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CALCULATING A VALUE FOR DOMINANT BATTLESPACE AWARENESS

THESIS

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THESIS

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Air Education and Training Command

In Partial Fulfillment of the Requirements for

the Degree of Master of Science in Operations Research

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March 1998

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The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the U. S. Government

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Finally, this experience has renewed my faith. I now realize that the same Sentience that first said, "Do unto others as you would have others do unto you," also said "2 + 2 = 4." I have found the Divine in every textbook I have studied, and I feel blessed to have been able to see that.

Eric A. Beene

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ABSTRACT

In times of ever-tightening military budgets, methodologies are required that can compare the contributions of various systems involved in the warfighting process. While many tools are in use that directly measure the effects of greater numbers of enhanced hardware, and even improved processes, no validated methodology exists to measure elements that contribute to Command, Control, Communications, and Computers (C4); Intelligence, Surveillance, and Reconnaissance (ISR); or to analytically compare these elements with more traditional hardware.

This thesis develops a methodology for mathematically quantifying awareness in a military command and control (C2) environment. This methodology begins with the Observe-Orient-Decide-Act Loop to show the connections between levels of command and control, and to show influences. Entropy, in an information theory context, is modified to reflect not only how much is known at any level, but to show how well that information is known, producing a mathematically quantified measure of awareness. The awareness capability for various systems is calculated, and the rate of awareness loss is shown over time. Finally, an awareness curve is developed that shows the awareness of the C2 system throughout the process of attacking a ground target from the air.

CHAPTER 1

1.1. INTRODUCTION

"[I]t is said that one who knows the enemy and knows himself will not be endangered in a hundred engagements." [Sun-Tzu, 1994: 179]

Clearly Sun-Tzu understood the importance of awareness on the battlefield many centuries ago. He understood many key elements about warfare that we still hold dear today. Sun-Tzu's understanding, however, pre-dates technologies and insights we think we have only recently discovered and have used to prove his theories. Today we are in an age with such new and vast capabilities it is easy to be drawn to the theory that we operate in a different way than armies of Sun-Tzu's time operated. While the technologies are different and the speeds are faster, many of the basic principles are the same.

In times of ever-tightening military budgets, methodologies are required that can compare the contributions of various systems involved in the warfighting process. Many tools are in use that directly measure the effects greater numbers of enhanced hardware and improved processes, but no validated methodology exists to measure elements that contribute to Command, Control, Communications, and Computers (C4); Intelligence, Surveillance, and Reconnaissance (ISR); or to compare these elements with more traditional hardware. This thesis creates such a methodology.

1.2. BACKGROUND

Dr. Roy Rice, in a 1996 monograph, contended that the fundamental nature of warfare has changed, due in large part to the exponential rise in the capabilities of intelligence-gathering assets [Rice, 1996]. He defined the problem of analyzing warfare

in the modern age as an inability to quantify battlespace awareness, its mapping, or the capabilities to gather awareness. He proposed the following approach to such combat analysis:

1) Define and quantify battlefield awareness.
2) Determine required data for this quantification.
3) Develop the mathematical functional relationships of these measure to warfare.
4) Develop the mathematical algorithms for combat modeling.
5) Interpret the context of such analysis methodologies. [Rice, 1996]

The thesis effort reported here was prompted by a request from Col. Gary Crowder, AFSAA/SAG. In his letter, he stated that the Department of Defense currently has no way to capture the effect of Command, Control, Communications, Computers, Intelligence, Information, and Electronic Warfare (C4I2EW) or Intelligence, Surveillance, and Reconnaissance (ISR) and the contributions of the elements of each. He requested a measure or method that captured this phenomenon in order to determine whether expenditures on awareness technologies are cost-effective with respect to the numerous other programs fighting for budget dollars.

This thesis accomplishes the objective requested by Crowder, and it does so along the lines Rice suggested.

1.3. RESEARCH APPROACH

The methodology used in this study begins with the Observe-Orient-Decide-Act Loop to show the connections between levels of command and control, and to show the influences on decision-making. Entropy, in an information theory context, is modified to reflect not only how much is known at any level, but to show how well that information is known, producing a mathematically quantified measure of awareness. The awareness

capability for various systems and at different levels of command and control is calculated, and the rate of awareness loss is shown over time. Finally, an awareness curve is developed that shows the awareness of the C2 system throughout the process of attacking a ground target from the air.

1.4. SCOPE AND LIMITATIONS

This thesis creates a new methodology that quantifies awareness and allows its comparison for acquisition of different systems at different levels of command and control. This is a completely new and unique algorithm with applications not only to basic military operations analysis but to warfare modeling and simulation, command and control analysis, information warfare, and theater warfighting requirements analysis. Furthermore, this methodology is easily adapted to any non-military decision-making organization, including automated systems and robotics, civilian management, and decision support systems.

This methodology is developed from accepted mathematical principles and applied to a notional but conceivable operational scenario. It is developed step-by-step to show the underlying theory in understandable terms. The theory is used to analyze the notional scenario and to show the insights to be gained by such analysis. Finally, the methodology is extended and applications to other fields are suggested.

1.5. OVERVIEW

Chapter 2 is a stand-alone item describing the research accomplished, including: a theoretical model of command and control based on Boyd's [1987] Observe-Orient-Decide-Act decision cycle; a mathematical variation of Shannon's [1948] basic formulation for entropy in an information theory context; a methodology to calculate a

time-valued entropy; the awareness curve, which traces system awareness throughout a notional attack; and further uses of the new formulation for entropy as a measure of awareness. It first presents a notional scenario describing a dynamic battlefield, providing a step-by-step explanation of the processes involved in executing an attack. It then describes these steps in terms of awareness. This awareness curve is used to draw insights into the awareness required, the awareness available, and the awareness desired. Chapter 3 provides extensions to the basic research, including modifications of the original research and possible applications. These include more complex analyses of the basic scenario presented in Chapter 2 and techniques to better analyze more complex scenarios. Appendix A is a formal literature review, including prior research done in pursuit of a value for battlespace awareness. It also reviews literature containing a theoretical basis for much of the research accomplished in this paper. Appendix B is a review of the command and control process from the point of view of Boyd's O-O-D-A cycle. It details the theoretical basis for linking O-O-D-A loops, and offers some insight into the C2 process based on this construct. Appendix C reviews the mathematical and spreadsheet models used in the development of the awareness curve.

CHAPTER 2

2.1. INTRODUCTION

The military operations research community is constantly searching for new tools, techniques, and methodologies to gain insight into present and future problems. Battlespace awareness is a burgeoning field that is ripe for such products. This paper presents a new methodology for quantifying battlespace awareness and measuring its capability. The research is illustrated through the following simple scenario.

A KH-series satellite maps an enemy controlled battlefield in search of targets. As the imagery is developed and processed, it is discovered that the enemy has deployed a mobile missile launcher; adjacent tire tracks indicate it is moving. Analysis reveals the make of the launcher, and prior intelligence reveals the maximum movement speed to be 1 km per hour. Based on the imagery and the launcher's lethal implications, it is decided at the command level to attack the mobile missile launcher.

The optimal weapon is determined to be a 2,000-lb gravity bomb delivered by a deployed F-15E fighter. The battlefield is known to be hostile and protected, but air superiority is available on a limited basis. Support assets are required to reduce the threat, but the threat reduction package can only maintain air superiority for 10 minutes in the target area.

Since the satellite image development and processing requires two hours, by the time the decision is made to attack the launcher, the location information is two and a half hours old. Due to logistic constraints, the aircraft cannot be on station over the target until three and a half hours after the target was imaged. With target position information three and a half hours old, the target could have moved as far as three and a half

kilometers. Because of this possible difference in position, coupled with the limited time on station for the air package, a target position update is required prior to bomb delivery. Since the satellite is no longer available in the time allotted, an E-8 Joint Surveillance Target Attack Radar System (JSTARS) is launched with the package to provide target position tracking.

Due to deployment basing and alert status, the JSTARS cannot arrive on station until imagery time + 3:30 hours, further delaying the attack time 15 minutes due to its mapping and analysis requirements. At imagery + 3:40, JSTARS begins to map the target area. After five minutes of mapping and analysis, the crew determines the location of the target to within 42 feet.

At imagery + 3:45 hours, the fighter arrives at the area of operations and receives the updated position from JSTARS, 15 minutes prior to bomb release. Three minutes prior to bomb release, the fighter, using its radar, is able to map on two different scales and refine the position to within 8.5 feet, resulting in a successful attack.

Table 2-1: Information on Assets described

System	Map Size	Resolution
KH satellite[1]	7.5 x 40 miles	6' (1.83 m)
JSTARS[2]	3.3 x 3.3 nm	42' (12.8 m)
F-15E[3]	1.3 x 1.3 nm	17' (5.18 m)
	0.67 x 0.67 nm	8.5' (2.59 m)

Operationally, this is a facile scenario for today's US airpower assets, and one that could occur tomorrow in Bosnia, Iraq, or anywhere in the world where a threat to US interests exists. The decisions made and aircraft employed seem natural, given the target and the resources available. Even this simple scenario, however, has complex information and awareness requirements which have not been quantified and compared.

This scenario will be used to show how we might quantify and compare battlespace awareness, its acquisition, and its use within a theater. Its simplicity, while contrived, will demonstrate several basic principles of awareness analysis that can have far-reaching results and applications to very complex situations. This thesis documents a conceptual model that simplifies such analysis, converts awareness to a single scale and quantifies it, and measures awareness over time, revealing insight into how much awareness is *available* and how much awareness is *required.*

2.2. COMMAND AND CONTROL

As early as 1978, Col. John Boyd described the need for a decision-cycle dominance for victory in battle. His concept of the Observe-Orient-Decide-Act cycle of decision making has become the standard tool for enunciating the phases on which each decision depends. He advises us that "getting inside an adversary's O-O-D-A loops magnifies the adversary's friction," and by doing that we can "produce paralysis and get [his] system [to] collapse." [Boyd, 1978]

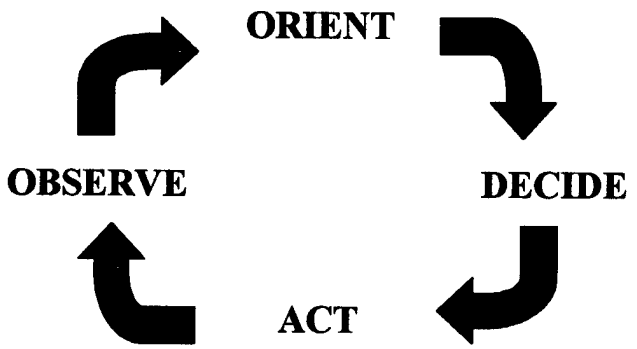

Figure 2-1: Boyd's Observe-Orient-Decide-Act (O-O-D-A) Loop

Both friendly and adversary forces operate according to this decision structure. The scenario presented directly references the decision process used by the commander and his staff. An important element of decision-making systems, however, that is not always enunciated is the fact that the decision cycle operates at all levels of the organization. In terms of a military organization, every level of command and control (C2) operates according to its own O-O-D-A loop. The key to this arrangement is that at only one level, the battlefield level, is the "act" step direct military action, such as shooting bullets, dropping bombs, firing missiles, or defending against such actions. At all other levels, when a decision is made to "act," the act is the issuance of orders, provision of information to a linear level, or commands directing that the battlefield level of the C2 chain performs its directed operation.

The C2 system thus operates as a chain of O-O-D-A loops connected to one another. Each loop is able to perform each function (indeed, each must), but the functions of one level do not necessarily mimic the functions of another. For example, a commander observes the entire battlefield while the infantry soldier observes, among other things, the target in his binoculars. The key to this chain is the connection between the O-O-D-A loop links. A superior level "acts" by influencing the "orient" phase of the subordinate.

Before proceeding further, it is beneficial to associate each of the actions in the scenario with its location in the O-O-D-A chain of command. Clearly, the command level in figure 2-2 "observes" the information via satellite imagery. It does not, however, know there is a target to be struck until the imagery is processed and analyzed. Processing, including downloading the imagery information from the satellite and

translating the information to an image either onto a computer screen or onto paper, belongs to the "observe" phase, while analysis of the resultant image falls under the category of "orient." The target is observed on the imagery, but it is not deemed a threat until it is analyzed, identified as a launcher, and seen to be moving in a troublesome direction. This phase, on the non-battlefield level, might be better thought of as the "Analyze" phase, instead of "Orient," as Boyd offered originally. It is these first two phases that require two hours in the scenario.

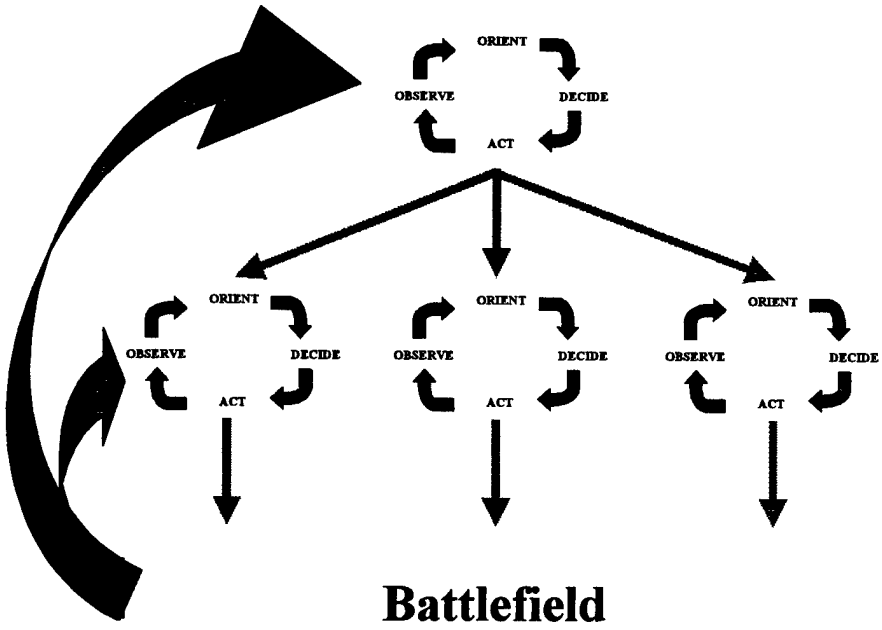

Figure 2-2: Linked O-O-D-A Loops. The "Act" phase in a superior's loop is an input to the "Orient" phase of a subordinate loop. All levels "Observe."

Once the threat is perceived, the commander "decides" to attack it in this scenario. Part of that decision is to decide which weapon systems will be employed.

This requires analysis of available assets by the commander, which takes another 30 minutes in this example. The commander "acts" by sending an order to subordinate units to launch an attack on the target at the imaged location. That order is transmitted instantaneously to the field units.

A similar decision cycle occurs at the line level. The fighter crew receives orders in the "orient" or "analyze" phase, decides the best route of flight, the go/no-go systems to prosecute the attack, and anything else not explicitly directed by the commander's order. The "act" is fly the mission and attack the target.

No matter the categorization, however, each item accomplished by each entity involves a time element. Obviously, each of the phases of the decision cycle takes a non-trivial amount of time. What is not so obvious is the fact that simple movement from one part of the decision cycle to another can also take a material amount of time. For instance, once the commander has decided to launch an attack, and the order is given, it may not always be received immediately, depending on how it is communicated. Electronic transmissions may result in near instantaneous communications, but in the event such channels are compromised, a courier may take minutes or even hours, depending on the distance to be traveled. Each of these elements of the decision making process and the decision communicating process must be included in a measure of the time required. This will be critical in calculating a value for awareness at key points in the scenario.

