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Advances in Numerical Methods

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This report summarizes work conducted over a two year period to improve numerical methods. The report has three main sections. Section 2 covers work on synthetics for layered earth models. Section 3 reports on three-dimensional (3D) elastic finite difference calculations and the relationship between one-dimensional (1D) (layered earth) and 3D laterally heterogeneous wavefields. Section 4 reports on development of a finite difference program to model the response of permeable hoses to atmospheric pressure fluctuations from wind turbulence.
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1.0 Introduction

This report summarizes work conducted over a two year period to improve numerical methods. The report has three main sections. Section 2 covers work on synthetics for layered earth models. Section 3 reports on three-dimensional (3D) elastic finite difference calculations and the relationship between one-dimensional (1D) (layered earth) and 3D laterally heterogeneous wavefields. Section 4 reports on development of a finite difference program to model the response of permeable hoses to atmospheric pressure fluctuations from wind turbulence.

Section 2 reports on work conducted to port a wavenumber integration synthetics code to the Parallel Virtual Machine software and tests of this software. Numerical experiments were performed with random layering added to suites of layered crustal earth models. It was verified that random layering does make synthetics look more like observed seismograms without significantly altering the overall attenuation and spectra of Pg and Lg waveforms. The wavenumber integration code was modified to permit frequency dependent Q(f) models. Numerous studies have found that Lg Q(f) is proportional to frequency to some power ranging between 0 and 1. Such studies commonly parameterize Q(f) = Q₀ * f^α. Numerical experiments were performed with a suite of crustal models to compare synthetics with three different Q models.

In Section 3, we report on analysis of the 3D scattered wavefield computed using 3D elastic finite differences with modal summation and wavenumber spectra on phase screens. We compute scattered waves in randomly heterogeneous 3D velocity models. The total elastic wavefield (3 components of particle velocity) is saved on vertical planes at selected distances from the source. The modal spectra and wavenumber spectra is estimated on those planes and compared to computations for a layered structure in order to gain insight into the scattering process.

In Section 4, we report on development of a finite difference program, Maxhose, to compute the response of permeable hoses to atmospheric pressure fluctuations (noise) and signals. Permeable hose arrays are planned as noise reduction measures for the deployment of infrasound stations in the CTBT International Monitoring System. These noise reduction systems are critical to the expected performance of the entire infrasound system. Maxhose was developed to provide a useful tool for understanding and predicting the performance of hoses given the frequency-wavenumber characteristics of turbulent wind noise.
2.0 Layered Earth Model Wavenumber Integration Synthetics

2.1 Wavenumber Integration Synthetics Using A Parallel Virtual Machine

In the first year of this project a wavenumber integration program, *Prose* (Apsel and Luco, 1983), was ported from a single processor application to a parallel processing environment using the Parallel Virtual Machine (PVM) software environment. This software allows the user to harness a network of UNIX workstations to perform calculations in parallel. A PVM based parallel wavenumber integration program was developed that computes Green’s functions in CSS 3.0 format readable with SAC. Near linear speed-up over a single processor has been realized with a heterogeneous network of SUN OS4/Solaris, HP, SGI, and DEC workstations tested over WAN and LAN. The software requires PVM 3.3.11, a FORTRAN F77 compiler, and a C compiler. The Gnu-make program is recommended on SGI, HP, and DEC workstations. Interested users should contact Keith L. McLaughlin (scatter@maxwell.com). Current information on the availability of the software can be found on the Internet at http://www.maxwell.com/products/geop. The interested reader is referred to McLaughlin and Shkoller (1996) for detailed descriptions of the load balancing algorithms employed.

2.1.1 What is PVM?

All parallel computer algorithms are composed of modules which are executed on multiple processors, messages that must be sent between these modules, and strategies for coordinating the modules and processors to work in parallel. The modules are typically distributed over a set of processors that are connected in some sort of communications network. Much of the programming work to develop a parallel algorithm is focused on passing messages between these modules and synchronizing their work. Commercially available massively parallel processors (MPP) provide custom compilers and libraries to facilitate this kind of programming. However, most research institutions already have the makings of a parallel machine; they typically have several workstations connected in a local area network. The aggregate computing power of their workstations often exceeds the CPU power that might be available at a supercomputer center. Furthermore, these machines are often idle much of the day or night.

Parallel Virtual Machine (PVM) is a public domain software system for turning a network of computers into a virtual parallel computer (Geist et al. 1994). The software supports heterogeneous networks including nearly all UNIX workstations and many parallel and single processor supercomputers. The software is based on widely used TCP/IP message passing protocols and therefore functions over a wide variety of local area networks (LAN) and wide area networks (WAN). The user interface libraries are both *FORTRAN* (F77 and F90) and C callable from a user’s program. PVM frees the user from the arcane details of TCP/IP message passing between programs (processes) by providing a high level C or *FORTRAN* user interface and providing buffering and routing daemons for to the TCP/IP packets on the user’s computer. This is done by running daemon process on each processor that serve as a relay posts for all messages passed between the user’s programs running on each processor. PVM may be obtained by anonymous ftp from Oak Ridge National Lab and University of Tennessee at
http://www.ornl.gov/pvm/pvm_home.html. Installation of PVM 3.3.11 requires an ANSI C compiler and can be accomplished in less than an hour.

2.1.2 Wavenumber Integration Synthesis - "The Perfect Parallel Algorithm"

Wavenumber integration calculations (Apsel and Luco, 1983) are computationally bound by the time it takes to compute the complex response between a source and a receiver for a given frequency. Wavenumber integration is also the "perfect parallel algorithm" because computation of one frequency 1) is independent of all other frequencies, 2) requires only a few input values, 3) results in only a few output values, and 4) there are many frequencies to compute. These programs are well suited to a master-slave architecture (Figure 1). The master program performs all file input/output and organizes the work of the many identical slave programs running on multiple processors in the network. We treat each slave program like a function call for each complex Green's function response at a fixed frequency and fixed source and fixed receiver. Figure 2 illustrates the concept behind the use of the master-slave modules on a virtual parallel machine using PVM. The master program sends messages to the slave programs telling them which frequencies to compute and then waits for the results to return from the slaves in the form of messages. When the slave programs are not computing the response at a specific frequency, they are waiting for instructions from the master program. The slave programs return the numerical results as well as timing information that is useful in load balancing the calculation.
Parallel Virtual Machine

The master program sends and receives messages through the PVM daemon (PVMD) which routes the messages to other daemons on the way to and from slave modules. There may be multiple slaves on a single host and there is often both a slave and master program running on the same host.

Key to success is balancing the computational load between machines (hosts or processors) in the network. A heterogeneous network of workstations may contain a varied mix of slow and fast machines. Furthermore, as a computation proceeds, the load on each machine will change with time as other users start-up and terminate other processes. Also, some machines may be on a local network and communications may be almost instantaneous while others may be further away on a wide area network and messages may require a greater time for delivery. Therefore, it is necessary to keep a concise database within the master program of the relative performance of each processor. We use a very simple algorithm for load balancing; a list of processors is ranked by speed and a list of tasks (frequencies) is maintained. Tasks are checked off the list as they are completed and the next task is always sent to the fastest available host. Near the end of the computation, if we have sent each task out and we have available hosts, we do not wait for slow hosts to complete their tasks but rather re-send those tasks out to the fastest available host. The goal is to keep all slave modules working. The resulting algorithm is therefore robust with respect to changes that do happen in the network(s) and on the various parts of the parallel virtual machine. The reader is referred to McLaughlin and Shkoller (1996) for details.
The master and slave programs, prosem and prose, are based on the wavenumber integration algorithm of Apsel and Luco (1983). File formats and program options are discussed in the UNIX style manual page included with the software distribution. The master program, prosem, starts up the parallel virtual machine, spawns the slave programs, prose, on each host and saves the frequency domain output in a file. A second program, gseis, is then run that computes the time domain Green's functions and stores them in CSS 3.0 format with a wfdisc file (see Anderson et al. 1990a and 1990b for definitions of CSS 3.0 format). This seismogram format can be read and manipulated using a number of programs including the popular Seismic Analysis Code (SAC, 1995).

2.2 Random Layering

It has been suggested that introduction of random velocity variations into a layered crustal model produce more realistic looking regional phases (Harvey, 1992). Real regional seismograms tend to exhibit extended Pg and Lg wavetrains while layered Earth synthetics often have very simple regional waveforms consisting of several strong isolated spikes. One way to reduce these isolated spikes and generate more extended waveforms is to introduce interfaces that produce many small internal reflections between the up-going and down-going waves trapped in the crust. We have experimented with introducing 1 km thick layers and random velocities distributed about the mean background model. The modified Mooney et al. (1997) layered Earth models, and details of the synthetic computation procedures are described in Bennett et al. (1997), and the frequency and depth dependent attenuation model, \( Q(f,z) = Q_0(\beta, z)^n \), is described in Section 2.3 of this report. Figures 2 and 3 compare Green's functions model D2 with and without random layers of 5% variation. The Pn and Sn waveforms in the two sets of Green's functions are larger and more impulsive with the random layering than without. Also, some of the isolated spikes in the Pg and Lg wavetrains are reduced.

