# Signal Processing for Optical Networks:

## Abstract

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## Subject Terms

- Optical Fiber Communication
- Filterbanks
- Source and Channel Coding

## Distribution / Availability Statement

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SIGNAL PROCESSING FOR OPTICAL NETWORKS

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Keywords: Optical Fiber Communication, Filterbanks, Source and Channel Coding
1 Project Overview and Objectives

The field of lightwave communication (optical fiber communication) has developed at an astounding rate over the past quarter century. Innovations and advances in fiber bandwidth and transparency have been answered by exciting developments in photonic devices used for signalling and receiving, so that today it is possible to envision the advent of "all-optical" communications networks. These offer the promise of very high digital data and telecommunication rates, with low losses, high reliability, and low error rates. One can now speak of an improvement of several orders of magnitude over communication rates of more traditional channels, which obviously makes optical fiber networks a very attractive alternative.

It should be expected that these features can be further enhanced by the use of modern signal processing techniques in order to take full advantage of the broad bandwidth and stable characteristics of the optical fiber channels. These techniques can be used to modulate and multiplex digital information for multiple users in a manner which makes efficient and adaptive use of the channel. Compression algorithms can be used to help fit the flood of high dimensional data into reasonable bandwidth. Error correcting codes enhance the robustness of modulation schemes.

Our project examined the application of advanced signal processing concepts and techniques to various applications associated with optical fiber communication systems. We have performed theoretical, simulation, and experimental studies of novel lightwave communication schemes and protocols in an effort to address the challenge to high performance represented by extremely fast signalling rates, nonlinearities, dispersion, and attenuation. We designed, analyzed, and began construction of all optical hardware for modulation, demodulation, and matched filtering. We made exciting progress in the areas of source and channel compression (compression and error correcting codes) for communication systems. Finally we developed interesting extensions and generalizations of matched filtering and detection algorithms and studied applications beyond communications.

More specifically, the main areas of study for our project were:

- **Modulation and Multiplexing:** We investigated the possible use of soliton pulse trains for non-dispersive signalling in the nonlinear regime of optical fiber operation [48, 49]. In the lower power linear regime, we concentrated on novel signal multiplexing techniques based on time-frequency representations (multi-rate filterbanks) for the efficient representation and transmission of information [28, 47]. A key concern here is efficient use of the fiber’s very large bandwidth while simultaneously allocating bandwidth resources to more or fewer channels adaptively, based on load. An important issue is the significant mismatch between the fiber channel bandwidth and the limited modulation rates available in current hardware.

- **Transceiver Design and Analysis:** Implementation of these ideas motivated design and implementation studies of architectures for real-time, optical time-frequency transforms for the purposes of modulation and for coherent detection and demodulation [1, 29, 47]. Re-
lated work touches on various generalizations of matched filtering with applications that go beyond communications [3, 16, 6, 21] to pattern recognition in minefield detection, digital mammography, and directional data analysis.

- **Source and Channel Coding:** Beyond the physics and hardware considerations for optical networks, we have developed various novel approaches to the source and channel coding issues present in any communication system. These include compression and coding algorithms for stochastic signals and images, based on time-frequency decompositions and group Fourier transforms [36, 15, 16, 42]. Beyond the applications in communications, all of these techniques have many corollary uses in many other areas of signal and image processing, including medical imaging.

- **Extensions:** We also performed some fundamental development work in the area of lapped orthogonal transforms associated with polygonal/polyhedral tilings of spaces of two dimensions and higher [42]. We anticipate that this work will find applications in image processing and scientific computation. We developed fast algorithms for image processing based on non-abelian harmonic analysis [22]. A multiresolution elastic image registration algorithm was developed and applied [20]. We also studied various limited spectral data problems, including limited angle tomography [61]. Potential DoD applications include imaging radars and reduced ambiguity in waveform diversity radars.

As this short introduction briefly indicates, research performed under this project has produced advances in a variety of optical and digital signal processing disciplines. These are laid out in more technical detail in Section 2 below, along with references to the relevant publications produced under this research program. The developments have produced several interesting technology transfers, which are presented in Section 4 below. Several of these results promise utility in several areas of interest to the Department of Defense, as evidenced by follow-on development funding of certain hardware and software products of the work by AFOSR, BMDO, ONR, NATO, and NSF. Other products described in Section 4 include various software libraries which we have made available on the net. Section 4 also summarizes highlight presentations by the members of our team.
2 Technical Summary of Research performed under AFOSR F49620-93-1-0567

The Dartmouth Optical Signal Processing Project made substantial progress in the targeted research areas described in our original proposal, as well as in several other domains. In this section we will give brief technical descriptions of the particular developments in:

- Compression and Coding for communication applications:
  - Joint source/channel coding for network application.
  - Tree structured quantization for transform coders (Significance Tree Quantizers, Subtree self quantization)
  - Adaptive image compression.

- Modulation and Multiple Access protocols and hardware for optical communications networks:
  - Time-Frequency bases for adaptive multiplexing in optical communications systems.
  - Optical real-time multi-rate filterbanks with applications to transmultiplexer and receiver design.
  - Design and hardware implementation of dispersive optical filterbanks.
  - Theoretical studies of modulated soliton pulse trains.

- Advanced mathematics for optical and digital signal processing
  - Non-abelian signal processing for coding, data analysis, and matched filtering:
  - Time-Frequency algorithms for image processing and data inverse problems.
  - Spatial statistics algorithms for image processing and target detection.

2.1 Compression and Coding for communication applications:

**Joint Source and Channel Coding:** Practical transmission of large amounts of data, such as that associated with images and video, often requires efficient representations of digital signals. We have studied various aspects of efficient signal representations and applications to compact signal storage, fast transmission of signals over lossy packet networks, rapid medical image acquisition, and feature-preserving denoising and restoration [2, 30, 31, 37, 38, 39, 40, 41, 50, 43].

