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Stimulated Backscattering From Relativistic Unmagnetized Electron Beams

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Analysis of stimulated scattering of a high frequency incident pump wave from an unmagnetized relativistic electron beam is presented. The backscattered radiation frequency can be enhanced by the factor $4\gamma^2$ over the incident pump frequency where γ_o is the relativistic factor of the electron beam. The linear growth rates associated with the wave-wave and wave-particle modes of scattering are examined for a number of different pump amplitude regimes. Estimates for the scattering efficiency are presented for the wave-wave scattering process.

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CONTENTS

STIMULATED BACKSCATTERING FROM RELATIVISTIC UNMAGNETIZED ELECTRON BEAMS

Section I. Introduction

Stimulated emission of backscattered radiation from intense relativistic electron beams has received considerable interest in the past few years. The primary reason for this interest lies in the fact that radiation backscattered from relativistic electron beams can undergo a dramatic frequency increase and is readily tunable over a wide frequency range. Hence, these scattering mechanisms, which rely on relativistic electron beams, may soon lead to a new class of submillimeter and infrared generating devices which could find application in such areas as radar, plasma heating, diagnostics, isotope separation and laser pellet fusion.

Analyses of the scattering phenomena have been carried out using both, a quantum mechanical formalism[1–4] as well as a classical approach.[5–9] In these theories, the incident pump field has taken various forms such as periodic static fields and traveling electromagnetic waves. Numerical simulations of the scattering processes have shown that the efficiency of converting electron kinetic energy into electromagnetic energy can be as high as 30% under certain conditions.[10,11] The frequency enhancement can be viewed as a double doppler upshift of the incident pump wave. An incident electromagnetic pump field at frequency ω_o, propagating antiparallel to a relativistic electron beam with speed v_o will backscatter into a frequency $\sim (1 + v_o/c)^2 \gamma_o^2 \omega_o$ where $\gamma_o = (1 - (v_o/c)^2)^{-1/2}$. In the case of a periodic static pump

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field of period l, the frequency of the backscattered wave will be approximately given by $(1 + v_o/c)\gamma_o^2 v_o(2\pi/l)$. The upshifted frequency can be easily varied by changing the energy of the electron beam. Experiments at Stanford University have demonstrated laser action at wavelengths of 10.6 μm as well as 3.4 μm using a static periodic pump. The gain in these experiments was relatively low: 7% increase in power was achieved in a 5.2 m interaction length at 10.6 μm and in an oscillator experiment only 0.01% of the electron beam energy was converted into radiation. Recent experiments at the Naval Research Laboratory using a traveling electromagnetic pump field have produced power levels of 1.5 MWs at 0.5 mm with an overall efficiency of 0.01%. At Columbia University experiments[12,13] employing a static periodic magnetic pump have resulted in megawatts of scattered radiation at wavelengths in the neighborhood of 1 mm. Scattering experiments using relativistic electron beams are also in progress at the Ecole Polytechnique in France.[14]

The two principal types of scattering processes in which an incident pump field is backscattered off an electron distribution into a transverse wave are wave-wave (Raman) and wave-particle (Compton) scattering.[15–19] In general, these two scattering modes are present simultaneously; however, the wave-wave process dominates if the incident pump wavelength in the electron beam frame is much greater than the Debye wavelength. Scattering then takes place off collective plasma oscillations. On the other hand, wave-particle scattering dominates when the pump wavelength is comparable to or smaller than the Debye wavelength. In this situation, scattering takes place off shielded or "dressed" particles . This paper will address both, wave-wave and wave-particle scattering.

The physical mechanism responsible for the instability of the backscattered electromagnetic wave, i.e., stimulated emission, can readily be described classically in the beam frame. In what follows, quantities in the beam frame will be written with primes. In the beam frame we stipulate that the existing electron equilibrium is perturbed by a low frequency density wave in the absence of an external magnetic field. Only waves propagating along the z axis, i.e., direction of beam velocity in the laboratory frame, will be considered. The electrostatic perturbation at frequency and wavenumber $(\omega_{||}', k_{||}')$ need not be an eigenmode of the electron distribution. The introduction of a large amplitude high frequency incident pump, E_o', at (ω_o', k_o') forces the electrons to oscillate at a frequency ω_o' in the direction along E_o' with a maximum velocity given by $v_{os}' = |e| E_o'/(m_o\omega_o')$. This transverse oscillation velocity, $\mathbf{v}_{os}'$ perpendicular to $\mathbf{k}_o'$, couples to the density wave, thus inducing transverse currents at frequency $\omega_{\pm}' = \omega_{||}' \pm \omega_o'$ and wave numbers $k_{\pm}' = k_{||}' \pm k_o'$. These currents now generate new electromagnetic waves at $(\omega_{\pm}', k_{\pm}')$. The generated or scattered electromagnetic field consists of backscattered waves propagating antiparallel to the incident pump wave. Forward scattered waves are also induced, but will not be considered because they are down shifted in frequency and also have a much smaller growth rate than the backscattered radiation. The pump and backscattered wave couple through the $\mathbf{v}' \times \mathbf{B}'$ term in the Lorentz force equation resulting in a longitudinal force at $(\omega_{||}', k_{||}')$. This induced longitudinal force, also called the ponderomotive or radiation pressure force, if properly phased will reinforce the originial density wave. The backscattered electromagnetic wave is, therefore, unstable resulting in stimulated emission of radiation. It should be noted that in the beam frame, the pump frequency is usually much greater than the frequency of the longitudinal wave, $|\omega_o'| >> |\omega_{||}'|$.

For a cold electron beam the electrostatic wave is an eigenmode of the system, $|\omega_{||}'|$ is roughly equal to the electron plasma frequency, $\omega_p' = (4\pi|e|^2 n_o' m_o)^{1/2}$ and the scattering process is referred to as Raman scattering. However, if the pump strengh is sufficiently strong, the frequency of the electrostatic wave is modified by the pump field and is greater than the plasma frequency. In this regime the scattering process is called modified Raman scattering. In either case, the phase velocity of the electrostatic wave is far removed from the electron velocity, $|\omega_{||}'/k_{||}'| >> v_{th}'$, where v_{th} is the electron thermal velocity; therefore, they are referred to as nonresonant, wave-wave or collective scattering modes. If the electron beam is sufficiently thermal so that the phase velocity of the electrostatic wave is comparable to the electron velocity, a resonance between the wave and particles results. This regime is called Compton scattering, resonant wave-particle scattering or inverse nonlinear Landau damping. Here the nonlinear coupling between the pump wave and scattered electromagnetic wave induces a longitudinal wave with a phase velocity comparable to the electron thermal velocity, $|\omega_{\|}'/k_{\|}'| \simeq v_{th}'$.

