ANALYTICAL MODELS FOR MOBILE SENSOR (UAV) COVERAGE OF A REGION

Institute for Joint Warfare Analysis
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Summary

The effective use of mobile sensors (UAVs for example, but not exclusively) to investigate own and opponent force status in geographical regions requires planning to compensate for their limitations. These limitations include finite endurance, and realistic mission unreliability: the failure propensities of platform and sensing packages, and the consequent need for maintenance and logistics support.

This paper supplies analytical (mathematical) models (AMM) to assist planners, and the acquirers and operational testers of mobile sensor assets. The results are formulas (that can be quickly evaluated numerically) for the expected time on station (fraction of time a region has sensor presence) of several cooperatively operating systems supported by several maintenance facilities. These results can be used to estimate the number of sensors of a particular type needed to cover a region adequately.

Limited evidence indicates usefully satisfactory agreement between analytical model estimates and those from more detailed and elaborate Monte Carlo simulations (MCS).
Even with modern fast computers the exercise of AMMs is more rapid than of MCSs, so initial exploratory investigation by AMMs is indicated.

1. A Generic Situation

A platform carrying sensing equipment, e.g. JSTARS, AWACS, or an RPV/UAV is used to carry out surveillance of a region. The platform has finite endurance: it requires a time $T$ to transit outbound from home base (airbase on land, Navy carrier) to the target region, then can remain on-station over that region, performing surveillance, for time $S$, and finally transit back again for time $T$. Once back on base it spends a time $D$ for refueling, maintenance, etc.

Often several such platforms must operate cooperatively in order to achieve adequate coverage of a relatively distant region. These may be serviced at home base by several maintenance facilities; in some cases there are fewer maintenance “paths” than platforms, in which case queues can form and the coverage correspondingly degenerates. We provide models that describe and predict the long-run consequences in such cases.

1.1 A Basic Formula: Consider that a cycle begins each time the UAV takes off from its home base, and ends when it is again ready to do so. Let $Y(t) = 1$ if the platform is on station and $Y(t) = 0$ otherwise. \( \{Y(t); \ t \geq 0\} \) is a degenerate alternating renewal process. The long-run proportion of time the platform is on station if no systems fail is seen to be

$$\lim_{s \to \infty} \frac{1}{s} \int_{0}^{s} Y(t) dt = \pi = \frac{S}{2T + S + D}.$$ \hspace{1cm} (1.1)

However, it is realistic, and essential, to account for failures during the mission, and their effect on time on station. Failures, and their repair times, make the on-station availability predicted by (1.1) degenerate, and our analytical models efficiently show how this occurs, although approximately. The Monte Carlo simulations of Pendergast and Stoneman (1998) are potentially more versatile and accurate, but require more setup and
computer time. The analytical models (here) are convenient and adequate for explorations.

2. Single System Models

VARIATION 1: Assume mission-affecting failures (MAF) occur according to a Poisson process with rate \( \lambda \). Once a single MAF occurs, the UAV immediately returns to base (crashes are not modeled here). If additional failures occur when returning to base, these influence the total repair time. Model the repair times as independent and random, with mean \( 1/\mu \) and let \( C \) be the duration of a cycle: total time from one takeoff to the next.

Let \( X \) be the time until failure. Let \( A \) be a random on-station time, and let \( B \) be the time from platform take-off until return to base, then it is shown in Appendix I that

\[
\pi = \frac{E[A]}{E[C]} = \frac{e^{-\lambda T} \frac{1}{\lambda} \left[1 - e^{-\lambda S}\right]}{\frac{2}{\lambda} \left[1 - e^{-\lambda T}\right] + e^{-\lambda T} \frac{1}{\lambda} \left[1 - e^{-\lambda S}\right] + D + \left[1 - e^{-(2T+S)}\right] \frac{1}{\mu}}
\]  

(2.1)

VARIATION 2: Suppose the need for unscheduled maintenance activities (UMA) occur according to a Poisson process with rate \( \alpha_U \) during the platform flight. These (UMA) do not shorten the flight but each requires a random time to perform with mean \( 1/\beta_U \) hours. In addition there is one Administrative logistic down time (ALDT) with mean \( 1/\beta_A \). Assume that scheduled maintenance activities (SMA) occur at a rate of \( \alpha_S \) per flight hour. Each scheduled activity requires \( 1/\beta_S \) hours. Model \( E[D] \) as follows

\[
E[D] = \frac{\alpha_U E[B]}{\beta_U} + \frac{\alpha_S E[B]}{\beta_S} + \frac{1}{\beta_A}
\]

with expected number of UMA's to occur in a cycle and approx. number of SMA's to occur in a cycle.
Numerical Example.

