**Title:** Acoustic Scattering for Buried Objects at High Frequencies - A Ray Theoretic Approach

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**Abstract:**
The goals of the proposed research were as follows:

1. Develop a stochastic model that can be used to determine the presence of a buried object in ocean sediments
2. Develop a method for obtaining the bottom properties from analysis of normal incidence data as in the case of chirp sonar.
Acoustic Scattering from buried objects at high frequencies

Subramaniam D. Rajan

Goals of research

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Detection of objects buried in ocean sediments

Technical approach

One of the important issues in detecting objects buried in ocean sediments is the ability to distinguish scattering caused by buried objects from scattering due to the rough ocean bottom surface and the ocean surface. In a series of recent papers [1 - 3] Professors L. Tsang and Y. Kuga and their group of researchers at the University of Washington have proposed a new approach based on angular correlation function for discriminating the scattered field caused by the rough surfaces and volume inhomogeneities from that caused by buried objects. This group is at present engaged in the evaluation of this method for the detection of buried objects in sandy soil using EM waves. We studied the acoustic counter part with application to the detection of buried objects in ocean sediments.

Angular correlation function is defined as the correlation of the scattered fields as the incident field changes direction. Let $\psi(\theta_1, \theta_{s1})$ be the scattered field in the direction $\theta_{s1}$ for a field incident in the direction $\theta_1$. We now change the direction of the incident field to $\theta_{i2}$ and measure the scattered field in the direction $\theta_{s2}$. The angular correlation function is defined as $<\psi(\theta_1, \theta_{s1})\psi^*(\theta_{i2}, \theta_{s2})>$ where $<$ represents an averaging operation and $*$ the complex conjugate. Note that all the angles referred are with respect to the horizontal (Figure 1). If the scattering medium is statistically homogeneous, then it has been shown [1] that the angular correlation function has large values for certain combination of the incident and scattering angles. Outside this region the angular correlation is relatively small. This effect was first observed when propagation of optical waves through a random media was studied [4] and was found to exist even when the scattered field underwent multiple scattering. This effect has been termed “memory effect” because the high correlation for a certain combination of the incident and...
scattering angles can be thought of as being due to the field at these scattering angles having a memory of the incident directions. The region of high correlation satisfies the equation

\[ \cos(\theta_{11}) + \cos(\theta_{s1}) = \cos(\theta_{12}) + \cos(\theta_{s2}) \]  

(1)

The averaging operation referred to above is done over a number of realizations of the random medium through which the wave propagates. The equation given above arises out of a phase matching condition similar to that of Snell's law except that it is applicable for a random medium and is the result of the averaging process [1]. This relation is valid as long as the random medium is homogeneous. If into this random medium a man made medium object is introduced, the statistically homogeneous nature of the medium is destroyed. This alters the character of the angular correlation function and this change in the character of the angular correlation function can be used to infer the presence of a man made object in the medium. The appealing features of this approach are:

1. The memory effect is observed in propagation through a random medium even when the scattered field has undergone multiple scattering. This approach will therefore be suitable in shallow water environment where multiple scattering occurs.

2. Whatever be the shape, size and nature of the surface of the man made object it is still a single scattering object and will destroy the homogenous nature of the medium. These factors will therefore have no impact on our ability to detect the object.

Preliminary results of simulation

A. Perfectly reflecting rough surface (Realization and frequency averaging)

In order to study the behavior of angular correlation function, we conducted simulations for the case of scattering from a rough interface. We consider an one dimensional problem for which a rough boundary with Gaussian spectrum was generated using the method outlined in [5]. This boundary was assumed to be perfectly reflecting in order to reduce the computational load. In Figure 2 we plot the normalized angular correlation function as a function of \( \cos(\theta_2) \) and \( \cos(\theta_{s2}) \) with the angles \( \theta_{11} = \theta_{s1} = 20^\circ \). Normalization was done by dividing the function by its maximum value with the result the normalized function has a maximum value of 1. A shallow grazing angle was assumed for the incident field to represent the commonly needed requirement of large stand off distance. The source frequency was 150 kHz and the averaging was done over 40 realizations of the rough surface. We note from the figure that there exists a region of high correlation and outside this region the correlation has a small value. In real field situations we have only one realization of the bottom from which we have to
compute the correlation function and averaging over realization is not feasible. However with a broadband source or with a number of sources transmitting at different frequencies we can obtain the scattered field at a number of frequencies and average over frequency. The result of averaging over frequency is shown in Figure 3 in which the averaging was done over 50 kHz to 150 kHz in 40 steps. We note that, as in the case of Figure 2, there is a region of high correlation and this region Eqn. (1) is satisfied. However, this region appears to be broader than in the previous case (i.e. realization averaging). It is presumed that the performance will improve if averaging is done over a larger set of frequencies. We also note that the angular correlation function has a high value in the specular direction while a similar feature is not seen when averaging over realizations. These issues will be investigated in the proposed work.