Section II. Dispersion Relation

In this section equations describing the coupling of the incident pump wave and the scattered electromagnetic and scattered electrostatic waves are derived. The large amplitude incident pump field is assumed to be linearly polarized in the x direction with frequency ω_o, and wavenumber $k_o = k_o\hat{e}_z$. Only spatial variations along the z axis will be considered. The pump field is incident upon a system of electrons which are electrostatically as well as magnetically neutral. The model is depicted in Fig. (1) and the analysis is fully relativistic and is performed in the laboratory frame of reference. The electromagnetic field of the incident pump wave is chosen to be of the form

$$\mathbf{E}_o(z,t) = E_o \cos(k_o z - \omega_o t)\hat{e}_x,$$

$$\mathbf{B}_o(z,t) = \frac{ck_o}{\omega_o} E_o \cos(k_o z - \omega_o t)\hat{e}_y, \tag{1a}$$

where E_o is the electric field amplitude and the direction of the axial Poynting flux along the z axis is given by the sign of ω_o/k_o. The form of the scattered electrostatic wave is

$$\mathbf{E}_{\|} = E_{\|} \cos(k_{\|} z - \omega_{\|} t + \phi_{\|})\hat{e}_z, \tag{2}$$

where $\phi_{\|}$ is the phase of the longitudinal field with respect to the pump field. The scattered electromagnetic field is chosen to be of the form

$$\mathbf{E}_s = \sum_{+,-} E_\pm \cos\,(k_\pm z - \omega_\pm t + \phi_\pm)\hat{e}_x,$$

$$\mathbf{B}_s = \sum_{+,-} \frac{ck_\pm}{\omega_\pm} E_\pm \cos\,(k_\pm z - \omega_\pm t + \phi_\pm)\hat{e}_y, \tag{3a}$$

where $k_\pm = k_{||} \pm k_o$, $\omega_\pm = \omega_{||} \pm \omega_o$, $\phi_\pm$ is the phase with respect to the pump wave and the sum is taken over +, −.

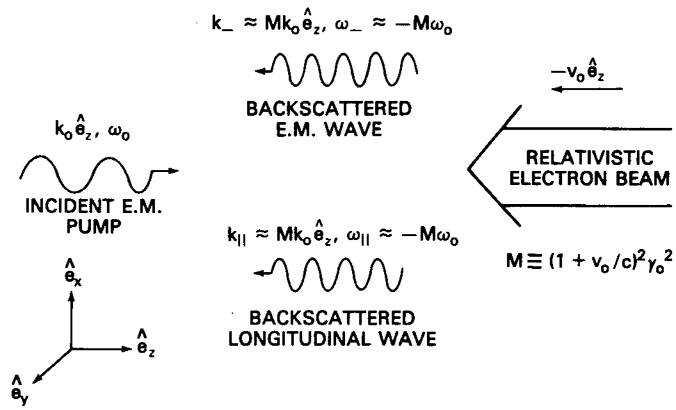

Figure 1. Schematic of Backscattering Off a Relativistic Electron Beam.

The evolution of the electron distribution is described by the relativistic Vlasov equation

$$\left(\frac{\partial}{\partial t} + \mathrm{v}_{||}\frac{\partial}{\partial z} - L\right) f(z, \mathbf{v}, t) = 0, \tag{4}$$

where $\mathrm{v}_{||} = \mathbf{v} \cdot \hat{e}_z$ is the component of velocity along the z axis, $L = |e|/m_o(\mathbf{E} + \mathbf{v} \times \mathbf{B}/c) \cdot \partial/\partial\mathbf{u}$, $\mathbf{E} = \mathbf{E}_o + \mathbf{E}_{||} + \mathbf{E}_s$, $\mathbf{B} = \mathbf{B}_o + \mathbf{B}_s$, $\mathbf{u} = \gamma\mathbf{v}$ is the normalized momentum, $\gamma = (1 - \underset{\sim}{\beta}^2)^{-1/2}$ and $\underset{\sim}{\beta} = \mathbf{v}/c$. In order to obtain the currents which drive the scattered fields we use a perturbation expansion to find the distribution function $f(z, \mathbf{v}, t)$ in terms of the scattered fields. Since the operator L consists of the perturbing fields, which include the pump, we may expand f in powers of the perturbating field amplitudes, that is

$$f = f^{(0)} + f^{(1)} + f^{(2)} + \cdots\ , \tag{5}$$

where

$$\frac{\partial f^{(0)}}{\partial t} = 0, \tag{6}$$

$$\left(\frac{\partial}{\partial t} + v_{\|} \frac{\partial}{\partial z}\right) f^{(n)} = L f^{(n-1)}, \tag{7}$$

and $n = 1, 2, \ldots$. In what follows the equilibrium distribution function described in (6) is chosen to have the form

$$f^{(0)}(u) = n_o \delta(u_x) \delta(u_y) g_o(u_{\|}), \tag{8}$$

where n_o is the ambient electron density, $\delta(u_i)$ is a delta function and $\int g_o(u_{\|}) du_{\|} = 1$. That is, the equilibrium distribution function is chosen to be cold in momentum space transverse to the direction of wave propagation while having a velocity spread parallel to the direction of wave propagation. It proves convenient to write the operator, L as the sum of two terms, one involving the pump field and the other scattered fields, that is, $L = L_o + L_s$ where

$$L_o \equiv \frac{|e|}{m_o} [E_o \cos(k_o z - \omega_o t) \vartheta_o],$$

$$L_s \equiv \frac{|e|}{m_o} \left[E_{\|} \cos(k_{\|} z - \omega_{\|} t + \phi_{\|}) \frac{\partial}{\partial u_{\|}} + \sum_{+,-} E_{\pm} \cos(k_{\pm} z - \omega_{\pm} t + \phi_{\pm}) \vartheta_{\pm} \right]$$

and

$$\vartheta_o \equiv \left(\frac{\psi_o}{\omega_o} \frac{\partial}{\partial u_x} + \frac{v_x k_o}{\omega_o} \frac{\partial}{\partial u_{\|}} \right),$$

$$\vartheta_{\pm} \equiv \left(\frac{\psi_{\pm}}{\omega_{\pm}} \frac{\partial}{\partial u_x} + \frac{v_x k_{\pm}}{\omega_{\pm}} \frac{\partial}{\partial u_{\|}} \right),$$

$$\psi_o \equiv \omega_o - v_{\|} k_o, \quad \psi_{\pm} \equiv \omega_{\pm} - v_{\|} k_{\pm},$$

$$\psi_{\|} \equiv \omega_{\|} - v_{\|} k_{\|}. \tag{9a-g}$$

The perturbing density and currents which drive the backscattered waves are given by

$$n_e = \sum_{n=1} n_e^{(n)}(z, t),$$

and,

$$\mathbf{J} = \sum_{n=1} \mathbf{J}^{(n)}(z, t),$$

where $n_e^{(n)} = \int f^{(n)}(z, \mathbf{u}, t) d^3u$ and $\mathbf{J}^{(n)} = -|e| \int (\mathbf{u}/\gamma) f^{(n)}(z, \mathbf{u}, t) d^3u$. The response current $\mathbf{J}$ drives the fields in Eqs. (1), (2) and (3) through the wave equation:

$$\nabla^2\mathbf{E} - c^{-2}\partial^2\mathbf{E}/\partial t^2 = 4\pi c^{-2}\partial\mathbf{J}/\partial t + \nabla(\nabla \cdot \mathbf{E}).$$