2.3. AWARENESS

Finally, we identify the missing element in this discussion: What is transmitted from one part of the decision process to the next? What is transmitted from a commander

to the fighting unit? Awareness is what is transmitted: awareness that a target is at a particular location, awareness that a launcher is moving, awareness that a target needs to be destroyed (and the order to destroy it). Awareness is a measure not only of what is known, but how well that information is known.

In the example given, the key element of awareness passed from the command level to the attack aircraft is the target location. It passes through various elements of awareness: units or individuals in the C2 system that use awareness and make decisions. This awareness, however, is reduced over time, so much so that it must be updated prior to the actual bomb delivery. This suggests two key points: (1) the value of awareness of a dynamic battlefield decreases over time, and (2) there are threshold levels of awareness applicable to each element of awareness, above which the awareness-gathering or -using element is ineffective. In other words, the location of the launcher was certain at the time it was imaged, even if it was not known to anyone in our chain of command. By the time aircraft were in a position to act on that information, the information was no longer as precise. The information was not wrong, but it was no longer precise enough, since the target may have moved a significant distance, to employ the selected weapon/aircraft combination against it armed with position awareness nearly four hours old. (This is dependent on the weapon/aircraft combination. A more powerful weapon would likely require less precise target position information.) The position must be more current, which in this case means the target cannot have moved from the last known position more than the limits of effective employment of the weapon system.

Considering the latter point first, the implication is an awareness requirement for effective weapon employment. The target position must be known to within specified

limits for the bomb to be effective. For purposes of discussion, suppose the bomb in the scenario has an effective kill radius against a missile launcher of 50 meters anywhere on the battlefield. As long as the target has not moved more than 50 meters away from the initial target coordinates prior to detonation (assuming the weapon impacts precisely at the target coordinates), the target can be effectively attacked by the F-15E and its crew. This awareness requirement is a threshold; with less awareness, the bomb is not effective. (In reality, the bomb loses effectiveness gradually over distance. A kill radius is the radius beyond which the bomb effectiveness falls below a predetermined level for the given damage mechanism.)

2.4. ENTROPY

To quantify this awareness, we need a metric that captures not just what we know, but how well or precisely we know it. A very useful measure of information, especially as a quantity passed between entities, is entropy. Shannon [1949] developed this first widely accepted measure of information. He defined entropy in an information theory context as[4]:

$$H = -K\sum_{i=1}^{n} p_i \log(p_i)$$

Where K = a scaling constant

p_i = probability of outcome i

This derivation measures the amount of information passed along a channel without a need for reference to the information itself. It is not necessary to know what information is being passed along a channel, only what kind of information (how many bits, or how many possible characters in each transmission). The resultant number is a

measure of the uncertainty, or what information is not received. The higher the entropy, the greater the uncertainty, and thus the lower the informational value of the message. The lower the entropy, the lesser the uncertainty, and thus the greater the informational value of the message.

Sherrill and Barr [1996] are responsible for one of the most recent works on the effects of intelligence on battle results. They developed entropy as a measure of information in a battlefield environment by quantizing the battlespace into a grid and determining, from the commander's perception, the probability a particular enemy asset is located in each grid cell. The entropy scale, as a measure of awareness, is reversed, such that a higher value for entropy implies less awareness, and a lower value implies more awareness. At the extremes, perfect information, knowing the precise grid cell an enemy tank is located, results in an entropy value of zero, while knowing only that the enemy has a tank, with an equal probability of it being located in any single cell, results in maximal entropy, in this case the natural logarithm of the number of cells in the battlespace grid. (In such a case, $p_k = p = 1/n$, where n is the number of cells.)

As in Shannon's derivation, the above methodology is scenario independent. If the battlespace can by quantized into states, then a measure of uncertainty, transformed to a measure of awareness, can be calculated. The scale of the battlespace is inconsequential; though it quickly becomes computationally intractable, the procedure for measuring the uncertainty in the location of a sizable enemy force spread over a large battlefield is technically the same as the procedure for measuring the uncertainty in the location of a single enemy tank. As long as the quantization is consistent, all elements of awareness can be measured on the entropy scale.

A problem quickly appears when considering awareness on different scales. Compare the awareness of the JSTARS and the satellite, as indicated in figure 2-3. Assuming JSTARS is able to locate the target, and map to the limits of its capability as annotated, it can give coordinates for the target accurate to within 42 feet. The satellite, on the other hand, can give position information accurate to within 6 feet. Quantizing the battlefield into square cells, the side of which corresponds to the limits of resolution for each system, and determining the entropy for each system, we find each to have an entropy of 0. This implies that both systems know precisely where the target is, which is true within respective resolution limits, and that they also have equal awareness. The latter implication is clearly not true: the size of the target notwithstanding, the satellite has a much more precise knowledge of the target's location than does JSTARS. Therefore we would say the satellite has better awareness. What is required is a metric that also indicates this difference in awareness.

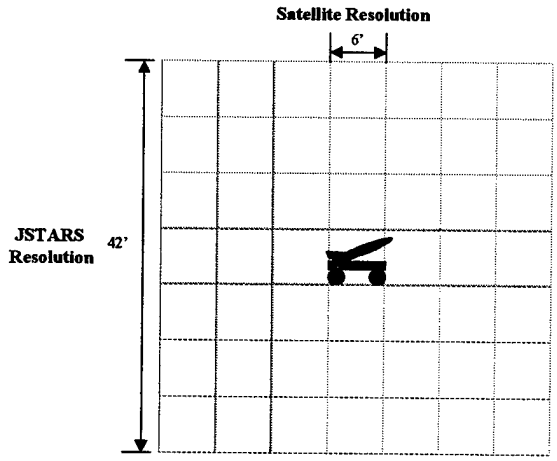

Figure 2-3: Relative resolutions for JSTARS and satellite imagery. The probability the target is located in a single cell is 1.0 for each, but JSTARS has a much larger resolution cell.

Sherrill and Barr divided the battlespace grid into regions based on the commander's assessment of the probability that a target was located in the grid, and further calculated the area associated with each of these regions. This calculation yielded a value for entropy based on the combination of how precisely a target location was known. A similar derivation can be used to create a common scale for all elements of awareness.

2.5. AN INFORMATION WARFARE ENTROPY MODEL

In Shannon's original entropy derivation, a sample space, Ω, was envisioned to include a finite number of mutually exclusive events E_k, whose probabilities p_k were assumed to be known. The random variable $X = -\log p$ was defined over the sample space. For each event E_k there corresponds a value x_k such that

$$x_k = -\log p[E_k] = -\log p_k$$

The quantity $-\log p_k$ was called the *self information associated with event* E_k. The resultant *average amount of information*, or *entropy*, displayed the highly desirable properties of continuity, symmetry, additivity, and the existence of an extremum. [Reza, 1994]

This formulation does not provide a coherent metric for different mappings of the same sample space. For instance, consider another division of sample space Ω into a finite number of mutually exclusive events $F_j \neq E_k$. Let that division be such that $F_1 + F_2 = E_1$. In this case, if $p[E_1] = 1.0$, then $p[F_1 \cap F_2] = 1.0$. Assuming it is equally likely that F_1 or F_2 occurs, $p[F_1] = p[F_2] = 0.5$. The average amount of information, using Shannon's calculation, is not equivalent, although it should be: information uncertainty

is equivalent in both cases. Using Shannon's formulation, the entropy for E is 0, while the entropy for F is log(0.5).

To create a coherent metric, redefine

$$X = -\ln\left[\frac{p}{S}\right],$$

where S is a measure of the relative size associated with E, which occurs with probability p. This variable better represents the actual limits of awareness, while including the uncertainty of Shannon's original formulation. The selection of the natural log does not change the fundamental nature of the calculation, it only serves to differentiate it from Shannon's formulation. This generates

$$x_k = -\ln\left[\frac{p[E_k]}{S_k}\right] = -\ln\left[\frac{p_k}{S_k}\right]$$

In the case of a two dimensional battlespace such as the flat battlefield in the scenario, S_k is simply the area, A_k, of the event E_k, the occurrence that the target is centered within that state, or grid cell. This is generally determined by the pixel size of a radar screen or image, or by coordinate resolution. (3-digit coordinates divide latitudinal nautical miles into thousandths, each thousandth representing nearly six feet.)

We may now define the average amount of information, or battlespace entropy, as:

$$E = -\sum_k p_k \ln\left[\frac{p_k}{A_k}\right]$$

Assuming all A_k are constant and equal, $A_k = A$, and

$$
\begin{aligned}
E &= -\sum_k p_k \ln(\frac{p_k}{A}) \\
&= -\sum_k p_k \left[\ln(p_k) - \ln(A)\right] \\
&= -\sum_k p_k \ln(p_k) + \sum_k p_k \ln(A) \\
&\text{Since } \sum_k p_k = 1 \\
\mathrm{E} &= -\sum_k p_k \ln(p_k) + \ln(A)
\end{aligned}
$$

Entropy is traditionally unitless. The addition of ln(Area) is technically a proportion that measures the ratio of the grid cell in question to a unit no larger than the smallest unit of grid area measure in the system. By selecting units of area small enough to describe all resolutions with values greater than one, we are assured of a positive value for the *ln(A)* term and thus a positive value for entropy. Using the scenario described, and selecting square meters (1 m^2) as the smallest area measurement, the entropy for the JSTARS awareness is $ln(12.8^2)$, or 5.1, and the entropy for the satellite awareness is $ln(1.83^2)$, or 1.21. This reflects the fact that the satellite has less entropy, and thus better awareness. That is, the satellite has a better knowledge of the launcher's location. It is not more correct, but it is more precise.

Considering the threshold requirement, if the weapon has an effective lethal radius of 50m, the awareness required to drop the bomb accurately must be at least as good as that corresponding to a circle with a radius of 50m. The corresponding entropy value is $ln(\pi * 50^2) = 8.97$. This indicates that awareness from either JSTARS or the satellite is "good enough," or precise enough, for accurate weapons employment.

Recall the fact that the target could be moving (this being a dynamic battlefield). While the probability that it was located within the specified cell 6 feet on a side when first imaged is 1.0, that probability is significantly reduced nearly four hours later when the weapon is to be delivered on the target. Exactly how much it is reduced over time, and the resultant increase in entropy and decrease in awareness, is the final piece of the awareness puzzle.

2.6. TIME VALUED ENTROPY

To derive a function for entropy over time, we require a formulation which defines the probability of being in a certain grid cell k as a function of time ($P_k = f(t)$). Over a quantized battlefield, each cell is a state the target may occupy. At time 0, a point of certainty, the target occupies a particular cell with probability 1.0. Over time, the target has a reduced probability of occupying that same cell, with a correspondingly increased probability of occupying adjacent cells, or states. This is essentially a Markovian process, with states, transition probabilities, and time steps. Indeed, the quantized battlefield can be modeled as such. The initial state vector for a 3 x 3 battlefield, pictured in figure 4, with the target located in the center (cell 5), would be represented by $I_0^T = [0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0]$, where each cell is numbered consecutively across then down. A transition probability matrix can define the possible directions of movement of the target in any single time step. In the simple case in which the target only travels in cardinal directions in any one time step, the transition matrix would be represented as shown in figure 2-4, where p_{ij} represents the probability of transition from cell i to cell j in one time step. The probability of the target being located in each cell

after one time step is calculated by multiplying $I^T \times \mathbf{P}$. To calculate the probability of the target being located in each cell after n time steps, multiply $I^T \times \mathbf{P}^n$. [Ross, 1993]

1	**2**	**3**
4	**5**	**6**
7	**8**	**9**

$$\mathbf{P} = \begin{Vmatrix} p_{11} & p_{12} & 0 & p_{14} & 0 & 0 & 0 & 0 & 0 \\ p_{21} & p_{22} & p_{23} & 0 & p_{25} & 0 & 0 & 0 & 0 \\ 0 & p_{32} & p_{33} & 0 & 0 & p_{36} & 0 & 0 & 0 \\ p_{41} & 0 & 0 & p_{44} & p_{45} & 0 & p_{47} & 0 & 0 \\ 0 & p_{52} & 0 & p_{54} & p_{55} & p_{56} & 0 & p_{58} & 0 \\ 0 & 0 & p_{63} & 0 & p_{65} & p_{66} & 0 & 0 & p_{69} \\ 0 & 0 & 0 & p_{74} & 0 & 0 & p_{77} & p_{78} & 0 \\ 0 & 0 & 0 & 0 & p_{85} & 0 & p_{87} & p_{88} & p_{89} \\ 0 & 0 & 0 & 0 & 0 & p_{96} & 0 & p_{98} & p_{99} \end{Vmatrix}$$

Figure 2-4: Three by three grid and corresponding transition matrix for movement in cardinal directions only.

The precise probability the target will transition from one cell to an adjacent cell depends on the target's speed, movement direction, and the precise location within the initial cell. The transition matrix presented assumes that each adjacent cell is equally likely to receive the target in one time step. The probability is then calculated by summing up all possible cell locations (cells with $p > 0$), assuming an equal likelihood for each, and calculating entropy. Only adjacent cells need be considered, and we can ensure this by reducing the time length of the transition step such that the target, given a maximum rate of speed, cannot have traversed more than one cell. At each time step, then, we can calculate the entropy of the system with said initial level of awareness to determine the loss of awareness over time. This corresponds to the certainty with which we can determine the target's position after one or more time steps have passed.

A somewhat simpler model for the spread of uncertainty is a diffusion model. During any single time step, the probability is allowed to diffuse to at most one adjacent cell. The rate of diffusion is controlled much as the transition rate in a Markov chain, based on speed and initial location within the specified cell. Graphically, the probability field would look like the representation in figure 2-5a. The position of the target is known with certainty (p = 1.0) at time 0. After 10 time steps (figure 2-5b) the target is not as likely to be located in the initial cell, while surrounding cells have an increasing probability of holding the target. After 20 time steps (figure 2-5c) the probability is highly diffused. The initial cell still has the greatest probability that the target is located there, but that probability has decreased until it is very nearly the same as the adjacent cells. Throughout the process of diffusion over time, however, the sum of the probabilities for all cells equals one.

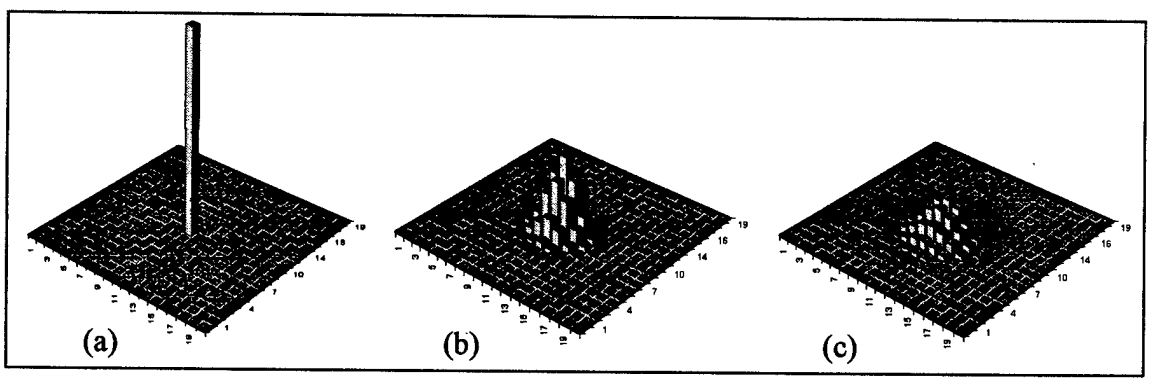

Figure 2-5: Probability that a moving target is located in a particular cell after (a) 0, (b) 10, and (c) 20 time steps. Probability is indicated along the vertical axis.

