**13. ABSTRACT (Maximum 200 words)**

Generalized EM wavelets: These are defined in a conceptually simple way using retarded Green functions. Models for their emission, reflection, and reception have been proposed. The ambiguity function formalism has been generalized to any number of independent transmitting and receiving platforms in arbitrary motion. When specialized to monostatic radar with a single target in uniform motion, this reduces to the usual wideband ambiguity function formalism, which is ordinary time-scale wavelet analysis. (Reduction to the standard time-frequency ambiguity functions is obtained in the well known way to considering the narrowband limit.) The general case is able to handle multiple targets and multiple reflections as well as general motions, being based directly on a physical model using Green functions.

**DTIC QUALITY INCREASED**

**14. SUBJECT TERMS**

Electromagnetics, wavelets, radar
FINAL REPORT
AFOSR GRANT#  F4960-95-1-0062

GRANT TITLE:
Application of Electromagnetic Wavelets to Radar Analysis and Synthesis

PROJECT PERIOD:
December 1, 1994 - November 30, 1997

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CO-PIs/SUBCONTRACTORS:
• Alexander Shvartsburg was paid as a consultant to visit Lowell during
  March 26-31, 1996 to discuss his time domain ideas.

• Arthur Yaghjian was hired as a consultant for work on electromagnetic
  scattering for the period Feb 1-March 31, 1997.

• Lou Rossi was subcontracted Jan 1-May 31, 1997 to analyze the ABL data using
  fast wavelet transforms.

OBJECTIVES:
Original objectives:
(1) Applications of EM wavelets and the Analytic-Signal Transform (AST) to the
analysis of radar signals

(2) Test the idea that the AST "decodes" the return, giving information about
targets.

(3) Possible application (of EM wavelets) to the design of new, wavelet-oriented,
radar systems

(4) Investigate the scattering properties of EM wavelets.

(5) Possible experimental study of laboratory-created EM-like wavelets. (This
possibility was less certain.)

(6) Attempt the construction of directed EM wavelets with good transversal
localization.

Revised objectives:
(1) Construction and study of generalized EM wavelets, emitted by sources in
arbitrary motion.
(2) Generalization of ambiguity function formalism to above wavelets.

(3) Study the application of Heyman-Felsen Complex-Source Pulsed Beams (CSPB) to radar. These can be interpreted as EM wavelets emitted by stationary point sources located in complex space-time. They provide a natural and simple model for directed EM wavelets, as proposed in point (6) above.

(4) However, I found that the CSPB are not Lorentz-covariant, hence they cannot be used to represent moving antennas and analyze the motion of moving targets, even at the nonrelativistic speeds needed for radar. Furthermore, the CSPB have no known 4-D Fourier representation, which is a great hindrance in using them for computations. I have therefore initiated a study of how to construct Lorentz-covariant alternatives to these pulsed beams with simple Fourier representations, together with Ehud Heyman.

(5) A new line of research, discovered since the original proposal, is scale filtering. This gives an alternate representation of filters (convolution operators) using wavelet transforms instead of Fourier transforms. The possibility of doing so with an efficient multiresolution algorithms is being studied.

(6) Starting June 1996, I have also worked on a wavelet analysis of data related to the Airborne Laser Program (see below).

(7) Recent work by Hansen and Norris shows that the field due to an arbitrary compact source distribution can be expressed as a superposition of CSPB centered at the origin and radiating in all directions, the strength in any given direction being governed by a direction-dependent weighting function. This amounts to integrating over a sphere in complex space, whose radius is the (common) size of the radiating source disks. I believe this work is very significant, and intend to develop it into a tool for radar with arbitrarily moving platforms by formulating a Lorentz-covariant version.

STATUS OF EFFORT:
(The numbers refer to the above description of revised objectives.)