Solving Eq. (7) for $f^{(1)}(z, \mathbf{u}, t)$, the first order particle and current density take the form

$$n^{(1)}(\omega_{||}, k_{||}) = -\frac{|e|}{m_o} \frac{n_o k_{||}}{\omega_p^2} \chi(\omega_{||}, k_{||}) E_{||} \sin(k_{||} z - \omega_{||} t + \phi_{||}),$$

$$J_{||}^{(1)}(\omega_{||}, k_{||}) = \frac{\omega_{||}}{4\pi} \chi(\omega_{||}, k_{||}) E_{||} \sin(k_{||} z - \omega_{||} t + \phi_{||}),$$

$$J_o^{(1)}(\omega_o, k_o) = -\frac{\omega_p^2}{4\pi < \gamma_{||} >} \frac{E_o}{\omega_o} \sin(k_o z - \omega_o t),$$

$$J_{\pm}^{(1)}(\omega_{\pm}, k_{\pm}) = -\frac{\omega_p^2}{4\pi < \gamma_{||} >} \frac{E_{\pm}}{\omega_{\pm}} \sin(k_{\pm} z - \omega_{\pm} t + \phi_{\pm}), \tag{10a-d}$$

where $\omega_p \equiv (4\pi|e|^2 n_o/m_o)^{1/2}$, $\chi(\omega_{||}, k_{||}) \equiv (\omega_p^2/k_{||}) \int du_{||} (\partial g_o(u_{||})/\partial u_{||})/\psi_{||}$ is the electron susceptibility, $< \gamma_{||} >^{-1} \equiv \int du_{||} g_o(u_{||})/\gamma_{||}$ and $\gamma_{||} \equiv (1 + u_{||}^2/c^2)^{1/2}$. For a cold electron beam, $g_o(u_{||}) = \delta(u_{||} - u_o)$, the electron susceptibility is then given by $\chi_{cold} = -(\omega_o^2/\gamma_o^{3/2})/(\omega_{||} - v_o k_{||})^2$. The arguments of the quantities on the left hand side of Eqs. (10) denote the frequency and wavenumber of the quantities. Using (10b-d) in the wave equation the linear dispersion relations for the pump, scattered electrostatic and electromagnetic waves are respectively:

$$\omega_o^2 - c^2k_o^2 - \omega_p^2/<\gamma_{||}> = 0,$$

$$1 + \chi(\omega_{||}, k_{||}) = 0,$$

$$\omega_\pm^2 - c^2k_\pm^2 - \omega_p^2/<\gamma_{||}> = 0. \tag{11a-c}$$

Evaluating the second order particle and current density gives:

$$n^{(2)}(\omega_{||}, k_{||}) = -\frac{k_{||}^2}{8\pi}\frac{E_o}{m_o\omega_o}\left[\frac{E_+}{\omega_+}\cos(k_{||}z - \omega_{||}t + \phi_+) - \frac{E_-}{\omega_-}\cos(k_{||}z - \omega_{||}t + \phi_-)\right]\tilde{\chi}(\omega_{||}, k_{||}),$$

$$J_{||}^{(2)}(\omega_{||}, k_{||}) = \frac{\omega_{||}k_{||}}{8\pi}\frac{|e|E_o}{m_o\omega_o}\left[\frac{E_+}{\omega_+}\cos(k_{||}z - \omega_{||}t + \phi_+) - \frac{E_-}{\omega_-}\cos(k_{||}z - \omega_{||}t + \phi_-)\right]\tilde{\chi}(\omega_{||}, k_{||}),$$

$$J_o^{(2)}(\omega_o, k_o) = \frac{k_{||}}{8\pi}\sum_{+,-}\frac{|e|E_\pm}{m_o\omega_\pm}\times E_{||}\cos(k_oz - \omega_ot \pm (\phi_\pm - \phi_{||}))\tilde{\chi}(\omega_{||}, k_{||}),$$

$$J_\pm^{(2)}(\omega_\pm, k_\pm) = \mp\frac{k_{||}}{8\pi}\frac{|e|E_o}{m_o\omega_o}\times E_{||}\cos(k_\pm z - \omega_\pm t + \phi_{||})\tilde{\chi}(\omega_{||}, k_{||}), \tag{12a-d}$$

where $\tilde{\chi}(\omega_{||}, k_{||}) \equiv (\omega_p^2/k_{||})\int du_{||}\,(\partial g_o(u_{||})/\partial u_{||})/(\gamma_{||}\psi_{||})$. Note the difference in the definitions of χ and $\tilde{\chi}$. It is necessary to find the third order transverse current density at $(\omega_\pm, k_\pm)$ in order to recover the wave-particle scattering. The third order current density at $(\omega_\pm, k_\pm)$ is

$$J_{\pm}^{(3)}(\omega_{\pm},k_{\pm}) = \frac{|e|^2 n_o}{4m_o}\left(\frac{|e|E_o}{m_o\omega_o}\right)^2 \frac{k_{||}^2\omega_o}{\omega_p^2\omega_{\pm}}\lambda_{\pm}\left\{\frac{E_+}{\omega_+}\sin(k_{\pm}z-\omega_{\pm}t+\phi_+)\right.$$

$$\left. - \frac{E_-}{\omega_-}\sin(k_{\pm}z-\omega_{\pm}t+\phi_-)\right\} \tag{13}$$

where

$$\lambda_{\pm} = -\frac{\omega_p^2}{k_{||}^2}\left(\frac{\omega_{\pm}}{\omega_o}\right)\int du_{||}\left[\frac{g_o(u_{||})}{\gamma_{||}^3\psi_{||}^2\psi_{\pm}^2}\left\{(k_{||}^2-\omega_{||}^2/c^2)\psi_{\pm}\psi_o + (k_ok_{\pm}-\omega_o\omega_{\pm}/c^2)\psi_{||}^2\right.\right.$$

$$\left.\left. + \psi_{||}\left[(k_{||}\omega_o+\omega_{||}k_o)\left(k_{\pm}+\frac{u_{||}}{\gamma_{||}}\frac{\omega_{\pm}}{c^2}\right) - (k_ok_{||}+\omega_o\omega_{||}/c^2)(\omega_{\pm}+u_{||}k_{\pm}/\gamma_{||})\right]\right\}\right]. \tag{14}$$

