A METHODOLOGY FOR
EVALUATING AND ENHANCING C4I NETWORKS

THESIS

Christine C. Davis, Captain, USAF

AFIT/GOR/ENS/97M-04

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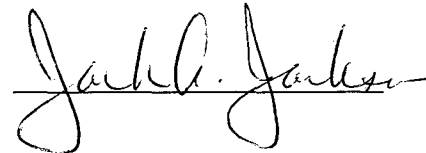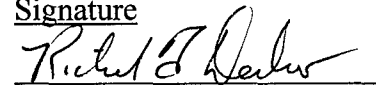
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Table of Contents

List of Figures

List of Tables

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Abstract

This effort focuses on expanding the bottleneck links of an existing C4I network to meet

projected demands. The minimum cost-maximum flow algorithm is used to identify bottlenecks.

A decision analysis approach enables the mapping of performance objectives for a network

expansion into measurable attributes within the structure of a value hierarchy. A multiattribute

value function combines these measures into an overall system effectiveness measure. This

allows for an evaluation of the impact of potential upgrade components on the system. A

knapsack model imposes budgetary constraints on the selection of components. This

methodology allows for the design of an optimized system upgrade based on system

effectiveness, while offering an indication of the "value" of the increased information flow

through the network.

A Methodology for
Evaluating and Enhancing C4I Networks

I. Introduction

Background

> "One of the surest ways of forming good combinations in war would be to order movements only after obtaining *perfect information* of the enemy's proceedings. In fact, how can any man say what he should do himself, if he is ignorant what his adversary is about?" [Jomini, 1862 : 268]

Jomini is considered one of the great military strategists of the 19th century. His writings were inspired by the historical study of and practical military experience with Napoleon, Frederick the Great, and other military revolutionists of that time. He recognized the critical importance of information in the hands of the warfighter. General John M. Shalikashvili, Chairman of the Joint Chiefs of Staff (JCS), echoes Jomini's words in Joint Publication 6-0:

> "A vast array of information . . . is utilized to employ combat power across the broad range of military operations. Command, control, communications, and computer (C4) networks and systems provide the means to synchronize joint forces. . . . The synthesis of advanced C4 capabilities and sound doctrine leads to battlespace knowledge essential to success in conflict." [Joint Pub 6-0, 1995: preface]

Jomini also recognized the obstacle in obtaining this information: "As it is unquestionably of the highest importance to gain this information, so it is a thing of the utmost difficulty, not to say impossibility; and this is one of the chief causes of the great difference between the theory and the practice of war." [Jomini, 1862: 268-9] While Jomini's 19th century quote is still true today, advances in information technologies

allow current military forces to conduct operations much closer to the theoretical construct of near-perfect information.

The art of war is now focused on the collection, processing, distribution and utilization of information. Forces that have the strongest combinations of technological capability, information systems, and quick, decisive reaction are victorious with the least loss of resources. Desert Storm serves as a relevant example. U.S. military operations were swift and successful, resulting in an incredibly small number of allied casualties. A major cornerstone of this success was our command, control, communication, computer and intelligence (C4I) systems [Freeman, 1992: 6].

Admiral Davis E. Jeremiah, the Vice Chairman of the JCS, states, "No matter where we fight in the future, no matter what the circumstances, we will fight as a joint team. . . . The days of single Service warfare are gone forever." [Joint Pub 6-02, 1996: IV.1] The issues of command, control, communication and computer (C4) network objectives, planning, management and expansion are central to Joint Warfare. The fundamental objective of C4 systems is to get the critical and relevant information to the right place at the right time [Joint Pub 6-0, 1995: viii]. C4 systems include both the communications and computer systems required to implement the command and control process. Components include: terminal devices, such as fax machines and computers; transmission media, i.e., radio, metallic wire and fiber optic cable; switches to route traffic through a network of transmission media; and control, providing management of local, regional, theater or global networks [Joint Pub 6-0, 1995: viii-ix]. This vast array of components combine to create complex, worldwide systems.