(1 & 2) Generalized EM wavelets: These are defined in a conceptually simple way using retarded Green functions. Models for their emission, reflection, and reception have been proposed. The ambiguity function formalism has been generalized to any number of independent transmitting and receiving platforms in arbitrary motion. When specialized to monostatic radar with a single target in uniform motion, this reduces to the usual wideband ambiguity function formalism, which is ordinary time-scale wavelet analysis. (Reduction to the standard time-frequency ambiguity functions is obtained in the well known way by considering the narrowband limit.) The general case is able to handle multiple targets and multiple reflections as well as general motions, being based directly on a physical model using Green functions.

(3) The wavelets in *1&2* are still emitted by point sources, hence are omnidirectional and not useful for imaging. They have been further extended by
replacing the retarded propagators with CSPB. The motion of a dish antenna along an arbitrary trajectory, with arbitrary time-varying orientation (tracking, scanning, etc) has been modeled by the trajectory of the complex source point, i.e. a simple curve in complex space-time. The AST has been proposed as a natural tool to model the reflection and reception of such "extended" EM wavelets. The efficacy of this approach is supported by the success of a similar method investigated by Zeroug, Stanke and Burrige (A complex-transducer-point model for finite emitting and receiving ultrasonic transducers, in Wave Motion, vol. 24, pp. 21-40, 1996). However, I believe the coupling between transmitting and receiving CSPB is somewhat different form that proposed by these authors.

(4) The problem of moving pulsed beams must still be addressed, as well as their lack of explicit 4-D Fourier representation. I have recently begun working on this with Ehud Heyman, and intend to continue doing so in the near future.

(5) I believe scale filtering may have applications to wideband Doppler filtering and to EM wave propagation through atmospheric turbulence. To make it computationally useful, a discrete multiresolution algorithm will be needed. Such a possibility is also being investigated.

(6) Since June 1996 I have been studying the application of wavelet analysis to the Airborne Laser Program. I spent much of June-October 1996 learning the issues (propagation and imaging through random media, atmospheric turbulence, adaptive optics, etc.), and participated in the workshops at Phillips Lab-Kirtland on adaptice optics and ABL (in October and December 1996, respectively). I then enlisted my colleague Louis Rossi to help with performing a wavelet analysis of ABL-related flight data. His knowledge of fluid dynamics and computing was an immense help. The results of our collaboration were presented in another workshop at Phillips Lab at Kirtland AFB on July 28-29, 1997. We are also writing a paper on this, jointly with Don Washburn of PLK, for the Journal of Mathematical Physics (special issue on Wavelets in Physics) to appear in 1998. Lou has spent summer 1997 as an AFOSR Faculty Research Fellow at Phillips Lab-Hanscom, and we are beginning a joint effort with Bob Beland to analyze balloon data.

ACCOMPLISHMENTS/NEW FINDINGS:
The "ambiguity functionals" formulation of radar using pulsed-beam wavelets is potentially useful because it is intuitively clear and physically correct (within the obvious limitations of the chosen model, e.g. dispersionless propagation through a homogeneous medium). The conceptual simplicity results from the fact that the theory is based directly in space-time, hence lends itself to intuitive interpretation. I believe this makes it accessible to a wide base of possible users. The usual time-frequency (narrowband) and time-scale (wideband) formalisms make unnecessary assumptions and are, moreover, less clear to the uninitiated (or unindoctrinated) because they depend on a "dictionary" related to these assumptions, for example, that the Doppler effect translates to a frequency shift (narrowband) or a scaling factor (wideband). In such a restricted context, it is not obvious how to deal with NEW situations not originally built into the model, such as accelerations, multistatic situations, multiple reflections, etc. The physics-based space-time modeling is especially helpful in this regard.
The present focus of the project is no longer the originally proposed analytic EM wavelets. These can be viewed essentially as waves emitted by uniformly moving point sources, since they are generated using the AST which involves integration along a straight "world-line" in space-time. Also, the originally proposed study of the return by its AST seems less effective than the present "ambiguity functional" formalism since it assumes a knowledge of the return in all of space, which is obviously impractical. By contrast, the present formalism only measures the return along the trajectory of the receiver.