Figure 4 shows more such random perturbations to the DA, GA, H2, and N4 models of Mooney et al. (1997). Variations of 0%, 2.5%, 5%, 7.5%, and 10.0% RMS random velocity variation have been introduced into both P and S wave velocities throughout each model. Green's functions for these models are compared in Figure 5 through Figure 8. The four background models were chosen for their diversity; the models represent a thick set of platform deposits (DA), a high-velocity crust with very thin sediments (GA), a moderate velocity crust with no sediments (H2), and a thin crust with sediments (N4). Smoothed Lg/Pg spectral ratios computed from these synthetic seismograms are shown in Figure 11.

It is ironic that while our principal goal was to generate more realistic Pg and Lg waveforms, introduction of random layering has made the Pn and Sn waveforms larger, more impulsive, and more realistic. These waves travel in the upper mantle waveguide and are sensitive to the velocity gradients just below the Moho. The background mantle PEM model has no gradient except for the "earth flattening" approximation. We presume that in the absence of the many interfaces, the waves consist of only a few rays or modes that propagate in the upper mantle waveguide. Addition of the random interfaces appears to proliferate the number of rays/modes that contribute to the Pn and Sn waveforms.
Figure 2. Comparison of synthetic transverse component Green's function, $G_{xy}$, at 400 km, source depth $h=15$ km, with (top) and without (below) 5% random velocity variations in the layered model D2. Note that the more impulsive Sn arrival from the random layered structure and that the reduced isolated "spikes" in the Lg signal.

Figure 3. Comparison of vertical component Green's function, $G_{zv}$, at 400 km, source depth $h=1$ km, with (top) and without (below) 5% random velocity variations in the layered model D2. Note the more impulsive Pn waveform and the reduced "spikes" in the Pg from the randomized structure.
Figure 4. P-wave velocities versus depth for layered models DA, GA, H2, and N4 with 0%, 5%, 7.5%, and 10% velocity variations.
Figure 5. Comparison of Green's functions for model DA with 0%, 2.5%, 5.0%, 7.5%, and 10% velocity variation (top to bottom).

Figure 6. Comparison of Green's functions for model GA with 0%, 2.5%, 5.0%, 7.5%, and 10% velocity variation (top to bottom).
Figure 7. Comparison of Green's functions for model H2 with 0%, 2.5%, 5.0%, 7.5%, and 10% velocity variation (top to bottom).

Figure 8. Comparison of Green's functions for model N4 with 0%, 2.5%, 5.0%, 7.5%, and 10% velocity variation (top to bottom).
Figure 9. Vertical seismogram at 600 km from an earthquake source (h=12.5km) for layered model DA with 0%, 2.5%, 5.0%, 7.5%, and 10% velocity variation.

Figure 10. Vertical component $L_g/P_g$ ratio at 600 km from an earthquake source (h=12.5km) for layered model DA with 0%, 2.5%, 5.0%, 7.5%, and 10% velocity variation.
Figure 11. Vertical component Lg/Pg spectral ratios for models DA, GA, H2, and N4 for an explosion source (h=1km) and 0, 2.5, 5, 7.5, and 10% RMS random velocity layers. Note that the general character of the Lg/Pg ratio while very different for each background model, is not altered much by the introduction of random velocity layers.

2.3 Frequency Dependent Attenuation, \( Q(f) = Q_0 f^\eta \)

We have examined three simple crustal attenuation models, Q1, Q2, and Q3 in detail.

Q1) \( Q_\mu(f,z) = Q_{0\mu} f^\eta, \eta = 0, Q_{0\mu} = \beta(z)/10 \) for \( z > 0 \) m,

Q2) \( Q_\mu(f,z) = Q_{0\mu} f^\eta, \eta = 0, Q_{0\mu} = \beta(z)/5 \) for \( z > 3 \) km, low Q surface layers

Q3) \( Q_\mu(f,z) = Q_{0\mu} f^\eta, \eta = 0.5, Q_{0\mu} = \beta(z)/5 \) for \( z > 3 \) km, low Q surface layers.

The choice of model Q3 was based on three requirements.

- Low Q surface layers are required to attenuate unwanted short-period low-group velocity higher modes and \( R_g \) that propagate in the low-velocity near surface layers. We chose \( Q_{0\mu} = 25 \) for \( 0 < z < 1 \) km, \( Q_{0\mu} = 75 \) for \( 1 \) km \( < z < 2 \) km and \( Q_{0\mu} = 150 \) for \( 2 \) km \( < z < 3 \) km.

- Average shear and compressional Q values in the crust must generally produce Lg/Pg ratios greater than unity near 1 Hz for earthquake mechanisms. We found
that $Q_{0z} = \beta(z)/5$ for $z > 3$ km would yield reasonable $Lg/Pg$ ratios near 1 Hz, while the model $Q_{0z} = \beta(z)/10$ attenuated $Lg$ too much.

- Short-period earthquake $Lg/Pg$ spectral ratios as a function of frequency should generally remain above or near unity from 1 to 5 Hz. We chose $Q$ proportional to $f^{0.5}$ in order to keep $Lg/Pg$ ratios from declining too steeply as a function of increasing frequency.

Figures 12 and 13 show compilations of $Lg/Pg$ spectral ratios from Bennett et al. (1997). Earthquake $Lg/Pg$ ratios generally slowly decrease with increasing frequency.

Synthetics from a variety of crustal models suggested that in order to produce earthquake-like $Lg/Pg$ ratios greater than unity near 1 Hz the higher $Q_0$'s of models Q2 and Q3 are preferred to those of model Q1. Also, to keep $Lg/Pg$ ratios relatively flat as a function of frequency, an increase in $Q$ as a function of frequency ($\eta > 0$) is needed. The $Lg/Pg$ ratios above 1 Hz for models Q1 and Q2 were too small and do not agree with observations. $Lg$ $Q$ increasing with increasing frequency is commonly observed (Nutti, 1981; Goncz et al., 1986, Gupta and McLaughlin 1987; Campillo et al., 1985; Mitchell 1981) with $\eta$ between 0 and 1. It should be noted that numerous researchers have found a negative correlation between $Q_0$ and $\eta$; the higher $Q$ is at 1 Hz, the slower it increases with increasing frequency. Therefore, it may be possible to reproduce many of the observed results by assuming higher $Q_0$'s with somewhat smaller values of $\eta < 0.5$. Also, in tectonic regions with lower 1 Hz $Q$ values, the value of $\eta$ may be higher.

Figures 15-17 show explosion and earthquake Green's functions computed for the three $Q$ models. The velocity model D2 is well suited to show off the differences between Q1, Q2, and Q3. The earthquake $Lg$ amplitudes are too small in models Q1 and Q2 for both the broadband and the higher frequency seismogram (0.5 Hz highpass Figure 14 and 3.0 Hz highpass Figure 16). A shallow explosion excites very slow unrealistic waves in the near surface layers in model Q1 (Figure 15).
Figure 12. Compilation of average explosion $L_g/P_g$ spectral ratios from Bennett et al. (1997) for explosions in Asia and Western US. Note that the average $L_g/P_g$ is generally greater than unity for frequencies at and below 1 Hz.

Figure 13. Compilation of average $L_g/P_g$ spectral ratios from Bennett et al. (1997) for earthquakes in Asia, Western US, and Eastern US. Note that $L_g/P_g$ is generally greater than unity for frequencies below 5 Hz with the exception of the ARU data. The average earthquake curves are generally above the corresponding explosion curves at frequencies above 1 Hz.
Figure 14. Comparison of vertical component synthetics (high pass at 0.5 Hz) at 200 km for three Q(f,z) models and velocity model D2, Q1 (top), Q2 (middle), Q3 (bottom). A dip-slip double-couple source (Gzds) at 15 km depth is shown. The Lg amplitude is significantly increased relative to the Pg amplitude for $\eta=0.5$, $Q_\varphi=\beta/5$, model Q3.

Figure 15. Comparison of vertical component synthetics (high pass at 0.5 Hz) at 200 km for three Q(f,z) models and velocity model D2, Q1 (top), Q2 (middle), Q3 (bottom). An explosive source (Gzi) at 1 km depth is shown. The high Q surface layer model does not sufficiently attenuate the late arriving fundamental surface waves excited by the shallow explosive source. Also, it is clear that the Q1, high Q surface layer model contains too much shallow propagating energy in the Lg window that is absent in the low Q surface layer models, Q2 and Q3.
Figure 16. Comparison of vertical component synthetics (high pass at 3.0 Hz) at 200 km for three Q(f,z) models and velocity model D2, Q1 (top), Q2 (middle), Q3 (bottom). A dip-slip double-couple source (Gzds) at 15 km depth is shown. The Lg amplitude is significantly increased relative to the Pg amplitude for $\eta=0.5$, $Q_0=\beta/5$, model Q3.

Figure 17. Comparison of vertical component synthetics (high pass at 3.0 Hz) at 200 km for three Q(f,z) models and velocity model D2, Q1 (top), Q2 (middle), Q3 (bottom). An explosive source (Gzi) at 1 km depth is shown below. The Lg is almost non existent in model Q2.
2.4 Acknowledgments

We wish to thank Al Geist and Bob Manchek of ORNL and UT for an excellent PVM tutorial at Supercomputing '95. Steve Day of SDSU provided many useful suggestions in the implementation of the wavenumber integration code.

