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AUTHOR(S): Michael E. Jones and Lester E. Thode

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DIPO QUALITY INSPECTED
INTENSE ANNULAR RELATIVISTIC ELECTRON BEAM GENERATION IN FOILLESS DIODES

Michael E. Jones and Lester E. Thode

Intense Particle Beam Theory Group
Los Alamos Scientific Laboratory
Los Alamos, NM 87545

ABSTRACT

The generation of intense annular electron beams in cylindrical foilless diodes is analyzed. A beam equilibrium model is used to determine the diode current. At low voltage the current is very nearly that of the space-charge limited beam. However, at higher voltages the current can be substantially less than the space-charge limit. The analysis is compared to detailed particle-in-cell simulations.
A foilless diode consists essentially of a coaxial transmission line in which the center conductor is truncated to form a cathode. The outer conductor of the transmission line serves as the anode as well as a drift tube for the beam. A large magnetic field applied axially insulates the diode causing field emitted electrons to form a beam as shown in the insert of Fig. 1.

For the generation of intense beams, foilless diodes have several advantages over the more conventional foil diodes.\(^1\)\(^{-3}\) Impedance collapse associated with plasma formation from the foil is eliminated. Also, the beam is not scattered as a result of passing through a foil. An electron beam generated by a foilless diode is highly annular which is the optimal profile to avoid problems of space-charge for transport of intense beams in a vacuum drift tube. Moreover, the foilless diode has the potential of being used in repetitive operation.

Previously suggested applications of intense electron beams generated by foilless diodes include microwave generation, collective-ion acceleration, and magnetically confined plasma heating. In addition, foilless diodes have been used in radial pulse line accelerators.\(^4\) It has also been suggested that an intense, annular relativistic electron beam can be used to produce a \(10^{17}\)\(-\)\(10^{20}\) cm\(^{-3}\), multi-kilovolt plasma with sufficient power density to implode a small cylindrical liner.\(^5\)

Theoretical models of beam generation in foilless diodes have been of two types. In one case, Poisson's equation is solved in the infinite applied magnetic field and ultrarelativistic beam limits.\(^6\)\(^,\)\(^7\) Although these models yield scaling laws, they do not provide quantitative results for realistic geometries. The other model assumes that the diode current can be determined by the equilibrium that the beam
reaches in the hollow anode-drift tube.\textsuperscript{2,8} The present analysis is of the latter type. Previous studies have assumed the diode current is equal to the space-charge limited current based on low-voltage experiment evidence. However, there is no a priori reason to believe that the equilibrium that the beam obtains is the one which gives the space-charge limit. Indeed it is found in the present study that the diode current is the space-charge limited current only at low voltage while at higher voltages the equilibrium of the beam yields a current substantially less that the space-charge limit.

Most applications require that the electrons in the beam exhibit laminar flow. It is found in the present analysis that very intense and highly laminar beams may be achieved by arranging for the electrons to always flow along a large applied magnetic field. Thus, to a good approximation, the beam can be modeled as a relativistic cold fluid. The equilibrium equations in this model consist of the radial force balance equation together with Ampere's law and Poisson's equation. These equations, together with two more constraints, determine the equilibrium.\textsuperscript{9} One constraint comes about naturally for any beam that is generated by a diode. Because all the electrons are produced from an equipotential cathode, they must all have the same total (kinetic plus potential) energy in equilibrium. This condition can be expressed as $d\gamma/dr = -E_r$ where $\gamma$ is the relativistic factor of an electron at the radial position $r$. The radial electric field is denoted by $E_r$ and the system of units is chosen so that the speed of light and the charge and mass of the electron are all one. An exact analytical equilibrium solution can be obtained if, as a second constraint, all
the electrons in the beam are required to have the same axial velocity. The longitudinal and transverse energies for this model are defined as $y_{||} = (1 - \beta_z^2)^{-\frac{1}{2}}$ and $y_{\perp} = y/y_{||}$ where $\beta_z$ is the constant axial velocity.