Such a basic model has been implemented in a spreadsheet for the case of the two dimensional battlefield. Simplifying assumptions for such a model include:

- Limited travel direction—restricting movement to one of four cardinal directions during any single time step, and movement of not more than the width of one cell during a single time step.
- Probability of movement out of the initial cell—for this example, the target was assumed to have a fixed probability of remaining within a cell (p_{ii}). This probability is a function of the maximum speed of the target.

Simulating the target movement on the battlefield over 30 time steps produces the curve shown in figure 2-6.

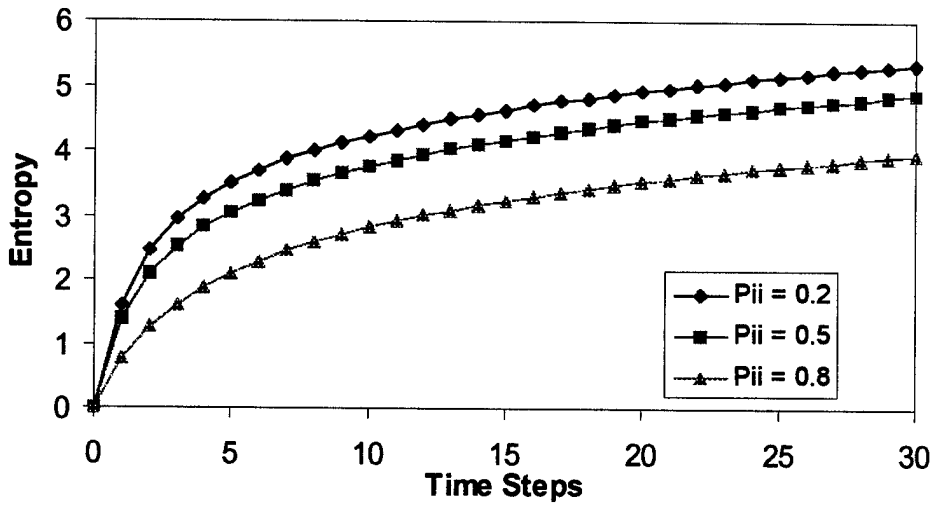

Figure 2-6: Entropy increase over time with four directions of movement at P_{ii} = 0.2, 0.5, 0.8.

The probability of movement into or out of a cell is difficult to calculate. On a flat battlefield with no roads, no water hazards, no vegetation, presumably the target is equally likely to travel to any adjacent cell. With a random probability the target travels in any of the four cardinal directions, the only remaining uncertainty is the actual speed of the target. If the target is assumed to travel only at maximum speed (the worst case),

the remaining uncertainty is its location in the initial cell. As figure 2-6 shows, the greater the probability the target will remain in a cell (the more likely it is to travel at a speed slower than its maximum), the greater the value for P_{ii}, and the slower the increase in entropy.

The time-dampened exponential curve in figure 2-6 shows the growth in entropy for a moving target, modeled using cardinal directions of movement. By assigning different time values to each time step, the curve can be used to represent the entropy change for any moving target. The faster the target moves, the less time each time step represents. More precisely, the faster the target can transit the cells of the observer's resolution, the less time each time step represents.

For example, using the illustrative scenario with a target capable of speeds of 1 km per hour and the JSTARS quoted resolution of 42 feet (12.8 m), the target can traverse one cell in approximately 45 seconds. Using that time for each time step, the curve in figure 2-6 reflects the passage of approximately 22.5 minutes. Using the satellite resolution of 6 feet (1.83 m), the target can traverse a cell in about 6.5 seconds, making the time scale of the curve approximately 3 minutes 15 seconds. For actual entropy calculation, the logarithm of the cell area must be added, as previously discussed. For the JSTARS example the initial entropy is 5.1, for the satellite example, the entropy begins at 1.21, as calculated previously.

Figure 2-6 represents a nonhomogeneous distribution of probability, centered in the initial cell, as depicted in figure 2-5. While many cells contribute to the entropy calculation, each contributes an unequal amount, based on the respective probability the target is located within the cell. This is indeed valuable information, but it is difficult to

imagine a method by which this type of information could presently be quickly communicated to a subordinate unit. It is essentially a bivariate distribution, in this case a nearly bivariate normal distribution.

What might more intuitively be passed, for example, from the command center to JSTARS, is a search radius or volume within which the target exists with certainty. After any period of time following initial detection, it is presumed the target cannot have traveled further than its maximum speed will allow. In 23 minutes, for example, the target cannot be more than approximately 385 meters from its initial position. That equates to an entropy of $ln(\pi * 385^2) = 13.05$. Intuitively, this formulation is very useful for entropy calculation. Practically, however, it offers little value, since there is no particular sensing system associated with the number 13.05, and nothing with which to compare it.

What is really calculated using the volume calculation is the inherent entropy of the target. This is evident in the calculation at the time of initial awareness, which is without area. While this defines a limit of awareness, it does not measure real awareness for real systems, or how aware a system is. On the other extreme, consider a system with a very course resolution, a cell size of 1 km on a side. After 23 minutes, its entropy is essentially unchanged. It started with an entropy of $ln(1000^2) = 13.82$, and that value remains until the first opportunity the target has to exit the cell. As the goal of the research is to supply a methodology to measure awareness for any awareness gathering or using system, the use of actual target-probable area appears of limited value[5].

The expansion of the probable target location area can be better quantified by the calculation of the number of resolution cells the target could possibly inhabit. For such a

representation, all cells n in which the target could be located (with any probability) are summed, the probabilities are homogenized ($p = 1/n$), and entropy is calculated. Using the four-direction movement assumption, and assuming the target center is initially located in only one cell, the number of cells at time step i can be calculated using the formula $n_i = n_{i-1} + 4i$, for $i > 1$. [See Appendix C.]

2.7. GENERATING AN AWARENESS CURVE

We can now generate an approximate awareness curve for the scenario given. This awareness curve, shown in figure 2-7, depicts the C2 system's awareness in the course of the attack on the mobile missile launcher. We start the time at 0:00, at which point the mobile missile launcher is imaged by the satellite. Prior to this time, entropy is at a maximum, with the target location equally likely throughout the battlefield[6]. Entropy is instantly reduced from a maximum to 1.21. Over the course of the next three hours and forty minutes, this value gradually increases. Based on the size of the resolution cell of the satellite image, this amounts to approximately 2000 time steps. (1 km per hour x 3.66 hours = 3.66 km ≈ 2000 x 6' increments.) JSTARS images the target and again instantly reduces the entropy to 5.1 by fixing the target location to the limits of its resolution. During the following 13 minutes while the fighter arrives on station, that value increases. Based on the 42 feet, or 12.8 meters, cell resolution, this amounts to 17 time steps. The fighter arrives and maps, reducing the entropy again, and maps twice more in even more detail, further reducing the entropy. These high resolution updates are required to keep the position uncertainty below the entropy threshold of the bomb. The awareness curve shows the progression of entropy, and thus awareness, during this scenario. In figure 2-7, the black line represents the awareness of the attacking command

and control system over the time from initial detection of the launcher until it is destroyed. The sharp spikes down represent awareness updates. The spikes end at the resolution limit of the updating system. The horizontal lines represent thresholds of effectiveness for different weapon systems. JSTARS must have entropy reduced below 17.4 to be effective, the F-15E must have an entropy value below 15.6 to be effective, and the entropy must be reduced below 9.0 for effective employment of the bomb. The smaller graph is a time-expanded view of the fighter's awareness updates, with the bomb entropy threshold included.

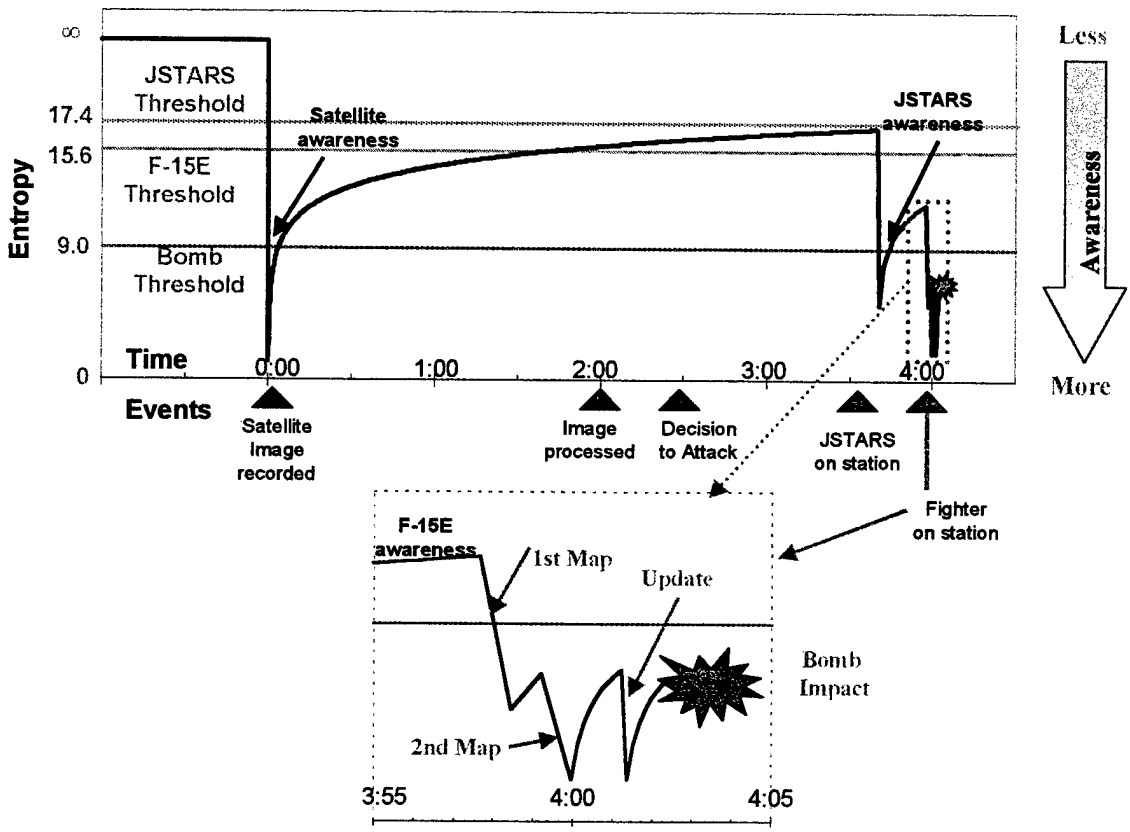

Figure 2-7: The awareness curve for attacking a mobile missile launcher. The top curve displays the entire four-hour process. The bottom curve is an expanded view of the final ten minutes. The system awareness is represented by the dark black line.

The initial awareness of the launcher, at the time it is first detected, is enough to effectively drop the weapon. However, because the commander's decision cycle, when including the time required for processing the satellite imagery, is so long, that awareness cannot be acted upon prior to its degradation. By the time the image is processed, the entropy is almost too great for effective use of the F-15E. By the time the decision is made to attack, the F-15E threshold has been exceeded. The system entropy, however, is well below the JSTARS threshold throughout the process.

The assumptions used to generate the awareness curve, and their effects, bear review:

- The target has a maximum speed of 1 km per hour. A target with a faster speed would result in a faster entropy increase following each awareness update.
- The target can travel in any direction, but for modeling purposes, the movement was restricted to one of four cardinal directions in any single time step. The area approximation using such a model is significantly less than the actual area the target could possibly occupy. When measured on the natural logarithm scale, however, the maximum error in entropy is less than 0.5. In the curve above, the system awareness curve actually crosses above the F-15E threshold slightly sooner than indicated.
- Each of the four directions of movement is equally likely in any single time step. Actual battlefield conditions would in all likelihood be considerably different. Prior intelligence, particularly Intelligence Preparation of the Battlefield (IPB), would improve the predictive abilities of this model. Construction of such a transition probability matrix, using past patterns of movement or trafficability and

mobility codes, would be cumbersome, but potentially quite helpful in determining particular awareness requirements[7].

- The target is assumed to occupy all possible cells with equal likelihood. Based on known terrain factors and geographic features, and even on characteristic movement routines, this is overly simplistic. It does, however, provide an upper bound for system entropy. That is, it is likely the system would know even more about the likely location of the moving target than is represented by the curve.

2.8. ANALYZING THE AWARENESS CURVE

Given this scenario and the resultant awareness curve, we are now able to estimate the effects of changes to resources and capabilities. In effect, we can now "what-if" the scenario, bearing in mind the limitations of the simplifying assumptions.

If we can introduce processes into the commander's O-O-D-A cycle that reduce the time from initial imagery to the decision to attack, we likely would still be unable to function effectively without JSTARS. The time required from the decision until aircraft on station (a delay due to physical and geographic factors) is approximately 1 hour and 30 minutes, by which time the system entropy is at or above the F-15E effectiveness threshold. To remove JSTARS from the process requires aircraft on airborne alert with the proper ordnance and with adequate fuel, along with a decrease in the commander's decision cycle time. In effect, JSTARS allows a commander to base assets further from danger without reducing effectiveness or forcing a quick decision.

Without employing moving target tracking technologies, the fighter must update position no more than approximately two minutes prior to bomb release to keep entropy below the threshold value for effective employment. The effort required to maintain such

a frequently updated ground position reduces the ability of the crew to maintain an awareness of other threats in the area, increasing their reliance on support assets for protection. The awareness update requirement effectively dictates the support assets necessary for this mission: assets that can reduce the threat of surface-to-air missiles and anti-aircraft fire in the target area, allowing freedom for maneuvering to map along appropriate axes; and assets that can reduce the airborne threat, since the F-15E cannot track air-to-air threats while ground mapping. The use of reliable moving target tracking technologies could conceivably reduce the F-15E's task saturation in the target area, thus increasing self-protection capability and decreasing the need for support assets for force protection.

Finally, we can see benefits that can be obtained by changing threshold values for specific equipment. If the F-15E threshold were raised, it might be possible to operate in this scenario without updates from JSTARS. If JSTARS capability were slightly downgraded, another awareness source would be required prior to JSTARS's arrival on station. This implies that any replacement aircraft in this scenario must have an awareness threshold at least as high as that of JSTARS. The awareness scale alone allows for direct comparison of awareness systems in such scenarios. Such modeling could have direct impact on systems planning and acquisition and force construction.

2.9. CONCLUSION

This simple scenario demonstrates the power of this awareness formulation to quantify battlespace awareness, map the awareness of systems with different awareness capabilities and requirements onto one scale, and determine constraints and excesses in battlefield awareness. This is a new capability. We have not previously been able to

directly compare different awareness-gathering or awareness-using elements quantitatively. We have not previously been able to quantify awareness mathematically as a measure of how well we know something. We have not previously been able to mathematically state awareness requirements to meet a warfighting objective. With this methodology, we can.

On one scale we can now measure the levels of awareness that permit an attack such as the one described, and with this measure we are now able to compare different elements of awareness. Using this tool, we can compare a faster satellite image analysis processor, for example, to an airborne platform, such as JSTARS. We can compare the effect of a more streamlined command and control process to a new fire-and-forget weapon that needs only a very general awareness update prior to release. We can compare our awareness capabilities to an adversary's and maximize our dominance by selecting the best weapons and weapon systems. Not only have we been unable to compare these systems in the past, we have never been able to merely quantify the effects of some of these systems.