A particularly important application concerns efficiently encoding signals for transmission over noisy channels. An instance of this is the problem of rapidly transmitting images over lossy packet networks such as the Internet. This is a very timely issue, since world wide web data transmissions
are the single largest consumer of Internet bandwidth, and these transmissions are dominated by image data.

Traditionally, this has involved an error correcting encoding (channel coding) which is independent of previous data compression (source coding). In contrast, for certain packet network applications, joint source/channel coding methods of the type considered by our group [2, 38, 50] have the potential to greatly increase the throughput of network-based multimedia applications and to reduce network congestion. This algorithm applies redundancy selectively to the compressed representation of the data to be transmitted. The algorithm uses an optimal bitrate allocator to explicitly minimize expected image distortion given a bit-budget and an expected transmission error rate by allocating appropriate numbers of information and redundancy bits to the different bands in a wavelet sub-band decomposition of the input image. A preliminary joint source/channel scheme has already attracted commercial interest, as detailed in Section 4 below.

In current practice, images are usually transmitted across the Internet using a lossless protocol such as TCP/IP. Lossless protocols require retransmission of lost packets, which can substantially increase transmission time. We take an alternative approach to transmitting images: we add error-correcting codes and transmit them under the assumption that some packets will be lost in transit. Channel codes increase the amount of data that must be transmitted, however, so they must be used judiciously. We have proposed algorithms for performing joint source and channel coding of images to reduce the channel-coding overhead. Our subband-level and bitplane-level optimization procedures give rise to an embedded channel coding strategy. Source and channel coding bits are allocated in order to minimize an expected distortion measure. More perceptually important low frequency subbands of images are shielded heavily using channel codes, and higher frequencies are shielded lightly. The result is a more efficient use of channel codes that reduces channel coding overhead. This reduction is most pronounced on bursty channels for which the uniform application of channel codes is particularly expensive. We derived optimal source/channel coding tradeoffs for our block erasure channel. We described an implementation of this algorithm and compared its performance on the Internet to lossless TCP/IP transmission of the same images. In our experiments our lossy image transmission scheme delivered images significantly faster than TCP/IP during periods of heavy traffic.

This work has been presented at a variety of venues, including the IEEE Image Processing Conference and the ACM multimedia conference [2, 38, 39].

Significance Tree Quantization: Our work on significance tree quantization provides a framework for understanding and enhancing a number of important recent wavelet-based and DCT-based image coders. We show that Shapiro's well-known EZW scheme, Said and Pearlman's SPIHT scheme (currently a contender for the JPEG-2000 standard), and Xiong et al.'s EZDCT scheme are all forms of a single family of algorithms. We call the members of this family significance tree quantizers (STQ's). Each of the aforementioned coders has been selected from a large family
of potential STQ's based on empirical work and a priori knowledge about transform coefficient
behavior. We demonstrate an algorithm for optimizing a significance tree quantizer for a given
image or class of images. For example, we can find an STQ that is optimized for SAR images, for
motion compensation residuals, for multispectral images, and so on. We demonstrate our algorithm
by applying our optimization procedure to the task of quantizing 8x8 DCT blocks. Our algorithm
yields a fully embedded, low-complexity DCT coder with performances from 0.7 dB to 2.5 dB better
than baseline JPEG for standard test images. This work has been featured at many important
and influential meetings, including the annual Data Compression Conference at Snowbird, and the
annual Asilomar conference [40, 41, 43].

**Self-Quantization of Subtrees:** Fractal image compression was one of the earliest compres-
sion schemes to take advantage of image redundancy in scale. The theory of iterated function
systems motivates a broad class of fractal schemes but does not give much guidance for implement-
tation. Fractal compression schemes do not fit into the standard transform coder paradigm and
have proven difficult to analyze. We introduce a wavelet-based framework for analyzing fractal
block coders which simplifies these schemes considerably. Using this framework we find that fractal
block coders are Haar wavelet subtree quantization schemes, and we thereby place fractal schemes
in the context of conventional transform coders. We show that the central mechanism of fractal
schemes is an extrapolation of fine-scale Haar wavelet coefficients from coarse-scale coefficients. We
use this insight to derive a wavelet-based analog of fractal image compression, the self-quantization
of subtrees (SQS) scheme. We obtain a simple SQS decoder convergence proof and a fast SQS
decoding algorithm which simplify and generalize existing fractal compression results. We describe
an adaptive SQS compression scheme which outperforms the best fractal schemes in the literature
by roughly 1 dB in PSNR across a broad range of compression ratios and which has performance
comparable to some of the best conventional wavelet subtree quantization schemes. This work has
been presented in several venues [2, 51, 52], including the IEEE Transactions on Image Processing.

**Adaptive Compression:** Other work in compression focuses on improving the image models
used in image compression. Transform coding, a technique which takes advantage of first-order pixel
correlations in images, is currently the image coding method of choice in industry. An enormous
amount of effort has been put into refining transform coders, and JPEG, the current standard
transform coder, very efficiently removes redundancies due to first-order interpixel correlations.
Considerable additional redundancy exists within images, however, in the form of higher order rela-
tionships between pixels. The next generation of coding standards, suitable for very low-bandwidth
applications, will require as yet undiscovered efficient methods of utilizing this redundancy. Our
work addresses this problem by focusing on improving the image models used in eliminating redu-
dancy. This work has been presented in several conference and journal papers and includes various
adaptive schemes including a wavelet analysis of fractal image coding [2], which demonstrates that
2.2 Modulation and Multiple Access for Optical Communications:

Time-Frequency Bases for Modulation and Multiplexing: We have studied the use of a class of time-frequency orthogonal and almost orthogonal bases, and in particular the use of wavelet packet bases, for modulation and multiplexing in optical fiber communication networks. These custom-made bases are particularly suited to the characteristics of optical fiber channels and the limitations of current modulation devices. Signalling schemes based on these signals permit flexible Time/Frequency Division Multiple Access (T/FDMA) to optimally utilize the bandwidth of a fiber, while being robust to dispersion and harmful non-linear side effects such as Brillouin scattering and self phase modulation.

