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13. ABSTRACT *(Maximum 200 words)*

The main goal of the proposed research was to study forefront problems in nonlinear dynamics and chaos, particularly in the direction of controlling chaos, and to explore applications to lasers in which the Air Force researchers are greatly interested. During the funding period, all specific objectives in the original plan were accomplished (see report). These achievements have advanced our understanding of how a chaotic system can be manipulated by utilizing small perturbations to achieve desirable system performance. Researchers at the Air Force Laboratories are investigating applying small perturbations to chaotic lasers to encode messages for nonlinear digital communication. Our results are closely related to this line of investigation.

During the funding period, the PI also provided a large amount of scientific consultation to the Nonlinear Optics Center at the Phillips Laboratory, Kirtland AFB, NM. Frequent contacts (including two visits to the Phillips Lab) were made with the Kirtland group. At present, the PI is collaborating with Air Force Researchers on laser chaos, communication, and chaos in electronic circuits.

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INTRODUCTION

This report summarizes our activities under the Air Force Office of Scientific Research (AFOSR) Grant No. F49620-96-1-0066. The report is organized under the following categories:

I. Objectives.

II. Professional accomplishments.

III. Description of scientific findings.

IV. List of publications.

V. List of invited talks.

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I. OBJECTIVES

1. Investigate methods to stabilize desirable chaotic state or unstable periodic orbits in optical systems so as to prevent sudden collapse of the system which may be induced by dynamical events such as boundary crises.

2. Develop tools and algorithms for analyzing the chaotic dynamics of solid-state and semiconductor lasers.

3. Investigate new theoretical methods to achieve desirable system performance by using properly chosen, driving chaotic signals.

Brief Summary of Effort

Main Objectives

All three objectives have been accomplished. Results have been published in nine refereed-journal papers:

1. Y. Nagai, X. D. Hua and Y.-C. Lai, "Controlling on-off intermittent dynamics," *Physical Review E* **54**, 1190-1199 (1996).

2. Y.-C. Lai, "Driving trajectories to a desirable attractor by using small control," *Physics Letters A* **221**, 375-383 (1996).

3. C. Grebogi, Y.-C. Lai, and S. Hayes "Control and applications of chaos," Joint Special Issue of *International Journal of Bifurcation and Chaos and Journal of Franklin Institute* **334B**, 1115-1146 (1997).

4. C. Grebogi and Y.-C. Lai, "Controlling chaos in high dimensions," *IEEE Transaction on Circuits and Systems* **44**, 971-975 (1997).

5. T. Yalcinkaya and Y.-C. Lai, "Phase characterization of chaos," *Physical Review Letters* **79**, 3885-3888 (1997).

6. Y.-C. Lai, "Synchronism in symmetric hyperchaotic systems," *Physical Review E (Rapid Communications)* **55**, R4861-4864 (1997).

7. E. Bollt, Y.-C. Lai, and C. Grebogi, "Coding, channel capacity, and noise resistance in communicating with chaos," *Physical Review Letters* **79**, 3787-3790 (1997).

8. E. Bollt and Y.-C. Lai, "Dynamics of coding in communicating with chaos," *Physical Review E*, in press.

9. L.-Y. Cao and Y.-C. Lai, "Anti-phase synchronism in chaotic systems," *Physical Review E*, in press.

Related Significant Scientific Results

During the funding period, we have made contribution to one of the frontier research areas in chaotic dynamics: riddling and related phenomena. Various results have been published in a number of papers in *Physical Review Letters* and *Physical Review E.* We have also explored some issue of chaotic data analysis utilizing delay-coordinate embedding.

II. PROFESSIONAL ACCOMPLISHMENTS

1. Publications

Twenty-five (25) refereed-journal papers (5 papers in *Physical Review Letters*, 13 papers in *Physical Review E*, 2 papers in *Physics Letters A*, and 5 others).

2. Ph.D. Theses Supervised

Two - both theses are available through the Graduate School, University of Kansas.