This methodology will extend to all manner of battlefield analysis. We can use a three-dimensional diffusion model to easily quantify and map air war awareness and calculate theater air support asset requirements. Even traditionally less quantifiable measures can be mapped and analyzed on an awareness scale. We can determine enemy posture, for instance, to within a measurable resolution. Based on prior knowledge about the enemy's actions and affiliations, we can estimate the probability that other assets will be activated in a known amount of time, and we can measure the amount of awareness of enemy posture required to determine when it is necessary to commit more forces.

The awareness curve and its analysis also highlight critical points and weaknesses in the enemy's command and control processes. If we know the processes involved in the adversary's C2 processes, not only can we better "get inside the adversary's O-O-D-A loops," as Boyd suggested, by reacting faster than the adversary can anticipate and respond, but we can "get inside" his decision loops in a figurative sense and more accurately target information warfare attacks where they are likely to have effects most detrimental to the enemy. Whether by luck or by malice aforethought, a classic example of this is the Argentineans' exploitation of the British awareness update cycle to render significant destruction to the British fleet during the Falkland Islands war. [Ganley, 1984] By using low-flying Super Etendards to update their awareness of British ship position without giving the British the opportunity for the same type of information, the Argentineans were able to maintain their own awareness below an employment threshold for their Exocet missiles practically undetected. By getting inside the British decision cycle and keeping their awareness (entropy) below employment thresholds, the Argentineans were able to use their five Exocets with surprising effect[8]. Just as we can analyze our own awareness curve to improve our performance, we can analyze an adversary's awareness curve to deteriorate his performance.

Even farther afield, an awareness-based entropy methodology has clear applications in robotics and automated systems, which require an element of awareness about the operating environment. We can model the operating environment, even very generally, and determine minimum update rates and appropriate times for human intervention. In the case of a relatively low bandwidth information transmission environment, such as the Mars Pathfinder mission, if we know the general terrain and the

speed of the rover, we know how frequently we need to send position updates and can maximize data transfer.

Any process or system that can sense and can sense with a measurable precision, will benefit from an entropy awareness analysis. The actual information is not required, only the nature of the information, its dimensionality, and its rate of change are required to generate insights into everything from minimum awareness required to update rates to necessary supporting assets. This methodology can be implemented within a model or simulation, used in a descriptive analytical environment, or even used as a prescriptive tool to point out discrepancies and requirements. Anywhere information and awareness dominance is required, this methodology can help calculate it.

Note 1: Data from http://www.milnet.com/milnet/declass.htm, the source of declassified military satellite capabilities. Data is based on KH-6 satellite information.

Note 2: Data from "F-15E Strike Eagle," World Airpower Journal, 21: 70 (Summer 1995). Article described JSTARS capability as similar to F-15E capability. Specific capabilities are highly classified.

Note 3: Data from "F-15E Strike Eagle," World Airpower Journal, 21: 49 (Summer 1995).

Note 4: Shannon's original formulation used $\log_2 p_i$. The base 2 is a natural choice when dealing with binary information variables.

Note 5: The volume calculation might well have a use in calculating a time to acquire, or re-acquire, given an area to search. Acquisition models that calculate a probability of acquisition or time to acquire might generate more insight into the time required in the "observe" phase for each asset.

Note 6: Assuming, of course, the existence of the mobile launcher was known. Without knowledge of the target's existence, arguments can be made that entropy is infinite and that entropy is zero. Using entropy as a measure of awareness, it is more logical to presume that lack of awareness equates to infinity.

Note 7: As mentioned previously, though it might be highly valuable in determining the actual awareness of a particular awareness- gathering system, the transmission of this type of information between systems is difficult to envision. The time spent transmitting the parameters of a multivariate probability field could perhaps better be used re-acquiring the target given a known area of equal probability.

Note 8: Though there is some dispute on the actual techniques employed by the Argentineans in updating the Super-Etendard's avionics prior to missile launch, and the distance at which the missile was launched, the fact remains that by the time the HMS Sheffield had an awareness that crossed a threshold for action, they had approximately 5 seconds to react prior to missile impact. By anyone's determination, the Argentineans successfully operated within the HMS Sheffield's decision

CHAPTER 3

3.1. INTRODUCTION

The research presented in Chapter 2 is complete in itself. It explains the mathematical development of the concept of battlefield awareness and intuitively demonstrates its use in a simple scenario. The basic research, however, raises other obvious research questions. This chapter outlines several extensions to the basic research, including a modified diffusion representation, awareness of multiple targets, and a battlefield scenario that requires confirmation from multiple sources prior to committing forces.

3.2. IMPROVED DIFFUSION MODELING

As mentioned in Chapter 2, the use of the diffusion model with the limitation of four cardinal directions of movement in any single time step, as shown in figure 3-1(a), underestimates the area, on the two-dimensional battlefield, a moving target can occupy with any probability. In the case of grid cells of 1m x 1m, assuming a maximum speed of 1m per time step, a four-direction diffusion model covers 20201 cells after 100 time steps, or 20201 m^2, while the actual area possibly occupied is approximately 31415 m^2. This discrepancy is magnified over time.

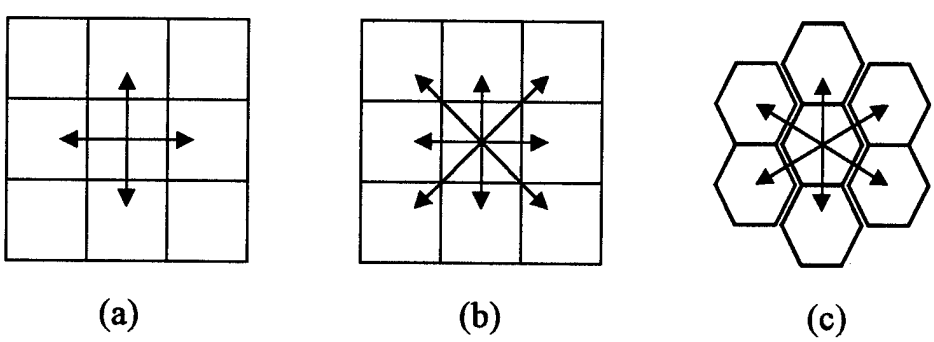

Figure 3-1: Three movement models: (a) four-direction, (b) eight-direction, and (c) hexagonal.

Using logarithms in Chapter 2 to calculate entropy, the differences were reasonably small when using the diffusion model. For comparison, the entropy in the example above is 9.91 for the four-direction model vs. 10.36 for the actual area calculation, a difference of approximately 0.45. After 1000 time steps, this discrepancy in entropy is essentially the same, 14.51 for the four-direction model vs. 14.96 for the area calculation. These values are shown in Table 3-1. The relative difference is 4.3% after 100 time steps, and even lower after more time steps. Furthermore, when considering different movement rates, the area term is scaled so that it will always add to the entropy value, no matter the target's speed or rate of transition. Therefore, the worst case relative error will never be more than 4.3% after 100 time steps.

Depending on the awareness elements involved and the actual time scale, this small difference could result in analytical errors in an examination of awareness transfer times and threshold crossing opportunities. In a simple scenario as presented in Chapter 2, this discrepancy need not tarnish the insights provided by the awareness curve, but in a complex analysis, perhaps a discrepancy of even this small magnitude could cause larger errors through compounding. For these reasons, other diffusion models are considered.

Table 3-1: Area and Entropy associated with different diffusion models. (Areas shown in square meters.)

Time Steps	Four-direction model		Eight-direction model		Hexagonal cell model		Actual Area	
	Area	Entropy	Area	Entropy	Area	Entropy	Area	Entropy
100	20201	9.91	40401	10.61	26241	10.17	31416	10.36
1000	2.00×10^6	14.51	4.00×10^6	15.20	2.60×10^6	14.77	3.14×10^6	14.96

The obvious alternative model of diffusion is a contiguous model, in which a moving target can migrate to any of the eight adjacent cells with equal probability. In

such a diffusion pattern, the area of possible occupation increases much faster than that for the four-direction model. This comparison is also shown in table 3-1. As we see, the error, while an overestimation, is actually closer in value to the area calculation, both in terms of area and entropy. This model overestimates the diffusion probability by giving equal probability to the corner cells, occupation of which would require a faster average speed than for the shared-side cells. After 1000 time steps, entropy is 15.20 compared to 14.96 for the area calculation. The error is approximately 1.6%,.

The result of this model is a greater value for entropy over time. This might be a better choice for a worst case analysis of awareness requirements. However, it results in awareness requirements greater than necessary.

Another model for diffusion is the hexagon model. Hexagons are widely used in ground war models to depict terrain cells [Hartman, 1997]. A ground unit or piece of equipment is said to occupy one or more hexagonal cells at a certain time. At each time step in a discrete simulation we assume the unit or equipment can traverse terrain up to the width of one hexagonal cell. This equates to six directions of movement, one perpendicular to each side of the hexagon.

Applying such a diffusion model, the width of each hexagonal cell is 1 m from side to opposite side. The resulting area of such a cell is approximately 0.866 m^2, slightly smaller than the area of the cells in the other two diffusion models. The number of cells of possible occupation in each time step, however, falls between the numbers for the other two models. The area, shown in table 3-1, is calculated by multiplying the number of cells by the area of each cell.

Intuitively, this should yield a reasonable compromise between the other two area estimation models. In fact, the estimation is an underestimation, much closer to the actual area calculation than the four-direction model. The entropy error value, however, is only slightly better in absolute magnitude than the error for the eight-direction model, albeit an underestimation. After 1000 time steps, the entropy error is 0.19, or 1.3%.

This model has inherent benefits and drawbacks. Clearly, in a ground combat simulation, this diffusion model is the best choice, since it exactly models the type of perception in the model: the resolution cell in such a model is frequently hexagonal. The relatively small entropy underestimation is probably still notably more accurate than either of the other two models.

On the downside, few real awareness-gathering elements perceive in hexagons. Typically position information is passed in terms of coordinates, either latitude and longitude or Universal Transverse Mercator (UTM) coordinates. Therefore, the effective resolution cell is much nearer the shape of a square than a hexagon. Furthermore, using a four-sided resolution cell makes modeling six possible directions of movement extremely unintuitive.

Perhaps the best compromise, in terms of fidelity, is a hybrid model combining the four- and eight-direction movement models. In such a hybrid, side-sharing cells have a greater probability of occupation after a time step than corner-sharing cells. To keep a homogenous probability field, a threshold probability value can be set into the model, whereby cells with probabilities of occupation falling below this threshold are not considered. This threshold would probably be a proportion of the side-sharing cell probability values.

3.3. AWARENESS OF MULTIPLE TARGETS

The scenario in Chapter 2 considered the existence of only one target for the sake of discussion. Clearly this is an oversimplification and one that requires investigation before any practical use can be made of the awareness methodology. It is far more likely an attack would be waged on a battlefield with multiple targets.

Sherrill and Barr [1996] used a conditioning argument in their treatment of the topic, whereby the subjects in their experiment were given location information on 6 of 22 obstacles. They calculated the average entropy based on the two possibilities—whether or not the obstacle was one of the six obstacles with a known position. Knowing the position of more obstacles obviously increased awareness and decreased entropy.

More applicable to the dynamic battlefield is the scenario in which the number of obstacles or targets is not known. Given that we know the enemy has 10 targets, we can calculate entropy based on how well we know the targets' locations. For the targets of which we are unaware, however, entropy is at a maximum and awareness is at a minimum.

Assume we later become aware of the existence of 10 more targets. By simply adding the entropy associated with how well we know the new targets' locations, the value for system entropy is instantly increased. Clearly, however, we are *more aware*, which implies a *lower value for entropy*. There are several different approaches to this conundrum, each with separate benefits and drawbacks.

The first approach is one that mimics the scenario in Chapter 2. That approach calculates the awareness acquired and required *given* there exists a target. More targets, as shown in figure 3-2, do not translate into more or less awareness, but more awareness

curves. This analysis considers the events that must take place to act on a given amount of awareness once it is acquired.

The awareness curve in Chapter 2 could have been generated only for the F-15E part of the scenario, given that JSTARS had located the missile launcher. Had JSTARS located another launcher, a new curve could be calculated for the awareness required to destroy that launcher.

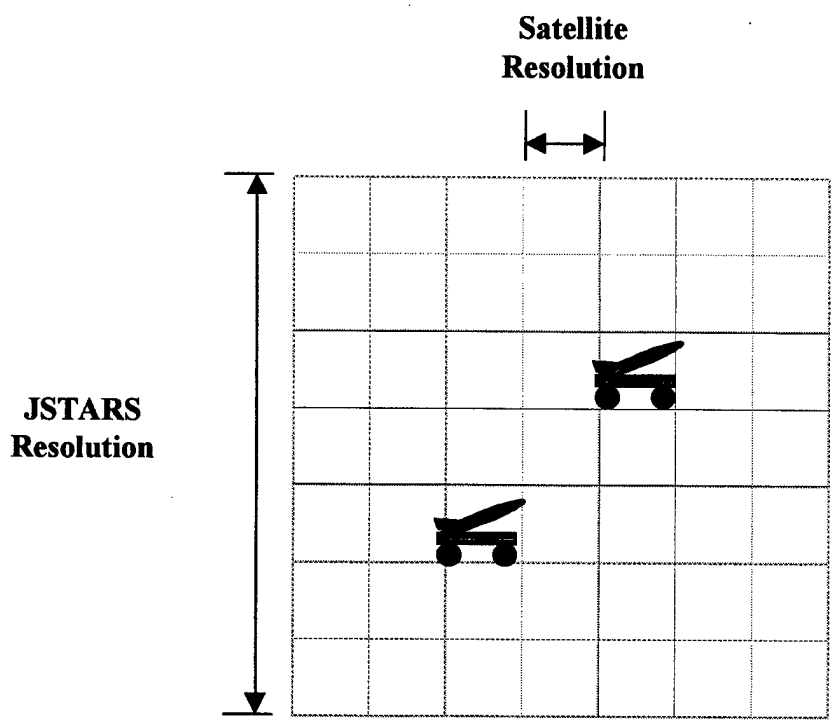

Figure 3-2: Awareness of multiple targets.

Another aspect of the multiple target awareness analysis is the concept of aggregation. The missile launcher in the scenario was technically a combination of a launcher and missiles, aggregated as one target. The missile launcher could have been part of a mobile force that occupied a large part of the battlefield, and the objective may have been to destroy the force, or perhaps its mobility. Given a target, an objective is

defined, a desired "act" to be performed on that target. The awareness curve can be calculated for that act on that target.

An interesting aspect of the multiple targets problem is highlighted by figure 3-2. This depicts two targets that fall outside the limits of resolution of the satellite, but inside the resolution limits of JSTARS. Initially the system has awareness of two targets and presumably acts on that awareness. JSTARS attempts to re-locate both targets but, because they are now both within a single JSTARS resolution cell, it cannot update position on both. In this case, there is only a limited awareness update, on one of the targets. Depending on relative confidences in awareness-gathering systems, this can either reduce confidence in the initial awareness or lead to a false confidence in the most current update. In either case, we can predict the effects of an awareness mismatch, and design a decision policy to handle such an event. This is yet another benefit of using awareness analysis.

Starting with only JSTARS awareness as shown in figure 3-2, the decision cycle would have focused on attacking the single target known to exist. When the F-15E detected the existence of two launchers (as it inevitably would, its resolution cell being nearly as small as the satellite's), system awareness would have increased, and entropy decreased, but due to the nature of the decision cycle, the additional target would likely go untargeted. This highlights the nature of the decision cycle that requires the appropriate information at the right level at the right time. This is clearly appropriate awareness, and the F-15E is clearly the right level, but two minutes from bomb release is definitely not the right time to become aware of the existence of another target. Thankfully, however, due to the ordnance to be delivered, both targets would be

destroyed by a properly dropped weapon. Again, awareness analysis highlights instances of awareness-gathering and awareness-using mismatches.