Assume MTBMAF=25 hours. There is scheduled maintenance of 7 hours at 50 hr. intervals. UMAs occur at a rate of $\lambda = 1/5$ per hour and each requires $1/\beta_U = 1.9$ hours of maintenance; the mean ALDT time $1/\beta_A = 0.5$. The mean repair time for a MAF is 1.9 hours +0.5 ALDT. The total time the platform can be in the air is 20 hours. $T$ is the ingress/egress time.

<table>
<thead>
<tr>
<th>$T$</th>
<th>Analytic ETOS</th>
<th>Simulation ETOS (95% Confidence Interval)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.52</td>
<td>0.5200 - 0.5332</td>
</tr>
<tr>
<td>2</td>
<td>0.45</td>
<td>0.4389 - 0.4511</td>
</tr>
<tr>
<td>3</td>
<td>0.38</td>
<td>0.3697 - 0.3849</td>
</tr>
<tr>
<td>4</td>
<td>0.32</td>
<td>0.3121 - 0.3284</td>
</tr>
<tr>
<td>5</td>
<td>0.26</td>
<td>0.2526 - 0.2667</td>
</tr>
<tr>
<td>6</td>
<td>0.21</td>
<td>0.1978 - 0.2094</td>
</tr>
</tbody>
</table>

3. Many-System Models: Ample Service

**VARIATION 3:** Assume there are $N$ platforms. Assume there is ample service capacity, here $N \leq s$, the number of maintenance paths or servers so that no platform needs to wait for maintenance or repair. The platforms are launched independently of one another when they are available; a pessimistic assumption. Then an approximation for the long-run proportion of time the region is under surveillance by at least one platform is $1 - (1 - n)^N$.

For the parameters of the previous numerical example, wherein $N = 4$, the long-run proportion of time the region is under surveillance by at least one platform is
Fraction of Time at Least One UAV is On Station

<table>
<thead>
<tr>
<th>$T$</th>
<th>Analytical Model Fraction of Time</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.95</td>
<td>0.9565 – 0.9642</td>
</tr>
<tr>
<td>2</td>
<td>0.91</td>
<td>0.9161 – 0.9247</td>
</tr>
<tr>
<td>3</td>
<td>0.85</td>
<td>0.8694 – 0.8946</td>
</tr>
<tr>
<td>4</td>
<td>0.79</td>
<td>0.8279 – 0.8451</td>
</tr>
<tr>
<td>5</td>
<td>0.70</td>
<td>0.7535 – 0.7949</td>
</tr>
<tr>
<td>6</td>
<td>0.60</td>
<td>0.6921 – 0.7093</td>
</tr>
</tbody>
</table>

The simulation results are from the MASS model; Pendergast and Stoneman (1998).

The analytical model gives smaller fractions of time the region is covered. This behavior is because the simulation includes scheduling of the UAVs, which the analytical model does not include.

4. A Model With Limited Service: Markovian Approximation

Suppose there are $s$ maintenance paths: servers in queuing theory parlance. Let the times between maintenance/repair for a UAV be iid exponential with mean $1/\gamma$ and assume the times for maintenance/repair are iid exponential with $1/\rho$. Let $X(t)$ be the number of UAVs down (receiving or awaiting maintenance/repair) at time $t$.