Theoretical and simulation studies of these novel lightwave communication schemes and protocols demonstrated their ability to address the challenge to high performance represented by channel nonlinearities, dispersion, and systems constraints such as timing jitter in the demodulator. The various stages of this work have been presented over the span of the contract, at conferences [28, 29], at a BMDO sponsored workshop/kickoff at the University of Maryland in 1996, more recent results in [23], and a journal article is ready for submission [39].

As in most current optical fiber communications applications, our approach concerns the transmission of digital information by modulating an optical carrier wave with a sequence of short pulses at regular time intervals. The resulting pulse train represents the original discrete time, discrete state digital information stream as an analog waveform. In general, the process of modulation may be viewed as the encoding of the digital information stream into a waveform suitable to the transmission channel. The particulars of this process are strongly determined by the spectral characteristics of the channel, the need to transmit information at high rates, and the desire to keep errors to a minimum. Of course, this transmitted information must eventually be read off, or demodulated, by the user. This process must recover the original digital information stream from the received waveform which may have been contaminated by noise and channel distortions. The goal is to recover each of the information symbols with a low probability of error. Roughly speaking, standard receivers examine the incoming signal to determine the information content. Often this is done by a correlation or matching operation between the signal and templates of the pulses representing the possible different symbol states.

Because of the enormously high bit rates and broad bandwidths available in the optical fiber channel, the resource is generally allocated over many users. Multiplexing, or sharing of the
channel bandwidth among many users is required. Multiplexing schemes must be considered with a view to the flexible use of the resource in multi-access applications, where the number of channels needed and the bitrate requirements of the users may vary in time. The problem of separating these concurrent channels with sufficient signal to noise ratio at the detector places constraints on the modulation scheme and on the receiver design. Our proposed approach meets these requirements.

In our approach, multiple data streams are multiplexed and modulated using an adaptive time-frequency channelization. Modulation and multiplexing of the digital information is performed using an extremely high speed hardware implementation of synthesis and analysis wavelet packet filter banks. We determined this may be done all optically in the time domain through the use of dispersive optical elements. This is in marked contrast to previous optical implementations of the wavelet transform, which have used spatial, diffractive architectures, and which would not be suitable for very high speed application. A dual architecture is used for demodulation and demultiplexing in the receiver.

In designing efficient and flexible modulation and multiplexing schemes, we take into account the special features and limitations of optical networks. Since the introduction of efficient Erbium-doped fiber amplifiers into optical fiber communication networks, signal attenuation over the course of long distance transmission has ceased to be a major limitation. Currently, the limiting factors to obtaining high bandwidth are dispersion and optical nonlinearities of the glass fiber. Dispersion refers to the tendency of the various frequency components comprising the signal pulses to propagate at different rates, leading to the spread of the pulse shapes after a long distances of propagation. This has the effect of making the pulses harder to recognize and disentangle at the receiver. Similarly, nonlinearities can cause other degradations of the pulses.

To address these issues, we considered network design for multiple user communications based on an all-optical implementation of a joint time-frequency division multiple access scheme. We concentrated on signal modulation and multiplexing techniques based on adaptive orthogonal time-frequency representations for the adaptive and efficient transmission of information. This design allows access to a wide variety of users, and a potentially high utilized bandwidth. In particular, we studied waveforms related to wavelet packets for modulating schemes to carry digital information on the optical channel. In this approach, we partition the fiber bandwidth into many channels with frequency characteristics defined by a simple variant of a smooth local trigonometric basis associated with the partition [47]. These channels are orthogonal without the use of guard bands and without physically meaningless ideal bandpass character. In a given channel, each of a number of users is assigned one of a sequence of temporal slots corresponding to a sequence of waveforms given as inverse Fourier transforms of various local trigonometric functions for that channel. The orthogonality of the waveforms corresponding to the various users in that channel follows from the orthogonality of the local trigonometric functions. A schematic view of the modulation scheme is shown in Figure 1.
Figure 1: A schematic diagram of the modulation/multiplexing scheme.
The key component of this network is a optical matched filterbank (OMF). This filter is utilized to multiplex and demultiplex the optical-bandwidth trunk fiber into a large number of electronic-bandwidth optical signals. These signals can then be transformed into electronic signals which can be further demultiplexed and processed via standard digital signal processing techniques. Compared with existing modulation techniques our approach most closely resembles quadrature amplitude modulation (QAM) where we use orthonormal time-frequency bases to shape the digital pulse to reduce intersymbol interference (ISI), jitter sensitivity and increased utilization of the bandwidth.

**Optical Filterbanks:** The implementation of the schemes described above requires the modulation of bit streams onto certain complex waveforms, and subsequent demodulation. With these waveforms the phase information is as important as the amplitude information, but it is harder to handle optically. We have completed design study of the hardware issues associated with real-time all-optical filterbank architectures to be used for the construction and detection of these complex waveforms.

This design uses diffraction gratings and amplitude and phase masks for all optical, real time dispersive pulse shaping, or dually, for matched filtering. This is inspired by efforts over the last two decades which produced some ability to control the temporal shape of short light pulses by means of filtering the angularly dispersed of frequency components of the pulse. The potential temporal resolution is higher now due to the development of femtosecond techniques and attendant pulse bandwidth increases. We developed and analyzed a modification of the techniques developed by Weiner and co-workers, indicated schematically in Figure 2. This approach is inspired the analogy between diffraction in spatial Fourier optics, and dispersion in the time domain.