This example points to a concept very basic to the quantification of awareness: the resolution of the awareness gathering system. Had one of the two targets been a surface-to-air missile system (SAM), one highly effective against the F-15E, that information is clearly critical to the success of the mission. Furthermore, that level of resolution, or awareness threshold, that can differentiate between targets to tell a mobile missile launcher from a SAM is critical to mission success. That awareness, however, is a separate quality and quantity from the awareness of the target's location. Effective analysis of awareness requirements likely depends on a study of an awareness vector that includes not only target location but also target type. This is not only critical in assessing the threat to the bombing aircraft, but in determining the effectiveness of the munition used against the threat.

This analytical approach to the multiple target problem divides awareness into different qualities: How many targets are there? Where are they located? What types of targets are there? In Chapter 2, we were concerned only with the second question—location related to performing the act of destruction. The other questions were never asked, but would have represented separate but similar problems. The awareness being sought is the number of enemy targets of specific types. This is highlighted in figure 3-2, when the different resolutions produce different awareness. The probability field considers states that also represent the number and types of targets. These can be single-dimensional, or part of a multi-dimensional problem. The dimensions, more than likely,

will not be independent. Given that we have discovered X targets, what is the probability that Y will be of a certain types?

This also corresponds to a multi-dimensional awareness curve analysis. The awareness begins with a value of entropy corresponding to the number of targets known (perhaps a percentage of the total targets in the enemy's possession), and is assessed over time to obtain a minimum value for each quality, below thresholds necessary to make a reasoned decision for action: how many weapons to employ and what types. In such a case, the action may be retreat because the enemy outnumbers the friendly force, or do not attack the mobile missile launcher with the SAM present. (The action arising from increased awareness is not always good or better, but the decision based on increased awareness will be more likely to lead to successful accomplishment of a particular action.) The analysis of multiple targets and non-homogenous targets obviously bears extended study.

3.4. A REQUIREMENT FOR CONFIRMATION OF AWARENESS

In many real-world operational scenarios, no attack is prosecuted without confirmation of intelligence. For the Chapter 2 scenario, no attack would have been ordered without first confirming, through another source, the existence of a mobile missile launcher or the intent of the enemy to move it within range of a friendly border. Such confirmation might come from human intelligence channels, reporting on the enemy's command decisions, or from a later satellite pass showing another image of the same launcher even closer to the friendly border. Such a requirement for confirmation can be analyzed by adding entropy values from two awareness-gathering resources.

Simple addition of entropy values from each source does not meet the needs of such analysis, however. Consider the simple scenario depicted in figure 3-3. There are two sources of awareness, each mapping the same battlefield. Figure 3-3a depicts a source with finer resolution than that of figure 3-3b. Both observe the same target in the same location simultaneously. For discussion, assume the area of each cell in 3-3a is two units, and the area of each cell in 3-3b is four units. Each source locates the target to within a single cell (p = 1.0). The corresponding values for entropy are 0.693 (= 1.0*ln[1.0/2]) for 3-3a and 1.386 (= 1.0*ln[1.0/4]) for 3-3b.

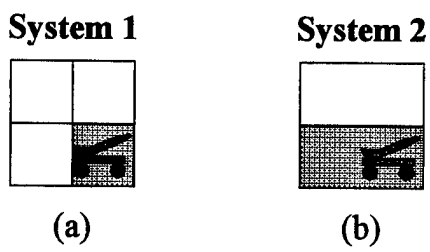

Figure 3-3: Confirmation of awareness. Shaded areas represent cells with probability = 1.0.

When both systems simultaneously map the target in their corresponding cells, we would expect awareness to be increased and entropy decreased. Simple addition of the two entropy values results in a greater value for combined entropy, reflecting reduced awareness. We can sum the probabilities, however, to create a new entropy metric with the following formulation:

$$E = -\sum_{\text{all } k}\left[p_{k_{sys1}} + p_{k_{sys2}}\right]\ln\left[\frac{p_{k_{sys1}}}{A_{sys1}} + \frac{p_{k_{sys2}}}{A_{sys2}}\right]$$

where k refers to the cell number, sys1 and sys2 refer to the two awareness gathering systems, and A refers to the area of the grid cells for each system.

There must be a coherent mapping of the two systems, such that a target located in cell 1 or 2 for system 1 corresponds to a target located in cell 1 for system 2. This formulation is analogous to the formulation used in basic information theory for communications networks, when two systems operate independently. The implication in this formulation is that the two systems gain awareness independently.

The resulting value for awareness in this formulation is 0.575, which is less than the value from either system independently. This implies an awareness that is greater than from either system singly.

If the target is located erroneously by one system as the other system locates it correctly at the same time, as in figure 3-4, we would expect entropy to be increased. This formulation indeed produces such a result, giving an entropy value of 2.079, essentially the sum of the two entropy values.

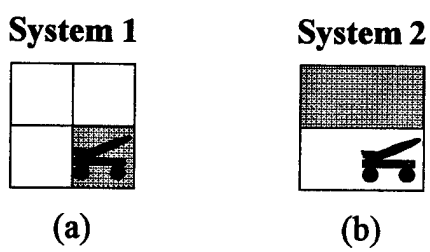

Figure 3-4: Erroneous awareness. The shaded areas represent cells with probability = 1.0.

This formulation, while provocative, is not flawless, however. When the formulation is applied directly to the scenario in Chapter 2, the resultant values do not provide the same insight. If, for instance, the satellite and JSTARS image the target simultaneously, each locating the target correctly within the respective cells, as in figure 3-2, the entropy for the satellite is 1.21, as calculated in Chapter 2, and the entropy for

JSTARS is 5.1. When we add the two probabilities using the formulation above, however, the resultant entropy is 2.38, *greater than the entropy for the satellite alone.* This implies the awareness from two sources is not as good as awareness from the satellite alone.

This bears discussion over the nature of awareness. As defined in chapter 2, awareness is not only what is known, but how well it is known. No mention is made of veracity; it has essentially been assumed throughout this thesis. Thus the differentiation between accuracy and precision is made clear. The satellite may be precise, but it may be precisely wrong. Entropy does not differentiate.

Of more utility is the multi-dimensional entropy surface, in which a threshold for action requires not just a value but a vector of values, including entropy values from each source. In the case presented in figure 3-4, in which one source had misrepresented the location of the target, the threshold for the attack could be represented by a vector of the form $[E_{sys1}, E_{sys2}, E_{sys1+sys2}]$, which demands that each determines the location to within the appropriate resolution of the system, and that both agree on that position. Actual location is thus verified, *per se*, when individual sensors reduce entropy below threshold values, but the combined value is not below the appropriate value. When the value for the combined entropy is below a separate threshold, not necessarily lower than either one singly, as in the scenario, that determines the accuracy of the awareness. A disagreement between the awareness from the different sources may simply prescribe that more awareness be obtained prior to pursuing an attack. In any case, more work needs to be done on the requirement for confirmation of awareness, but it is likely to be of the vector form mentioned above.

Perhaps an even more valuable result of such analysis falls under the heading of information warfare. If we know the enemy's awareness threshold for action on the battlefield, we can determine how and when to introduce uncertainty into his awareness-gathering system such that it will either prevent him from taking action or require him to gather more awareness. This is precisely the desired effect Boyd spoke of when he suggested that by getting "inside the adversary's O-O-D-A loops, [we could] deny [the] adversary the opportunity to cope with events/efforts as they unfold." [Boyd, 1987]

3.5. SUMMARY

The awareness analysis methodology developed to analyze the simple scenario in Chapter 2 has a robust capability to generate insights into much more complex awareness systems. The examples presented here only touch the surface of the power of this methodology. The underlying quantization of the battlefield is open to many different interpretations, depending on its use, all with small relative errors. An analysis of the awareness of multiple targets on a battlefield points to the complexities inherent in real-world analysis of the decision cycle and awareness mismatches in a C2 system It is likely best analyzed with a vector of awareness values. A confirmation of awareness requirement, a real world limitation, is another extension ripe for further research, likely best handled with a combination of entropy addition and awareness vectors.

The awareness analysis methodology developed in Chapter 2 can be extended well beyond the areas outlined here. Further extensions to the research presented may include implementing the methodology in an aggregated combat simulation that uses different elements and resolutions of awareness to help determine optimum update requirements; analysis of an actual combat scenario using classified data to determine

possible awareness constraints and decision cycle inefficiencies; and analysis of enemy capabilities to determine opportunities for misinformation and masking in an information countermeasures scenario.

3.6. CONTRIBUTIONS

The basic ability to compare elements of awareness of different resolutions and different time schedules on one scale is a new capability itself. The added ability to compare requirements for awareness users can be used to develop unforeseen insights into combat operations. The interaction of different levels of command and control is crucial to the warfighting effort. This methodology quantifies that interaction with regard to a critical resource, awareness; it demonstrates opportunities to increase performance in the C2 system; and it can help show areas most vulnerable to attack. The only limitation to the insights to be gained by using this methodology is in the resolution or detail we seek from the analysis.

APPENDIX A: LITERATURE REVIEW

A.1. PROBLEM BACKGROUND

Typically, armies of the earliest warfighting eras were physically led by their commanders in a show of moral support that included little in the way of essential communication and delegation. [Van Creveld, 1985: 17-18] Awareness applied to a man as it applied to the army, so little formal command and control (C2) structure was required. The growth of C2 (and accompanying letters in the acronym) is due to a number of factors:

> (a) the increased demands made on command systems by present-day warfare;
> (b) technological developments that have multiplied the means at the disposal of command systems;
> (c) changes in the nature of the command process, resulting from the interaction of factors (a) and (b);
> (d) the appearance of new weapons systems that, when coupled with structural changes inside command systems themselves, have increased the vulnerability of command systems;
> (e) the rise in costs, caused by factors (a) through (d). [Van Creveld, 1985: 1-2]

It is this last characteristic of the modernization of command that has produced the question to which this research can provide insight. In an age of fixed and even decreasing defense spending, some elements of the force structure will be sacrificed in order to afford only those elements with the largest contribution to mission success. As Gen. Howell M. Estes, III, Commander-in-Chief, US Space Command, said,

> "Hard choices need to be made between investments in information infrastructure [and] the combat systems themselves. This is an extreme dilemma, because combat systems, without timely, relevant information, are useless. On the other hand, you can't take out an enemy tank with just information." [Scott, 1998: 59]

The evolution of this fight for defense dollars is explained quite well by Bjorklund [1995: 16-48]. To summarize, as Van Creveld mentioned, increased technological capability translates into increased cost. The elements of command and control, though essential to all branches and divisions of the Department of Defense, are nevertheless owned by no one party. The Air Force fights for newer and more capable fighters, the Navy fights for newer and more capable ships, and the Army fights for newer and more capable tanks, but, until recently, no one fought for newer and more capable computer monitors or faster CPUs. If one were to fight for these elements of C2, one would be hard-pressed to show unequivocally how more enemy tanks or ships or fighter would be destroyed with a faster CPU or a larger radar monitor. Furthermore, the fight to introduce new and previously undemonstrated technologies into the battle is an even more difficult challenge with no historical proof of force enhancement.

The development of the US Air Force Space Command has taken over control of the space-based information systems, providing advocacy for their benefits to the battlespace. Still other elements of awareness have few direct advocates and thus force awareness is difficult to advance. The only way to attract support for such technologies is to show how they increase success in battle.

A.2. EARLY ATTEMPTS TO MEASURE THE EFFECTS OF COMMAND, CONTROL, COMMUNICATIONS, COMPUTERS, INTELLIGENCE, SURVEILLANCE, AND RECONNAISSANCE (C4ISR)

Long the basis for combat models, the Lanchester equations offer an analytical solution to many combat scenarios. In his explanation of his square law, Lanchester explained:

> With modern long-range weapons—fire-arms, in brief—the concentration of superior numbers gives an immediate superiority in the active combatant ranks, and the numerically inferior force finds itself under a far heavier fire, man for man, than it is able to return...consequently, the number of men knocked out per unit time will be directly proportional to the numerical strength of the opposing force." [Lanchester, 1916: 2139-2140]

Schreiber [1964: 507-510] later used Lanchester's square law equations to measure the effect of having a certain level of awareness of the opponent. He reasoned that if a fighting force had better information on the position of the enemy, it could better direct its fire:

> The effectiveness of the intelligence and command and control systems in this type of battle can be measured by the fraction of the enemy's destroyed units from which fire has been redirected. If this fraction is one, fire is always directed only at the enemy's surviving units and no "overkilling' results; if it is zero, fire is directed all during the battle against the original enemy positions, and much of it is wasted in 'overkilling.' This fraction will be called the 'command efficiency,' and is assumed to be constant throughout the battle. [Schreiber, 1964: 507]

In this method, a 'command efficiency' can be input into the mathematical model, reflecting some level of awareness of the battle damage inflicted on the enemy and a capability, by the C2 system to re-target firepower. A measure can then be made of the effect of this level of command efficiency. Studies indicate this measure of 'command efficiency' can result in an increase in effectiveness corresponding to a numerical strength increase of up to approximately 41%. [Schreiber, 1964: 510] No method exists of directly measuring 'command efficiency' of an existing C2 system prior to its use, however. [For a brief overview of Lanchester's linear and square laws, see DARCOM Pamphlet 706-102, Engineering Design Handbook: Army Weapon Systems Analysis, Part Two.]

Schutzer [1982: 119-144] used a form of Lanchester's equations to measure the effects of improved C2 systems, both enemy and friendly, on attrition. He compared exchange ratios with and without C2 enhancements to determine a force multiplication factor. This factor is a measure of the relative strength of the C2-enhanced force compared to a non-C2-enhanced force, and can be indirectly used to determine initial force requirements necessary to attain a given measure of effectiveness (MOE). While the rigorous mathematics generally prove that a force with C2 enhancements is better than one without, he gives little detail of the necessary means by which to attain these enhancements.

Dupuy [1985] took great pains in developing his Quantitative Judgment Method of Analysis of Historical Combat Data. This involved a calculus by which he could measure a host of influential effects quantitatively, insert them into a series of equations to ultimately calculate Combat Power Potential Values for each side, and compare the ratio of the two values to determine a likely victor. He exercised his calculations for a number of historical battles and compared his results to each battle's outcome. He concluded, with some qualitative analysis, that this method could be quite useful in determining dominant forces in future battles.

Dupuy's influencing factors, however, fall short of being able to account for elements of awareness. He lists 73 factors, but highlights important, "intangible" factors, such as Intelligence, Technology, Time, Space, and Leadership as being either "probably individually incalculable" or "probably incalculable; not yet calculated." [Dupuy, 1985: 22] These are perhaps the essential elements that produce awareness: Intelligence, a measure of the enemy's position and capability; Technology, a measure of the capability

of the links that transmit information to and within a C2 structure; Time and Space, measures of friendly location and capability; and Leadership, perhaps the most vital measure of a friendly forces capability and direction. Clearly, without a way to quantify these values for an existing or postulated force, such equations are hard-pressed to calculate a measure of awareness.

Bjorklund [1995: 197-220] used an Analytical Hierarchy Process as a means to analyze a commander's or decision-maker's preference for different types of equipment (e.g., fighters or tanks vs. C2 improvements) in wartime scenarios with varying degrees of uncertainty. The assumption is that with greater uncertainty, a higher value is placed on the increased information that can be obtained with the C2 assets. Consequently, C2 assets tend to be preferred in a higher risk scenario than in a lower risk scenario. Bjorklund went on to put this value calculation into a budgeting problem to determine the best use of defense dollars to optimize performance in a given risk scenario.