2.5 References


3.0 Finite Difference 3D Regional Scattering Calculations

3.1 Introduction

Numerical methods for 3D elastic wave propagation have made significant advances in the last decade due in part to better software, but largely due to exponential growth of computer capabilities. For a fixed price, computer memory and speed roughly double every 18 months (popularly known as "Moore's Law"). Provided such technological advances continue it will be another decade or more before 3D computations are as economical as 1D computations today. 3D computations require 8 times more memory and 16 times more CPU time than comparable 1D computations. For example, computation by wavenumber integration of a record section of 0.5 Hz regional Green's functions from 0 to 200 km at 0.5 km intervals requires about 1/20th the time as a section of 3D waveforms for a single source (hours versus days). While the suite of 3D seismograms is far richer than the 1D record section, the bandwidth and range of the 3D computation is far more limited than the 1D computation. With this in mind, it is beneficial to leverage 1D numerical methods as much as possible in understanding 3D results and devising hybrid methods for seismogram calculation.

Seismologists have developed extensive insight into the nature of wave propagation in layered earth models using 1D methods. Layered earth model synthetics correctly predict many of the gross features of regional seismograms. Distinct regional phases such as Pn, Pg, Sn, Lg, and Rg propagate as loosely coherent wave packets with well defined group velocities for large distances in crustal and upper mantle waveguides. 3D methods aim to predict how these phases are affected by scattering. For some cases we can view the 3D scattering as a perturbation upon the waveguides. From the point of view of modal summation methods, we can examine how the "modes" defined by the average 1D layered structure exchange energy with increasing range. Alternatively, we can view the outgoing wavefield from the point of view of wavenumber integration or phase screen methods. We can view the scattering as converting energy from one wavenumber into energy at a different wavenumber.

In this section, we report on work we have done to analyze the scattered wavefield computed using 3D elastic finite differences with modal summation and wavenumber spectra on phase screens. We compute scattered waves in 3D velocity models constructed by adding random perturbations to a mean crustal model. The total elastic wavefield (3 components of particle velocity) is saved on vertical planes at selected distances from the source. We then estimate the modal spectra and the wavenumber spectra on those planes. We compare these spectra with computations for a layered structure in order to gain insight into the scattering process.

Tres3D with recursive grid refinement was used to compute complete 3D elastic wave propagation in a 3D laterally heterogeneous model. Tres3D is a 2nd order explicit finite difference code for 3D rectangular meshes. It requires a minimum of 10 cells/wavelength and the grids were designed to provide a bandwidth from 0 to 0.6 Hz. Sources are inserted as moment tensors. Recursive grid refinement (RGR) described in McLaughlin and Day (1995) uses nested grids to decrease memory and CPU time. Fine zoning is used
in the crust (250 m), coarser zoning is used in the upper most mantle (500 m) and the
cor subtle zoning (1000 m) is used in the deepest mantle. Moderate to high attenuation is
used in the mantle to suppress reflections from the bottom of the grid and the coarser
grids. The sources were placed at $X = 0$, $Y = 0$ at depths of $Z = -0.5$ km and $Z = -12.5$
k m. A reflection symmetry axis was used as a boundary condition on the $Y = 0$ and $X = 0$
planes to reduce the size of the problem by a factor of 4. The geometry is diagrammed in
Figure 3.1-1.

The computation was performed over a 200 by 100 by 100 km volume and run to
durations of about 100 seconds. The recursive grid refinement was used with 21 grids and
a total of 10 million cells. Each computation required about 3 CPU days on a DEC 2100
5/250 workstation to complete the 7.3*10^{11} cell-cycles.

A list of computations is given in Table 1. Two source types were modeled: an
explosion source with $Mxx = Myy = Mzz$ centered at a depth of 0.5 km and a "double-
couple" source with $Mxx = -Mzz$, $Myy = 0$ at a depth of 12.5 km. Lateral heterogeneity
was limited to the crust and upper most mantle. The mantle was a homogeneous half-
space and a simple linear gradient was chosen for the background crustal structure. The
background velocity model is shown in Figure 18. In order to introduce lateral
heterogeneity we used the following procedure:

1) Generate a 3D random array, $r(x,y,z)$, uniformly distributed between -1 and 1,
sampled at $x = 0$, ..., $x_{max}$, $y = 0$, ..., $y_{max}$, $z = z_{min}$, ..., 0 to fill the volume $x_{max} = 128$ km, $y_{max} = 32$ km, $z_{min} = -32$ km, with sampling intervals $dx = 1.0$ km, $dy = 1.0$ km, and $dz = 0.5$ km.

2) Smooth the random numbers in the X and Y directions with a 3-point smoothing
operator $(1,1,1)$. Re-scale the array to an RMS value of 1.

3) For a given $(x,y,z)$ location in the crust $(z > z_{min})$ set the S-velocity to $\beta^{-1}(x,y,z) = \beta^{-1}(0) \cdot (1 + RMS \cdot r(x,y,z))$. For $x > x_{max}$ or $y > y_{max}$ or $z > z_{min}$, the velocity is
set to the background model. Run #1 was formulated with fluctuations specified in
velocity instead of slowness.

This procedure introduces an-isotropic heterogeneity with characteristic lengths of about 1
km in the Z and X directions and 2 km in the Y direction. Figures 20 and 21 show two
two ways to visualize this lateral heterogeneity.

Snap shots of vertical particle velocity from Run#1 (5% RMS velocity variation,
exlosion source) along the $Y = 0$ plane are shown in Figures 22 through 27. Complexity
of the wave propagation is immediately evident. Development of the refracted P and Pn
can be seen at $T = 15$ seconds as well as development of the moho reflection, PmP. Lg can
be best seen at later times $T = 20$, 25, and 30 seconds as an interference of up-going and
down-going waves in the crust. The short-period fundamental Rayleigh wave, $Rg$, can be
seen running along the surface at about 2.5 km/s. The disorganized wavefield behind the
Rg wavefront is simply labeled "coda". Waveforms from Run#1 are shown in Figure 28 to
illustrate that Rg is the most significant arrival at these distances on the surface, and it is
systematically delayed and attenuated by the random fluctuations.
Table 1. Computational Runs.

<table>
<thead>
<tr>
<th>Run #</th>
<th>Source Type</th>
<th>RMS Variation</th>
<th>Source Depth (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Run 1</td>
<td>Mxx = Myy = Mzz</td>
<td>5% velocity</td>
<td>0.25</td>
</tr>
<tr>
<td>Run 2</td>
<td>Mxx = Myy = Mzz</td>
<td>5% slowness</td>
<td>0.25</td>
</tr>
<tr>
<td>Run 3</td>
<td>Mxx = Myy = Mzz</td>
<td>5% slowness</td>
<td>0.25</td>
</tr>
<tr>
<td>Run 4</td>
<td>Mxx = Myy = Mzz</td>
<td>0</td>
<td>0.25</td>
</tr>
<tr>
<td>Run 5</td>
<td>Mxx = Myy = Mzz</td>
<td>5% slowness</td>
<td>0.25</td>
</tr>
<tr>
<td>Run 6</td>
<td>Mxx = Myy = Mzz</td>
<td>7.5% slowness</td>
<td>0.25</td>
</tr>
<tr>
<td>Run 7</td>
<td>Mxx = -Mzz, Myy = 0</td>
<td>7.5% slowness</td>
<td>12.5</td>
</tr>
<tr>
<td>Run 8</td>
<td>Mxx = -Mzz, Myy = 0</td>
<td>0</td>
<td>12.5</td>
</tr>
</tbody>
</table>

Figure 18. Diagram of 3 levels of nested grids; 1 grid on level 1, 4 grids on level 2, and 16 grids on level 3 for a total of 21 grids. Each grid contains 64 x 64 x 128 = 524,288 cells for a total of 11 million cells. The root grid (102.4 x 102.4 x 204.8 km) has 1 km cells. The 4 intermediate level grids fill the space to a depth of 51.2 km with 0.5 km cells. The 16 finest level of grids fill the space to a depth of 25.6 km with 0.25 km cells. A reflection symmetry axis was placed on the Y = 0 and X = 0 planes. The Z = 0 plane is a free surface.
Figure 19. Plot of the test structure used in all computations. P- and S-wave velocities in the crust are simple gradients with Poisson's ratio of 0.25. The Moho consists of two steps at 25 and 26 km.

Figure 20. Shear velocity along a Y = Constant slice of the model. The horizontal anisotropic variation is clearly evident.
Figure 21. Visualization of the 3D random variation. Isosurfaces with absolute variation greater ±1σ are shaded in the top image. Regions within ±1σ are transparent. The lower image shows the same with ±3σ. These images help visualize the cores of the scatterers. Note the elongation of the scatter cores in the Y direction.
Figure 22. Run #1 snap shots of vertical particle velocity on the $Y = 0$ plane, at $T = 5$ seconds.

Figure 23. Run #1 snap shots of vertical particle velocity on the $Y = 0$ plane, at $T = 10$ seconds.

Figure 24. Run #1 snap shots of vertical particle velocity on the $Y = 0$ plane, at $T = 15$ seconds.
Figure 25. Run #1 snap shots of vertical particle velocity on the $Y = 0$ plane, at $T = 20$ seconds.