The initial pulse is dispersed off a grating, whereupon its various frequency components diffract into different angles. The lens focuses the different components onto corresponding points along a mask, placed at the focal plane of the lens. The amplitude and phase of a given one of these components is modified by the mask’s complex transmittance at the position that the component passes through, see Fig.2. The second lens and grating recombines all the frequency components to produce a new temporal profile with properties determined by the mask.

For arbitrarily shaped pulses the mask has to be able to modify both the amplitude and the phase of the incoming signal. Analysis revealed that the *spatial* intensity profile of the signal beam interacts with its temporal frequency components in an interesting way as it traverses the device. Indeed, we demonstrated that the output of the device described in Figure 2 may be described as an inverse windowed Fourier transform (Heisenberg wavelet transform) involving the functions describing spatial beam shape and the temporal profile of the signal. The analysis of the dispersive pulseshaper as a spatio-temporal filterbank was given in [29, 47, 24].
Figure 2: A schematic diagram of the temporal optical Fourier transform and correlation device, comprised of a grating pair and a transmittance mask.
We have also experimented with chirped local cosine filterbanks. While local cosines are just cosines multiplied by special windows, there is some freedom in choosing the window values. Small timing error jitter will affect the orthogonality of adjacent local cosines. However, preliminary results obtained by injecting oscillations into the window, show improved robustness of orthogonality to jitter.

Further research considered the problem of numerous issues associated with the practical implementation of the optical filterbank, and undertook a careful study of several important design issues such as beam size effects and mask resolution. The results of the design study, including beam size effects and mask resolution is covered in a journal article which has just appeared [1]. This device is now being implemented in hardware by Professor Ulf Österberg of Dartmouth’s Engineering School and researchers from Purdue University under new NSF/AFOSR support.

**Construction of Modulated Soliton Pulse Trains:** The previous items assume low-power short-distance transmissions, for which the optical fiber is essentially a linear system. We have also looked carefully at high-power long-distance transmissions, in which nonlinear effects come into play. In particular, we are studying the possibility of optically modulating trains of extremely short soliton pulses. The signal processing of such pulse trains will require an entirely different analysis.

The propagation of solitons in optical fibers is governed by the nonlinear Schrödinger equation. To study this, we have been adapting the work of Prosser and Moses on the use of inverse scattering methods to analyze the solutions of the KdV equation. In particular, their use of the Gelfand-Levitan-Marchenko equation and certain of its generalizations can be modified for the study of the nonlinear Schrödinger equation.

Reese Prosser spent the last year of his life studying the decay of transients associated with soliton solutions for the KdV and Nonlinear Schrödinger equations. A quantitative understanding of the decay of transients is important in the study of the generation of solitons, since in practice the initial conditions required for the generation of soliton solutions can be met only approximately, and the errors in the approximation must decay fast enough to allow the soliton solution to survive.

This question seems very hard, and only partial results have been obtained so far. These results depend upon the eigenvalue expansions associated with the Gelfand-Levitan and related integral equations obtained previously by Prosser. They are summarized in preprints [48, 49], listed below.

While there may still be some utility in modulated soliton trains for optical communications, this concept has been somewhat overtaken by developments in the hardware. With the introduction of efficient Erbium-doped fiber amplifiers into optical fiber communication networks, it is now possible to create all optical active regeneration of the signal in the fiber at intervals as needed. This at least somewhat reduces the pressing need for solitons. Indeed, the limiting factors to obtain high bandwidth are now seen to be dispersion and optical nonlinearities. For this reason, we have steered most of our efforts towards theoretical and simulation studies of novel lightwave communication
schemes and protocols described previously.

2.3 Advanced mathematics for optical and digital signal processing

Techniques developed in this project for communications have found many other applications of potential DoD and dual use interest. These include spatial statistics for pattern detection in imagery [3], the use of multiscale edge representations for denoising and contrast enhancement [58], adaptive signal representations for fast MRI and CT imaging [31, 32, 34], and several others. Many of these extensions arose out of research on generalizations of matched filtering and detection which we had promised in our original proposal.

This work has produced much attention, including some transitions and potential technology transfers, detailed in Section 4 below. We now review some of these developments.

Statistical Image processing for detection and classification: Recent application of spatial statistics and segmentation algorithms to antipersonnel mine detection and target recognition shows great promise. The work [3], which is joint with Carey Priebe of The Johns Hopkins University, concentrates on statistical detection and classification of non-homogeneous regions within an image. In this paper, an exact closed form test for homogeneity, under a uniform spacings hypothesis is developed. It has been noted by Noel Cressie, an acknowledged leader in the field, that such tests in the higher dimensional cases had been considered by the authors have been elusive in the past. Reviews of the paper include the following comments: "This paper presents a nice addition to the literature on scan statistics which appears to offer the possibility of more powerful tests in a variety of situations...the authors are studying an interesting and important problem...This is an interesting paper. It deals with an area of many potential applications."

This work has transitioned into a development project led by Tim Olson, jointly funded by Power Spectra Inc. of Sunnyvale, CA, and the National Science Foundation via their joint University-Industry initiative.

Another research direction includes recent work by team members on discriminant analysis. This work has been motivated in part by the work of R. Coifman and N. Saito on the construction of optimal wavelet discriminant bases. The background theme is a classic one, which can be traced to the work of R.A. Fisher in 1936, and is popularly known as Linear Discriminant Analysis (LDA). Much work has been done in this area, and LDA has become a standard engineering tool for a variety of applications, including Automatic Target Recognition (ATR).

Generally, LDA is a model-based tool, however. LDA provides a Bayes optimal decision statistic when the noise is uncorrelated Gaussian, of equal variance. We have worked to extend this type of analysis to more general situations, where the noise is unknown. Thus we optimize not only over the structure of the targets, but also over the noise, i.e. double optimization. We have developed this approach as a constrained optimization problem. We are currently in the process of finishing
the academic work on the project, and are currently producing software to test the concept in the near future.