Bjorklund's analysis hinges on eliciting relative value responses from assorted commanders at various levels on the command structure and technical experts. The analysis is specific to certain force-on-force scenarios which are postulated beforehand. He makes no measure of the probability of the likelihood of a particular risk scenario, implying all are equally likely. He further conducts his analysis as a measure of the complete force structure, not just a measure of incremental improvement. His conclusions are illuminating, and his analysis is provocative, but his results are hypothetical and without real-world rigor.

Finally, Lawson [1982: 64-69] modeled C2 as "a cybernetic system which is attempting to control the environment around it." He created a loop in which the system

"Senses" its environment, "Compares" it with a desired state, "Decides" what must be done to change the system to make it more like the desired state, and then "Acts" on this decision. He amplified the process by adding another step between "Sense" and "Compare," that of "Processing" the information "Sensed." He used this to analyze C2, but he never realized concrete results. He makes suggestions for the use of his theoretical model for future research.

A.3. BOYD'S OBSERVE-ORIENT-DECIDE-ACT (O-O-D-A) LOOP

Boyd [1987: 23] defines a decision loop as being a four stage process: the system first observes; it then orients itself based on this observation; it decides on a optimal course of action; finally it acts. The process begins again. He further calls this process a command and control loop, since these are the actions that take place during the process of command and control.

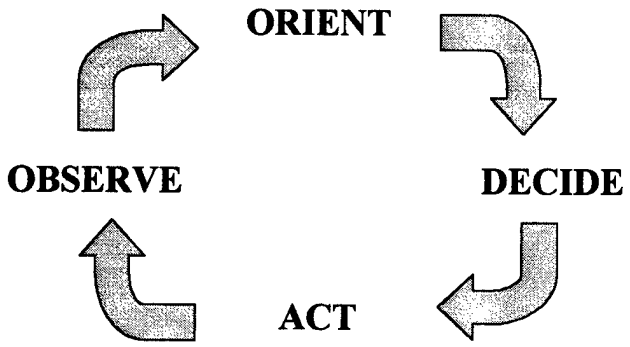

Figure A-1: Boyd's O-O-D-A Loop

Boyd's intent was to emphasize the human element in modern command and control systems over hardware. He claimed that by either having a system that could execute the processed involved in the O-O-D-A loop faster than the enemy, or inundating

the enemy with too many possibilities and bring the enemy's decision loop to a standstill, the friendly force could emerge victorious. This amounted to "[getting] inside the adversary's O-O-D-A loop." [Boyd, 1987: 23]

Boyd's O-O-D-A loop and command and control theory is discussed extensively in Appendix B.

A.4. ENTROPY

Shannon [1949] first developed the equation for entropy that became the basis for information theory. He established it as a measure of the information content of messages and the capacities of channels for transmission. His original equation is

$$H = -K\sum_{i=1}^{n} p_i \log(p_i)$$

Where K = a scaling constant
p_i = probability of outcome i

The equation was based on the thermodynamic concept of entropy, but it has been applied in different ways. *K*, therefore, is not Boltzmann's constant, but takes on various values depending on the application. Shannon's original formulation used $\log_2 p_i$. The base 2 is a natural choice when dealing with binary information variables.

This formulation has several features that make it particularly suited for its use. [Reza, 1994: 80-84]

(1) *Continuity.* If the probabilities of the occurrence of events E_k are slightly changed, entropy varies accordingly in a continuous manner.

(2) *Symmetry.* The entropy function is functionally symmetric for every combination of probabilities, p_k. That is, the order of the events is immaterial to the value for entropy.

(3) *Extremal Property.* When all events are equally likely, implying the most uncertainty in the outcome, the value for entropy is a maximum.

(4) *Additivity.* The total entropy for a sample space is equal to the sum of its parts, even when one or more possible outcomes are further divided into disjoint sets. (This property does not imply nor contest the comparative entropy formulation developed in Chapter 2.)

Sherrill and Barr have most recently applied the entropy concept to a quantized battlefield in a study of the contribution of awareness to battlefield results. [Sherrill and Barr, 1996: 17-33] Their research used entropy as a measure of the information subjects participating in a wargame had about the location of enemy forces. Their research goal was to show a direct effect that having more information had on battle outcomes, and entropy provided a convenient and mathematically proven method to measure the information. They developed the equation to describe a quantized battlefield cut into discrete grid cells for analysis. At maximal awareness, the value for entropy is 0: the enemy force is known to be at a specific location, or in a known grid. The probability the target is in the known cell is 1, and all other probabilities are zero. At minimal awareness, the enemy could be anywhere on the battlefield, with equal probability. The corresponding value for entropy in that case is ln(n), where n is the number of cells in the battlespace grid. Sherrill and Barr calculated the entropy (which is inversely proportional to awareness) at each stage in their experiment, and used that to determine an effect on battle outcomes.

Sherrill and Barr scaled their entropy formulation to a battlefield of known dimensions with the inclusion of an area term in their entropy calculation. By scaling

entropy in this way, they were better able to describe the awareness of the subjects at each phase of their experiment. They also developed a combinatorial argument for the calculation of entropy for a group of elements of which information for only a limited number was provided.

A.5. THE MULTIPLICATIVE LAW OF PROBABILITY

Essential to both Sherrill and Barr's combinatorial argument and this paper's development of a formulation of entropy when awareness confirmation is required is the multiplicative law of probability. The multiplicative law states that for two events A and B, the probability of both A and B occurring is equal to the probability of A occurring times the probability of B occurring given A has occurred. [Wackerly, et al., 1996: 50] The conditioning can be on either A or B. Figure A-1 shows the Venn diagram for the intersection of the two sets of events A and B. Mathematically, the formula is:

$$P(A \cap B) = P(A)\, P(B \mid A)$$

$$= P(B)\, P(A \mid B)$$

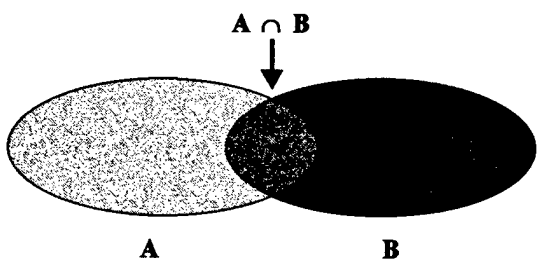

Figure A-2: The Multiplicative Law of Probability: The intersection of A and B.

A.6. MARKOV CHAINS

Markov chains are a highly useful tool for describing the conditional distribution of the future state of a system given only the present state.

...[C]onsider a stochastic process $\{X_n, n = 0, 1, 2, \ldots\}$ that takes on a finite or countable number of possible values. Unless otherwise mentioned, this set of possible values of the process will be denoted by the set of nonnegative integers $\{0, 1, 2, \ldots\}$. If $X_n = i$, then the process is said to be in state i at time n, We suppose that whenever the process is in state i, there is a fixed probability P_{ij} that it will next be in state j. That is we suppose that

$$P\{X_{n+1} = j \mid X_n = i, X_{n-1} = i_{n-1}, \ldots, X_1 = i_1, X_0 = i_0\} = P_{ij}$$

for all states $i_0, i_1, \ldots, i_{n-1}, i, j$ and all $n \geq 0$. Such a stochastic process is known as a *Markov chain*. [The equation above] may be interpreted as stating that, for a Markov chain, the conditional distribution of any future state X_{n+1} given the past states $X_0, X_1, \ldots, X_{n-1}$ and the present state X_n, is independent of the past states and depends only on the present state.

The value P_{ij} represents the probability that the process will, when in state i, next make a transition into state j. Since probabilities are non-negative and since the process must make a transition into some state, we have that

$$\mathrm{P_{ij}} \geq 0, \qquad \mathrm{i, j} \geq 0; \qquad \sum_{\mathrm{j}=0}^{\infty} \mathrm{P_{ij}} = 1, \qquad \mathrm{i} = 0,1,\ldots$$

Let **P** denote the matrix of one-step transition probabilities P_{ij}, so that

$$\mathbf{P} = \begin{Vmatrix} P_{00} & P_{01} & P_{02} & \cdots \\ P_{10} & P_{11} & P_{12} & \cdots \\ \vdots & & & \\ P_{i0} & P_{i1} & P_{i2} & \cdots \\ \vdots & \vdots & \vdots & \end{Vmatrix}$$

[Ross, 1993: 137-138]

The Markov chain is used to represent the transition from state to state within a system over a discrete time step. The initial state is described by a row vector with the probability spread across all possible states. The total probability always sums to one. In describing the diffusion of the location probability, when the location is known with certainty to within a single grid cell (or state), the location or state vector is represented as $\mathrm{I_0}^{\mathrm{T}} = [\ldots\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ \ldots]$. The state vector after one time step is calculated by multiplying $\mathrm{I_0}^{\mathrm{T}} \times \mathbf{P}$.

This concept can also be used to represent the transition over multiple time steps.

We now define the n-step transition probabilities P_{ij}^{n} to be the probability that a process in state i will be in state j after n additional transitions. That is

$$P_{ij}^{n} = P\{X_{n+m} = j \mid X_m = i\}, \quad n \geq 0, \quad i, j \geq 0$$

Of course $P_{ij}^{1} = P_{ij}$. The *Chapman-Kolmogorov equations* provide a method for computing these n-step transition probabilities. These equations are

$$P_{ij}^{n+m} = \sum_{k=0}^{\infty} P_{ik}^{n} P_{kj}^{m} \qquad \text{for all } n, m \geq 0, \text{ all } i, j$$

and are most easily understood by noting that $P_{ik}^{n} P_{kj}^{m}$ represents the probability that starting in i the process will go to state j in $n + m$ transitions through a path which takes it into state k at the nth transition. Hence, summing over all intermediate states k yields the probability that the process will be in state j after $n + m$ transitions.

If we let $\mathbf{P}^{(n)}$ denote the matrix of n-step transition probabilities P_{ij}^{n}, then [the Chapman Kolmogorov equations assert]

$$\mathbf{P}^{(n+m)} = \mathbf{P}^{(n)} \bullet \mathbf{P}^{(m)}$$

Where the dot represents matrix multiplication. Hence, in particular,

$$\mathbf{P}^{(2)} = \mathbf{P}^{(1+1)} = \mathbf{P} \bullet \mathbf{P} = \mathbf{P}^{2}$$

And by induction

$$\mathbf{P}^{(n)} = \mathbf{P}^{(n-1+1)} = \mathbf{P}^{n-1} \bullet \mathbf{P} = \mathbf{P}^{n}$$

That is, the n-step transition matrix may be obtained by multiplying the matrix $\mathbf{P}$ by itself n times. [Ross, 1993: 140-141]

To calculate the probability of the target being located in each cell after n time steps, multiply $I_0^{T} \times \mathbf{P}^{n}$

APPENDIX B: COMMAND AND CONTROL AND THE OBSERVE-ORIENT-DECIDE-ACT LOOP

Colonel John Boyd, in his Discourse on Winning and Losing [1987], founded a fundamental theory of decision making that formalized the process into four connected elements: Observation, Orientation, Decision, and Action (O-O-D-A). This decision cycle, represented in figure B-1, has become the model for the Department of Defense's command and control system representation. The O-O-D-A loop appears in Joint Publication 3-13.1, Joint Doctrine for Command and Control Warfare.

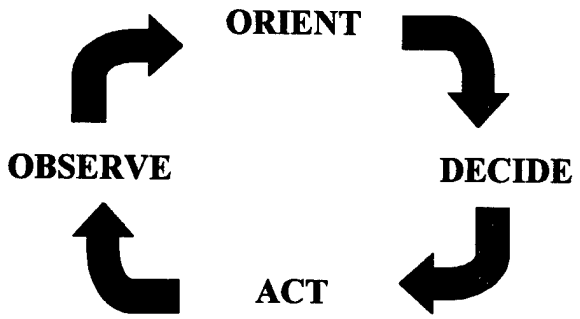

Figure B-1: Boyd's Observe-Orient-Decide-Act (O-O-D-A) Cycle

The Publication further goes on to define each phase of the cycle in detail, even moreso than Boyd did himself. The definitions and explanations appear in figure B-2. As stated by Col Boyd and JP 3-13.1, the decision cycle is applicable to all command and control (C2) systems, friendly and enemy alike.

Though it is stated only implicitly by both Boyd and JP 3-13.1, a subtle but essential point to the decision cycle theory is the fact that this cycle takes place not just within the organization, but within each element of the organization. Every unit and every person tasked with decision and action proceeds along this cycle. Col Boyd says

a. Observation. In the observation portion of the decision cycle, the commander gathers information from the reconnaissance, surveillance, and target acquisition (RSTA) apparatus and from status reports of friendly forces. Much of a commander's RSTA capability and knowledge of the status of friendly forces will come from the control portion of the friendly force C2 system — that is, from subordinate commanders.

b. Orientation. In the orientation phase of the decision cycle, information about the opposition's status received in the observation portion of the cycle is converted into intelligence through the commander's intelligence staff. Based upon this intelligence and knowledge of the status of friendly forces, the commander will make an assessment of the "reality" of the operational area.

- The "reality" of the operational area is the actual situation in the operational area including, but not limited to, the disposition of forces on both sides, casualties to personnel and equipment suffered by both sides, the weather in the area, and morale on both sides.

- The commander's assessment of the "reality" of the operational area is based on the input of the commander's intelligence system, sensors and lower echelon commanders in the observation portion of the cycle. Since these sources of input are imperfect and subject to manipulation by the opposing side, the commander's assessment of "reality" will invariably be something other than the actual "reality" of the operational area.

c. Decision. The commander will make military decisions based on the assessment of the "reality" of the operational area. The decisions made by the commander will be communicated to subordinate commanders as orders via various communications methods.

d. Action. Subordinate commanders at all lower echelons, the control portion of the friendly force C2 system, will cause the commander's decisions to become actions that impact the "reality" of the operational area.

e. Continuity of the Cycle. Since the decision cycle is a continuous process rather than a step-by-step process, all parts of the cycle are active simultaneously. The commander will be gathering information, forming appraisals, and making decisions for future operations at the same time that current orders are being executed as actions by subordinate commands. The same cycle is occurring simultaneously for all opposing sides in an operation. The same cycle is also occurring at all subordinate levels at a scope commensurate with the responsibilities of the commander at that echelon. All of these decision cycles, on all sides and at all levels will impact the "reality" of the theater of operations on a continuous basis.

f. Size of the Cycle. The amount of time taken to observe, orient, decide and act is represented by the length of the arc between portions of the cycle. Consistent with classic military doctrine, the commander that can gather and process information and initiate action to affect the theater of operations quickest will have a decided military advantage. Conceptually, the ability to process information into action via the cycle at a quicker pace than the opposition can be thought of as getting "inside" the adversary's decision cycle by making the friendly force cycle smaller than the opponent's.

Figure B-2: Excerpt from Joint Publication 3-13.1, Joint Doctrine for Command and Control Warfare, Appendix A, pp A-1, 2.

explicitly that good strategy seeks to "get inside [the] adversary's O-O-D-A loops," but nowhere does he or JP 3-13.1 indicate the arrangement of these loops, or the interaction process by which elements within a C2 system operate.