- Yoshihiko Nagai, Ph.D. (Physics, 1998). Thesis: *Controlling chaos, blowout bifurcation, and periodic-orbit theory in chaotic dynamics.* Dr. Nagai is now a Research Associate at the Center for Nonlinear Dynamics and Physiology, McGill University, Canada.

- Tolga Yalcinkaya, Ph.D. (Physics, 1998). Thesis: *Phase characterization and controlling chaos in deterministic flows.* Dr. Yalcinkaya is now a scientist and team leader at the Advanced Research Division, U.S. Sprint.

3. Awards

1997 Air Force Presidential Early Career Award for Scientists and Engineers.

4. Consultative And Advisory Functions To Air Force Laboratories

During the funding period, a large amount of scientific consultation was provided to the Nonlinear Optics Center at the Phillips Laboratory, Kirtland AFB, NM. Frequent contacts (including two visits to the Phillips Lab) were made with the Kirtland group during the funding period. Currently I am collaborating with Air Force Researchers on projects on laser dynamics, communication, and chaos in electronic circuits.

III. DESCRIPTION OF SCIENTIFIC FINDINGS

1. *Driving trajectories to a desirable attractor by using small control* (Objectives 1 and 3) Driving trajectories to a desirable attractor for dynamical systems with multiple coexisting attractors has been a challenging problem in the field of chaos control. An algorithm is developed to steer most trajectories to the desirable attractor by using only small feedback control. The idea is to build a hierarchy of paths to the desirable

attractor and then to stabilize trajectories around one of the paths in the hierarchy. A substantial improvement in the probability for a random trajectory to asymptote to the desirable attractor has been achieved when there are fractal basin boundaries or riddled basins in the phase space. [Y.-C. Lai, "Driving trajectories to a desirable attractor by using small control," *Physics Letters A* **221**, 375-383 (October 1996).]

2. *Control and applications of chaos* (Objective 1)
A procedure is proposed for stabilizing a desirable chaotic orbit embedded in a chaotic attractor of dissipative dynamical systems. The key observation is that certain chaotic orbits may correspond to a desirable system performance. By carefully selecting such an orbit, and then applying small feedback control to stabilize a trajectory from a random initial condition around the target chaotic orbit, desirable system performance can be achieved. As applications, three examples are considered: (1) synchronization of chaotic laser systems; (2) conversion of transient chaos into sustained chaos; and (3) controlling symbolic dynamics for communication. The first and third problems are potentially relevant to communication in engineering, and the solution of the second problem can be applied to electrical power systems to avoid catastrophic event such as the voltage collapse. [C. Grebogi, Y.-C. Lai, and S. Hayes "Control and applications of chaos," *International Journal of Bifurcation and Chaos* **7**, 2175-2198 (1997); C. Grebogi and Y.-C. Lai, "Controlling chaos in high dimensions," *IEEE Transaction on Circuits and Systems* **44**, 971-975 (1997).]

3. *Controlling on-off intermittent dynamics* (Objective 3)
On-off intermittent chaotic behavior occurs in physical systems with symmetry. We demonstrate that, by using arbitrarily small feedback control to an accessible parameter or state of the system, the "on" state can be eliminated completely. This could be practically advantageous in situations where the desirable operational state of the system is the "off" state. Relevant issues such as the influence of noise and the time required to achieve the control are addressed. It is found that the average transient time preceding the control obeys a scaling law that is qualitatively different from the algebraic scaling law which occurs when one controls chaos by stabilizing unstable periodic orbits embedded in a chaotic attractor. A theoretical argument was provided for the observed scaling law. [Y. Nagai, X. D. Hua and Y.-C. Lai, "Controlling on-off intermittent dynamics," *Physical Review E* **54**, 1190-1199 (1996).]