Voronin, et. al. have used this equilibrium solution to model beam production in foilless diodes. In their analysis they assume that the inner radius of the beam is equal to the outer radius of the cathode, and the outer radius of the beam is chosen to give the maximum beam current. Thus, their model is that of a diode operating at the space-charge limit.

In the present analysis, the free parameters in the equilibrium are determined by assuming the electron flow is laminar and along the self-consistent magnetic field lines. Then conservation of canonical angular momentum for this equilibrium gives the relation

$$
(y_{e}^2 - 1)^{\frac{1}{2}} = (r_e B_{ze} - r_c B_0/r_e)/2
$$

(1)

where the "e" subscript denotes quantities evaluated at the outside edge of the beam. The cathode radius is $r_c$. The applied magnetic field is $B_0$ and $B_{ze}$ is the axial magnetic field at the outer beam radius. In most experimental situations, the applied magnetic field penetrates the conductors but the flux is unable to diffuse out on the time scale of the beam. Thus for flow along the field lines, the magnetic flux enclosed by the beam in equilibrium is equal to the flux enclosed by the cathode and the flux between the outer beam radius and the anode radius $r_a$ must be equal to the flux between the cathode and the anode. These conditions are expressed as
\[ r_c^2 B_0 = 2 r_e (\gamma_{le}^2 - 1)^{1/2} + r_i (1 - C^2)^{1/2} \quad (2) \]

\[ B_{ze} (r_a^2 - r_e^2) = B_0 (r_a^2 - r_c^2) \quad (3) \]

where \( r_i \) is the inner beam radius and \( C \) is a constant. Equations (1)-(3) together with Eqs. (4)-(7) of Ref. 8 constitute a "laminar flow" model for the foilless diode. They have been solved numerically to find the diode current as a function of voltage, anode and cathode radii, and applied magnetic field.

In the infinite magnetic field limit \( \gamma_i = 1 \) and the beam becomes infinitesimally thin with radius \( r_c \). The energy of the beam is given by \( \gamma = \gamma_a / (1 + 4 \ln r_a / r_c) \) where \( \gamma_a \) is related to the diode voltage \( V \) by \( \gamma_a = 1 + V \). The diode current, \( v \), measured in units of the Alfvén current (17 kA), in this limit is

\[ v = 2\left\{ [\gamma_a / (1 + 4 \ln r_a / r_c)]^2 - 1 \right\}^{1/2} \quad (4) \]

From the numerical solution of the equations, it was found that Eq. (4) is accurate to within about 10% for all magnetic fields greater than the critical field required to insulate the diode. Similarly, the infinite magnetic field limit for the space-charge limit model of Voronin, et al. also gives an infinitesimally thin beam but with \( \gamma = \gamma_a^{1/3} \) and thus, the familiar space-charge limited current formula\[11\]

\[ v = (\gamma_a^{2/3} - 1)^{3/2} 2 \ln r_a / r_c \quad (5) \]