Figure 26. Run #1 snap shots of vertical particle velocity on the $Y = 0$ plane, at $T = 25$ seconds.

Figure 27. Run #1 snap shots of vertical particle velocity on the $Y = 0$ plane, at $T = 30$ seconds.
Figure 28. Comparison of finite difference and wavenumber integration synthetics for Run #1 (5% RMS, explosion source) at 100 km for lowpass filters 0.2, 0.4, 0.6, and 0.8 Hz (top to bottom). Note that the Rg is unaffected in the lowest bandwidth and is increasingly attenuated and delayed with increasing frequency. Frequencies above 0.6 Hz are affected by grid dispersion.

3.2 Modal Spectra

Modal summation is a common approach to either analyze or compute wave propagation in a layered structure (Aki and Richards, 1980). The methods for seismogram synthesis have a long and fruitful history. In particular, Lg can be analyzed as groups of the P-SV and/or SH modes. Several researchers have described weak scattering as mode-mode conversion, where energy is transferred from one mode to another. We wish to take advantage of the machinery developed for modal summation and estimate a "modal spectra" or a "modal decomposition". We write the vertical seismogram at frequency, f, depth, z, and distance, Δ, as a mode sum, \( S_Z(f,z,Δ) = \sum_{j=1,N} C_{Zj}(f,Δ) E_{Zj}(z,f) \), where \( E_{Zj}(z,f) \) is the jth mode vertical eigenfunction at frequency, f (see Figures 29 and 30). Likewise for the radial seismogram, \( S_R(f,z,Δ) = \sum_{j=1,...,N} C_{Rj}(f,Δ) E_{Rj}(z,f) \). We can then examine the modal spectra, \( C_j(f,Δ) \), as a function of frequency and distance. In order to estimate this modal spectra, we save the vertical and radial velocities at some distance for each depth
in the finite difference grid. Since the eigenfunctions are orthogonal, we should only need to compute an inner product between \( S(f, z, \Delta) \) and \( E_{f}(z, f) \) to compute the coefficients, \( C_{j}(f, \Delta) \). However, the finite difference seismograms are only sampled at specific depths from the surface to a maximum depth and they contain wave types that are not present in the pure P-SV modes, such as Pn and Pg as well as scattered body waves. In short, the modes are not exactly orthogonal under the numerical quadrature and leakage can occur of the nonmodal waves into the modes. It helps to use both the vertical and radial motion to constrain the modal volumes. We use a stripping procedure described below.

1) start with mode \( j = 1 \) and set
   \[
   S'_{Z}(f, z, \Delta) = S_{Z}(f, z, \Delta) \\
   S'_{R}(f, z, \Delta) = S_{R}(f, z, \Delta)
   \]

2) estimate the normalized inner product over depth from \( z_{1} \) to \( z_{nx} \)
   \[
   E_{Zj0}(f) = \Sigma_{k=1}^{n_{z}} E_{Z}(z_{k}, f) \\
   E_{Rj0}(f) = \Sigma_{k=1}^{n_{z}} E_{R}(z_{k}, f) \\
   C_{j}(f, \Delta) = (1/2) \Sigma_{k=1}^{n_{z}} (S'_{Z}(f, z_{k}, \Delta) E_{Z}(z_{k}, f)/E_{Zj0}(f)+S'_{R}(f, z_{k}, \Delta) E_{R}(z_{k}, f)/E_{Rj0}(f)) dz_{k}
   \]

3) strip the \( j \)th component from the observed seismogram
   \[
   S'_{Z}(f, z, \Delta) = S'_{Z}(f, z, \Delta) - C_{j}(f, \Delta) E_{Zk}(z, f) \\
   S'_{R}(f, z, \Delta) = S'_{R}(f, z, \Delta) - C_{j}(f, \Delta) E_{Rk}(z, f)
   \]

4) if \( j < N \) then increment the mode number \( j \) to \( j + 1 \) and go to step 2.

We find this stripping procedure is relatively insensitive to whether we start with mode \( j = 1 \) and increment to mode \( j = N \), or start with mode \( j = N \) and decrement to mode \( j = 1 \). Amplitude of the residual seismogram is small in comparison to the original seismogram. Generally, about 90% of the seismogram energy is accounted for by the modal spectra. Obviously, this simple quadrature rule could be refined, but we believe that the modal spectra estimated in this way provide insight into the scattering processes. Figure 31 shows modal spectra, \( C_{j}(f, \Delta) \), for Run\#2 (5% RMS) and Run\#4 (0% RMS) at a range of 120 km. Note the modal cut-off where the number of modes increase with increasing frequency. Mode 1 corresponds to the fundamental Rayleigh wave. Modes 2 through 8 are the higher modes that exist in the test structure for frequencies up to 0.6 Hz. The modes are normalized to unit vertical amplitude at the free surface. Therefore, the spectral amplitudes can be interpreted in terms of the amplitudes of the modes of an observed surface vertical seismogram. The modes, however, may not necessarily arrive at their customary group velocities nor as isolated modes in the time domain. Clearly, the fundamental is the largest amplitude mode from 0.1 to about 0.4 Hz for both the laterally homogeneous and heterogeneous models. The fundamental is the largest mode for the laterally homogeneous model (Run\#4) at all frequencies, but has been reduced to an amplitude comparable to the higher modes at frequencies greater than about 0.4 Hz in the laterally heterogeneous model (Run\#2). We interpret this as scattering from the fundamental into higher modes by lateral heterogeneity. A simple way to illustrate this is shown in Figure 32 where we form a ratio between the modal spectra from Run\#2 and Run\#4. This modal spectral ratio demonstrates the enhancement of energy in the higher
modes at nearly all frequencies at the expense of the fundamental. Likewise, we show a ratio of modal spectra at 60 km to spectra at 120 km in Figure 33.

Figure 29. Plots of P-SV modes at 0.5 Hz (right) and 0.75 Hz (left) for the test structure shown in Figure 19. Note that the fundamental mode has very little motion below 4 km (at 0.75 Hz) and below 5 km (at 0.5 Hz). The higher modes sample the entire crust.

Figure 30. Diagram of Modal Spectra. $C(J,F)$. $C(J,F) = 0$ to the right of the modal cut-off line.
Figure 31. Modal spectra at 120 km (top) and 60 km (bottom) for models with 0% RMS (left) and 5% RMS (right) lateral heterogeneity.
Figure 32. Modal Spectral Ratio at 120 km 5% RMS model / 0% RMS model. Yellow and Red shades are modes enhanced w.r.t. the background model. Blue and purple shades are modes deficient w.r.t. the background model.

Figure 33. Ratio of 120 km to 60 km. This spectral ratio highlights modes that have lost or gained energy between 60 and 120 km.
3.3 Wavenumber Spectra

While the modal spectra are useful descriptions of the crustal wavefield, they do not provide a complete description. In order to obtain this more complete description of the wavefield, we have taken a cue from other numerical methods for computing wavefields such as wavenumber integration or phase screen methods. We save the 3 components of motion on a 32 km by 32 km vertical plane perpendicular to the nominal direction of propagation (X = constant) and compute a 3D FFT of the seismograms to transform them from the (y, z, t) domain to the wavenumber-frequency (F_y,F_z,f) domain. F_z = f s_z, and F_y = f s_y are the wavenumbers for propagation in the vertical (Z) and transverse (Y) directions respectively; s_z and s_y are the slowness components in the vertical and transverse directions respectively.

\[ S(F_y,F_z,f) = \sum e^{-iF_yt} \sum e^{-iF_zz} \sum e^{-if\sigma} \delta(y, z, t) \]

The diagram in Figure 34 can be of help in understanding these wavenumber-wavenumber spectra. Note that because the finite difference calculations are performed with a symmetry axis at Y = 0, the phase plane spectra are also symmetric about the Z axis, S(F_y, F_z) = S(-F_y, F_z). Waves traveling straight at the phase plane are plotted at the origin of the diagram. Upping waves arriving perpendicular to the plane plot along the positive F_z axis, while downgoing waves arriving perpendicular to the phase plane plot along the negative F_z axis. Off azimuth waves (side-swipe) plot away from the F_z axis. Since there is a minimum velocity in the crustal waveguide, all propagating waves must plot inside a circle given by \((F_z^2 + F_y^2)^{1/2} = f / \beta_{\text{min}}^2\). Waves with \(F_z = f s_z < f/\beta_{\text{mantle}}\) are trapped in the crust where \(\beta_{\text{mantle}}\) is the shear wave velocity of the mantle below the moho. Because the phase plane has finite dimensions, there are side lobes. Also, all outgoing waves are not planar and they have curvature as they impinge upon the phase plane; even in the absence of lateral heterogeneity, the spectra are not confined strictly to the F_z axis. Therefore, for each wavenumber spectra for a laterally heterogeneous model we show the corresponding wavenumber spectra for the strictly layered structure for comparison. And, in order to gain insight into the side lobes and impulse response to a curved wavefront at a selected distance, we plot the impulse response of some cylindrical waves incident upon the 32 km by 32 km phase plane at 120 km from a point source in Figures 35 and 36.
Figure 34. Diagram of wavenumber-wavenumber spectra at frequency, $f$. All wavenumber spectra in this paper are symmetric about the vertical wavenumber axis because of a reflection symmetry on the $Y = 0$ plane in the finite difference calculations.