Model: \{\(X(t); t > 0\)\} is a continuous time Markov chain with limiting distribution

$$
\psi(x) = \lim_{t \to \infty} P\{X(t) = x\} = \prod_{i=0}^{x-1} \frac{(N-i)\gamma}{\min(i+1,s)\rho} \psi(0)
$$

(4.1)

for $x = 1, \ldots, N$ where $N$ is the number of UAVs. $\psi(0)$ is found by solving

$$
1 = \sum_{i=0}^{N} \psi(i)
$$

(4.2)

Let $\pi_u = \frac{E[A]}{E[B]}$ be an approximation for the fraction of time a flying UAV is on station. Model the long-run fraction of time the region is covered by a UAV by
\[
\pi_s = \sum_{x=0}^{N} [1 - (1 - \pi_0)^{N-x}] \nu(x)
\] (4.3)

**Numerical Example**

Take \(1/\gamma = E[B]\) and \(1/\rho = E[D] + \left(\frac{1}{\lambda} + \frac{1}{2T+S}\right) \frac{1}{\mu}\). For the parameter values of the previous numerical example with 4 UAVs, \(T = 3\) and a variable number of servers:

<table>
<thead>
<tr>
<th>Number of Servers</th>
<th>Fraction of Time</th>
<th>Area Covered</th>
<th>Simulation 95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.62</td>
<td></td>
<td>0.8299 - 0.8601</td>
</tr>
<tr>
<td>2</td>
<td>0.80</td>
<td></td>
<td>0.8754 - 0.8947</td>
</tr>
<tr>
<td>3</td>
<td>0.85</td>
<td></td>
<td>0.8713 - 0.8951</td>
</tr>
<tr>
<td>4</td>
<td>0.85</td>
<td></td>
<td>0.8750 - 0.8891</td>
</tr>
</tbody>
</table>

If there are 4 UAVs, one maintenance path and variable ingress/egress time

<table>
<thead>
<tr>
<th>(T)</th>
<th>Model Fraction of Time</th>
<th>Simulation 95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.73</td>
<td>0.9529 - 0.9604</td>
</tr>
<tr>
<td>2</td>
<td>0.68</td>
<td>0.8977 - 0.9194</td>
</tr>
<tr>
<td>3</td>
<td>0.62</td>
<td>0.8299 - 0.8601</td>
</tr>
<tr>
<td>4</td>
<td>0.55</td>
<td>0.7678 - 0.7916</td>
</tr>
<tr>
<td>5</td>
<td>0.48</td>
<td>0.6540 - 0.6836</td>
</tr>
<tr>
<td>6</td>
<td>0.40</td>
<td>0.5230 - 0.5493</td>
</tr>
</tbody>
</table>

**5. A Model with Scheduling: Replacement UAVs Always Available**

In this section we describe a model that accounts for scheduling the UAVs so that one is always on station. Initially assume that there are always UAVs to replace on-station UAVs. Let \(T\) be the ingress/egress time and \(S\) be the time on station. We assume \(2T < S\).

Assume a UAV has just arrived on station at time 0. Its replacement must take off at time \(S - T\) in order to relieve it when the first UAV starts its trip back to base. The egress time is \(T\).
Assume a UAV experiences a mission affecting failure after an exponential length of time having mean $1/\lambda$.

Assume a replacement UAV is launched $T$ time units before the on-station UAV is to start to return to base or upon failure of the on-station UAV whichever comes first. Let $L$ be the length of time a replacement UAV is late to the region; during this time the region is not covered.

\[
E[L] = e^{-\lambda T} 0 + \int_0^T \lambda e^{-\lambda x} \left( x + \frac{E[L]}{\text{replacement launched}} \right) dx
\]

\[
= \frac{1}{\lambda} \left[ 1 - e^{-\lambda T} - (\lambda T) e^{-\lambda T} \right] + (1 - e^{-\lambda T}) E[L].
\]

Thus,

\[
E[L] = \frac{1}{\lambda} \left[ 1 - e^{-\lambda T} - (\lambda T) e^{-\lambda T} \right] e^{-\lambda T} = \frac{1}{\lambda} \left[ e^{-\lambda T} - 1 - \lambda T \right].
\]

Notice that the expected lateness increases exponentially with the transit time to station, which argues for getting closer to the region.