This work shows great promise for applications, and the concept has been approved for testing as a transition on the Longbow Fire Control Radar of the Apache helicopter, via Lockheed Martin Electronics and Missiles of Orlando, Florida. Olson and Lockheed Martin are applying for funding for a major initiative to develop our methods for this program. We have the support of the Longbow program office, and we are very excited about this potential transition.

The goal of this project is to develop an optimized detector, for the Longbow Fire Control Radar on the Apache Helicopter, currently the world's premier attack helicopter. We have on file letters of support from the Project Head for the Apache-Longbow project and from executives of Lockheed Martin Electronics and Missiles.

**Non-Abelian Signal Processing Algorithms:** We have developed the theory and applications of efficient algorithms for computational harmonic analysis [4, 15, 17, 19, 54]. Applications include various forms of data analysis, including the statistical analysis of directional data. This arises in many fields, including quality control for computer manufactured parts [6, 16, 21].

Researchers working on problems which can be cast on the sphere typically need the use of fast spherical transforms. Examples include the terrestrial problem of atmospheric modeling and the astronomical problem of studying the cosmic microwave background. In order to achieve fast spherical transforms, one needs to be able to perform fast Legendre transforms. Continuing the work we reported earlier, we are currently implementing, testing and benchmarking computationally efficient versions of fast spherical transforms which are based on the work of Driscoll, Healy and Rockmore [12, 4] on Legendre transforms. Our current implementation of the full spherical harmonic expansion is faster than Spherepack, a standard code produced by NCAR (National Center for Atmospheric Research), at band widths of 256 and above.

Applications include deconvolution density estimation in statistics of directional data and has appeared in prestigious journals such as the Annals of Statistics [6, 21]. An application to CAD/CAM is described in [16]. Being able to perform fast Legendre transforms is also of benefit in our limited angle tomography work.

We have made web-available our "Spharmonic Kit," a collection of C routines which implement Legendre and spherical harmonic transforms, and spherical convolutions. This is available at the web site: http:///www.cs.dartmouth.edu/ geelong/sphere.

More recently, we have been examining the possible application of non-abelian transform algorithms in positive characteristic to the development of nonabelian convolutional codes. This generalizes the usual spectral approach to cyclic codes. This work has already gained attention in the computer science community [26, 27]. This paper presents a sub-quadratic complexity encoding algorithm for a class of error-correcting codes based on expander graphs that was recently developed by Sipser and Spielman. For codes built on Cayley graphs of the group $PSL_2(p)$, the algorithm
has complexity $O(n^{4/3})$, where $n = O(q^3)$ is the block length. Our methods exploit the symmetries of the graphs using the representation theory of the underlying group and its Fourier analysis, and are parallel to the use of spectral techniques for classical cyclic codes. Our results suggest that this new class of codes may combine many of the strengths of cyclic codes with those of the low-density parity-check codes first proposed by Gallager. We continue to study the development of this idea.

We have considered the general problem of efficient computation of expansions of functions defined on compact groups in terms of irreducible matrix coefficients of their representations. These algorithms are a natural generalization of the famous Cooley-Tukey FFT. In analogy with the use of the classical FFT for analyzing time series, we have been able to apply these techniques to the analysis of data on experimental designs by viewing the design as a homogeneous space for an appropriate symmetry group. In many cases of interest, the corresponding spectral decomposition of the data generalizes the usual Analysis of Variance (ANOVA) approach [16, 52].

We have been investigating the applicability of Fourier transforms on various nonabelian groups, in particular wreath product groups, for possible use in ATR. We view the pixel locations as a homogeneous space for the group. This yields an accompanying Fourier expansion of any image in which the Fourier coefficients now exhibit collective invariance under the new symmetry group. In particular this can yield a certain amount of rotational invariance, a key property for ATR. The group theoretic setting allows us to define an associated convolution operator which has potential for pattern matching which takes into account the accompanying group symmetry. The wreath products have a hierarchical structure which produces a multiresolution analysis in the spectral domain. In fact, we are able to recover Haar wavelets in any dimension by this approach, thereby providing a new link between group theoretic based analysis and wavelet analysis [36, 22, 57].

**Image Registration:** In many DoD and other settings, one is faced with a problem of change detection: i.e., when comparing current images to past images, a primary goal is to detect any changes that may have occurred in the intervening period. For example, in mammograms one would look for any potential tumorous growth that could have occurred between successive examinations. Simple subtraction of the current image with the past image would yield inaccurate results due to the positioning differences between the two examinations. There are many ways that positioning differences can arise. As two examples, a chest film may be taken during a different amount of inspiration one year from the next, and in mammography, it is common that the patient would be compressed differently from one mammogram to the next. Such positioning differences would result in non-rigid deformations of common features. Therefore, one would like to elastically align the two images and then subtract one from the other. Elastic alignment would compensate for the non-rigid deformations, and hence prove useful as a diagnostic tool for Radiologists.

We have developed a technique to perform elastic alignment that is based on the multiscale representations of the images [20]. Most other techniques typically involve the computationally intensive problem of solving non-linear differential equations (and hospitals may lack the massively
parallel machines that are best suited for such tasks). Our algorithm first performs a global, rigid (simple rotation and translation) alignment of the two images. Then, in successive stages, it refines the initial alignment on smaller and smaller regions of the images until some minimum region size is finally reached. The rotation-translation parameters determined at the final stage are then smoothly interpolated out over the entire image by the technique of "thin-plate splines". This technique minimizes (with respect to a certain measure) the amount of "bending" that can occur when molding one image to fit another.

The efficient algorithm we have developed is robust to noise and is fully automated. (Some methods require the identification of a limited number of "landmarks", features common to both images.) The radiologist or image analyst can forget about the choice of fiducial points and simply focus on the result of elastic alignment. Examples of this algorithm may be found in Figure 3 and Figure 4.
Figure 3: Stages in multiscale elastic alignment of test image in upper left panel with reference image in upper right.