Figure 35. Phase plane wavenumber impulse response for a cylindrical wave, 120 km from the source at 0.4 (left) and 0.6 Hz (right).
Figure 36. Phase plane wavenumber impulse response for an off-axis cylindrical wave, 120 km from the source at 0.6 Hz.

Wavenumber-wavenumber spectra at 120 km and 0.6 Hz from Run # 2 (5% RMS explosive source) and Run #4 (0% RMS, explosive source) are compared in Figures 37-39. In the absence of scattering, on-azimuth up-going and down-going waves with vertical slownesses near 0.3 sec/km (apparent vertical velocity about 3 km/sec) are particularly strong on the vertical and radial components of motion (Z and X). Presumably these correspond to the developing Lg wavetrain. Energy with vertical slownesses less than 0.2 sec/km is presumably related to the developing Pg wavetrain as well as steeply incident P and S waves that should escape into the mantle. For all three components of motion, we see that scattering has stretched out the spectra in the Fy direction indicative of the off-azimuth energy. Scattering has generally lowered the on-azimuth energy. Note that the transverse particle motion (Y component) is not identically zero for the laterally homogeneous case since the X = 120 km plane is only perpendicular to the direct waves at Y = 0. The scattered Y component wavenumber spectra become very complicated and fill a larger wavenumber region.
Figure 37. Wavenumber spectra of the vertical component velocity field on a $Y = 120$ km phase plane at 0.6 Hz for the 5% RMS (right) and 0% RMS (left).

Figure 38. Wavenumber spectra of the radial component velocity field on a $Y = 120$ km phase plane at 0.6 Hz for the 5% RMS (right) and 0% RMS (left).
Figure 39. Wavenumber spectra of the transverse component velocity field on a Y = 120 km phase plane at 0.6 Hz for the 5% RMS (right) and 0% RMS (left).

3.4 Conclusions

Modal and wavenumber spectra of phase planes provide tools to analyze finite difference simulations of regional scattering. Modes serve as a convenient set of basis functions and have a straightforward interpretation. Modal spectral ratios can be used to estimate scattering attenuation of each mode and the general transfer between modes. Wavenumber spectra are perhaps more difficult to visualize but they provide means to visualize off-azimuth waves and waves that are not necessarily trapped in the crustal waveguide.

Fundamental mode dispersion looks significantly different for the randomized models than predicted by the average model. The scattering both attenuates the Rg and introduces significant group delay. If scattering makes a significant contribution to fundamental Rayleigh attenuation, seismologists should be careful interpreting the apparent group velocities and the inferred shallow velocities.

Conversion from the fundamental Rg becomes increasingly important with increasing frequency. With only moderate heterogeneity (5% RMS) the P-SV modes are rapidly approaching equilibrium at distances less than 100 km for frequencies above 0.5 Hz. Likewise, both off-azimuth P-SV and SH energy is significant at 60 km.
3.5 Acknowledgements

We would like to thank Ru-Shan Wu for stimulating discussions which led to the F-K analysis of phase planes.

3.6 References


4.0 Permeable Hose Characteristics and Noise Reduction
   For Infrasound Monitoring

4.1 Summary

The proposed CTBT infrasound monitoring network consists of between 50 and 60 4-station microbarograph arrays. Many of the infrasound stations will be co-located or adjacent to seismic systems and work in concert. Each infrasound station is intended to consist of a broadband microbarograph equipped with several hundred meters of noise reduction hose. The permeable hose design replaces Daniels microphone pipes for the purposes of spatially averaging wind eddy generated pressure fluctuations. Useful detection thresholds for infrasound stations will be directly related to the effectiveness of the noise reduction hose arrays. We present an analysis of the differential equations that describe the acoustics of infrasound recording with a permeable hose as opposed to the discrete set of coupled equations that have traditionally been used to describe a Daniels pipe. It is shown that a hose may be characterized by a characteristic time constant, \( \tau_0 \), and a characteristic length, \( \lambda_0 \). The time constant is related to permeability of the hose and the characteristic length is related to both flow resistance and permeability of the hose. Signal-to-noise improvement is directly proportional to the characteristic length of the hose. The low pass filter corner frequency of the system is determined by the characteristic time. Wavelengths of the pressure field shorter than characteristic length are averaged over the length of the hose. A finite difference code, Maxhose, is described that computes response of a simple linear permeable hose. The finite difference code is used to model both operational hose designs as well as calibration configurations. Simulations of a single hose to atmospheric pressure fluctuations are presented for a white noise and a fractal noise model. A simple experimental calibration is described to measure the characteristic times and lengths of permeable hoses. Calibration results are shown for commercially available soaker hose. Typical measured characteristic times are between 10 to 20 milliseconds, while characteristic lengths are between 50 and 200 meters. Of particular note are the effects of hose degradation during a typical San Diego winter as demonstrated by a reduction in characteristic length of the hose by a factor of 2. An operational system would have experienced a comparable degradation of signal-to-noise over time. Simple calibration systems can be designed to track such hose characteristics.

4.2 Introduction

The proposed IMS infrasound monitoring system will contain approximately 50 to 60 primary infrasonic arrays scattered around the globe. These arrays, composed of three or more sensors, will often be co-located or near an IMS seismic station. The arrays will be designed to provide arrival time and azimuth of arrival of low-frequency 0.01 to 10 Hz sound waves propagating in the atmosphere. While the primary mission of the infrasound network will be to detect nuclear explosions detonated in the atmosphere, these stations may also serve in an ancillary mission to help identify large chemical blasts recorded at seismic stations. The final design of sensors and operational procedures for detection, location, and identification of infrasonic events are as yet still in flux. Uncertainties still
exist in the areas of 1) excitation as a function of source type (explosion, quarry blast, meteor, volcanic explosion) and size 2) propagation attenuation as function of distance and frequency, and 3) optimum sensor design, and data processing.

Figures 40 and 41 show projected detection capabilities in Log10 (Yield in Kt) for atmospheric explosions by the proposed IMS infrasound network. These network simulations show the proposed network should be able to provide 90% assurance of detection at 2 or more infrasound stations at the 1 Kt level for most of the world. The 90% detection capability rises above 1 Kt in the South Pacific for 3 or more detections. In order to make projections such as Figure 41, we require models for 1) the excitation of infrasound as a function of frequency, yield, and height of burst, 2) the attenuation of infrasound as a function of frequency and distance, and 3) the noise and signal recording responses of the recording systems as a function of frequency. However, our understanding of all three of these important factors are far from complete. Critical to the ability of the proposed networks to monitor near the 1 Kt threshold are the permeable hose noise reduction systems. Detection thresholds are inversely proportional to the signal-to-noise improvements expected by the hose systems.

**Figure 40.** Contours of detection threshold in Log(Yield in Kt) at 90% probability for 2 infrasound detections at stations of the proposed IMS. Event scaling, attenuation relations, and noise levels based on Blandford and Clauter (1995) have been used. Thresholds are below 1 Kt nearly everywhere.
Figure 41. Contours of detection threshold in Log (Yield in Kt) at 90% probability for 3 infrasound detections at stations of the proposed IMS. Event scaling, attenuation relations, and noise levels based on Blandford and Clauter (1995) have been used. Thresholds reach 2 Kt in the South Pacific.

4.3 Theory of Noise Reduction Hoses and Their Calibration

The concept behind the use of a noise reduction hose relies on the fact that wind generated turbulent eddies have shorter apparent wavelengths than an infrasound signal arriving from great distance (Cook, 1971; Mack and Flinn, 1971; McDonald et al. 1971). The hose "averages" the local atmospheric pressure along its length and the incoherent eddies are "averaged" out.

Figure 42. Wind generated eddies create pressure fluctuations and hence noise. The permeable hose acts to spatially "average" the pressure fluctuations.
The original Daniels pipe (Daniels 1959) was conceived as a discrete set of inlet ports along a length of pipe. Grover (1971) presents results for pipe arrays with hypodermic inlet ports, while Burridge (1971) presents a numerical solution to this discrete system of inlet ports, with a propagator method (a system of coupled equations). However, with the advent of soaker hose, the system is analogous to a transmission line or antennae problem and requires the solution of coupled partial differential equations.

An element of hose of length $dx$

![Diagram of a hose element](image)

Figure 43. A hose element: length $dx$, radius $a$, flow in the $x$ direction $qx$, flow in/out $qa$.