Consider a cycle to start when a new UAV arrives on station. The expected length of a cycle is

\[
E[C] = e^{-\lambda S} [S + E[L]] + \int_0^{S-T} \lambda e^{-\lambda x} dx \left[ x + E[L] + T \right] + \int_{S-T}^S \lambda e^{-\lambda x} dx \left[ S + E[L] \right]
\]

\[
= E[L] + e^{-\lambda (S-T)} S + \frac{1}{\lambda} \left[ 1 - e^{-\lambda (S-T)} - (\lambda (S-T)) e^{-\lambda (S-T)} \right] + T \left[ 1 - e^{-\lambda (S-T)} \right]
\]

\[
= E[L] + \frac{1}{\lambda} \left[ 1 - e^{-\lambda (S-T)} - \lambda (S-T) e^{-\lambda (S-T)} \right] + e^{-\lambda (S-T)} S + T \left[ 1 - e^{-\lambda (S-T)} \right]
\]

\[
(5.2)
\]

\[
(5.3)
\]

\[
(5.4)
\]

\[
(5.5)
\]
Let $V$ be time the region is not covered during a cycle.

\[
E[V] = (T + E[L]) \left( 1 - e^{-\lambda (S-T)} \right) + e^{-\lambda (S-T)} \int_{0}^{T} \lambda e^{-\lambda x} \left[ T - x + E[L] \right] dx
\]

\[
= (T + E[L]) \left( 1 - e^{-\lambda (S-T)} \right)
\]

(5.6)

The long run fraction of time the region is not covered is

\[
\alpha = \frac{E[V]}{E[C]}. \quad (5.7)
\]

6. A Model with Scheduling: Limited Repairs

Let $X_i$ be the number of failures during a cycle for the ingressing UAV. If there is always a replacement UAV available, then

\[
E[X_i] = \left( 1 - e^{-\lambda T} \right) + \left( 1 - e^{-\lambda T} \right) E[X_i]
\]

(6.1)

\[
E[X_i] = \frac{\left( 1 - e^{-\lambda T} \right)}{e^{-\lambda T}} = e^{\lambda T} - 1.
\]

(6.2)

Let $X_o$ be the number of failures for the UAV on station.

\[
E[X_o] = \left( 1 - e^{-\lambda (S+T)} \right)
\]

(6.3)

where $T$ is the egress time.

The long run average failure rate is

\[
\lambda_0 = \frac{E[X_o] + E[X_i]}{E[C]}. \quad (6.4)
\]

Consider a queue with arrival rate $\lambda_0$ and a maximum number of customers waiting or being served equal to $N$, the total number of UAVs and $s$ servers (maintenance paths,
each with service rate \( \rho \). Let \( \phi(x) \) be the long run proportion of time there are \( x \) UAVs either waiting for or being repaired.

\[
\phi(x) = \prod_{i=0}^{x-1} \frac{\lambda_0}{\min(i+1,s)\rho}\phi(0)
\]

where \( \phi(0) \) is determined so that the sum is one and \( N \) is the total number of UAVs.

Approximate the long run fraction of time during which there is at least one UAV on station as

\[
\alpha_s(1) = \left(1 - \phi(N)\right) \alpha \quad \leftarrow \text{From (5.7)}
\]

long-run fraction of time at least one UAV is up from queuing model

or

\[
\alpha_s(2) = \left(1 - \phi(N) + \phi(N-1)\right) \alpha
\]

long-run fraction of time at least 2 UAVs are not waiting for or being repaired

and \( \alpha \) is the long run fraction of time there is a UAV on station for the schedule model of Section 5.

**Numerical Example**

Take \( \frac{1}{\rho} = E[D] + \left(\frac{1}{\lambda} + \frac{1}{2T+S}\right)\frac{1}{\mu} \). Suppose there are 4 UAVs, one maintenance path and variable ingress/egress time. For the parameter values of the previous examples

<table>
<thead>
<tr>
<th>Fraction of Time at Least One UAV is On Station</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T )</td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>6</td>
</tr>
</tbody>
</table>

The simulation results are from Stoneman (1998).
REFERENCES


APPENDIX I

$B$ is the time between takeoff and return to base. Then

$$B = \begin{cases} 
2X & \text{if } X < T \text{ (fails on flight out)} \\
X + T & \text{if } T < X < T + S \text{ (fails during surveillance time)} \\
2T + S & \text{if } X > T + S \text{ (fails on flight back or does not fail)}
\end{cases} \quad (AI.1)$$

Thus,

$$E[B] = \int_0^T 2s\lambda e^{-\lambda s} \, ds + e^{-\lambda T} \int_0^S (s + 2T)\lambda e^{-\lambda s} \, ds + e^{-\lambda (T+S)}(2T+S)$$

$$= \frac{2}{\lambda} \left[1 - e^{-\lambda T}\right] + e^{-\lambda T} \frac{1}{\lambda} \left[1 - e^{-\lambda S}\right] \quad (AI.2)$$