Now substituting the currents $J_{||}(\omega_{||},k_{||}) = J_{||}^{(1)} + J_{||}^{(2)}$, $J_{\pm}(\omega_{\pm},k_{\pm}) = J_{\pm}^{(1)} + J_{\pm}^{(2)} + J_{\pm}^{(3)}$ and $J_o(\omega_o,k_o) = J_o^{(1)} + J_o^{(2)}$ into the wave equation for $\mathbf{E}_{||}$, $\mathbf{E}^{\pm}$ and $\mathbf{E}_o$ we obtain

$$(1+\chi(\omega_{||},k_{||}))E_{||}e^{i\phi_{||}}e^{i(k_{||}z-\omega_{||}t)} = -\frac{i}{2}\left(\frac{|e|E_o}{m_o\omega_o}\right)k_{||}\tilde{\chi}(\omega_{||},k_{||})$$

$$\times\left(\frac{E_+}{\omega_+}e^{i\phi_+} - \frac{E_-}{\omega_-}e^{i\phi_-}\right)e^{i(k_{||}z-\omega_{||}t)},$$

$$D_{\pm}(\omega_{\pm},k_{\pm})E_{\pm}e^{i\phi_{\pm}}e^{i(k_{\pm}z-\omega_{\pm}t)} = \pm\frac{i}{2}\left(\frac{|e|E_o}{m_o\omega_o}\right)k_{||}\omega_{\pm}$$

$$\times\left(\tilde{\chi}(\omega_{||},k_{||}) \pm \lambda_{\pm}\frac{(1+\chi(\omega_{||},k_{||}))}{\tilde{\chi}(\omega_{||},k_{||})}\frac{\omega_o}{\omega_{\pm}}\right)E_{||}e^{i\phi_{||}}e^{i(k_{\pm}z-\omega_{\pm}t)},$$

$$D_o(\omega_o,k_o)E_oe^{i(k_oz-\omega_ot)} = -\frac{i}{2}\frac{|e|}{m_o}k_{||}\omega_oE_{||}\tilde{\chi}(\omega_{||},k_{||})$$

$$\times\left(\frac{E_+}{\omega_+}e^{i(\phi_+-\phi_{||})} + \frac{E_-}{\omega_-}e^{-i(\phi_--\phi_{||})}\right)e^{i(k_oz-\omega_ot)}. \tag{15a-c}$$

Combining Eqs. (15), the following dispersion relation for the scattered waves is obtained

$$(1+\chi) = \left(\frac{|e|E_o}{2m_o\omega_o}\right)^2 k_{||}^2 \tilde{\chi}$$

$$\cdot\left\{\frac{(\tilde{\chi} - (\lambda_+/\tilde{\chi})(1+\chi)\omega_o/\omega_+)}{D_+} + \frac{(\tilde{\chi} + (\lambda_-/\tilde{\chi})(1+\chi)\omega_o/\omega_-)}{D_-}\right\}. \quad (16)$$

The dispersion relation in Eq. (16) describes the relationship between $\omega_{||}$ and $k_{||}$ in the laboratory frame. It is convenient, however, to transform Eq. (16) to the beam frame where the average electron momentum is zero; $< u_{||} > = 0$. Beam frame quantities will be denoted by primes. In order to simplify the dispersion relation in the beam frame we assume that the frequency of the electrostatic wave is much smaller than either the pump or scattered electromagnetic wave, $|\omega_{||}'| << |\omega_o'|$, and hence $\omega_o' \simeq \pm\, \omega_\pm'$. It is easy to see that this is an excellent approximation in the beam frame. The expression for $\lambda_\pm$ and $\tilde{\chi}$ in the prime frame can be approximated by

$$\lambda_\pm' \simeq \chi'$$

$$\tilde{\chi}' \simeq \chi' \quad (17a,b)$$

By using Eqs. (17a, b) and assuming the electron thermal velocity is nonrelativistic in the beam frame, Eq. (16) reduces to the rather simple form

$$(1+\chi') = -(v_{os}'/2)^2 (k_{||}')^2 2\chi'\left(\frac{1}{D_+'} + \frac{1}{D_-'}\right) \quad (18)$$

where

$$\chi' \equiv \frac{\omega_p'^2}{k_{||}'} \int dv_{||}' \frac{\partial g_o'(v_{||}')/\partial v_{||}'}{\omega_{||}' - v_{||}' k_{||}'}, \qquad v_{os}' \equiv \frac{|e|E_o'}{m_o\omega_o'},$$

$$D_\pm' \equiv (\omega_\pm')^2 - c^2(k_\pm')^2 - \omega_p'^2, \qquad \omega_p' \equiv (4\pi|e|^2 n_o'/m_o)^{1/2}.$$

Note that m_o is the electron rest mass and hence is the same in all frames. Equation (18) is the dispersion relation for waves scattered parallel or antiparallel to the incident pump wave off a cold (Raman Scattering) or thermal (Compton Scattering) distribution of particles in the beam frame. Before examining the different scattering modes given by Eq. (18), the electron susceptibility is written in terms of the standard plasma dispersion function, $Z(\xi') \equiv \pi^{-1/2} \int_{-\infty}^{\infty} dx \exp(-x^2)/(x - \xi')$ for $Im\xi' > 0$. In terms of $Z(\xi')$, the suceptibility is

$$\chi' = \left(\frac{k_D'}{k_{||}'}\right)^2 (1 + \xi' Z(\xi')) = -\frac{1}{2}\left(\frac{k_D'}{k_{||}'}\right)^2 \partial Z'/\partial \xi' \tag{19}$$

where $k_D' \equiv \omega_p'/v_{th}'$ is the Debye wavenumber, $\xi' \equiv (\omega_{||}'/k_{||}')/(\sqrt{2}\, v_{th}')$, and v_{th}' is the thermal electron velocity defined by $g_o'(v_{||})' \equiv (\sqrt{2\pi}\, v_{th}')^{-1} \exp(-v_{||}'^2/(2v_{th}'^2))$.

The temporal linear growth rates for Raman and Compton scattering can now be obtained for the backscattered electromagnetic wave in the beam frame. The energy flux of the incident pump will be assumed to propagate towards the right, i.e., $\omega_o' > 0$ and $k_o' > 0$, as shown in Fig. (1).

Raman Scattering (Wave-Wave Scattering)

We first consider scattering off a cold electron distribution such that $|\omega_{||}'/k_{||}'| >> v_{th}'$ or $\xi' >> 1$. In the case of a small amplitude pump field the electrostatic mode is very close to being an eigenmode of the pump-free system. That is, for a small amplitude pump $\omega_{||}'$ and $k_{||}'$ approximately satisfy the dispersion relation $1 + \chi'(\omega_{||}', k_{||}') = 0$. If the pump amplitude is large enough, the eigenmodes of the electrostatic wave are modified and no longer satisfy the relationship given by $1 + \chi'(\omega_{||}', k_{||}') \simeq 0$. This strong pump regime will be discussed later. The dispersion relation in Eq. (18) leads to unstable roots if D_-' or D_+' vanish simultaneously along with the left hand side of the equation. Figure (2) shows the general form of the dispersion relation in Eq. (18) for a cold electron system and small amplitude pump field. The situation where both D_-' or D_+' vanish simultaneously will not be considered here since this case does not correspond to stimulated backscattering; and hence, will not lead to the proper frequency enhancement in the laboratory frame. Furthermore, the linear growth rate is substantially smaller for this instability. Because the frequency of the longitudinal wave is much less than the frequency of the pump wave, the quantity $D'_\pm(\omega'_\pm, k'_\pm)$ can be approximated. With this assumption we find that

$$D'_\pm(\omega'_\pm, k'_\pm) \simeq \pm 2\omega'_o \left(\omega'_{||} - \frac{k'_o k'_{||} c^2}{\omega'_o} \mp \frac{c^2 k'^2_{||}}{2\omega'_o}\right). \tag{20}$$