In a military C2 system, an obvious action, or output, of the system is attacking a target by dropping a bomb, firing a bullet, or launching a torpedo. That is the action of both the entire C2 system and of the line element of that system: the airman, soldier, or sailor. The commander, clearly the driver of this action, however great his part in bringing arms to bear on a target, does not himself typically drop the bomb, shoot the bullet, or launch the torpedo. He depends on subordinates to perform this action. He observes the location of the enemy, orients himself as to the disposition of his and the enemy's forces, decides the best plan of attack, and *issues orders to attack.* The line unit or individual observes the location of his target, orients himself based on his disposition and the disposition of others around him, friend and foe, decides when best to pull the trigger (when he has the best opportunity to render the desired effect to the target), and acts by shooting. In light of this simple organization, it appears that a superior, non-line decision-maker "acts" by issuing orders that are received and interpreted in the "orientation" phase of the subordinate or line decision-maker.

The resolution of the O-O-D-A cycle—how many people or units constitute one O-O-D-A grouping—depends on the context of the decision and its scope. The Air Force can be said to operate on one decision cycle when the "act" is defend the skies. A ground soldier is a decision-maker when the "act" is shoot at the target.

Considering the fact that the commander may have several and varied units under his command, the organization or alignment of these decision cycles might best be

represented as in figure B-3. All levels observe the battlefield, each orients and decides, but superior-level decision makers act by influencing the orient phase of subordinates.

As JP 3-13.1 states, "all parts of the cycle are active simultaneously." This applies to each decision cycle as well as the hierarchy of decision cycles. Just as the line soldier holds the target centered in his gunsight, he continues to orient and decide. Based on his observations and his orientation, or guidance via orders from the commander, he decides the optimal time to shoot. His orders may be to shoot when the target comes out from behind cover, but when the target begins shooting at the shooter, and is observed

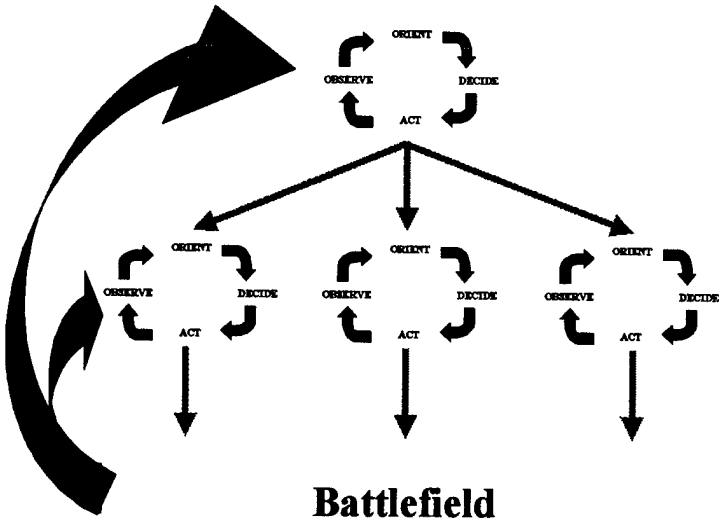

Figure B-3: Linked O-O-D-A Loops

doing so, the shooter re-orients himself, based on prior training and orders emphasizing self-protection, and decides to return fire immediately. (In this case, the decision might be said to be dictated by the enemy's action. In fact, a decision is always influenced by observation.)

Furthermore, as the line soldier is shooting at the enemy shooter, the commander is observing the pace of the battle, if not the individual engagements, is re-orienting himself and his forces, and is making future and follow-on decisions. All of the elements of all the decision cycles happen simultaneously. Each element, however, and each link between elements requires time.

The arrangement of the processes in the O-O-D-A loop, and the arrows that connect them, imply a flow of something from one process to another, and an action, for lack of a better term, at each process. Presumably, value is added at each step in the loop, and that value is transmitted to the next step. The difficulty is in ascertaining what the valued quantity is that is being transmitted.

At least part of this quantity is information. From the "observe" to the "orient" phase, that information is the raw imagery or the data from intelligence reports. The product of the commander's "orient" process is an assessment of "reality," to use the Joint Staff nomenclature [JP 3-13.1, App. A]. The decide phase produces knowledge about how to act, and the act itself produces a response from the enemy (or even friendly forces), which is observed in the next phase. Furthermore, the time taken to go from phase to phase causes the information about a dynamic battlefield to lose value.

In the conduct of warfighting operations, a more esoteric quality of that information is awareness. As defined in this paper, awareness is a measure of how well something is known, be it enemy location, enemy intent, or even friendly posture. This paper concentrated on the use of awareness in the context of enemy position on a two-dimensional battlefield, but clearly anything that can be known can be known to some measurable degree, and that quality is awareness as measured by entropy.

In a warfighting decision process, though the final act is rarely known at the time the battlefield is initially observed, the process that begins with observation and ends with line action can be modeled using the linked O-O-D-A construct. For the analysis accomplished in this paper, the awareness that precipitated an attack on a mobile missile launcher was tracked throughout the process. Initially, the target was imaged by satellite and the resulting awareness was made available to the commander. It was not always thus, however.

In the earliest days of combat, rank only, not geographical position, differentiated leaders from the led. As the fighting troops (which often included the commander) observed, so did the commander. Division of labor and the complexity and massiveness of warfare led to specialization, which more frequently found a commander physically separated from his command. In such a system, the commander depended on his force to observe the enemy as they were engaged, and to report back to him these observations.

In such a system, the linked O-O-D-A construct is still valid. In this case, however, additional connections need to be made from the "observe" phases of lower levels to the "observe" phases of the commanding levels, creating a chain of O-O-D-A loop links. In combat, front-line troops made observations on the enemy, the battlefield, and even friendly forces and forwarded those observations "up the chain" to the decision-maker. This connection that fed awareness to the commander, however, required no small amount of time to traverse, meaning the information the commander had to make decisions had lost significant value. By the time his act, or commands, reached the front-line troops, they were likely ill-suited to a dynamic battlefield. His best hope was to

arrive on the battlefield with the element of surprise, giving undiminished value to his awareness and therefore his commands.

The fundamental change in warfare [Rice, 1996] is not such a change after all, in this light. With the newest technologies, commanders have access to near-real-time battlespace awareness. The change is that now the commander can again observe the battlefield as his troops observe the battlefield, and in some cases, in even more detail. The nature of warfare itself has not changed, only the way it is perceived by the fighting force.

Vincent [1993] touched on this concept in his discussion of a cybernetic command and control system. He submitted that the C2 system of the future would be a "massively parallel design [that] creates Boyd loops that run in parallel and never overlap or run in sequence. Each loop is small and compact, minimizing the 'friction' in the entire system." (Friction as defined by Clausewitz.) This design, however, is not the goal of future C2 systems, but the implicit goal of C2 systems of the past.

With the time disadvantage of the pre-modern (less technologically enhanced) battlefield, which separated the commander from his command yet depended on others for awareness, it was crucial that the fighting troops be so rigorously trained and drilled that little chance could exist that they would be faced with a situation for which they needed immediate guidance. The pre-modern fighting soldier was forged in a pattern that required him to always act and react in a logical and calculated manner. A dependence on an orientation from above based on current information about a rapidly changing battlefield meant paralysis. Given the goal of his battle, and a clear vision of the enemy and the means of combat, the pre-modern fighting soldier needed little battlefield

guidance, only expert training and access to usable battlefield awareness. The commander's job was to ensure the soldier was trained and equipped to carry out his mission. Indeed, through the early 1990s, the mission of the US Air Force was to "organize, train, equip, and provide forces for the conduct of prompt and sustained combat operations in the air," *not* to win aerial battles. [Department of the Air Force, 1992]

Training and drill is the peacetime linkage between the commander and subordinates. The link still is the commander's "act" phase and the subordinates' "orient" phases, but it is a link exercised in peacetime to foster subordinate decision-making abilities and to practice acting on a simulated battlefield. The importance of this was not lost on Boyd in his original work: "The second O, orientation—as the repository of our genetic heritage, cultural tradition, and previous experiences—is the <u>most important part</u> (Boyd's emphasis) of the O-O-D-A loop since it shapes the way we observe, the way we decide, the way we act."

In today's technologically enhanced environment, in addition to organizing, training, and equipping forces, commanders are better able to actually command during combat, armed with more timely information and closer (time-wise) ties to the battlefield soldier. The current Air Force mission statement is now far more broad: "To defend the United States through control and exploitation of air and space." [Department of the Air Force, 1996] This does not preclude the need for rigorous training, but it gives commanders the ability to better manage (some would argue micro-manage) forces in a rapidly changing battlefield environment. It brings warfare back to its earliest days, before the complexity of combat, when the commander observed through the troops.

Today's technology overcomes the obstacle of complexity that previously formed a boundary around the commander.

Vincent suggested that his version of the cybernetic C2 system is "a case of using high technology to unleash lower-level initiative." In fact this organic system has its roots in Boyd's original work:

> A review...suggests that, for success over the long haul and under the most difficult conditions, one needs some unifying vision that can be used to attract the uncommitted as well as pump up friendly resolve and drive and drain-away or subvert adversary resolve and drive. In other words, what is needed is a vision rooted in human nature so noble, so attractive that it not only attracts the uncommitted and magnifies the spirit and strength of its adherents, but also undermines the dedication and determination of any competitors or adversaries. Moreover, such a unifying notion should be so compelling that it acts as a catalyst or beacon around which to evolve those qualities that permit a collective entity or organic whole to improve its stature in the scheme of things. Put another way, we are suggesting a need for a supra-orientation or center-of-gravity that permits leaders, and other authorities, to inspire their followers and members to enthusiastically take action toward confronting and conquering all obstacles that stand in the way. [Boyd, 1987: 143]

Prior to the high-technology revolution, Boyd foresaw the need for a unifying vision to "improve fitness as an organic whole." Without the vocabulary we now possess, he was suggesting the ideal force become a complex adaptive system, with the ability to organize and guide itself in response to any situation. The implications of this structure for command and control, which exists today in various forms, are huge and ripe for exploitation, but far beyond the scope of this research.

The links represented in figure B-3 are not the only links between O-O-D-A loops, however. In the Chapter 2 scenario, JSTARS arrives on station with orders to update the target's position. Its act is image the target. It then observes the display. The observation phase is obvious, but it's orient phase is again one that requires processing, to

confirm the existence of the target and its location. After deciding that the information is accurate enough to relay, the new act JSTARS performs is relaying the position to the fighter carrying the bomb.

This relationship between JSTARS and the fighter links the act phase of one line unit with the orient phase of another line unit, a lateral connection. The actual arrow on a chart may tend to blur the rank in the C2 hierarchy on paper, just as this rank is blurred operationally. It is not uncommon, from operational experience, for those in possession of a "bigger picture," or a critical awareness link, to lord over those without, in effect using awareness-gathering ability as a measure of battlefield rank. The immediate reaction to this is to create an equal level of awareness available to all members of the hierarchy, but this response is not beneficial to the C2 system.

The precipice on which we find ourselves, having been carried by the new technologies, overlooks a battlespace of which we can be aware far faster than ever before and in stunning detail. The temptation is to try to gain awareness of all that we can, in all the detail we can, and pipe that awareness through all the links in the command and control system in an attempt to create the ultimate organic fighting machine. The mismatch in capabilities and requirements, however, all but guarantees failure for such a system, whose means do not match its needs.

The goal of effective command and control, in Boyd's theory, is to "operate at a <u>faster tempo or rhythm</u> than our adversaries—or, better yet, get inside [the] adversary's <u>Observation-Orientation-Decision-Action time cycle or loop</u>." [Boyd, 1987: 5] By gaining awareness to the limits of awareness and distributing it throughout the chain of command and control, the C2 system inevitably provides awareness to entities that

cannot use it. Since each element of the O-O-D-A process requires time, and the transition between phases also takes time, that increased time translates to a slower tempo or rhythm, thereby directly obliterating the desired effect of operating inside the enemy's O-O-D-A loops. The new goal, therefore, is more effective C2 in light of the new technologies.

Boyd further stated a tactic of

> Operat[ing] inside the adversary's O-O-D-A loops... to create a tangle of threatening and/or non-threatening event/efforts as well as repeatedly generate mismatches between those event/efforts adversary observes, or anticipates, and those he must react to, to survive. [Boyd, 1987: 131]

This goal of this tactic is not only to operate inside the adversary's O-O-D-A loop by operating at a faster tempo than the adversary, but also to get inside and become aware of the operation of the adversary's decision cycle, and use such knowledge to create the "tangles" and "mismatches." Boyd further stated that the effect of this would be to "disrupt his operations,...overload his system, [and] produce paralysis." [Boyd, 1987: 133] By knowing when the adversary updates his awareness and the level of awareness the adversary requires, we are better able determine how to create mismatches between what the adversary needs to know and what he can know by observing our forces. If he consistently assumes he needs to reduce entropy below a certain threshold value, we can work to ensure he cannot maintain that level of entropy. We can do this by either making him think he needs more awareness than he does (causing him to set a lower entropy threshold than necessary), or by introducing uncertainties and increased entropy values into his awareness-gathering mechanisms, prohibiting him from attaining or maintaining

adequate awareness. His constant quest for more and better awareness produces the mandated paralysis.

By looking inside our own decision loops and analyzing the way we update our awareness, we can better prepare our forces for quick action and reaction. Examining awareness requirements for specific elements of command and control lets us better focus efforts on obtaining the appropriate levels of awareness at the appropriate time. In the scenario in Chapter 2, the realization that entropy needed to be reduced below the threshold of the fighter for its arrival meant JSTARS was not required on station until just prior to the fighter's arrival. That kind of analytical capability, common sense and experience notwithstanding, is crucial to focusing awareness gathering and using efforts efficiently.

The discrepancies in coordination during war in the past were based largely on the time that awareness took to filter down to line-level units who used the awareness to "act" on the enemy. During this time, the awareness itself lost value so that commanders were effectively isolated from the real-time or tactical war and forced to exert influence on a more strategic level. Now links are such that the commander can fight the real-time war. The same level of influence, however, does not abide by Boyd's principles.

Awareness needs to be focused commensurate with the mission, or the "act" to be accomplished. Given an enemy, a battlespace, and resources within the theater, the commander needs to determine what he needs to know, how well he needs to know it, and, most importantly, how the awareness will be used. The methodology presented here gives a tool for analyzing the resources and requirements of awareness to maintain dominance in a dynamic and highly technical battlespace.

APPENDIX C

This appendix explains in detail the methodologies involved in producing the results shown throughout this paper.

C.1. MODELING THE PROBABILITY FIELD AS A MARKOV CHAIN

A Markov chain can be used to represent the probability field as it changes over time, given a fixed probability transition matrix. For a 3 x 3 matrix, as depicted in Figure C-1, the probability transition matrix is a 9 x 9 matrix. The sparsity depends on the assumptions for modeling target movement. The cardinal movement assumption produces a matrix whose rows have at most four non-zero values, as chown in figure C-1. The eight-direction movement model produces a matrix whose rows have at most eight non-zero values.

1	**2**	**3**
4	**5**	**6**
7	**8**	**9**

$$\mathbf{P} = \begin{Vmatrix} p_{11} & p_{12} & 0 & p_{14} & 0 & 0 & 0 & 0 & 0 \\ p_{21} & p_{22} & p_{23} & 0 & p_{25} & 0 & 0 & 0 & 0 \\ 0 & p_{32} & p_{33} & 0 & 0 & p_{36} & 0 & 0 & 0 \\ p_{41} & 0 & 0 & p_{44} & p_{45} & 0 & p_{47} & 0 & 0 \\ 0 & p_{52} & 0 & p_{54} & p_{55} & p_{56} & 0 & p_{58} & 0 \\ 0 & 0 & p_{63} & 0 & p_{65} & p_{66} & 0 & 0 & p_{69} \\ 0 & 0 & 0 & p_{74} & 0 & 0 & p_{77} & p_{78} & 0 \\ 0 & 0 & 0 & 0 & p_{85} & 0 & p_{87} & p_{88} & p_{89} \\ 0 & 0 & 0 & 0 & 0 & p_{96} & 0 & p_{98} & p_{99} \end{Vmatrix}$$

Figure C-1: Three by three grid and corresponding transition matrix for movement in cardinal directions only.