The new results on scale filtering show that operations usually performed in the frequency domain (filters or convolutions) can also be performed in the wavelet domain: Take the wavelet transform of the signal, multiply it by a scale-dependent "transfer function," then apply the inverse wavelet transform. A one-to-one correspondence was established between certain "admissible" convolution operators and such scale transfer functions, based on Mellin convolutions and Mellin transforms. An application has been proposed to the analysis of atmospheric turbulence effects on EM waves: It turns out that the "canonical objects" in scale filtering are spectral power laws. Precisely such laws enter into turbulence analysis, for example as occurs in connection with the Airborne Laser Program. Also, I believe I can extend my one-dimensional results to space-time scale filtering. One possible objective would be to generate models of (extended) EM wavelets propagating in a turbulent environment by building the background (Kolmogorov) power spectrum directly into the propagator.

Most of my effort during the period August 96-August 97 was spent learning issues related to the Airborne Laser Program and applying wavelet methods to deal with the associated data. I enlisted Lou Rossi, whose specialty is computational fluid dynamics, to perform an analysis of the ABLE ACE temperature data using Haar wavelets. (Continuity is not an important issue since the data is quite fractal; thus higher-order multiresolution analyses, for example ones using Daubechies wavelets, are not necessary. They have been tried by George Papanicolaou and found to give results similar to ours.) My initial suggestion was to use wavelet averages at successive resolutions to find intervals over which the data can be regarded as being stationary. Previous wavelet analyses of atmospheric data (for example, by Hudgins et al., Phys. Rev. Lett. vol. 71, pp. 3279-3282, or Katul et al., in "Wavelets in Geophysics," E. Foufoula-Georgiou and P. Kumar, eds, Academic Press, 1994) used exclusively wavelet detail coefficients to compute the spectrum and structure functions. We found that wavelet averages give a very effective way to segment the data into roughly stationary intervals. At about the 10-th level (where each step involves \(2^{10}\) original data points), it becomes clear that most of the data stream is roughly stationary, with the exception of a few brief intervals consisting of very singular spike clusters representing intermittent events or transition regions. When restricted to such stationary segments, or even to within any extended burst, the data gives spectra and structure functions with rather well-defined scaling exponents, as is necessary in order to estimate coefficients such as \(C_n n^2\) giving the order of magnitude of the local turbulence. Otherwise, the scaling laws are a bad fit. By looking at wavelet averages of the derivative-squared of the temperature (representing the local energy dissipation rate), Lou also discovered a square-wave pattern embedded in the data, completely unexpected and previously undetected. The interpretation of this wave is still unknown, but its discovery proves the utility of our analysis whatever the final verdict. Lou has also noticed some correlation between the computed values
of $C_n^2$ and the square wave. Thus, if the square wave is a contamination of the data, it may have a nonnegligible effect on the important estimation of the overall turbulence level. Although we have only analyzed data gathered on a single day, George Papanicoleau has applied a similar analysis to other available data and found the square wave in each data set. This would seem to rule out the suggestion that the square wave is due to a SAR radar operating in the area on the particular day the data was gathered. Since the periods of the square waves vary with height, they also don’t seem to be due to instrumentation electronics either. (This possibility has also been dismissed on other grounds by the Phillips Lab instrumentation experts.)

PERSONNEL SUPPORTED:
• Lou Rossi, Mathematical Sciences Dept., UMass Lowell, supported as subcontractor Jan 1-May 31, 1997.

• Alexander Shvartsburg, supported as consultant March 26-31, 1996.

• Arthur Yaghjian, supported as consultant Feb 1-March 31, 1997.

• Danielle White, supported as secretary during September-October 1996 and August-October 1997.

PUBLICATIONS
Submitted:

• The Fast Haar Transform: Gateway to Wavelets, paper submitted by invitation to IEEE Potentials, the undergraduate student magazine.

Published:


• Wavelet filtering with the Mellin transform, Applied Mathematics Letters B9, 69--74, 1996.

• Wavelet filtering in the scale domain, invited paper, in Wavelet Applications III, SPIE Conference Proceedings #2762, Orlando, FL, April, 1996.


• Wavelet filtering in the scale domain, in Wavelet Applications in Signal and Image Processing, SPIE Conference Proceedings #2825, Denver, CO, August, 1996.