4. *Phase characterization of chaos* (Objective 2)
The phase of a chaotic trajectory in autonomous flows is often ignored because of the wide use of the extremely popular Poincaré surface-of-section technique in the study of chaotic systems. We present evidence that, in general, a chaotic flow is practically composed of a small number of intrinsic modes of proper rotations from which the phase can be computed via the Hilbert transform. The fluctuations of the phase about that of a uniform rotation can be described by fractional Brownian random processes. Implications to nonlinear digital communications are pointed out. [T. Yalcinkaya and Y.-C. Lai, "Phase characterization of chaos," *Physical Review Letters* **79**, 3885-3888 (1997).]

5. *Communication with chaos* (Objectives 1-3)
Recent work has considered the possibility of utilizing symbolic representations of controlled chaotic orbits for communicating with chaotically behaving signal generators. The success of this type of nonlinear digital communication scheme relies on partitioning the phase space properly so that a good symbolic dynamics can be defined. A central problem is then how to encode an arbitrary message into the waveform generated by the chaotic oscillator, based on the symbolic dynamics. We argue that, in general, a coding scheme for communication leads to, in the phase space, restricted chaotic trajectories that live on nonattracting chaotic saddles embedded in the chaotic attractor. The symbolic dynamics of the chaotic saddle can be robust against noise when the saddle has large noise-resisting gaps covering the phase-space partition. Nevertheless, the topological entropy of such a chaotic saddle, or the channel capacity in utilizing the saddle for communication, is often less than that of the chaotic attractor. We present numerical evidences and theoretical analyses which indicate that the channel capacity associated with the chaotic saddle is generally a non-increasing, devil-staircase like function of the noise-resisting strength. There is usually a range for the noise strength in which the channel capacity decreases only slightly from that of the chaotic attractor. The main conclusion is that nonlinear digital communication using chaos can yield a substantial channel capacity even in noisy environment. [E. Bollt, Y.-C. Lai, and C. Grebogi, "Coding, channel capacity, and noise resistance in communicating with chaos," *Physical Review Letters* **79**, 3787-3790 (1997); E. Bollt and Y.-C. Lai, "Dynamics of coding in communicating with chaos," *Physical Review E*, in press.]

6. *Synchronization of high-dimensional chaotic systems* (Objectives 1-3)
We demonstrate that for symmetric dynamical systems with an invariant subspace in which there is a chaotic attractor, synchronism between the transverse subsystem and its replica can be achieved in wide parameter regimes. The synchronism occurs in situations where the interaction between the invariant subsystem and the transverse subsystem can be either uni-directional or bi-directional, and the full system can possess more than one positive Lyapunov exponent. The idea is illustrated by a numerical example. [Y.-C. Lai, "Synchronism in symmetric hyperchaotic systems," *Physical Review E (Rapid Communications)* **55**, R4861-4864 (1997); L.-Y. Cao and Y.-C. Lai, "Anti-phase synchronism in chaotic systems," *Physical Review E*, in press.]

7. *Riddling, intermittency, blowout bifurcation, complexity, and strange nonchaotic attractors* (related research)
We have published 12 papers addressing consequences of symmetry and invariance in chaotic systems. Particular research topics have been riddling, intermittency, blowout bifurcation, hyperchaotic synchronism, complexity, and strange nonchaotic attractors. One Physical Review Letter investigates the phenomenon of noise-induced riddling and its universal scaling laws. Other papers concern on-off intermittency, symmetry-breaking bifurcation, and characterization of blowout bifurcation by unstable periodic orbits. Another Physical Review Letter describes a new route to strange nonchaotic attractor in physically realizable symmetric systems. We have also worked out the unstable periodic-orbit theory of the blowout bifurcation. More recently, a method for

systematically designing hyperchaotic synchronism has been devised by utilizing the principle of symmetry and invariance.