Depending on the movement assumptions, the matrix may be quite difficult to construct. Unlike the scenario in Chapter 2, real world battlefields may not have a homogeneous probability transition matrix. In combat modeling, this information is

readily available, depending on the model. Regular grid terrain models, one of two major types of battlefield representation which are widely used in aggregated combat simulations, overlay a map of the battlefield with a grid of cells [Hartman, 1997]. Typically the cells are rectangles or hexagons. Each cell has attributes that describe the characteristics of the contained terrain. Movement speed through each cell is determined by a weighted calculation using the attributes of the cell. Different units and equipment use different weights or factors in each movement calculation. These calculations could be used to generate a transition probability matrix for each type of enemy unit for which awareness is sought.

In operational awareness calculations, a more accurate and more complicated representation relies on Intelligence Preparation of the Battlefield, which precisely models each grid cell with respect to transitability. Analogous to the attributes for the terrain grid cells in combat models, attributes can be estimated for sections of enemy-held or transited territory that describe probable movement routes and rates. Based on these attributes, transition probabilities can be calculated to fill a transition probability matrix.

The benefit of using a Markov representation for any battlespace transition is that it is always a two-dimensional representation of the state transition for a battlefield of any dimensionality. The cells themselves may be tagged with multi-dimensional identifiers, but as long as each can be represented with a single and unique identifier, it can be represented as one row and one column in a two-dimensional matrix. Furthermore, the field distribution after any number n of time steps can be directly calculated by raising the transition probability matrix to the *nth* power.

The drawback to using a Markov representation is its computational complexity. The matrix itself grows as the square of the number of cells, as in figure C-1. Each time step calculation requires a multiplication of a matrix and a row vector, or n^2 multiplications, where n is the number of cells. For future time step calculations, the matrix must be raised to a power equal to the number of time steps. Squaring a matrix results in individual multiplications summing to the number of cells cubed. For the 3 x 3 matrix in figure C-1, one time step calculation requires 81 separate multiplications, while a two time step calculation requires an additional 729 multiplications to raise the transition probability matrix to the second power. An accurate representation of a battlefield needs much more than 9 cells, producing a computationally very cumbersome mathematical overhead. The sparsity of the initial matrix can be exploited in the earliest time step calculations, but this advantage quickly disappears as the probability spreads throughout the matrix.

Another disadvantage to this representation, disregarding the existence of absorbing states, is the requirement to know before beginning the calculations the final size of the probability field. This ultimately corresponds to the number of time steps for which entropy is desired. Since the target can traverse adjacent cells only, this translates to a grid representation of $2n+1$, where n is the number of time steps, which requires a transition probability matrix of size $2n+1$ by $2n+1$.

For reasons of computational complexity, and for ease of spreadsheet modeling, a diffusion model was selected for probability calculations.

C.2. DIFFUSION MODELS

Diffusion can most easily be thought of as the spread of a fixed quantity of a substance. In the case of probability diffusion, this refers to the spread of the probability throughout the state space. At all times throughout the process, the probability sums to one. [Envision It!, 1998]

A diffusion model represents the rate of transition of a quantity. A simple one-dimensional model begins with the known quantity or probability at a certain point along a line and models its rate of movement along the line in both directions. For a fixed time step representation, the single dimension is divided into segments, and each segment is sampled at each time step to determine the amount of the original probability it possesses.

Basic to this representation is a rate of diffusion, or transition from one cell to another, or one state to another, at each time step. To simulate the rate of diffusion, we use a fixed rate of absorption within each discrete cell. This is expressed as a percentage that remains within a cell at each time step. The remaining percentage is distributed equally to adjacent cells. Just as a quantity can travel either to the right cell or the left in a one-dimensional diffusion model, so each cell absorbs from the right and the left. With the probability of 1.0 starting at a single cell, after one time step, the initial cell will have the specified percentage remaining, while the two adjacent cells will have half of the remainder each. Mathematically, the formula for calculating the probability contained in cell 3 (in figure C-2) after one time step is $p_{ii} \times x_3 + (1 - p_{ii}) \times (x_2/2 + x_4/2)$, where p_{ii} is the proportion remaining within a cell.

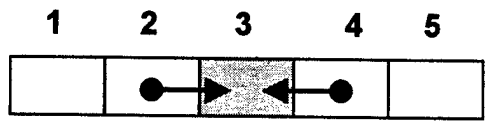

Figure C-2: One-dimensional diffusion. A cell absorbs from itself and the two adjacent cells in each time step.

In the two-dimensional model, with four cardinal movement directions, each non-exterior cell absorbs from four adjacent cells. With an equal probability of absorption from each adjacent cell, we can vary the rate of absorption into the cell by again specifying the rate or probability with which it leaves each cell. Since the probability in each cell in the interior diffuses into four other cells, the probability absorbed from each adjacent cell is one-quarter of the absorbable probability, if you will, or the probability amount that does not remain in the cell. In this manner, using figure C-3 as an example, the calculation for the probability absorbed into cell 13 after each time step is defined by the equation

$$p_{ii} \times x_{13} + (1 - p_{ii}) \times (x_8/4 + x_{12}/4 + x_{14}/4 + x_{18}/4)$$

Where p_{ii} is the probability, or rate, of staying within a cell in one time step, and x_j is the amount contained within cell j.

Correspondingly, the probability for each cell in the grid is described by the same type of formula. All interior points depend on absorption from themselves and four adjacent cells. Side cells absorb from themselves and three adjacent cells. Corner cells absorb from themselves and two adjacent cells.

To calculate multiple time steps, the grid is reproduced once for each time step, and each diffusion calculation refers to the grid for the previous time step. This can

become tedious and cumbersome, but the calculation is mathematically faster than using matrices and transition probabilities: each cell requires only six multiplications per time step for the four-direction movement model. At each step entropy can be calculated for each cell and the total for all cells summed.

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

Figure C-3: 5 x 5 Grid with all cells labeled. In a four-direction diffusion model, cell 13 absorbs from cells 8, 12, 14, and 18, and itself.

Intuitively, after an infinite number of time steps, the probability should have diffused such that it is a single value for all cells in the grid. In practice, this model does not generate such an ending state. Instead the edge cells all have slightly reduced probabilities. What is essential for a battlefield calculation, however, is not the existence of edge cells or states, but a measure of the area the target could possibly occupy, in terms of a number of resolution cells. By creating a grid large enough so as to avoid limiting the diffusion of probability in the time considered, we can avoiding the need for edge cells and avoid the errors they create. In essence it is equivalent to creating an

infinitely large battlespace. In practice it is limited by the length of time in question and the speed of the target, or its rate of state change.

The result of such calculations is a bivariate probability distribution, or a two-dimensional near-normal surface. This is a powerful tool that represents the probable location of a randomly moving target whose movement is restricted to one of four directions in any single time step. This somewhat complex surface, while theoretically useful, is in practice not transferred as awareness from one observer to another. Only a location or state and a time when that state was known are transferred. This surface, however, can be used to measure an estimated time to acquire, using a common search model such as the random search model or the glimpse model [Hartman, 1997].

This surface and its entropy was calculated in a spreadsheet in preliminary investigations of entropy as a function of p_{ii}, as described in Chapter 2, the probability the target will remain in a given cell from one time step to the next, and a function of time in steps. The resultant curves, shown in figure 2-6, demonstrate the increased rate of awareness loss with a faster moving target. The data were generated by creating thirty cell grids measuring 100 cells by 100 cells, each grid representing a single time step. Each cell used the formula of the form described previously for the two-dimensional diffusion model. The grid size enabled diffusion over a maximum of 50 time steps to be calculated, in the unbounded case. The rate of entropy increase appears to be of the form of a time-dampened exponential, though no specific equation could be fit with any reasonable degree of approximation.

A confined 20 x 20 grid was also modeled using this same algorithm, with similar results. When calculating more than 10 time steps, however, the diffusion reaches the

boundaries and the entropy calculation becomes bounded, reaching a maximum value in a finite number of time steps, depending on the value for p_{ii}. In this case, the approximating equation appears to be of the form

$$E_{approx}(t) = \ln(n) - \frac{\ln(n)e^{ct}}{(a\sqrt{t}+1)},$$

where t is time in steps, n is the total number of cells in the grid (400 in the case of the 20 x 20 grid), and a and c are both functions of p_{ii}. As mentioned in Chapter 2, this appears to be useful information, especially with regard to a confined battlespace. This effectively describes the entropy for a confined, bivariate near-normal distribution.

To simplify awareness calculations, to more accurately model what a C2 system knows, and to generate the awareness curve in figure 2-7, the actual surface was not used. Instead an unbounded, homogenized approximation of that surface was created, in which each cell that had a non-zero probability was given a single probability value. That probability was equal to the inverse of the number of cells with non-zero probabilities. That is, if the probability had diffused to 25 cells, so that each had non-zero probability values (all summing to one, of course), each was said to have a probability of 1/25, or 0.04.

Mathematically, this is a worst case for the entropy calculation, which reaches its extremum when all cells *k* have equal probability. Analytically, this is desirable. This gives an upper bound to entropy, or a lower bound to awareness, at each time step. In reality, it is unlikely that a moving target, having occupied a cell at time step *i*, would occupy that cell again after another *n* time steps. We assume, in effect, that if it is

moving, it is going somewhere. We do not need to know where it is going, although if we did we could better model its movement. We only need to know how fast it can move, or how frequently it changes states.

To create a homogenous surface, the only requirement is the number of cells that could contain the target. This can certainly be accomplished by the use of the diffusion model, counting all cells with non-zero probability values at each time step. This can more easily be accomplished, however, by observing the flow from time step to time step and creating a characteristic equation for the number of cells with non-zero probability at each time step.

Using the four-direction model, starting with one cell at time zero, the number of cells with non-zero values at each time step is shown in figure C-4. At each time step, the probability can diffuse in four directions. Since some cells already have a non-zero probability, they do not increase the total number of cells with non-zero probability values.

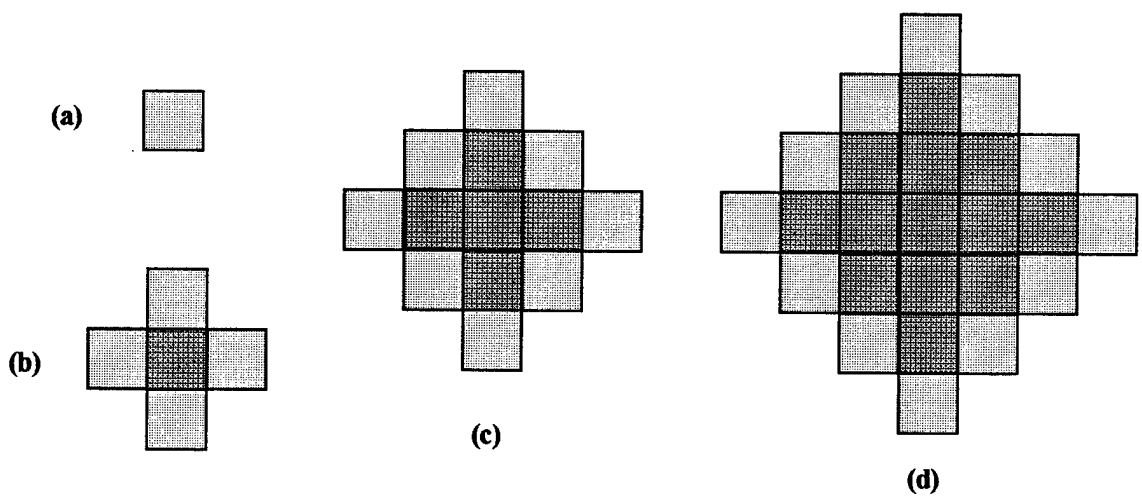

Figure C-4: Probability diffusion in four directions. Each grouping represents the number of cells with a non-zero probability value (a) initially (0 time steps) and after (b) one, (c) two, and (d) three time steps.

After one time step, the number of non-zero cells is five. After two time steps, it is 13, and after three time steps, it is 25. This corresponds to an increase of 4, 8, and 12 at each time step, respectively, or $4i$, where i is the time step number. So at each time step the number of non-zero cells is $n_{i-1} + 4i$, where n_{i-1} represents the number of cells in the previous time step. Equivalently, the number of cells at each time step in the eight-direction movement model, as described in Chapter 3, is $n_{i-1} + 8i$, and in the hexagon model, $n_{i-1} + 6i$. The number of cells and the formulae are shown in table C-1.

Table C-1: Homogeneous probability cell counts

Time Steps	Four-direction	Eight-direction	Hexagon
0	1	1	1
1	5	9	7
2	13	25	19
3	25	49	37
4	41	81	61
5	61	121	91
i	$n_{i-1} + 4i$	$n_{i-1} + 8i$	$n_{i-1} + 6i$

C.3. TIME STEP CALCULATIONS

The limitation to the previous model is the requirement that the probability can only diffuse to an adjacent cell in any single time step. This limitation is largely based on a movement model, in which states represent location, and the only way to get from one state to another is by traversing intermediate states between the two.

To conform to this constraint yet still make use of this model to model movement on a battlefield, we define each time step to be the minimum time required to traverse a single cell from side to side. This amounts to the worst-case estimate for the travel distance from the original position, which inflates the entropy calculation to its highest

value over time. Since the goal is to quantify how well we can know location, or the worst case boundary, this is a reasonable estimate.

In the case of the satellite imagery, the resolution cell is 6 feet on a side, or 1.83 meters. The mobile missile launcher, traveling at a maximum speed of 1 km per hour or 0.278 m per second, can traverse the resolution cell in 6.59 seconds. Every 6.59 seconds, therefore, equates to one time step for the reduction in awareness from an update from the satellite imagery. In 3 hours and 40 minutes, 2003 time steps have passed.

In the case of JSTARS awareness, each resolution cell is 42 feet on a side, or 12.8 meters. The missile launcher can traverse that range in 46.1 seconds. The 13 minutes from JSTARS imagery to the first F-15E map is therefore 17 time steps. In both cases, the entropy at each time step is calculated by counting the number of cells with a non-zero probability for containing the target, assuming an equal probability among all such cells, and applying the entropy formulation.

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13. ABSTRACT *(Maximum 200 words)*

In times of ever-tightening military budgets, methodologies are required that can compare the contributions of various systems involved in the warfighting process. While many tools are in use that directly measure the effects of greater numbers of enhanced hardware, and even improved processes, no validated methodology exists to measure elements that contribute to Command, Control, Communications, and Computers (C4); Intelligence, Surveillance, and Reconnaissance (ISR); or to analytically compare these elements with more traditional hardware.

This thesis develops a methodology for mathematically quantifying awareness in a military command and control (C2) environment. This methodology begins with the Observe-Orient-Decide-Act Loop to show the connections between levels of command and control, and to show influences. Entropy, in an information theory context, is modified to reflect not only how much is known at any level, but to show how well that information is known, producing a mathematically quantified measure of awareness. The awareness capability for various systems is calculated, and the rate of awareness loss is shown over time. Finally, an awareness curve is developed that shows the awareness of the C2 system throughout the process of attacking a ground target from the air.

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