B. Penetrable surface(Frequency averaging)

To study the behavior of the angular correlation function to the specific problem at hand, we computed the angular correlation function for the case where the ocean bottom interface was modeled as a rough interface. This interface therefore separates two regions one representing the water column with sound speed = 1500 m/s and density = 1.0 gm/cc and the other representing the sediment with sound speed of 1600 m/s and density = 1.5 gm/cc. The angular correlation function was calculated with $\theta_{i1} = \theta_{s1} = 20^0$ and the averaging was done over frequencies (50 kHz to 150 kHz). The angular correlation function is shown in Figure 4 and has a region of high correlation as in other examples. We now consider the case where a cylindrical object is introduced into the sediment. If we assume the cylinder to be of infinite length, then in the 1D problem that we are solving, the cylinder will be represented by a circle. We also assume that the surface of the cylinder is rigid. The cylinder was assumed to be 0.2 m in diameter and buried with its center 0.101 m from the mean surface of the rough interface. As before $\theta_{i1} = \theta_{s1} = 20^0$ and averaging was done over frequencies. The angular correlation function for this case is shown in Figure 5. The character of the correlation function has undergone a dramatic change. Over all the angular correlation function has a much higher value. Further the correlation function has large values even in regions that do not satisfy Eqn. (1).

The change in the character of the angular correlation function is more clearly seen in Figure 6 (a) which is the angular correlation function for $\theta_{i1} = \theta_{s1} = \theta_{i2} = 20^0$. A display which accentuates the differences between the two cases is shown in Figure 6(b). This is a plot of the magnitude of the FFT performed on the correlation function in 6(a). The FFT is plotted with a shift. The curve for the case where an object is buried is easily distinguishable from the case where there is no object. This characteristic can therefore be used to discriminate between clutter and scattering by the buried object.
Layer Statistics

For an acoustic pulse that is incident normally at the sediment, the travel time of the pulse to the layer interfaces can be obtained from the backscattered signal. The variability of the travel time to a given interface is related to the variability in the layer speeds and layer thickness. In a layered model where the properties in a layer is assumed constant, the covariance of travel time \( <t(x)t(x')> \) is related to the covariance of the sound speed in the layer and the covariance of the layer thickness under the assumption that the layer thickness and the distance to the layer interfaces from the source are uncorrelated.

The same normal incidence data can be used to obtain the impedance structure in the sediment layers. In Table 1 we give the results of simulation done for a single layer model where the unknown are the layer impedance and attenuation. The algorithm computed the reflection coefficient as a function of frequency and estimated the unknown parameters by minimizing in a least squares sense the difference between the calculated reflection coefficient and the reflection coefficient from data. Therefore the data obtained for different pings along the track of the source one can estimate the impedance profile and hence the horizontal variability in impedance. Since the variability in density can be related to the variability in sound speed, one can obtain the estimate of the variability. in sound speed from the impedance estimates. Having determined the variability in sound speed, we can then estimate the variability in layer thickness.

<table>
<thead>
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<th>TRUE</th>
<th>INITIAL</th>
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<th>SNR</th>
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<td>( c=1500 )</td>
<td>( c=1873 )</td>
<td>No noise</td>
</tr>
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<td>( \rho=1.0 )</td>
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<tr>
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<tr>
<td>( \rho c=2880 )</td>
<td>( \rho c=1500 )</td>
<td>( \rho c=2880.64 )</td>
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</tbody>
</table>

\[ \begin{align*}
\text{c}=1873 \\
\text{\rho}=1.538 \\
\alpha=0.094 \\
\rho c=2880.64
\end{align*} \]

\[ \begin{align*}
\text{c}=1872 \\
\text{\rho}=1.536 \\
\alpha=0.096 \\
\rho c=2875.4
\end{align*} \]

Table 1
The results of the work described above were presented at the Acoustical Society of America meeting held in Honolulu Hawaii, in December 1996.

References


Figure 1: Incident and scattered field

Figure 2: Angular correlation function (Realization averaging)
Figure 3: Angular correlation function (Frequency averaging)

Figure 4: Angular correlation function (Penetrable rough interface)
Figure 5: Angular correlation function (Penetrable rough interface with a buried object)

Figure 6: (a) Angular correlation function for $\theta_{i1} = \theta_{i2} = \theta_{l2} = 20^\circ$ (b) FFT of functions in Figure 6(a)