Let $A$ be the time on station

$$A = \begin{cases} 
0 & \text{if } X < T \text{ (Platform fails on ingress)} \\
X - T & \text{if } T < X < T + S \text{ (Platform fails during surveillance)} \\
S & \text{if } X > T + S
\end{cases} \quad (AI.3)$$

Thus,

$$E[A] = 0\left[1 - e^{-\lambda T}\right] + e^{-\lambda T} \int_0^S s\lambda e^{-\lambda s} \, ds + e^{-\lambda (T+S)}S$$

$$= e^{-\lambda T} \frac{1}{\lambda} \left[1 - e^{-\lambda S}\right] \quad (AI.4)$$

The expected length of a cycle is

$$E[C] = E[B] + D + (1 - e^{-\lambda (T+S)}) \frac{1}{\mu} \quad (AI.5)$$

\[\text{prob of failure during cycle} \quad \text{expected repair time}\]
The long-run proportion of time the platform is performing surveillance:

\[ \pi = \frac{E[A]}{E[C]} \]

\[ = \frac{e^{-\lambda r} \frac{1}{\lambda} [1 - e^{-\lambda s}] + e^{-\lambda r} \frac{1}{\lambda} [1 - e^{-\lambda s}] + D [1 - e^{-(\lambda + s) \frac{1}{\mu}}]}{2 \frac{1}{\lambda} [1 - e^{-\lambda r}] + e^{-\lambda r} \frac{1}{\lambda} [1 - e^{-\lambda s}] + D [1 - e^{-(\lambda + s) \frac{1}{\mu}}]} \]

\[ (A1.6) \]
APPENDIX II
A Closed Queueing Network Model

Consider a closed queueing network with 4 “service centers” or states labeled as follows:

<table>
<thead>
<tr>
<th>Center</th>
<th>Number of Servers</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>Down</td>
</tr>
<tr>
<td>I</td>
<td>Ingress</td>
</tr>
<tr>
<td>S</td>
<td>On Station</td>
</tr>
<tr>
<td>E</td>
<td>Egress</td>
</tr>
</tbody>
</table>

Consider a closed migration process in the sense of Kelly (1979). The state of the process is \( \{(X_D(t), X_I(t), X_S(t), X_E(t)); t > 0\} \) where \( X_D(t) \) is the number of platforms down at time \( t \); \( X_I(t) \) is the number traveling to the region at time \( t \); \( X_S(t) \) is the number on station at time \( t \); and \( X_E(t) \) is the number traveling from the region at time \( t \). Assume \( \{(X_D(t), X_I(t), X_S(t), X_E(t)); t > 0\} \) is a continuous time Markov chain with

\[
P((X_{i}(t+h) = x_{i} - 1, X_{j}(t+h) = x_{j} + 1, X_{k}(t+h) = x_{k} \text{ for } k \neq i, j | X_{\ell}(t) = x_{\ell} \text{ for all } \ell) = \lambda_{ij} \phi_i(x_i)h + o(h)
\]
Put

\[ \lambda_{DI} = E[D] + \left(1 - e^{-\lambda(2T+S)}\right) \frac{1}{\mu} \]
\[ \lambda_{Dj} = 0 \text{ for } j = S, E \]
\[ \varphi_D(x_D) = \min(x_D, s) \]
\[ \lambda_{ID} = \lambda \]
\[ \lambda_{IS} = \frac{1}{T} \]
\[ \lambda_{Ij} = 0 \text{ for } j = E \]
\[ \varphi_I(x_I) = x_I \]
\[ \lambda_{SE} = E[A] \]
\[ \lambda_{Sj} = 0 \text{ for } j = D, I \]
\[ \varphi_S(x_S) = x_S \]
\[ \lambda_{ED} = \frac{1}{T} \]
\[ \lambda_{Ej} = 0 \text{ for } j = I, S \]
\[ \varphi_E(x_E) = x_E \]