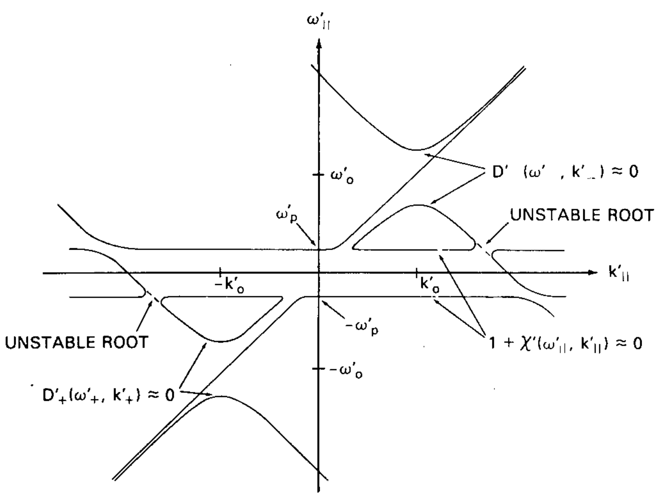

Figure 2. Dispersion Relation in the Beam Frame, for a Cold Electron Beam, Showing Stimulated Growth of the Scattered Radiation.

In obtaining (20) the fact that the pump wave satisfies the dispersion relation, $D'_o(\omega'_o, k'_o) = 0$, was also used. Since we are considering scattered waves such that $|\omega'_{||}/k'_{||}| \gg v'_{th}$, the susceptibility can be expanded to give $\chi' \simeq -\omega'^2_p/\omega'^2_{||} - 3\omega'^2_p v'^2_{th} k'^2_{||}/\omega'^4_{||} + i Im(\chi')$ where $Im(\chi') = \pi^{1/2}\xi' \exp(-\xi'^2)$ is the imaginary part of the electron susceptibility. From here on we will take the wave at (ω'_-, k'_-) to be resonant, i.e. $D'_-(\omega'_-, k'_-) \simeq 0$, and the (ω'_+, k'_+) wave to be nonresonant, i.e. $D'_+(\omega'_+, k'_+) \neq 0$. Therefore, we consider the case where $\omega'_0/k'_o > 0$ and $\omega'_-/k'_- < 0$. Our choice for the resonant backscattered wave, $D'_- \simeq 0$, is completely arbitrary, since it is easy to see that choosing $D'_+ \simeq 0$ and $D'_- \neq 0$ leads to the same results. The dispersion relation in (18) now becomes

$$(\omega'^2_{||} - \omega'^2_l(1 - i Im(\chi')))(\omega'_{||} - \Omega') = -\frac{\omega'^2_p}{8}\frac{v'^2_{os} k'^2_{||}}{\omega'_o}, \tag{21}$$

where

$$\Omega' \equiv k_o' k_{||}' c^2/\omega_o' - c^2 k_{||}'^2/(2\omega_o') \text{ and } \omega_l'^2 \equiv \omega_p'^2 + 3\mathrm{v}_{\mathrm{th}}'^2 k_{||}'^2 \simeq \omega_p'^2 .$$

Equation (21) is the approximate form of the dispersion relation when $1 + \chi' \simeq 0$, $D_-' \simeq 0$, $D_+' \neq 0$, $\xi' >> 1$ and $\omega_p' << \omega_o'$. From Fig. (2) we note that the unstable roots occur for $k_{||}' \simeq 2k_o'$, which corresponds to stimulated backscattering.

To obtain the growth rate from Eq. (21) we set $\omega_{||}' = \omega_l' + \delta\omega'$ where ω_l' is set equal to Ω' and $|\delta\omega'|$ as well as $|\omega_l' Im(\chi')|$ are assumed much less than $|\omega_l'|$. Substituting $\omega_{||}' = \omega_l' + \delta\omega'$ in Eq. (21) gives the following expression for $\delta\omega'$

$$\delta\omega' = -i\frac{\omega_l'}{4} Im(\chi') + \frac{i}{4}\left[(\omega_l' Im(\chi'))^2 + \frac{\omega_p'^2 \mathrm{v}_{os}'^2 k_{||}'^2}{\omega_l'\omega_o'}\right]^{1/2} . \tag{22}$$

There are two cases to consider in Eq. (22), depending on the strength of the incident pump wave. If the incident pump amplitude is sufficiently weak to satisfy the inequality

$$\beta_{os}' << \frac{(\omega_l'^3\omega_o')^{1/2}}{\omega_p' c k_{||}'} Im(\chi') \simeq \left[\frac{\omega_p'}{2\omega_o'}\right]^{1/2} Im(\chi') \tag{23}$$

where $\beta_{os}' = \mathrm{v}_{os}'/c$, then the temporal growth rate is given by

$$\Gamma' = Im(\delta\omega') = \frac{1}{8}\left[\frac{\omega_p'\mathrm{v}_{os}'k_{||}'}{\omega_l' Im(\chi')}\right]^2 \frac{Im(\chi')}{\omega_o'} \simeq \frac{\beta_{os}'^2}{2}\frac{\omega_o'}{Im(\chi')} \tag{24}$$

and the real part of the frequency is $Re(\omega_{||}') = \omega_l' \simeq \omega_p'$. In the moderately strong pump regime where $\beta_{os}' >> (\omega_l'^3\omega_o')^{1/2} Im(\chi')/(\omega_p' c k_{||}')$, but small enough so as not to greatly modify the pump-free eigenmodes, the growth rate is

$$\Gamma' = Im(\delta\omega') = \frac{1}{4}\left[\frac{\omega_p'\mathrm{v}_{os}'k_{||}'}{\sqrt{\omega_l'\omega_o'}} - \omega_l' Im(\chi')\right] \simeq \frac{\beta_{os}'}{2}\left[\frac{\omega_p'}{\omega_o'}\right]^{1/2} , \tag{25}$$

the real part of the frequency is

$$Re(\omega_{\|}') = \omega_l' + \frac{1}{16}\beta_{os}'^2 \frac{\omega_p'^2\omega_o'}{\omega_l'^2} \simeq \omega_p'. \tag{26}$$