We consider an element of permeable hose of length $dx$ and radius $a$. From conservation of mass we write,

$$m(x,t) = \dot{\rho}(x,t) A dx = \dot{q}(x,t) = q_a(x,t) - (q_x(x + dx, t) - q_x(x, t)),$$

where $x$ is the distance along the hose, $t$ is time, $m$ is the mass of air in a length of hose $dx$, $\rho(x,t)=p(x,t)/(RT)$ is air density at temperature $T$, $A$ is the cross sectional area of the hose, $q(x,t)$ is the total mass flow in the hose, $q_a(x,t)$ is the mass flow into the hose from outside per unit length, and $q_x(x,t)$ is the lateral mass flow along the hose in the positive $x$ direction. We assume that air diffuses into and out of the hose proportional to the pressure difference, $(p_a(x,t)-p(x,t))$, where $p_a(x,t)$ is the pressure of the atmosphere outside the hose and $p(x,t)$ is the pressure inside the hose,

$$q_x = \rho A v = 2 \pi a e (p_a(x,t)) - p(x,t)dx,$$

where $e$ is a diffusion constant per unit length related to the permeability of the hose with radius, $a$. We next ignore inertial effects and assume that flow along the hose is steady state and proportional to the gradient of pressure along the hose,

$$q_x = \rho A v = (1/ \nu) \partial p(x,t)dx,$$

where $\nu$ is the hose flow resistance per unit length and $\nu$ is the flow velocity. Grover (1971), gives excellent evidence that flow in noise reduction pipes can be described by simple Poiseuille flow where the hose resistance is proportional to viscosity, $\nu=8\eta/\rho/\pi/\alpha'$, where $\eta$ is the viscosity of air. We combine these equations to write a
partial differential equation for the pressure along the hose in terms of the external atmospheric pressure,

\[ 2\pi \alpha e (p_s(x,t) - p(x,t)) - (1/v) \frac{\partial^2 p(x,t)}{\partial x^2} - \frac{\dot{p}(x,t) \Delta (RT)}{\partial x} = 0. \]

This equation describes the hose system driven by the atmospheric pressure along its length. We transform from the time-length domain, \((x,t)\), to the frequency and wavenumber domain, \((\omega,\kappa)\),

\[ p(x,t) = \int \hat{p}(\omega,\kappa) \exp(-i\omega t) \exp(-i\kappa x) \, d\omega \, d\kappa. \]

We can write the solution in the frequency-wavenumber domain,

\[ \hat{p}(\omega,\kappa) = \frac{1}{(1 + i\omega / \omega_0 + (\kappa / \kappa_0)^2)^2} \hat{p}_s(\omega,\kappa), \]

\[ \vec{p}(\omega, x) = \int \frac{1}{(1 + i\omega / \omega_0 + (\kappa / \kappa_0)^2)^2} \hat{p}_s(\omega,\kappa) \exp(-i\kappa x) \, d\kappa. \]

Now we observe that a far-field signal has the form \( \hat{p}_s(\omega,\kappa) = \delta(\kappa)\vec{p}_s(\omega) \), so the signal pressure in the hose is simply

\[ \vec{p}_{signal}(\omega) = \frac{1}{(1 + i\omega / \omega_0)} \vec{p}_s(\omega). \]

The typical soaker hose has a characteristic time constant, \( \tau_0 = 2\pi/\omega_0 = 2\pi \alpha/(2RT\varepsilon) < 0.2 \) seconds, so the pressure signal inside the hose is a faithful representation of the signal outside the hose for infrasound frequencies of interest. This characteristic time is determined by the permeability of the hose. A very permeable hose has a short time constant while an impermeable hose has a long time constant. If we write the atmospheric noise as \( \hat{p}_n(\omega, x) = \hat{p}_n(\omega, \kappa) \) then the observed noise signal is given by a Fourier integral over the wavenumber spectrum of the atmospheric noise,

\[ \vec{p}_{noise}(\omega, x) = \int \frac{1}{(1 + i\omega / \omega_0 + (\kappa / \kappa_0)^2)^2} \hat{p}_n(\omega,\kappa) \exp(-i\kappa x) \, d\kappa, \]

This integral in general cannot be evaluated in closed analytic form; however, by inspection we can note several interesting features. The incoherent atmospheric wavenumbers
are attenuated proportional to \((\frac{\kappa}{\kappa_0})^2\) where \(\kappa_0^2 = \frac{2\pi a e \nu}{\nu} = \frac{4\pi a \nu}{(RT)/(\tau_0)} = \frac{(2\pi / \lambda_0)^2}{\tau_0}\) is the characteristic wavenumber for the hose. The smaller the characteristic wavenumber, the more attenuated will be the incoherent wavenumbers of the noise field. Note that the characteristic wavenumber for the hose is inversely proportional to the characteristic time constant. Therefore, a long time constant means more noise attenuation for a fixed length of hose. Typical soaker hoses have characteristic lengths, \(\lambda_0\), of about 50-200 meters, but we have found that time constants and hence characteristic lengths and the potential noise reduction change with time as the hose ages.

Figure 44. A simple and inexpensive calibration configuration.

The diagram above (Figure 44) shows an experimental setup for measuring time constants of a soaker hose. The soaker hose is placed in the 44 Liter volume test chamber. The test chamber volume is then altered with a 12 cc step function volume decrease, producing a change in pressure of about 300 microbars. Pressure in the soaker hose is recorded using a differential pressure gauge (microbarograph). The experiment is repeated with different lengths of hose and different amplitude step functions to test for linearity. Pressure transients measured with this setup are shown in Figure 45 for a virgin soaker hose and the same brand of hose left outside for six months during a typical San Diego winter. The time constant for this aged hose has decreased by nearly a factor of two. Hence, noise reduction will have decreased by nearly the same amount.
Figure 45. Shows rise time calibration of two 2.082 m long hoses. The "aged" hose had been exposed to the elements for about 6 months during a typical San Diego winter. The rise time and hence the characteristic length of the aged hose is about 50% that of the virgin hose.

Other diagnostic tests are possible for hose systems. The diagram below (Figure 46) shows one such simple test. The results are shown for two identical lengths (about 10 m) of virgin and aged soaker hose (Figure 47). The change in response of the two hoses is clearly evident. In particular, the aged hose transmits nearly half of the pressure amplitude injected compared to the virgin hose. This again demonstrates the importance of controlling permeability of the hose system. This experimental test setup is not as easily analyzed as the previously described arrangement, but it is diagnostic. There is a tendency for a damped Helmholtz oscillation to occur. With shorter lengths of hose this Helmholtz oscillator can be used to measure other properties of the hose such as the flow resistance (damping).
Figure 46. A simple, inexpensive diagnostic configuration. The analysis is not as simple, but the test is sensitive to both the time and length constants of the permeable hose.

Figure 47. Two short sections of 5/8" permeable hose were tested using the end-on test geometry described in the text. The virgin sample shows a longer time constant and a higher transmittance than the hose exposed to the elements for about 6 months. Increased permeability of the aged hose has shortened the characteristic length and hence the transmittance of the hose by about a factor of 2 at long periods. A small Helmholtz oscillation is evident in the virgin hose but was not excited under identical condition for the aged hose.
4.4 Analysis of a Helmholtz Oscillator

If a manifold is connected to a short non-permeable hose and a step function in pressure is applied to the manifold, it is possible that a Helmholtz oscillator may be set up. This condition occurs when the slug of air within the hose protrudes into the reservoir without immediate mixing and the reservoir acts like a spring. The slug of air acts as a mass connected to the spring and a damped simple harmonic oscillator is set in motion. The viscous flow of air back and forth in the hose causes damping proportional to the flow velocity and the damped simple harmonic oscillator can be modeled to "measure" the damping constants used in the previous analysis.

\[ m\ddot{u} = \rho AL\dddot{u} = -L\zeta \dot{u} + A(P_i - P_r), \]

where \( P_r \) and \( P_l \) are the right and left manifold pressures. If the slug of air protrudes into the manifold with displacement \( u \), and does not mix with the air in the manifold, then the effective volume of the right and left manifolds is \( V_r = V_{or} - A\dot{u} \) and \( V_l = V_{ol} + A\dot{u} \), where \( V_{or} \) and \( V_{ol} \) are the original right and left manifold volumes. The right and left manifold pressures are given by \( \frac{\delta P_r}{P_0} = \frac{\delta V_r}{V_{or}} \) and \( \frac{\delta P_l}{P_0} = \frac{\delta V_l}{V_{ol}} \). With some algebra, we have the equation for a simple harmonic oscillator,

\[ \rho A L\dddot{u} = L\zeta \ddot{u} + A(P_l - P_r) = L\zeta \ddot{u} - A^2 P_0 u \left( \frac{1}{V_{or}} + \frac{1}{V_{ol}} \right) u, \]

\[ \dddot{u} = \frac{\zeta}{A\rho} \dddot{u} - \frac{AP_0}{L\rho} \left( \frac{1}{V_{or}} + \frac{1}{V_{ol}} \right) u = -\nu A \ddot{u} - \frac{ART}{L} \left( \frac{1}{V_{or}} + \frac{1}{V_{ol}} \right) u, \]

with natural frequency \( \omega_0 = \sqrt{ART(1/V_{or} + 1/V_{ol})/L} = \sqrt{A^2 P_0 (1/V_{or} + 1/V_{ol})/m} \) and damping constant \( \beta = \nu A/2 = \zeta/(2A\rho) \). The natural period is proportional to the square root of the manifold volumes and the square root of the hose length. It is desirable to keep manifold volumes small and hence keep parasitic Helmholtz oscillator frequencies in the hose array above the infrasound frequencies of interest. Likewise, it is desirable to
avoid large changes in the cross sectional area of the operational hose-manifold system to avoid the Helmholtz instability. If we assume Poiseuille flow in the hose then the damping constant only depends upon air viscosity, air density, and hose radius, $\beta = \frac{4\eta}{\rho a^2}$. The damping constant does not depend upon the length of the hose. These relations can be used to determine damping characteristics and hence drag terms as shown in Figure 49 below.