Theorem 2.3 of Kelly (1979) states that

\[
\lim_{t \to \infty} P\{(X_D(t), X_I(t), X_S(t), X_E(t)) = (x_D, x_I, x_S, x_E)\}
= K \prod_{j \in Q} \xi_j^{x_j} \prod_{r=1}^{x_E} \varphi_r(r)
\]

where \( K \) is a normalizing constant, \( Q = \{D, I, S, E\} \) and \( \xi_j \) is the unique solution to the system of equations.
\[ \xi_j \sum_k \lambda_{jk} = \sum_k \xi_k \lambda_{kj} \quad \text{for } j = D, I, S, E \]

\[ \sum_j \xi_j = 1 \]

Numerical Example

Suppose there are 4 UAVs; MTBMAF = 25; there is one maintenance path; and the other parameters are as before with the ingress/egress time \( T \) variable.

**Long-run proportion of time at least 1 UAV is on station**

**1 server**

<table>
<thead>
<tr>
<th>( T )</th>
<th>Limited Service Model</th>
<th>Queuing Network Model</th>
<th>Simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.76</td>
<td>0.79</td>
<td>0.93</td>
</tr>
<tr>
<td>2</td>
<td>0.70</td>
<td>0.74</td>
<td>0.88</td>
</tr>
<tr>
<td>3</td>
<td>0.64</td>
<td>0.68</td>
<td>0.83</td>
</tr>
<tr>
<td>4</td>
<td>0.58</td>
<td>0.61</td>
<td>0.75</td>
</tr>
<tr>
<td>5</td>
<td>0.50</td>
<td>0.54</td>
<td>0.60</td>
</tr>
<tr>
<td>6</td>
<td>0.42</td>
<td>0.45</td>
<td>0.48</td>
</tr>
</tbody>
</table>

If there are 4 servers

**Long-run proportion of time at least 1 UAV is on station**

**4 servers**

<table>
<thead>
<tr>
<th>( T )</th>
<th>Independent Alternating Renewal Process Model</th>
<th>Limited Service Model</th>
<th>Queuing Network Model</th>
<th>Simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.95</td>
<td>0.96</td>
<td>0.96</td>
<td>0.95</td>
</tr>
<tr>
<td>2</td>
<td>0.91</td>
<td>0.92</td>
<td>0.93</td>
<td>0.91</td>
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In the present case the multi-state queuing model approach appears from the limited tabulation to be *biased low* for a limited number of repair servers (1). This is not conclusive, but in view of the very good results obtained by the simpler Markovian
model of Section 4 it does not now seem desirable to use the present approach. Some further investigation may be worthwhile.
## App. III
Display of Spreadsheet Implementation of Analytic Models

**Analytical Models for Mobile Sensor (UAV) Coverage of a Region**

**Parameter Values**

- 1/\( \lambda \): mean time between mission affecting failure (MAF) (hours)
- \( T \): ingress-egress time
- \( E \): total endurance time
- \( \alpha_{au} \): rate of unscheduled maintenance activities (number per hour)
- 1/\( \alpha_{as} \): time between scheduled maintenance activities (hours)
- 1/\( \beta_{au} \): mean time to repair each unscheduled maintenance (hours)
- 1/\( \beta_{as} \): mean time to repair each scheduled maintenance (hours)
- 1/\( \mu \): mean time to repair mission affecting failure
- \( s \): number of maintenance paths
- \( N \): number of UAVs (max=10)
- \( S \): time on station if no failures
- 1/\( \beta_{aa} \): administrative and logistic down time (ALDT)

**MOEs**
- Long run proportion of time at least 1 UAV is on station: limited service model
- Long run proportion of time 1 UAV is on station: 1 UAV and alternating renewal process model
- For infinite repair capacity, the fraction of time at least one UAV is on station

**Calculated Parameters**

- exp model: prob of failure during flight
- \( \lambda \): MAF failure rate
- prob of MAF during flight
- prob of MAF during ingress
- prob of MAF during surveillance
- expected time of flight
- expected repair time (Alt. Renewal)
- expected time flying UAV is on station
- proportion of flying time spent on station
- gam: rate of return for repair

### Table

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### Table

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**E[repair time]:**

Model alt. renewal: \( E[\text{maintenance time}] + \frac{1 - e^{-\lambda E}}{\mu} \)

Model limited service: \( E[\text{maintenance time}] + \frac{\lambda}{\lambda + \frac{1}{E}} \frac{1}{\mu} \)