8. *Periodic-orbit theory of the natural measure in chaotic systems* (related research)
In studying chaotic systems, one is often interested in long term statistics such as averages, Lyapunov exponents, dimensions, and other invariants of the probability density or the measure. But these statistical quantities are physically meaningful only when the measure being considered is the natural measure, the one generated by a typical trajectory in phase space. It is therefore of paramount physical importance to be able to understand and to be able to characterize the natural measure in terms of fundamental dynamical quantities. And there is nothing more fundamental than to express the natural measure in terms of the periodic orbits embedded in a chaotic attractor. Previous work has indicated that the natural measure of a chaotic set in a phase-space region can indeed be related to the dynamical properties of the unstable periodic orbits embedded in that set. This result has been proven to be valid for hyperbolic chaotic systems. We test the goodness of such a periodic-orbit characterization of the natural measure for nonhyperbolic chaotic systems by comparing the natural measure of a typical chaotic trajectory with that computed from unstable periodic orbits. Our results suggest that the unstable periodic-orbit formulation of the natural measure is typically valid for nonhyperbolic chaotic systems. [Y.-C. Lai, Y. Nagai, and C. Grebogi, "Characterization of the natural measure by unstable periodic orbits in chaotic attractors," *Physical Review Letters* **79**, 649-652 (1997); Y.-C. Lai, "Characterization of natural measure by unstable periodic orbits in nonhyperbolic chaotic systems," *Physical Review E* **56**, 6531-6539 (1997).]

9. *An upper bound for the proper delay time in chaotic time-series analysis* (related research)
Analyzing chaotic time series generated in experimental situations where a detailed knowledge of the system is not known is critically important for understanding the system dynamics. The most often used tool is the delay-coordinate embedding technique. Whether good results can be obtained depends on the choice of many parameters such as the time delay used in the reconstruction of the phase space. We established an upper bound for the proper delay time. The derivation was based on analyzing the effective scaling regime in the computation of the correlation dimension using the Grassberger-Procaccia algorithm. Numerical results agreed with the theoretical prediction. [Y.-C. Lai, D. Lerner and R. Hayden, "An upper bound for the proper delay time in chaotic time series analysis," *Physics Letters A* **218**, 30-34 (1996); Y.-C. Lai and D. Lerner, "Effective scaling regime for computing the correlation dimension in chaotic time-series analysis," *Physica D*, in press.]

Brief Summary of Scientific Findings

- Accomplishments (1-6) have advanced our understanding of how a chaotic system can be manipulated by utilizing small perturbations to achieve desirable system performance and to perform digital communication, and how high-dimensional chaotic systems can be synchronized. The new tool to analyze the phase of chaotic systems has direct applications to characterization of the dynamics of chaotic lasers. Researchers at the Air Force Laboratories are investigating applying small perturbations to chaotic lasers to encode messages for nonlinear digital communication. Accomplishments (1-6) are closely related to this line of investigation.

- Accomplishment (7) has elucidated the role of symmetry and invariance in chaotic systems. It addresses several key fundamental issues in the dynamics of riddling and intermittency, and it has direct applications to a large variety of problems in physical sciences and engineering such as synchronization, prediction, complexity, coupled oscillators, and limits of deterministic modeling, etc.

- Accomplishment (8) fills the long-existing gap (since 1988) of a systematic check for the validity of the periodic-orbit theory of the natural measure for nonhyperbolic chaotic systems, systems to be expected in almost all practical situations. This is an important step forward in the development of the chaos theory.

- Accomplishment (9) deals with analyzing chaotic time series in experiments, which has been a field of intensive research in the past fifteen years. Nonetheless, how the time delay affects the performance of the delay-coordinate embedding algorithms remained uninvestigated. Our result fills this important gap. Analyzing chaotic time series has become a common practice for researchers who work on experimental optical and fluid systems. Our result is therefore relevant to the Air Force's mission.

IV. LIST OF PUBLICATIONS

1. Y.-C. Lai, "Symmetry-breaking bifurcation with on-off intermittency in chaotic dynamical systems," *Physical Review E (Rapid Communications)* **53**, R4267-R4270 (1996).

2. Y.-C. Lai, "Distinct small-distance scaling behaviors of on-off intermittency in chaotic dynamical systems," *Physical Review E* **54**, 321-327 (1996).