Figure 4: Alignment of test image in upper left panel with reference image in upper right. Lower left panel shows the warped test image with corresponding warped grid. Lower right shows same test image in rectangular grid for ease of comparison to reference image above it.
**Limited Angle Tomography:** We have worked extensively on technology to solve highly ill-conditioned problems. This work is currently showing great promise [61]. One example of this type of problem is a spectral recovery problem referred to as the limited angle tomography problem. This corresponds to taking measurements of the spatial Fourier transform of an object only within a cone. This situation is found in many real world situations, including X-ray radiography of welds in a reactor vessel wall. Traditional linear analysis has suggested that the accurate solution of this problem is not feasible in the presence of noisy data.

We have developed a non-linear method which exploits the positivity of the problem, in order to solve this problem. We have discovered that one needs to pay very close attention to the formulation of the problem in order to achieve satisfactory results. The problems which must be addressed are the formulation of the spectral limiting operator on a domain of compactly supported images, the discretization of the operator, and the range characterization of the data. We have addressed these problems and formulated an interior point optimization algorithm to solve this problem.

The initial testing of this algorithm has shown that in the presence of perfect data, essentially perfect reconstructions can be achieved. To our knowledge, this in itself had never been achieved before. More importantly, our initial tests have shown that very good reconstructions can be achieved from noisy data. We are continuing this work, and will have a major publication from this work within the year.

Applications of this work include the use of chirp diversity in radar to synthesize better ambiguity surfaces, as suggested by Bernfeld and Grunbaum. In this work, it is observed that a chirp radar effectively computes a projection of an extended target in range-doppler, with the angle of the projection determined by the chirp rate. Bernfeld and Grunbaum independently suggest tomographic techniques for combining projections at different angles, corresponding to different chirp rates. In any real system, there would be some severe constraints on the range of angles at which projections could be obtained. This forces one to perform a regularized limited angle tomographic reconstruction along the lines of the one we have studied.

**Local Tomography:** We have published numerous articles on Local Tomography, and we believe that we now have an algorithm which will produce an image of a localized region of the body, from a minimal amount of data. The major emphasis of this work is the equalization of the noise over the range of the Radon transform.

The range theorems for the Radon transform have caused the implicit oversampling of low frequency components of the Radon transform, with only critical sampling of the high frequency components. Since all components would seem of equal importance, we have developed an algorithm which equalizes the distribution of noise on the Radon transform, allowing localized images which utilize a minimum of radiation.

This work is currently of interest to the National Cancer Institute, and is one of the foci for a "Lung Cancer Imaging Workshop", in January of 1998, to which two of us were invited speakers.
Local trigonometric bases for arbitrary polygonal tilings: Our continued efforts in flexible multidimensional time-frequency localization methods have brought exciting results. Previous methods rely on one-dimensional local cosine basis techniques, as described by Coifman and Meyer. Using tensor products, these local cosine bases extend to images, volumes, and higher-dimensional data. However, this only permits spatial localizations with rectangular support regions. Recent related work has sought to generalize these techniques to include nonrectangular support regions with partial success. We have developed a unifying framework which permits nonrectangular support regions. For images, spatial localization using polygonal support regions is possible within this framework. In particular, images can be decomposed using hexagonally supported basis elements.

Our framework comes about from a careful analysis of the folding operator formulation of local cosine bases. Folding operators may be written in terms of a group action. This group action is fundamental in determining a folding operator's transition region geometry. The transition region geometry determines the support region shape near its corners. Earlier methods correspond to using involution group actions, forcing rectangular geometry on the transition region. Our framework permits nonabelian group actions, making nonrectangular geometry on the transition region possible. Thus we are able to construct spatial localizations with polygonal support regions.

A framework for flexible spatial localization must also provide the capability of preserving smoothness properties. For local cosine bases, this is a simple matter of using a smooth window. In techniques for nonrectangular localization, even preserving continuity can become difficult to ensure. Preserving continuity is possible within our framework and reduces to checking conditions on a function analogous to the window used in local cosine bases. The framework is currently being extended to preserve smoothness, although conditions guaranteeing this are somewhat more involved. Thus we are able to construct nonrectangular spatial localizations of images which preserve continuity, with the potential for preserving smoothness as well. Examples of this algorithm may be found in Figure 5 and Figure 6.
Figure 5: Unfolding operator for hexagonal tiling applied to the constant function.

Figure 6: Basis functions for rhombic components hexagonal tiling.
We have also developed a numerical C++ library called liblca. This library provides complete flexibility in constructing local cosine bases and best basis searching. It includes an interface to GNU Octave. A similar interface to Matlab is being written. It is available on the world-wide web at http://www.cs.dartmouth.edu/warnerd.

Limited Data and Problems in Magnetic Resonance Imaging: Magnetic Resonance Imaging (MRI) has become an essential tool in clinical medicine, producing exquisite contrast in images of soft tissue structures without the introduction of artificial contrast agents. However, it can be limited in the speed of image acquisition; on a standard scanner, imaging time can range from a second to as long as thirty minutes depending on the desired contrast. In past, researchers have devised expensive hardware to alleviate this problem. In contrast to this approach, we advanced a software based approach using prior knowledge of the class being imaged to design the sequence of measurements made by the scanner. These measurements amount to taking projections along the members of an approximate Karhunen-Loeve (K-L) basis corresponding to the image class. This method greatly reduces imaging time without compromising image quality. Our results indicate that this procedure yields substantial improvements over the conventional Fourier basis techniques. We have also developed clustering algorithms using wavelets for this purpose, and here the results are somewhat mixed. Also, our theory extends to the general problem of compressing images, and we described this extension [40, 43].