Equations. (4) and (5) give the same value for the diode current when \( \gamma_a \) equals \( \gamma_{cr} \equiv (1 + 4 \ln r_a / r_c)^{3/2} \).
These models have been extensively tested using a $2\frac{1}{2}$-dimensional time-dependent, fully relativistic, electromagnetic particle-in-cell simulation code CCUBE. By solving for the full time-dependent evolution of the beam, the convergence problems of steady-state diode codes are avoided for the particularly demanding characteristics of the foilless diode. The simulations in Fig. 1 were performed for $r_a/r_c = 1.5$. The results agree well with the space-charge limit model at low voltage. However, at voltages such that $\gamma_a > \gamma_{cr}$ the simulation results follow the laminar flow model. Thus, it is proposed that a useful and accurate model of the foilless diode is to use the space-charge limited model at voltages such that $\gamma_a < \gamma_{cr}$ and to use the laminar flow model for $\gamma_a > \gamma_{cr}$. The justification for this model lies in its agreement with experiments performed at low voltage$^{2,8}$ and in its agreement with the simulations of this study. A plausible reason for this type of dependence of diode current on voltage might be as follows. The beam in the foilless diode tries to obey the laminar flow model. In the infinite magnetic field limit, the beam energy is given by $\gamma = \gamma_a/(1 + 4 \ln r_a/r_c)$. For $\gamma_a < \gamma_{cr}$ this requires that $\gamma$ be less than $\gamma_a^{1/3}$. There has been unproven speculation that a beam with $\gamma < \gamma_a^{1/3}$ is unstable.$^{11}$ If this were true, then the space charge of the beam might adjust itself to try to increase $\gamma$ to a higher value which is stable. As $\gamma$ increases, it first reaches a stable value at $\gamma = \gamma_a^{1/3}$ resulting in a space-charge limited beam. It should be noted, as shown in Fig. 1, the composite space-charge limit laminar flow model for the foilless diode predicts an impedance, which is almost independent of voltage, making a foilless diode look like a purely resistive load.
In addition to voltage scaling in Fig. 1, simulations were performed for a wide variety of parameters. Figure 2 shows a series of simulations for several anode radii. These simulations also include voltage variation since the diode is connected to a 37 \( \Omega \) coaxial transmission line driver onto which is launched a voltage of 10 meV. The steady-state voltage, which results from the diode-transmission line impedance mismatch varies from 6.8 to 12 meV. The open circles correspond to diodes with an A-K gap \( \delta \) as defined in Fig. 1 of 0.4 \( r_c \) and an applied magnetic field given by \( r_c B_0 = 59 \). The triangle shows the current for a diode with the same magnetic field and \( \delta/r_c = 0.2 \). The inverted triangles represent diodes with \( \delta/r_c = 0.4 \) and \( r_c B_0 = 24 \). All these runs have \( \varepsilon \) (in Fig. 1) equal to 0.135 \( r_c \). For these large magnetic fields, the beams become highly annular so that the beam thickness, from the simulations, are typically 0.05 \( r_c \) or less. It is unlikely that \( \varepsilon \) can affect the diode current unless it is less than this value. In fact, the square indicates a simulation in which there is no lip on the cathode (\( \varepsilon = r_c \)) and no change in the anode radius (\( \delta \rightarrow \infty \)) with \( r_c B_0 = 32 \). In all the simulations \( \gamma_a \) is greater than \( \gamma_{cr} \). The simulations show the foilless diode current to be less than the space-charge limit and near the value given by the laminar flow model.

The simulation results in Fig. 3 show the effect of the A-K gap \( \delta \) on diode current for \( r_a/r_c = 1.5 \), \( r_e B_0 = 59 \), and a voltage of 10 meV. The upper dashed line represents the current from Eq. (4) with \( r_a/r_c = 1.5 \). The lower dashed line shows the current for \( r_a/r_c = 1.85 \), which is the size of the transmission line radius \( r_t \). Thus, it appears that, for large enough \( \delta \), the beam comes to an equilibrium before reaching the reduced anode radius region. Furthermore, except for a
narrow transition region, the diode current is given accurately by the equilibrium model provided the appropriate value for $r_d/r_c$ is used.

The simulations also provide information about beam density and temperature. The scaling is such that as the magnetic field strength is increased the beam becomes more annular making the current density increase roughly linearly with field strength while the angular scatter varies as the inverse of the field strength. For example, at 100 kG a diode with a 1 cm cathode radius can produce a beam with a current density exceeding 700 kA/cm$^2$ with an angular scatter of less than 25 mrad at 5 MV. Such a beam is of sufficient quality and intensity to carry out feasibility experiments in the high-density plasma-driven liner concept.\textsuperscript{13}

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FIGURE CAPTIONS

Figure 1. Foilless diode current as a function of voltage for \( \frac{r_a}{r_c} = 1.5 \). The dashed curve is from Eq. (5) and the chained curve is from Eq. (4). Inset shows foilless diode geometry.

Figure 2. Simulations and theory for wide parameter range at high voltage. The dashed curve is from Eq. (4) and the solid curve is from Eq. (5).

Figure 3. The effect of A-K gap on diode current.