SHORT COURSES GIVEN BY INVITATION:
Participants lists, comments, and evaluations available on request.


• 5/9/95: Physical Wavelets, with Applications to Radar and Sonar (3 hours), IEEE International Radar Conference, Alexandria, VA.

• 6/6-8/95: Mathematics and Physics of Wavelets (12 hours), Applied Technology Institute, College Park, MD.

• 4/8/96: Introduction to Wavelets and Physical Wavelets, with Applications to Remote Sensing (4 hours), SPIE Conference on Wavelet Applications III, Orlando, FL.

• 5/16/96: Applications of Electromagnetic Wavelets to Radar and Sonar (3 hours), IEEE 1996 National Radar Conference, Ann Arbor, MI.

• 6/4-6/96: Mathematics and Physics of Wavelets (12 hours), Applied Technology Institute, College Park, MD.

• 7/21/96: Electromagnetic Wavelets in Radar and Sonar (6 hours), IEEE-AP-S International Symposium and URSI Radio Science Meeting, Baltimore, MD.


• 12/17-19/96: Mathematics and Physics of Wavelets (12 hours), Applied Technology Institute, Brookline, MA.

• 2/10/97: Wavelet Fundamentals (6 hours), Internat. Soc. for Optical Engineering (SPIE) Photonics West, San Jose, CA.
• 3/21/97: Radar via Physical Wavelets (6 hours), Applied Computational Electromagnetics Society (ACES), Monterey, CA

• 4/21/97: Remote Sensing via Physical Wavelets (6 hours), Internat. Soc. for Optical Engineering (SPIE) AeroSense, Orlando, FL

• 7/30/97: Introduction to Radar via Physical Wavelets (6 hours), Internat. Soc. for Optical Engineering (SPIE) Annual Meeting, San Diego, CA

• 12/16-18/97: Mathematics and Physics of Wavelets (12 hours), Applied Technology Institute, Brookline, MA.

REVIEWS OF A FRIENDLY GUIDE TO WAVELETS:
• Chosen and reviewed in 1995 as a Book of the Month by the Library of science Book Club.

• Reviewed in the July 1995 issue of Physics Today.

• Reviewed in the September 1995 issue of Mathematical Reviews.

• Reviewed in the December 1995 issue of SIAM Review.


• To be reviewed in the Proceedings of the IEEE.

SEMINARS AND TALKS (Recent samples):
• Nov 8, 1996: Seminar on scale filtering at Stanford University Mathematics Dept.

• Jan 5, 1997: Talk on scale filtering in the wavelet domain at AFOSR Electromagnetics workshop in San Antonio, TX.

• Jan 31, 1997: Seminar on wavelet scale filtering at Tufts University Mathematics Dept.

• Feb 18, 1997: Seminar on wavelet analysis of atmospheric intermittency at UMass Amherst ECE Dept.

• Nov 20, 1997: Talk given at Schlumberger on using Complex-Source pulsed beams in radar.
CONSULTATIVE AND ADVISORY FUNCTIONS TO LABS AND AGENCIES

• Dec 9, 1996: Seminar given on wavelet analysis of atmospheric intermittency at NOAA. I was invited by Bob Weber, who is heading a project to develop clear-air radar for the Air force. The detection and analysis of intermittent events (such as the passing of a flock of birds) is an important issue.

• Dec 11, 1996: Talk given on wavelet analysis of atmospheric intermittency at the Atmospheric Modeling and Laser Propagation Workshop, Phillips Laboratory (PL/LI), Kirtland AFB. This work later developed into a joint project with Lou Rossi and Don Washburn.

• April 10, 1997: Seminar on wavelet analysis of atmospheric intermittency at MIT Lincoln Laboratory (invited by the IEEE Geophysics Remote Sensing Society).

• July 28, 1997: Talk given by Lou Rossi on our joint work on wavelet analysis of passive scalar spectra and structure functions from Argus data, at the ABL workshop, Phillips Laboratory (PL/LI), Kirtland AFB.