Figure 49. A 0.32 m section of 5/8" non-permeable hose was set up as a Helmholtz oscillator and subjected to a 0.5 millibar step function change in pressure of one manifold. Crosses are data, the dotted line is a damped sinusoidal fit to the data. The damping constant, $\tau = 1/\beta = 0.37 +/- 0.01$ sec, is a measure of flow resistance in the hose and is consistent with Poiseuille flow.

4.5 Finite Difference Modeling, Maxhose

We have developed an implicit finite difference code, Maxhose, to simulate the response of a permeable hose and manifold system. Our goal was to model calibration configurations such as those shown in Figures 46 and 47 as well as the response of a system to an arbitrary atmospheric pressure field specified as a function of position and time. We start with the partial differential equation for pressure in the hose as derived above,

$$\dot{p}(x,t) = \left(\frac{RT}{A}\right)2\pi a e(p_s(x,t)) - p(x,t) - \left(\frac{RT}{Av}\right)\frac{\partial^2 p(x,t)}{\partial x^2}$$

and define $c_1 = (RT/Av)$ and $c_2 = (RT/A)$ $2\pi a e$. We discretize the space and time variables, $x_j = x_{j-1} + dx_i$ and $t_n = t_{n-1} + dt_n$, and define a pressure vector of j'th hose
elements, $p_j^{(n)}$ for time $t_n$ and $p_j^{(n+1)}$ for time $t_{n+1}$. Pressure, $p_j$, is the pressure of the volume between $x_j = x_{j-1} + dx_j$ with cross sectional area $A_j = \pi a_j^2$ and volume $V_j = A_j \, dx_j$. We follow the Crank-Nicolson implicit method (Smith 1978; Davis 1986) to derive a set of second order finite difference equations for the pressure updates at time step $n+1$ from the pressures at time step $n$. The system of equations that must be solved is,

$$
p_j^{(n+1)} - \left( c_j^2 dt_n / 2 \right) \left[ p_j^{(n+1)} - 2p_j^{(n)} + p_j^{(n-1)} \right] =
+ \left( c_j^2 dt_n / 2 \right) p_j^{(n+1)}
-
\left( c_j^2 dt_n / 2 \right) \left[ p_j^{(n+1)} - 2p_j^{(n)} + p_j^{(n-1)} \right] - \left( c_j^2 dt_n / 2 \right) p_j^{(n)}
+ \left( c_j^2 dt_n / 2 \right) \left( p_{j+1}^{(n+1)} + p_{j-1}^{(n+1)} - 2p_j^{(n+1)} \right), \text{ for } j = 2, \ldots, N-1
$$

where $p_j$ is the pressure inside the $j$th element of the hose and $p_{aj}$ is the atmospheric pressure outside the $j$th element of the hose. Note that $c1$ and $c2$ may depend upon position along the hose and that the spatial differences must be altered if hose elements are not uniform lengths, $dx_j$. We use second order approximations for the spatial and temporal derivatives. We require two sets of boundary conditions at the ends of the hose at $x_1 = 0$ and $x_N = L$. The first useful boundary condition is a no-flow condition where the spatial pressure derivative is zero. The left-hand side no-flow boundary equation becomes,

$$
p_{j=1}^{(n+1)} - p_j^{(n+1)} = 0, \text{ where } j = 1
$$

and right-hand side no-flow boundary condition

$$
p_{j=N}^{(n+1)} - p_j^{(n+1)} = 0, \text{ where } j = N.
$$

The second set of useful boundary conditions are volumetric reservoirs or manifolds at either end of a hose. In the case of multiple hoses, they are connected by manifolds of finite volume. If we assume pressure changes in the manifold are at constant temperature, then we have the relationship $p/(\text{mass}/V) = p/\rho = RT = p_0/\rho_0$ for the manifold. The change in pressure is therefore proportional to the mass flux,

$$
\delta p = p \frac{\delta m}{m} = (p / m) q_x \, dt = (RT / V) \, dt (1 / v) \, \partial p(x,t) / \partial x.
$$
If we have volume reservoirs of volume $V_j$ at $j = 1$ and/or $j = N$, then using the Crank-Nicolson scheme we arrive at the left-hand side boundary condition equation,

$$p_j^{(n+1)} - \left( \frac{RT \Delta t}{2 \nu V_j \Delta x_j} \right) (p_{j+1}^{(n+1)} - p_j^{(n+1)}) = p_j^{(n)} + \left( \frac{RT \Delta t}{2 \nu V_j \Delta x_j} \right) (p_{j+1}^{(n)} - p_j^{(n)}), \text{ for } j = 1,$$

and the right-hand side boundary condition,

$$p_j^{(n+1)} - \left( \frac{RT \Delta t}{2 \nu V_j \Delta x_j} \right) (p_{j+1}^{(n+1)} - p_j^{(n+1)}) = p_j^{(n)} + \left( \frac{RT \Delta t}{2 \nu V_j \Delta x_j} \right) (p_{j+1}^{(n)} - p_j^{(n)}), \text{ for } j = N.$$

Equations documented up to this point assume momentum of the gas within the hose is insignificant or inertial forces are not significant. Under some circumstances this may not be the case, so we now derive equations for the velocities, $v_j$, along the hose. The rate of change of momentum in the x-direction in the hose element of length, $dx$, may be written,

$$\frac{d(mv)}{dt} = \dot{m}v + mv \dot{v} = (\rho A dx) \dot{v}$$

$$= F_{\text{drag}} - A (p(x + dx) - p(x))$$

$$= \zeta \Delta x v - A (p(x + dx) - p(x))$$

$$= - \rho v A^2 v - A (p(x + dx) - p(x)).$$

We must assume that $\dot{m}v << m \dot{v}$ or nonlinear terms (velocity times pressure) will appear in the response of the hose and we have,

$$\dot{v} = v A v - \left( \frac{1}{\rho \Delta x} \right) (p(x + dx) - p(x)).$$

Note that for constant velocity flow with a constant pressure difference along a section of hose we have $\dot{v} = \rho A \frac{\pi a^4}{8 \eta \Delta x} (p(x + dx) - p(x))$, and if this corresponds to simple Poiseuille flow, $v \rho A = \frac{\pi a^4}{8 \eta \Delta x} (p(x + dx) - p(x))$, and the flow resistance constants may be computed from the air viscosity, $\zeta = 8 \pi \eta$, $\nu = 8 \eta / \pi / \rho / a^4$.

Let $c_3 = -Av/dx$ and $c_4 = -A/mass = -1/(\rho dx)$. We let the discrete velocity vector, $v_j$, correspond to the velocities between the hose elements at $x = x_j$ at node $j$. Therefore, the velocities and pressures constitute a staggered grid. Pressures are defined between $x_j$ and $x_{j+1}$ while the velocities are defined at $x_j$. Again, we apply the Crank-Nicolson formalism to derive finite difference equations that couple velocities and pressures at time, $t_n+1$, and $t_n$. 

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\[
\left(1 - \frac{dt\,c^3_j}{2}\right) v_j^{(n-1)} \frac{dt\,c^4_j}{2} \left(p_j^{(n-1)} - p_j^{(n+1)}\right) = \\
\left(1 + \frac{dt\,c^3_j}{2}\right) v_j^{(n+1)} \frac{dt\,c^4_j}{2} \left(p_j^{(n+1)} - p_j^{(n)}\right), \text{ for } j = 2, N-1.
\]

Note the \(c^4\) coefficients depend upon local mass and therefore they are time dependent and must be updated with each time step using

\[
\delta m_j = \text{dt} a_e_j \left(2p_{n,j}^{(n)} - p_j^{(n)} - p_j^{(n+1)}\right) \\
+ \left(\frac{\text{dt}}{v_j}\right) \left(p_j^{(n)} - p_j^{(n-1)} + p_j^{(n+1)} - p_j^{(n+2)}\right) / (4\Delta x_j).
\]

For left-hand side no-flow boundary conditions we have, \(v_j^{(n+1)} = 0\), for \(j = 1\). For right-hand side no-flow boundary conditions we have, \(v_j^{(n)} = 0\), for \(N + 1\). If we have no-flow boundary conditions on both ends of the hose, then there are \(N\) hose elements (pressures) and \(N+1\) nodes (velocities).