3. Y.-C. Lai, D. Lerner and R. Hayden, "An upper bound for the proper delay time in chaotic time series analysis," *Physics Letters A* **218**, 30-34 (1996).

4. Y. Nagai, X. D. Hua and Y.-C. Lai, "Controlling on-off intermittent dynamics," *Physical Review E* **54**, 1190-1199 (1996).

5. Y.-C. Lai, "Driving trajectories to a desirable attractor by using small control," *Physics Letters A* **221**, 375-383 (1996).

6. Y.-C. Lai and C. Grebogi, "Complexity in Hamiltonian-driven dissipative chaotic dynamical systems," *Physical Review E* **54**, 4667-4676 (1996).

7. Y.-C. Lai, U. Feudel, and C. Grebogi, "Scaling behaviors in the transition to chaos in quasiperiodically driven dynamical systems," *Physical Review E* **54**, 6070-6073 (1996).

8. T. Yalcinkaya and Y.-C. Lai, "Blowout bifurcation route to strange nonchaotic attractors," *Physical Review Letters* **77**, 5039-5042 (1996).

9. Y.-C. Lai and C. Grebogi, "Noise-induced riddling in chaotic dynamical systems," *Physical Review Letters* **77**, 5047-5050 (1996).

10. Y. Nagai and Y.-C. Lai, "Characterization of blowout bifurcation by unstable periodic orbits," *Physical Review E (Rapid Communications)* **55**, R1251-R1254 (1997).

11. Y.-C. Lai, "Synchronism in symmetric hyperchaotic systems," *Physical Review E (Rapid Communications)* **55**, R4861-4864 (1997).

12. Y.-C. Lai, "Scaling laws for symmetry breaking by blowout bifurcation in chaotic systems," *Physical Review E* **56**, 1407-1413 (1997).

13. Y.-C. Lai, Y. Nagai, and C. Grebogi, "Characterization of the natural measure by unstable periodic orbits in chaotic attractors," *Physical Review Letters* **79**, 649-652 (1997).

14. T. Yalcinkaya and Y.-C. Lai, "Bifurcation to strange nonchaotic attractors," *Physical Review E* **56**, 1623-1632 (1997).

15. C. Grebogi, Y.-C. Lai, and S. Hayes "Control and applications of chaos," *Journal of Franklin Institute* **334B**, 1115-1146 (1997); also to be published in *International Journal of Bifurcation and Chaos* (1997).

16. Y.-C. Lai, "Scaling laws for noise-induced temporal riddling in chaotic systems," *Physical Review E* **56**, 3897-3908 (1997).

17. Y. Nagai and Y.-C. Lai, "Periodic-orbit theory of the blowout bifurcation," *Physical Review E* **56**, 4031-4041 (1997).

18. C. Grebogi and Y.-C. Lai, "Controlling chaos in high dimensions," *IEEE Transaction on Circuits and Systems* **44**, 971-975 (1997).

19. E. Bollt, Y.-C. Lai, and C. Grebogi, "Coding, channel capacity, and noise resistance in communicating with chaos," *Physical Review Letters* **79**, 3787-3790 (1997).

20. T. Yalcinkaya and Y.-C. Lai, "Phase characterization of chaos," *Physical Review Letters* **79**, 3885-3888 (1997).

21. Y.-C. Lai, "Characterization of the natural measure by unstable periodic orbits in nonhyperbolic chaotic systems," *Physical Review E* **56**, 6531-6539 (1997).

22. Y.-C. Lai and D. Lerner, "Effective scaling regime for computing the correlation dimension in chaotic time-series analysis," *Physica D*, in press.