We now consider the situation where the pump amplitude is so large that it modifies the eigenmodes of the electrostatic waves. That is, if $\beta_{os}' > \beta_{\rm crit}'$ where $\beta_{\rm crit}' = (2\,\omega_l'/\omega_o')^{1/2} \simeq (2\omega_p'/\omega_o')^{1/2}$. The frequency and wavenumber of the longitudinal wave, $(\omega_{\|}', k_{\|}')$, no longer satisfies the relationship $1 + \chi'(\omega_{\|}', k_{\|}') \simeq 0$. In this case $\omega_{\|}' >> \omega_l', \Omega'$ and the dispersion relation in (21) takes the form $\omega_{\|}'^3 = -(\beta_{os}'\omega_p')^2\omega_o'/2$ which gives the growth rate

$$\Gamma' = Im(\omega_{\|}') = \frac{\sqrt{3}}{2}((\beta_{os}'\omega_p')^2\omega_o'/2)^{1/3}. \tag{27}$$

The real part of the frequency in this case is

$$Re(\omega_{\|}') = \frac{1}{2}((\beta_{os}'\omega_p')^2\omega_o'/2)^{1/3}. \tag{28}$$

Equations (24), (25) and (27) are the expressions for the temporal growth rates for stimulated Raman backscattering in the beam frame. These expressions all have a different parametric dependence on the pump amplitude. Since $Im(\chi') << 1$, for a cold electron distribution, we set $Im(\chi') = 0$ and discuss only the moderately strong and strong pump regimes whose growth rates are given in Eqs. (25) and (27) respectively. The results of the linear theory for these two cases can easily be transformed back to the laboratory frame. The value of β_{os}, in the laboratory frame, which distinguishes the moderately strong and strong pump regime is $\beta_{\rm crit}$, and is given by

$$\beta_{\rm crit} \equiv \gamma_o^{-3/2}(1+\beta_o)^{-1/2}\left(2\frac{\omega_l}{\omega_o}\right)^{1/2}$$

$$\simeq \gamma_o^{-3/2}(1+\beta_o)^{-1/2}\left(2\frac{\omega_p/\gamma_o^{1/2}}{\omega_o}\right)^{1/2}, \tag{29}$$

where $\beta_o = v_o/c, \gamma_o = (1 - (v_o/c)^2)^{-1/2}$ and v_o is the axial speed of the beam in the laboratory frame. From Eq. (25) and (27) we find that the linear growth rate in the laboratory frame, for the moderately strong and strong pump regime, is given respectively by

$$\Gamma = \frac{1}{2}\beta_{os}\gamma_o^{1/2}((1+\beta_o)\omega_o\omega_l)^{1/2} \simeq \frac{1}{2}\beta_{os}\gamma_o^{1/2}((1+\beta_o)\omega_o\omega_p/\gamma_o^{1/2})^{1/2}, \tag{30}$$

when $\beta_{os} < \beta_{\text{crit}}$ and

$$\Gamma = \frac{\sqrt{3}}{2}((1+\beta_o)\beta_{os}^2\omega_o\omega_l^2/2)^{1/3} \simeq \frac{\sqrt{3}}{2}((1+\beta_o)\beta_{os}^2\omega_o\omega_p^2/\gamma_o/2)^{1/3}, \tag{31}$$

when $\beta_{os} > \beta_{\text{crit}}$. The frequency of the backscattered electromagnetic wave in both the above cases is

$$|\omega_-| \simeq (1+\beta_o)^2\gamma_o^2\omega_o. \tag{32}$$

In the beam frame of reference the phase velocity of the electrostatic wave is

$$\omega_{||}'/k_{||}' \simeq \omega_l'/2k_o', \tag{33}$$

when $\beta_{os}' < \beta_{\text{crit}}'$ and

$$\omega_{||}'/k_{||}' \simeq \frac{1}{4}((\beta_{os}'\omega_l')^2\omega_o'/2)^{1/3}/k_o' > \omega_l'/2k_o', \tag{34}$$

for $\beta_{os}' > \beta_{\text{crit}}'$. The growth rates in Eqs. (30) and (31) are valid as long as $|\omega_{||}'/k_{||}'|$ is much greater than the thermal velocity v_{th}'. The opposite limit is the Compton regime and will be discussed in detail later. The thermal velocity in the beam frame is related to the thermal velocity in the laboratory frame by the relation $v_{\text{th}}' = \gamma_o^2 v_{\text{th}}$. The total spread in the beam energy in the laboratory frame due to the thermal velocity spread v_{th} is $\Delta\epsilon_{\text{th}} = 2\beta_o\gamma_o^3(v_{\text{th}}/c)mc.^2$ Therefore, for thermal effects to be negligible and Eq. (30) and (31) applicable, the following conditions on $\Delta\epsilon_{\text{th}}$ must be satisfied in the moderately strong pump case

$$\Delta\epsilon_{\text{th}}/\epsilon_o << \frac{\beta_o}{(1+\beta_o)(\gamma_o - 1)}\left(\frac{\omega_p/\gamma_o^{1/2}}{\omega_o}\right), \tag{35}$$

and in the strong pump case

$$\Delta\epsilon_{th}/\epsilon_o << \frac{\gamma_o\beta_o/2}{(1+\beta_o)(\gamma_o-1)}\left[\beta_{os}^2\left(\frac{\omega_p/\gamma_o^{1/2}}{\omega_o}\right)\frac{(1+\beta_o)}{2}\right]^{1/3}, \tag{36}$$

where $\epsilon_o = (\gamma_o - 1)m_oc^2$ is the electron kinetic energy. These results are summarized in Table I for a highly relativistic electron beam. Estimates for the efficiency of converting electron beam energy into electromagnetic energy are also given in Table I and will be discussed shortly.

Compton Scattering (Wave-Particle Scattering)

We now consider the kinetic regime where the phase velocity of the longitudinal wave is of the order of the electron thermal velocity, i.e., $\omega_{\parallel}/k_{\parallel} = \vartheta(v_{th})$. In this regime the electrostatic waves are heavily Landau damped in the absence of the pump wave. This scattering mode is called stimulated Compton or inverse nonlinear Landau scattering because the electrostatic wave, resulting from the beating of the two electromagnetic waves, is resonant with the electrons. Since the longitudinal wave is not an eigenmode of the system, i.e., $1 + \chi' \neq 0$, the dispersion relation of the electrostatic wave for Compton backscattering takes the form

$$\omega' - \Omega' = (\beta_{os}'^2/2)\omega_o'\chi'/(1 + \chi'), \tag{37}$$

where Eq. (20) for D_- together with $k_{\parallel}' \simeq 2k_o' \simeq 2\omega_o'/c$ were used in obtaining Eq. (37). Taking the imaginary part of both sides of Eq. (37) and noting that

$$Im\left(\frac{\chi'}{1+\chi'}\right) = -Im\left(\frac{1}{1+\chi'}\right),$$

the growth rate for Compton scattering, in the beam frame, is found to be

$$\Gamma' = -(\beta_{os}'^2/2)\omega_o'Im\left(\frac{1}{1+\chi'(\Omega',2k_o')}\right), \tag{38}$$

where

$$Im[(1+\chi(\Omega,2k_o))^{-1}] = \frac{\frac{1}{2}\left(\frac{k_D}{k'}\right)^2 Im(Z_\xi')}{\left[1-\frac{1}{2}\left(\frac{k_D}{k}\right)^2 Re(Z_\xi')\right]^2+\left[\frac{1}{2}\left(\frac{k_D}{k}\right)^2 Im(Z_\xi')\right]^2}$$