Both the K-L basis and the approximate K-L basis fully approximate the first and second order statistics of the set of images we are trying to estimate. In the case of Gaussian distributed images, the K-L basis is optimal and the approximate K-L basis is near optimal. For other distributions, however, we can obtain better estimates given additional information about the distribution of the class of images. In our application we assume that the test set is a realization of a Gaussian mixture. Our goal then is to, given a set of data, identify the distribution that this sample is a realization of. This is the problem of clustering, and we have developed and examined several algorithms to this end.

K-bases Algorithm: This is an iterative algorithm which at each step minimizes the expected volume of the variance ellipsoid of the image class. It starts with an initial partition and the joint-best-basis for each partition. Then each sample is expanded into the joint-best-basis of the remaining partitions, and the new expected volume is computed for each such switch. A switch is made if the volume decreases. After each sample in the population is tried, the new basis and volume is computed for each partition and this process is iterated upon. The procedure stops when a minima is reached.

Joint-Best-Basis Clustering Algorithm (JBBC): The JBBC algorithm clusters data using the principal components of the joint-best-basis of a population. The heuristic used here is that high variance in these coordinates is the result of a Gaussian mixture in the population. Clustering is done by extracting a subset of the principal components, ranking them based on variance, and
then running the K-means algorithm on the data in this reduced feature space to obtain class assignments.

Approximate Distance Clustering (ADC): ADC is a technique that differs from the previous two algorithms in that it does not use the best-basis paradigm. In fact, it does not rely on any transform, instead working on the data in the Dirac basis. It works by projecting the high dimensional input data into a low dimensional space while approximately preserving the salient features in the higher dimensional space. ADC computes this projection by using a family of maps that result in non-linear projections. The K-means algorithm is then run on the data in the reduced space to obtain class labels.

Associated with clustering, is the problem of classification: Given a new patient in the scanner, we need to make a class assignment by taking a small set of measurements. We have examined and existing algorithm and devised a new one to this end.

Local Discriminant Bases (LDB): LDB is a variant of the best basis paradigm that rather than optimizing a cost function that measures the efficiency of the representation, measures the usefulness of a given coordinate system between various classes of images represented in that coordinate system. Once this coordinate system, or basis, is found, a ranking of coordinates, from most discriminant to least discriminant, is obtained. The metric used here is the Fisher’s class separability index. A subset of the most discriminant features are then extracted and used to train a classifier. We use Fisher’s Local Discriminant Analysis classifier.

Joint-Best-Discriminant Bases (JBDB): under this approach, the principal components in the joint-best-basis are assumed to be the most discriminating features. These features are ranked using Fisher’s class separability index, and then a subset of these ranked features are used to train the LDA classifier.

References

The project generated the following publications, cited in the overview:

Journal Articles


Journal Articles Submitted:


Articles in Conference Proceedings

[23] U. Osterberg, P.M.P. Gouvea, and D.M. Healy "Optical multi-user detection in a high bandwidth orthogonal waveform encoded fiber optic communications system," submitted to Optical Society of America’s annual meeting in Baltimore in October this fall.


**Journal Articles in Preparation**


[50] J. Danskin, G. Davis, and X. Song, "Optimal Source Channel Coding for Image Transmission".

[51] D. Rockmore, K.-S. Tan, and R. Beals. "Deciding finiteness for matrix groups over function fields"
[52] D. Rockmore, R. Bailey, P. Diaconis, and C. Rowley, "A spectral analysis approach to data on block designs"

[53] D. Rockmore, and D. Maslen, "Separation of Variables and the computation of Fourier transforms on finite groups, II"


[56] D. Rockmore and P. Doyle. "Ruffles and the turning point algebra"


Invited Book Chapters


3 Personnel

The following personnel received partial support from this grant.

Faculty members at Dartmouth College:

- Geoffrey Davis
- Dennis Healy
- Timothy Olson
- Reese T. Prosser
- Dan Rockmore

Postdoctoral and Graduate students at Dartmouth College:

- Sumit Chawla
- Manoj George
- Peter Kostelec
- Douglas Warner
4 Interactions and Presentations

Technology Transfer:

Multiscale Edge Representations for Image Processing and Recognition: These algorithms, developed originally under our DARPA grants, are being used by Dr. Jeff Solka at Naval Surface Warfare Center (NSWC) and by Dr. Carey Priebe of Johns Hopkins University for a variety of tasks including segmentation, enhancement, and feature extraction. The software was standardized in C with the help of development funding from the USMC and NSWC.

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Dr. Jeff Solka:
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Optical Real-Time Filterbanks: This concept is being implemented in hardware and tested by Professor Ulf Österberg of Dartmouth's Engineering School as part of a spin-off project. This project was first funded by BMDO as a joint project with researchers from Purdue University under grant BMDO/AFOSR F49670-95-1-0533. It was then picked up by AFOSR/NSF as

Joint Source/Channel Coding for Internet image transmission: A patent for some of these algorithms has been applied. Duplexx Software, makers of Net Toob and other fine products, is currently negotiating for the rights to distribute these algorithms. Individuals at Apple and Intel have also expressed interest.

Enhanced CT imagery for design and installation of prosthetics: Tim Olson will be moving to The Johns Hopkins University as of September 1996. Olson and Jerry Prince, of Electrical
Engineering at JHU have a tentative agreement with Integrated Surgical Systems and IBM for a joint research effort. This effort will involve a 60 K addition to a previously existing 90 K effort. The goal of this research is to produce enhanced CT images to assist in the design and installation of artificial joints. Techniques developed under the auspices of our DARPA sponsored research effort will be applied to solve beam-hardening difficulties associated with this new application area.