If we have a volumetric reservoir at \(j = 1\) then the \(j = 1\) velocity corresponds to the flow velocity between the reservoir and the first hose element, consistent with our previous pressure boundary condition equations, we have,

\[
\left(1 - \frac{dt\,c^3_j}{2}\right) v_j^{(n-1)} \frac{dt\,c^4_j}{2} \left(p_{j+1}^{(n-1)} - p_j^{(n+1)}\right) = \\
\left(1 + \frac{dt\,c^3_j}{2}\right) v_j^{(n+1)} \frac{dt\,c^4_j}{2} \left(p_j^{(n+1)} - p_j^{(n)}\right), \text{ for } j = 1.
\]

If we have a volumetric reservoir at \(j = N\), then the \(j = N-1\) velocity is between the last (\(N-1\)) hose element and the reservoir. The right-hand side boundary condition becomes,

\[
\left(1 - \frac{dt\,c^3_{j-1}}{2}\right) v_{j-1}^{(n+1)} \frac{dt\,c^4_{j-1}}{2} \left(p_{j-1}^{(n+1)} - p_j^{(n+1)}\right) = \\
\left(1 + \frac{dt\,c^3_{j-1}}{2}\right) v_{j-1}^{(n+1)} \frac{dt\,c^4_{j-1}}{2} \left(p_j^{(n)} - p_{j-1}^{(n)}\right), \text{ for } j = N.
\]

For both right-hand and left-hand side no-flow boundary conditions there are \(N\) pressure equations and \(N+1\) velocity equations. For both right-hand side and left-hand side volumetric reservoirs there are \(N\) pressure equations and \(N-1\) velocity equations. At each \((n+1)\) time step, linear equations are solved, \(Au^{(n+1)} = Bu^{(n)} + C\left(w^{(n)} - u^{(0)}\right) = b^{(0)}\), for the new vector, \(u^{(n+1)}\), of pressures and velocities \(W^{(n)}\) is the vector of atmospheric pressures along the hose. We construct the pressure-velocity vector components; \(u_{2j} = p_{j}, u_{2j+1} = v_{j}\), and \(w_{2j} = p_{2j}, w_{2j+1} = 0\), for \(j = 1\) to \(N\). \(C_{2j,1}, C_{2j,2} = 0\) and \(C_{2j,1} = 1\) for \(j = 1, N\). The \(A\) and \(B\) matrices are very sparse but not necessarily banded when multiple coupled hoses are
considered, therefore a general sparse matrix linear equation solver is used rather than a solver that assumes a strictly banded system.

4.6 Stability Conditions

While the Crank-Nicolson implicit scheme is generally stable regardless of time step size, we address one possible source of instability. Negative values of pressure and mass are valid solutions to the equations but physically meaningless. Mass and pressure are explicitly positive quantities; therefore, we wish to avoid large changes in the mass of any hose or manifold element, $|\Delta m| < m$ at any given time step. The lateral mass flux across each node should be small compared to the mass within an element during a time step,

$$ |\Delta m| = |dt \frac{\partial p}{\partial x} \frac{1}{v}| < \frac{p \text{ vol}}{RT} = p \frac{dx}{A} \frac{A}{RT}, $$

which leads to a restriction on the time step, $dt$, for each node,

$$ dt < \frac{p \frac{dx}{A} \frac{AV_j}{RT \partial p/\partial x}}{p_j \left| p_j - p_{j-1} \right|} \left( \frac{dx^2 A_j v_j}{RT} \right) = \frac{p_j}{p_j - p_{j-1}} \left( \pi \nu \frac{dx^2}{A_{0j}} \right), \text{ for all } j. $$

Likewise, the change in mass of the right- and left-hand volumetric reservoirs should remain small compared to the mass of these reservoirs as well as the mass of their adjacent hose elements. The conditions for these restrictions on $dt$ are easily derived. We write them here for completeness,

$$ dt < \frac{V_{\text{min}} \left( p_{j-1} dx_{j+1} A_{j+1} p_{j+1} V_{01} \right)}{\left| p_j - p_{j-1} \right|}, \text{ for } j = 1, $$

and

$$ dt < \frac{V_{\text{min}} \left( p_{j-1} dx_{j+1} A_{j+1} p_j V_{0e} \right)}{RT \left| p_j - p_{j-1} \right|}, \text{ for } j = N. $$

The diffusive flows into or out of the hose elements must also remain small for each time step,

$$ |\Delta m| = |dt 2\pi a e (p a - p)dx| < \frac{p \text{ vol}}{RT} = p \frac{dx}{A} \frac{A}{RT}, $$

$$ dt < \frac{p_j}{p_0 - p_{j-1}} \left( \frac{A_j}{RT 2\pi a e_j} \right) = \frac{p_j}{\left| p_j - p_{j-1} \right|} \left( \frac{\tau_{0j}}{2\pi} \right) \text{ for all } j \text{ with } e_j > 0. $$

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4.7 Hose Calibration Simulation

Figure 50 shows three Maxhose simulations of the calibration test configuration diagramed in Figure 44 compared to virgin hose data from Figure 6. A proper analysis of the calibration data requires that the pressure gauge manifold be included in the response. When the manifolds are included in the simulation, the value of $\tau_0 = 0.10$ sec. fits the data very well.

![5/8 Inch Soaker Hose - Rise Time Measurements](image)

Figure 50. Comparison of data and Maxhose finite difference simulation of the calibration test configuration of Figure 44.

4.8 Finite Difference Simulations of Wind Noise Response

We have demonstrated the use of the finite difference code, Maxhose, to model the hose response from atmospheric wind noise. An atmospheric pressure history is specified as a function of time and position along the hose, $p_a(t,x)$, and the pressure inside the hose is computed. Figure 51 shows the results for $p_a(t,x)$ specified as white noise on a 50 m hose with characteristic time of 0.01 second and a characteristic length of 100 m. While this is not a realistic wind noise situation, it illustrates several aspects of the permeable hose response. The numerical experiment was repeated for several hose lengths and the predicted noise reductions are summarized in Figure 52. RMS noise levels are reduced by nearly a factor 40 in the middle of a hose that is longer than 1/2 the characteristic length. Noise reduction saturates at about 1/2 the characteristic length. For such a hose under white noise conditions, noise levels at the end of the hose are roughly 30% higher than the levels in the center of the hose.
Figure 51. Pressure variation in the hose as a function of time (seconds) and position (X in meters) for random white noise pressure fluctuations in the atmosphere. Note that the hose smooths out the pressures along the hose.

Figure 52. RMS pressure fluctuations normalized to external atmospheric RMS fluctuations at the end of the hose and in the middle of the hose versus different hose lengths for a fixed characteristic length hose. Note that noise reduction is best in the middle of the hose.
Under more realistic wind conditions, atmospheric pressure fluctuations exhibit spatial correlation. Given an arbitrary correlation function or power spectrum we can synthesize a realization of the atmospheric noise. In the absence of a deterministic wind model, we have postulated that \( p_\alpha(t,x) \) appears as Brownian or fractal noise with a power spectrum that is inversely proportional to frequency and wavenumber, \( f^{-a} \) and \( k^{-b} \), where \( a \) and \( b \) are specified constants. Figure 53 shows the results of such a Monte Carlo simulation with \( a = b = 2 \). The same hose characteristics were used as in previous simulations. An ambient atmospheric pressure noise source is shown with a power spectrum approximately proportional to \( k^{-2} \) and \( f^{-2} \) is shown on the left and the internal hose pressure fluctuations are shown on the right. Note that the hose has spatially averaged the pressure field. High frequencies and spatial wavenumbers have been reduced. However, low spatial wavenumbers are still evident in the internal hose signal (Figure 54).

![Atmospheric Pressure Fluctuations Run 1](image1)

![Hose Pressure Fluctuations Run 1](image2)

Figure 53. An atmospheric pressure history as a function of space and time on the left and the resulting pressure fluctuation signal in the hose on the right. The 50 m hose reduces the atmospheric noise by averaging it over the length of the hose with characteristic length of 100 m.

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Figure 54. Pressure fluctuations inside and outside the middle of the hose simulation of Figure 53.

4.9 Conclusions

Noise reduction is critical for the successful utilization of microbaragraph arrays in a future CTBT system. Current plans for the IMS anticipate signal to noise increases from multiple hose arrays on the order of 100 for common wind conditions.

We present an analysis of the partial differential equations that govern the response of pressure within a permeable hose to external atmospheric pressures. We show that a hose may be characterized by a time constant and characteristic length. The characteristic time constant is determined by the permeability of the hose. The characteristic length is determined by the permeability of the hose and flow resistance along the hose. Methods for measurement of the time and length constant are proposed.

Noise signals in the hose will be attenuated for pressure fluctuations in the atmosphere with wavelengths shorter than the characteristic hose length. Infrasound signals with apparent wavelengths longer than the hose will not be so attenuated and the hose affords a net signal-to-noise ratio gain. Control of permeability of the hose is critical to the signal-to-noise ratio gains that can be realized. Measurements of the time constants of hoses have shown that the permeability can change by a factor of two due to natural weathering of a hose. Methods must be devised to protect hoses in the field and to diagnose hose degradation in operational systems.
Conversion of the partial differential equations to a finite difference program, Maxhose, is described. Maxhose simulates hose pressure response to an arbitrary atmospheric pressure defined as a function of time and position along the hose. This second order implicit Crank-Nicholson code runs on UNIX workstations and Windows 95 computers. Results are shown for simple calibration configurations and for two Monte Carlo realizations of wind noise. If wind noise is uncorrelated in space and time, then noise reduction for a single hose is predicted to be about a factor of 40 and maximized in the center of a hose at least as long as 1/2 the characteristic length. Noise reduction for a single hose in the case of fractal noise is somewhat less and depends critically upon the details of the atmospheric turbulence spectrum. Maxhose can be extended to simulate multiple hose arrays and wind noise with arbitrary correlation structures in space and time. Analysis of this kind coupled with an understanding of wind noise correlation structures will prove useful in understanding the potential for permeable hose noise reduction.

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4.11 References


