \( w_p \) is the weight of the projectile (oz),
- \( |\vec{v}_p| \) is the velocity of the projectile (ft/s x 1000),
\( \rho_t \) is the density of the wood target (lb/ft\(^3\)), and
\( h_t \) is the hardness of the wood target (lbs).

The simulator then determines the geometry at the point of impact by executing the Performer\(^2\) function pfNodeIsectSegs(). The data is stored in the following structure:

```c
typedef struct {
    long flags;
    pfVec3 point;
    pfVec3 norm;
    long tri;
    long prim;
    pfGeoSet *gset;
    pfGeode *geode;
    pfMatrix mat;
} HitInfo.
```

The fields include the point of intersection, the normal vector at this point, identification number of the triangle that was impacted, the number of the corresponding primitive, and various geometry parameters.

The penetration depth \( z \) for the impacted object is compared to the line-of-sight thickness.

\[
ILOS = \frac{t}{\cos(\gamma)},
\]

where \( \gamma \) is the angle between the direction vector of the projectile and the unit normal of the target satisfying \( 0 \leq \gamma < 90 \). If \( z > t_{LOS} \), then there is penetration and the hole() function is called (see Figure 6).

### 3.2. The Hole Library

The libhole library consists of the simulation interface module (SIM) and the polygon clipper module (PCM). The polygon clipper, which was described in section 2, produces the actual geometry modifications. The SIM generates the appropriate data structures needed by the PCM, and converts the output of the polygon clipper into simulation geometry; it uses Performer pfdBuilder functions to create this resultant geometry.

The calling syntax is

```c
hole(pfGroup *parent, HitInfo *hitinfo, float diameter, float gamma).
```

The parent geometry node may be passed as NULL. The HitInfo structure points to the original geometry set data structure, or pfGeoSet; this contains the actual vertex information needed by the PCM. The other two parameters define the shape of the perforation.

\(^2\)Performer is a registered trade mark of Silicon Graphics, Inc. (SGI).
4. Results and Future Work

A combat simulator for the training of soldiers should include events that may influence decision-making within that environment. This paper addressed one such event: a munition that penetrates an urban structure, which involves fracture of a polygonal object in the ARL Stealth simulator environment. The approach taken should also allow for easy integration into existing simulators as well.

An example is given in the appendix. Here an explosive had been placed on the wall, then detonated for tactical entry to the building. Real-time simulation of this scenario has been demonstrated at the Association of the United States Army (AUSA) annual meeting held in Washington D.C. on October 1996.

Since an impact may result in a large number of polygons, both in the target and fragments, decimation logic to reduce the polygon count is being implemented. Future work should include a more comprehensive database of material types. In addition, the effect of multiple perforations against a structure, which could result in structural failure, needs to be investigated.
References


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APPENDIX:

Wall Perforation
Appendix

These two figures represent successful penetration of the wall of a building at normal incidence. Note the objects inside the building and that the “hole” is somewhat regular.

(a) before penetration  
(b) after penetration

Figure A-1: Perforated urban structure for small $\Delta$ and $\delta = 22.5^\circ$. 

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