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13. ABSTRACT *(Maximum 200 words)*

This final report describes the results of a project to study the use of Bayesian networks to implement Levi's epistemic utility decision theory. The results of this project can be roughly categorized as falling into one of three areas: implementation of Levi's theory using Bayesian networks, development of Bayesian network updating algorithms for continuous valued network nodes, and application of continuous Bayesian networks to stochastic filtering problems. We found that although Bayesian networks do not preserve the convexity of sets of distributions, Levi's decision theory can still be implemented by computing extremal points in these sets. Also, we developed a method of implementing Bayesian networks containing continuous valued nodes using Gaussian sum approximations; this method is applicable in any context in which a Bayesian network may be applied and, in particular, is not restricted to networks used to implement Levi's theory. Finally, we investigated the the application of Bayesian networks to stochastic filtering problems and demonstrated this application through a simple angle-only target tracking problem. This report provides an overview of these results, which are fully documented in the PhD dissertation and papers referenced in Section 3.

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CONVEX BAYES DECISION NETWORKS

FINAL PROGRESS REPORT

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US ARMY RESEARCH OFFICE

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1 STATEMENT OF THE PROBLEM STUDIED

Isaac Levi, a philosopher at Columbia University, has developed an epistemological model of the process by which a rational decision agent may acquire new information. Levi's model addresses the goals and values of the rational agent, the agent's willingness to risk error, and the representation of incomplete and contradictory evidence. His model is based on the representation of knowledge and value systems using convex sets of probability and informational distributions and on the use of an epistemic utility function that balances the risk of error against the desire for new information; his model includes classical Bayesian decision theory as a special case.

A potential drawback to practical application of Levis theory is the computational complexity associated with computing convex sets of probability distributions. Bayesian networks, developed by Judea Pearl, address similar complexity problems in Bayesian decision theory and Dempster/Shafer evidential reasoning.

The problem studied in this work was the use of Bayesian networks to implement Levi's theory. Initially, the emphasis was both to develop the theoretical foundation necessary to apply Bayesian network structures to Levi's decision theory and to develop practical network updating algorithms. As the work progressed and issues associated with application of Levi's theory were settled, the emphasis shifted to methods of implementing Bayesian networks that contained both continuous-valued and discrete-valued nodes. The issue of continuous and mixed continuous-discrete networks is generally pertinent to decision problems, using either Levi's theory or classical Bayesian decision theory; the method of implementing continuous and continuous-discrete networks developed in the course of this project is applicable to both decision approaches.

2 SUMMARY OF RESULTS

The results of this project can be roughly categorized as falling into one of three areas:

- Implementation of Levi's theory using Bayesian networks,
- Development of Bayesian network updating algorithms for continuous valued network nodes, and
- Application of continuous Bayesian networks to stochastic filtering problems.

In the following sections, we describe each of these results in more detail.

2.1 BAYESIAN NETWORKS AND LEVI'S THEORY

A key point in Levi's development of his theory is that sets of distributions used in the decision process must be convex. Thus, we investigated whether the computations performed in Bayesian networks preserve the convexity of sets of distributions. We found that, in general, these computations do not preserve convexity; however, there are significant special cases in which convexity is preserved.

In light of these results, we addressed the question of whether Levi's theory could be implemented using Bayesian networks. To this end, we showed that application of the decision rule that follows from Levi's model (called the rule of expected utility) to non-convex sets of distributions gives the same decision as is obtained using the convex closure of these sets. Thus, the preservation of convexity by a network structure is not necessary when applying Levi's rule of expected utility. An important consequence of this result is that, to implement Levi's decision rule, a Bayesian network need compute only extreme distributions and not all distributions in a set.

2.2 CONTINUOUS AND CONTINUOUS/DISCRETE NETWORKS

Bayesian networks that contain continuous valued nodes (i.e. continuous valued random variables) are applicable to a much broader range of problems than networks containing only discrete nodes. Thus, we investigated methods by which Bayesian networks can be extended to continuous nodes. In particular, we investigated approximations of density functions associated with continuous random variables. The approach showing the most promise out of the several considered is to approximate density functions by weighted sums of Gaussian densities, and this is the approach that we adopted for further study.