$Z_\xi \equiv \partial Z/\partial\xi$, and $\xi = \omega'/(\sqrt{2}\,v_{th}k')$. The term $Im((1+\chi)^{-1})$ can be readily approximated in the limit that $\omega'/k' << v_{th}$, i.e., $\xi << 1$. In this domain the wavelength of the electrostatic disturbance, $|2\pi/k'|$, is much less than the shielding length, $2\pi/k_D$. Therefore, in the limit that $k_D/k' << 1$ and

$$Im\,[(1+\chi)^{-1}]\Big|_{k_D/k' << 1} \simeq \frac{1}{2}\left(\frac{k_D}{k}\right)^2 Im(Z_\xi') \simeq -\pi^{1/2}\left(\frac{k_D}{k}\right)^2 \xi \exp(-\xi^2). \tag{39}$$

Substituting Eq. (39) into (38), the Compton growth rate becomes

$$\Gamma = \frac{\pi^{1/2}}{2}\beta_{os}^2\omega_o\left(\frac{k_D}{k}\right)^2 \xi \exp(-\xi^2). \tag{40}$$

The term $\xi\exp(-\xi^2)$ has a maximum when $\xi = 1/\sqrt{2}$, *i.e.*, $\omega'/k' = v_{th}$, so the maximum value of Γ in Eq. (40) is approximately given by

$$\Gamma_{max} = .4\,\omega_o\beta_{os}^2(k_D/k')^2 = \frac{1}{10}\,\frac{\omega_p^2}{\omega_o}\left(\frac{v_{os}}{v_{th}}\right)^2. \tag{41}$$

In ref. (8) it was shown that the temporal Compton growth rate has the following transformation properties from the beam frame to the laboratory frame,

$$\Gamma' = \frac{\Gamma}{\gamma_o(1+v_o/c)}. \tag{42}$$

Substituting Eq. (41) into (42) and writing the beam frame quantities in terms of laboratory frame quantities we obtain

$$\Gamma_{max} = \frac{1}{10}\,\frac{\omega_p^2}{\gamma_o^5(1+\beta_o)^2\omega_o}\left(\frac{v_{os}}{v_{th}}\right)^2$$

$$= \frac{2}{5}\,\frac{\beta_o^2\gamma_o\omega_p^2}{(1+\beta_o)^2\omega_o}\,\frac{\beta_{os}^2}{(\gamma_o-1)^2}\left(\frac{\epsilon_o}{\Delta\epsilon_{th}}\right)^2$$

$$\simeq \frac{1}{10}\,\frac{\omega_p^2}{\omega_o}\,\frac{\beta_{os}^2}{\gamma_o}\left(\frac{\epsilon_o}{\Delta\epsilon_{th}}\right)^2, \tag{43}$$

where the last expression is valid for a highly relativistic electron beam.

Section III. Saturation Levels and Efficiencies

This section will deal primarily with the saturation and efficiency levels of Raman backscattering off a cold, i.e., $v_{th} = 0$, electron beam. Saturation of the backscattered electromagnetic wave may be due to either pump depletion or nonlinearities associated with the electrostatic wave (density wave). Pump depletion ocurrs when the amplitude of the pump is depleted by the scattering process. Nonlinearities result when the electrostatic wave, given in Eq. (2), grows to a level sufficient to trap electrons. Roughly speaking, for a small amplitude pump field, pump depletion occurs before the electron dynamics become nonlinear. However, for a large amplitude pump field electron trapping takes place before all the incident photons are scattered. Therefore, the magnitude of β_{os} determines the nature of the saturation mechanism.

In the beam frame the magnitude of the backscattered electromagnetic wave, when saturation is due to pump depletion, is given by

$$|E'_-| = (\omega'_-/\omega'_o)^{1/2}E'_o. \tag{44}$$

Equation (44) is just a statement of conservation of wave action. When the frequency of the scattered wave, ω'_-, is approximately equal to the pump frequency, ω'_o, in the beam frame, we find that $|E'_-| \simeq E'_o$ and virtually all the incident pump photons are backscattered. However, before this happens the level of the density wave may become comparable to the ambient density of the electrons. When this happens the electron dynamics become nonlinear and electron trapping occurs in the potential well associated with the total electrostatic field. The total longitudinal electrostatic field consists of the sum of the self consistent field given by Eq. (15a) and the ponderomotive field associated with the v $\times$ B/c axial force. The magnitude of the sum of these two fields is

$$|E_{total}| = \left| \frac{1}{2} \frac{k'_{||} \chi'}{(1 + \chi')} \frac{|e| E'_o E'_-}{m_o \omega'_o \omega'_-} \right| . \tag{45}$$

Associated with $|E_{total}|$ is a density wave, the magnitude of which is

$$|\delta n'| = \left| \frac{1}{8\pi} \frac{k'^2_{||} \chi'}{(1 + \chi')} \frac{E'_o E'_-}{m_o \omega'_o \omega'_-} \right| . \tag{46}$$

Equating $|\delta n'|$ to the ambient electron density n'_o, we find that electron trapping limits $|E'_-|$ to the value

$$|E'_-| = \left| 8\pi n'_o m_o \frac{(1 + \chi')}{\chi'} \frac{\omega'_o \omega'_-}{k'^2_{||} E'_o} \right| . \tag{47}$$

For the moderately strong pump regime we find that

$$\left| \frac{1 + \chi'}{\chi'} \right| \simeq |2\Gamma'/\omega'_p| \simeq \frac{1}{\beta_{os}} \left(\frac{\omega'_p}{\omega'_o} \right)^{1/2} , \tag{48}$$

where Eq. (25) was used for Γ'. In the case of a strong pump the magnitude of the susceptibility is much less than unity and therefore

$$\left|\frac{1+\chi'}{\chi'}\right| \simeq \left|\frac{1}{\chi'}\right| = \left|\left(\frac{\omega_{\|}'}{\omega_p'}\right)^2\right| \simeq \left(\frac{\beta_{os}^{2}\,\omega_o'}{2\omega_p'}\right)^{2/3}, \tag{49}$$

where Eqs (27) and (28) were used for $\omega_{\|}'$. Substituting these expressions for $\chi'/(1+\chi')$ into Eq. (47) we find that the amplitude of the backscattered electromagnetic wave, when saturation is due to electron trapping, for the moderate and strong pump case is respectively given by

$$|E_-'| = \frac{1}{2}\left(\frac{\omega_p'}{\omega_o'}\right)^{3/2} \frac{E_o'}{\beta_{os}'}, \qquad \beta_{os}' < \beta_{crit}' \tag{50a}$$

and

$$|E_-'| = \frac{1}{2^{5/3}}\left(\frac{\omega_p'}{\omega_o'}\right)^{4/3} \frac{E_o'}{(\beta_{os}')^{2/3}}, \qquad \beta_{os}' > \beta_{crit}'. \tag{50b}$$