Double optimization for detection and classification. This work by Olson and associates shows great promise for applications, and the concept has been approved for testing as a transition on the Longbow Fire Control Radar of the Apache helicopter, via Lockheed Martin Electronics and Missiles of Orlando, Florida. Olson and Lockheed Martin are applying for funding for a major initiative to develop our methods for this program. We have the support of the Longbow program office, and we are very excited about this potential transition. The goal of this project is to develop an optimized detector, for the Longbow Fire Control Radar on the Apache Helicopter. The Apache Helicopter is currently the world's premier attack helicopter. The contact person for this potential transition is Mr. Charles Stirman at 407-356-2573. We have on file letters of support from Colonel Steven Kee, Project Head for the Apache-Longbow project supporting this work, along with a letter signed jointly by Mr. John Bremer and Mr. Aaron Penkacik of Lockheed Martin Electronics and Missiles.

Detection and estimation of buried objects Tim Olson is currently working on a project jointly funded by Power Spectra, Inc. and the National Science Foundation, which is a direct outgrowth of some of the image processing work done under this grant. The major initiative of this work is the identification of underground structures from ground penetrating radar. We are currently looking into this technology for two purposes: 1) the civilian identification of buried utility cables, and 2) the military identification of buried ordinance and land mines. The contact person for this work is Mr. Jeff Oicles at 408-737-7977x1051.

Available Software:

Wavelet Image Compression Construction Kit This code implements a wavelet transform-based image coder for grayscale images. The coder is not the most sophisticated—it's a simple transform coder—but each individual piece of the transform coder has been chosen for high performance. The coder is quite effective, despite its lack of more sophisticated features such as zerotrees. It yields performance comparable to Shapiro's EZW coder (J. M. Shapiro, "Embedded image coding using zerotrees of wavelet coefficients", IEEE Trans. on Signal Processing, v. 41, no. 12, pp. 3445-3463, Dec. 1993). It is designed to be a foundation upon which more more sophisticated coders can be built—there's no need to reinvent the wheel if you're working on only one aspect of the coding process. The package includes an arithmetic coding library.
**k-bases algorithm:** all the files are in /usr/barbuda/mri/chawla/kbases/matlab. There is a readme file in this directory that describes the contents of the sub-directories. Data for use with this algorithm can be found in several sites. /usr/barbuda/mri/chawla/kbases/matlab/data contains the aligned training data. There is a sub-directory in here called synthetic with routines to create a gaussian (1-d and 2-d) training set.


**Spherical Harmonic Software:** We have made web-available our "Spharmonic Kit," a collection of C routines which implement Legendre and spherical harmonic transforms, and spherical convolutions. This is available at the web site: http://www.cs.dartmouth.edu/geelong/sphere

**Local Trigonometric Bases:** We have developed a numerical C++ library called liblca. This library provides complete flexibility in constructing local cosine bases and best basis searching. It includes an interface to GNU Octave. A similar interface to Matlab is being written. It is available on the world-wide web at http://www.cs.dartmouth.edu/warnerd.

**Invited Presentations on Project Research:**

Healy and Rockmore gave invited talks at the Workshop on Spectral Methods in Medical Signal Processing, Neuherberg, Germany February 1998


Healy gave an invited talk at SPIE annual meeting: Conference on Wavelet Applications in Signal and Image Processing IV, San Diego, CA. July 1997

Davis gave an invited talks at SPIE annual meeting: Conference on Image Compression, San Diego, CA. July 1997

Healy and Davis gave talks at the AMS annual meeting: Special Session on Wavelets, Multi-Wavelets, and their Applications, San Diego CA. January 1997

Healy gave an invited talk at the Workshop on Spectral Methods and Wavelets in Digital Signal Processing, Institute fur Informatik Mathematik, Medizinische Universität zu Luebeck, Germany. October 1996.

Rockmore gave an Invited Minisymposium Presentation at the SIAM Parallel Processing Meeting, March 1997
Rockmore gave invited talks at Brown University Applied Math Colloquium and the University of Connecticut Math Colloquium in the Fall of 1996.

Davis, Healy, Rockmore and Olson were all invited to give talks at the AMS annual meeting: Session on Computational Harmonic Analysis and Approximation Theory, Orlando, January 1996


Healy gave a colloquium talk on project research at Johns Hopkins Department of Mathematical Sciences in April 1996.

Davis was an invited speaker at SPIE Applications of Digital Image Processing XIX, and at SPIE Wavelet Applications in Signal and Image Processing IV, Denver 1996:

Healy and Olson were invited speakers at the second annual Workshop on New Advances in Biomedical Signal and Image Processing, at the IEEE-EMBS International conference on Engineering in the Medical and Biological Sciences in Montreal in September, 1995. Healy chaired a session.

Other Presentations:

Rockmore presented at a Special Session on Algebraic Combinatorics and Representation Theory at the American Math Society Northeast Meeting in October 1995.

Rockmore presented at a Special Session on Representation Theory of Associative Algebras at the American Math Society/Mexico Math Society Meeting in November 1995

Olson presented joint work with Healy and Osterberg at the BMDO "Kick-off" workshop on optical fiber communications at University of Maryland, December 19, 1995, College Park, Md.

Davis and Olson presented their joint work with Healy, Rockmore, Chawla, and Warner at IEEE IMDSP International Conference on Image and Multidimensional Digital Signal Processing, March, 1996, Belize City, Belize, CA.

Davis, Healy, and Rockmore presented work at IEEE ICASSP 96, May 1996, Atlanta.

Rockmore presented joint work with Healy at the CHAMP Conference on Numerical Solutions of PDEs in Spherical Geometry in June 1996

Davis, Healy, and Olson spoke at the SPIE conference on Wavelets IV, San Diego July 1995. Healy served on the program committee and chaired sessions on communications applications, biomedical applications, and denoising.
Other Interactions:

Davis and Chawla participated at the IEEE Data Compression Conference, Snowbird Utah, March 1996, 1997

Olson and Rockmore participated at the Symposium on Mine Detection and Real Time Retargeting, Naval Surface Warfare Center, May, 1996. Dahlgren, VA.

Rockmore participated at the IMA Workshop on Emerging Applications of Number Theory (July 1996), Minneapolis, Minnesota