Using Gaussian sum approximations, we derived network updating algorithms for nodes whose values depend on their parent nodes in both linear and non-linear relationships. Implementation of these update equations requires that the network compute weighting coefficients, means, and variances of Gaussian distributions; these computations are simply performed using the network structure. This approximation approach also allows the formation of hybrid networks with both continuous and discrete nodes without imposing restrictions on the relationships allowable between continuous and discrete variables, as is the case with previous work on hybrid networks. The use of Gaussian sum approximations for continuous nodes is applicable in any context in which a Bayesian network may be applied and, in particular, is not restricted to networks used to implement Levi's theory.

We also addressed the difficult problem of computing good Gaussian sum approximations to densities. We investigated several different possible techniques, including gradient descent, annealing algorithms, and expectation-maximization algorithms. We obtained the best approximations using a gradient descent method which incrementally increases the number of Gaussians to refine the approximation until a desired accuracy is obtained. This approach was also used to re-approximate densities after network computations to reduce the number of Gaussians in approximations.

2.3 STOCHASTIC FILTERING

We investigated the the application of Bayesian networks to stochastic filtering problems; these are problems for which the Kalman filter might be considered as a potential solution and include target recognition and tracking, sensor fusion, and terrain aided navigation. The application of Bayesian networks to stochastic filtering facilitates the fusion of classical stochastic filtering techniques with uncertain reasoning techniques, allowing symbolic as well as quantitative information to be used in the filtering process. A stochastic filtering problem can often be formulated as estimating the state of a Hidden Markov Model (HMM). Since a HMM is a particular form of a Bayesian network, our previously developed continuous Bayesian networks can be applied to this problem; however, the number of nodes in the network increases linearly with the number of observations processed (i.e. the network size increases as a function of time). In our work,we addressed this problem through a filter control system that stores densities for use in subsequent computations; this allow the Bayesian network size to remain fixed.

We applied this filtering approach to an angle only target tracking problem. In this problem, a moving target is tracked using a sequence of bearings-only measurements. This problem is a particularly difficult filtering problem because the observations do not provide enough information to completely locate the target and because the observations are a non-linear function of the target position; thus, classical approaches such as the extended Kalman filter cannot be successfully applied. We developed a Bayesian network and corresponding filter control system for this problem, and, using simulation, compared its performance (in terms of position estimate accuracy and filter convergence) to the extended Kalman filter. Our tracker was significantly more robust than the extended Kalman filter, and our tracker provided much more accurate estimates of the target position.

3 PUBLICATIONS AND TECHNICAL REPORTS

3.1 PHD DISSERTATION

E. Driver, *Implementation of Levis Epistemic Utility Theory Using Bayesian Networks*, PhD Thesis, Arizona State University, April 1996.

3.2 JOURNAL ARTICLES

E. Driver and D. R. Morrell, "A New Method for Implementing Hybrid Bayesian Networks Containing Both Continuous and Discrete Nodes," submitted to *IEEE Transactions on Pattern Analysis and Machine Intelligence,* April 1998.

E. Driver and D. R. Morrell, "Implementation of Bayesian Networks for Continuous Random Variables Using Weighted Sums of Gaussians," submitted to *IEEE Transactions on Pattern Analysis and Machine Intelligence,* April 1998.

D. R. Morrell and E. Driver, "Bayesian Network Implementation of Levi's Epistemic Utility Decision Theory," *International Journal of Approximate Reasoning,* Vol. 13, pp 127–149, 1995.

3.3 CONFERENCE PAPERS

E. Driver and D. R. Morrell, "Angle Only Target Tracking Using a Continuous-Valued Bayesian Network," *Proceedings of 30th Asilomar Conference on Signals, Systems, and Computers,* Nov. 1996, pp. 839–843.

E. Driver and D. R. Morrell, "Implementation of Continuous Bayesian Networks Using Sums of Weighted Gaussians," *Uncertainty in Artificial Intelligence 95,* Aug. 1995.

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