23. E. Bollt and Y.-C. Lai, "Dynamics of coding in communicating with chaos," *Physical Review E*, in press.

24. L.-Y. Cao and Y.-C. Lai, "Anti-phase synchronism in chaotic systems," *Physical Review E*, in press.

25. Y.-C. Lai and C. Grebogi, "Riddling in classical mechanical systems," *Applied Mechanics Review*, accepted.

V. LIST OF INVITED LECTURES DURING THE FUNDING PERIOD

1. "Unpredictability of asymptotic attractors in physical systems," Colloquium, Department of Physics, University of Missouri at St. Louis, April 5, 1996.
2. "Riddling bifurcation in chaotic dynamical systems," U.S.-China Conference on Recent Development of Differential Equations, Hangzhou, China, June 25, 1996.
3. "Symmetry-breaking bifurcation with on-off intermittency in chaotic dynamical systems," Nonlinear Optics Workshop, University of Arizona, October 10, 1996. (*30 minutes*)
4. "Critical exponent for gap-filling at crisis," Seminar, Kansas Center for Advanced Scientific Computing, October 18, 1996.
5. "Critical exponent for gap-filling at crisis," Colloquium, Max-Planck-Arbeitsgruppe "Nichtlineare Dynamik," Universität Potsdam, Germany, November 25, 1996.
6. "Is riddling observable?" Seminar, Kansas Center for Advanced Scientific Computing, February 14, 1997.
7. "Critical exponent for gap-filling at crisis," Colloquium, Department of Physics and Astronomy, Ohio University, February 17, 1997.
8. "Controlling chaos," Colloquium, Department of Mathematical Sciences, The U.S. Millitary Academy (West Point), March 6, 1997.
9. "Chaos - an interdisciplinary science," Introductory talk (50 minutes), Transition Studies Program, Institute for Public Policy and Business Research, University of Kansas, March 20, 1997.
10. "Statistical modeling of deterministic chaotic systems," Hungary Statistical Physics Conference, Budapest, March 26, 1997.
11. "Is riddling observable?" Colloquium, Institute for Theoretical Physics, Loránd Eötvös University, Budapest, Hungary, March 27, 1997.
12. "Fractal geometry and chaos," Short Course (3 hours), Seventh Midwest Geometry Conference held at University of Kansas, April 10-11, 1997.
13. "Riddling bifurcation, strange nonchaotic attractors, and control of multiple attractors," Lecture (*90 minutes*), Nonlinear Electronics Laboratory, University of California, Berkeley, April 22, 1997.
14. "Statistical modeling of deterministic chaotic systems," Lecture, Workshop *Physics via High Performance Computing: Approaches and Tools*, University of New Mexico, May 3, 1997.

15. "Chaos," Introductory Lecture, Gardner Edgerton High School, Gardner, Kansas, May 12, 1997.

16. "Coding, channel capacity, and noise resistance in communicating with chaos," Invited Minisymposium (*30 minutes*), 4th SIAM Conference on Dynamical Systems, Snowbird, Utah, May 18, 1997.

17. "Noise-induced riddling in chaotic systems," Invited Minisymposium (*30 minutes*), 4th SIAM Conference on Dynamical Systems, Snowbird, Utah, May 22, 1997.

18. "Statistical modeling of deterministic chaotic systems," Colloquium, Institüt for Theoretical Physics, Universität Potsdam, Germany, June 16, 1997.

19. "Scaling of crisis in chaotic systems," Lecture, Workshop on *Fundamentals, Modeling and Control of Semiconductor Lasers*, July 23, 1997, University College Cork, Ireland.

20. "Phase characterization of chaos," Nonlinear Optics Workshop, University of Arizona, September 25, 1997. (*30 minutes*)

21. "Phase characterization and fractal phase dynamics of deterministic chaos," Colloquium, Department of Physics, University of Missouri at St. Louis, October 3, 1997.

22. "Controlling chaos," Stochastic Control Seminar, Department of Mathematics, University of Kansas, November 14, 1997.

23. "Critical exponent for gap-filling at crisis," Condensed Matter Physics Seminar, University of Kansas, November 14, 1997.

24. "Critical exponent for gap-filling at crisis," Colloquium, Department of Physics and Astronomy, University of Southern California, January 30, 1998.