where we have used the fact that $k_{\|}' \simeq 2k_o' \simeq 2\omega_o'/c$ and $\omega_-' \simeq \omega_o'$. Comparing Eq. (50a) with (44) we find that if $\beta_{os}' < \beta_1'$, where $\beta_1' \equiv (1/2)(\omega_p'/\omega_o')^{3/2}$, then pump depletion saturates the backscattering process before electron trapping takes place. Since β_1 is always less than β_{crit} it is clear that for $\beta_{os} > \beta_1$ electron trapping is the saturation mechanism and it occurs before the pump is depleted. The level of the fields at saturation in the beam frame can then be summarized as follows

$$|E_-'| \simeq \begin{cases} E_o', & \text{for } \beta_{os}' < \beta_1', \text{ pump depletion} \\ (\beta_1'/\beta_{os}')\,E_o', & \text{for } \beta_1' < \beta_{os}' < \beta_{crit}', \text{ trapping} \\ \frac{1}{\sqrt{2}}\,(\beta_1'/\beta_{crit}')\,(\beta_{crit}'/\beta_{os}')^{2/3} E_o', & \text{for } \beta_{os}' > \beta_{crit}', \text{ trapping.} \end{cases} \tag{51a-c}$$

In order to obtain the efficiency it is necessary to transform the magnitudes of the backscattered fields in Eqs. (51) to the laboratory frame. Since the electric fields have the following transformation properties, $|E_-'| = |E_-|/(\gamma_o(1+\beta_o))$ and $E_o' = (1+\beta_o)\gamma_o E_o$, the amplitudes in Eqs. (51) when written in the laboratory frame become

$$|E_-| \simeq (1+\beta_o)^2\gamma_o^2 \begin{cases} E_o, & \text{for } \beta_{os} < \beta_1 \\ \dfrac{\beta_1}{\beta_{os}} E_o, & \text{for } \beta_1 < \beta_{os} < \beta_{\text{crit}} \\ \dfrac{1}{\sqrt{2}} \dfrac{\beta_1}{\beta_{\text{crit}}} \left(\dfrac{\beta_{\text{crit}}}{\beta_{os}}\right)^{2/3} E_o, & \text{for } \beta_{os} > \beta_{\text{crit}} \end{cases} \tag{52a-c}$$

where

$$\beta_1 = \frac{1}{2}\gamma_o^{-5/2}(1+\beta_o)^{-3/2}\left(\frac{\omega_p/\gamma_o^{1/2}}{\omega_o}\right)^{3/2} \text{ and}$$

$$\beta_{\text{crit}} = 2^{1/2}\gamma_o^{-3/2}(1+\beta_o)^{-1/2}\left(\frac{\omega_p/\gamma_o^{1/2}}{\omega_o}\right)^{1/2},$$

are the expressions for β_1' and β_{crit}' transformed to the laboratory frame.

The efficiency of stimulated backscattering can be defined as the ratio of the average electromagnetic energy density in the laboratory frame to the kinetic energy density of the electrons. Efficiency is then defined as $\eta \equiv \langle W_E + W_M \rangle / (n_o(\gamma_o - 1)m_oc^2)$ where $\langle W_E \rangle \simeq \langle W_M \rangle \simeq |E_-^2|/16\pi$ is the average electric field energy density. The electric energy density is very nearly equal to the magnetic energy density, since $ck_- \simeq \omega_-$. Using the expressions in Eqs. (52) for $|E_-|$, the efficiencies in the three regimes determined by the magnitude of β_{os} are given by

$$\eta = \begin{cases} \dfrac{1}{2}\dfrac{\gamma_o^5(1+\beta_o)^4}{(\gamma_o-1)}\left(\dfrac{\omega_o}{\omega_p/\gamma_o^{1/2}}\right)^2 \beta_{os}^2, & \text{for } \beta_{os} < \beta_1 \\ \dfrac{1}{8}\dfrac{(1+\beta_o)}{(\gamma_o-1)}\left(\dfrac{\omega_p/\gamma_o^{1/2}}{\omega_o}\right), & \text{for } \beta_1 < \beta_{os} < \beta_{\text{crit}} \\ \dfrac{1}{16}\dfrac{\gamma_o}{(\gamma_o-1)}\left[\dfrac{(1+\beta_o)^4}{2}\left(\dfrac{\omega_p^2/\gamma_o}{\omega_o^2}\right)\beta_{os}^2\right]^{1/3}, & \text{for } \beta_{os} > \beta_{\text{crit}}. \end{cases} \tag{53a-c}$$

Table I contains a summary of the results obtained for the Raman backscattering instability off a highly relativistic electron beam. It contains the pump amplitude regimes in terms of β_{os}, temporal growth rates, saturation mechanisms, efficiencies at saturation and energy spread requirements.

Table I. Summary of Collective Wave-Wave Scattering Results in the Laboratory Frame for a Highly Relativistic Electron Beam. The parameters are defined as $\beta_o \approx 1$, $\beta_{os} = |e| E_o/(\gamma_o m_o \omega_o c)$, $\beta_1 = (32)^{-1/2} \gamma_o^{-5/2} \xi^{3/2}$, $\beta_{crit} = \gamma_o^{-3/2} \xi^{1/2}$, $\xi = (\omega_p/\gamma_o^{1/2})/\omega_o$ and $\Delta\epsilon_{th}/\epsilon_o = 2\gamma_o^3 (v_{th}/c)/(\gamma_o - 1)$.

Regime	Growth Rate	Saturation Mechanism	Saturation Efficiency	Energy Spread Requirements
$0 < \beta_{os} < \beta_1$	$\Gamma = \frac{\beta_{os}}{\sqrt{2}} \gamma_o^{1/2} \omega_o \xi^{1/2}$	pump depletion	$\eta = \frac{8\gamma_o^5}{\gamma_o - 1} (\beta_{os}/\xi)^2$	$\frac{\Delta\epsilon_{th}}{\epsilon_o} << \frac{\xi}{2(\gamma_o - 1)}$
$\beta_1 < \beta_{os} < \beta_{crit}$	$\Gamma = \frac{\beta_{os}}{\sqrt{2}} \gamma_o^{1/2} \omega_o \xi^{1/2}$	trapping	$\eta = \frac{\xi}{4(\gamma_o - 1)}$	$\frac{\Delta\epsilon_{th}}{\epsilon_o} << \frac{\xi}{2(\gamma_o - 1)}$
$\beta_{crit} < \beta_{os}$	$\Gamma = \frac{\sqrt{3}}{2} \omega_o (\beta_{os}\xi)^{2/3}$	trapping	$\eta = \frac{1}{8} \frac{\gamma_o}{\gamma_o - 1} (\beta_{os}\xi)^{2/3}$	$\frac{\Delta\epsilon_{th}}{\epsilon_o} << \frac{\gamma_o (\beta_{os}\xi)^{2/3}}{4(\gamma_o - 1)}$

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