C4 systems provide continuous, uninterruptable flow and processing of voice, data, facsimile and video information to support operational planning, decision making and execution. Joint doctrine identifies seven principles of C4 systems which provide the foundation for building these networks [Joint Pub 6-0, 1995: II.4]. These systems must be interoperable, flexible, responsive, mobile, disciplined, survivable and sustainable to be useful to the warfighter.

Those who plan C4 networks must be fully aware of the mission the system is to support. They should clearly understand the capabilities and limitations of all potentially available strategic, operational and tactical C4 systems and equipment [Joint Pub 6-02, 1996: I.3]. Some key factors to be considered in the design of a system include the C4 principles listed above, the mission of the system, standardization, connectivity, spectrum management, and information protection. These factors, or network objectives, are most likely conflicting; that is, a completely mobile system may not have the receiving capacity that would make the system as 'responsive' as desirable. Planners may use a combination of manual design techniques in order to accurately estimate performance requirements. These include structural analysis, a statement of requirements for an existing network, traffic flow experience, and a 'rule of thumb,' relying on past experience with similar network users [Joint Pub 6-02, 1996: II.9-10]. The capacity requirement used in planning must meet the needs of a wartime system [Joint Pub 6-02, 1996: II.11].

Once systems are fielded or deployed, there are several reasons a system must be properly monitored. If a detailed network status is known, technicians are able to reconfigure the network, if necessary, to maintain adequate connectivity. In addition,

problem areas and solutions can be more easily developed. Complete monitoring should indicate critical circuits, *bottleneck components* and alternate routing, which are quite useful when planning a network expansion [Joint Pub 6-02, 1996: II.9-13]

A bare-bones network of an initial phase deployment or an existing network supporting daily operations will be incrementally expanded during contingencies to directly or indirectly support the build-up of forces in a specified area. The goals of the expansion include (in order of priority): continuous flow of information; increase system reliability and 'robustness,' provide acceptable performance with system degradation; automate flow and processing of information so as to be transparent to system users; and adapt system to changes in mission requirements and user demands [Joint Pub 6-02, 1996: III.1-2].

In order to achieve the network objectives of any C4 network, it is clear that C4 principles and planning factors must be considered in all decisions. Planners must be aware of the mission the system supports, and the capabilities and limitations of individual components and network structure. A fully characterized system status must be maintained to allow for accurate system analysis. This will aid in the optimal addition of components to enhance an existing or deployed C4 network.

Statement of the Problem

When faced with a communications network expansion, whether from changing mission requirements or the need to meet the increasing demands of end users, a quantitative method is needed that will incorporate C4 principles and planning factors into a mathematical model which will support an optimizing network expansion plan.

Coupled with this is the requirement for some type of network structural analysis to locate bottleneck components and critical network paths which aid in generating expansion plans.

Research Approach

This research uses multiattribute utility theory (MAUT) to balance the potentially conflicting objectives of a network expansion. A network planner, or decision maker, delineates objectives in a hierarchical structure down to measurable attributes which fully define each objective. For each of the attributes, individual utility functions are constructed which quantify the value tradeoff among various objectives. Assigned weights demonstrate each attribute's relative importance within the objectives hierarchy are also assigned. Both of theses tasks incorporate the decision maker's preferences.

The network model can be compared to a production flow shop. In a flow shop, "jobs" are processed through a series of machines, or operations, sequentially [Pinedo, 1995: 93]. In the communication network, messages are transmitted through a variety of stations, such as transmitters, switches and receivers. The throughput of a flow shop is the number of jobs that can be processed through the shop given its resources. To put it simply, the throughput is limited by the operation or operations whose capacity or resources are the first fully utilized. This limitation is known as a bottleneck. A bottleneck in a communication network is the component or station whose capacity is fully utilized. Therefore, capacity expansion should focus on the bottleneck locations. The minimum cost flow algorithm [Pinedo, 1992] is used to identify bottleneck

operations. The operations in the solution of this algorithm which have no excess capacity are the potential bottlenecks.

The characterization of the bottleneck components helps to identify possible expansion plans to increase system capacity. This may be done on either a long term or a short term basis. Once components which would potentially relieve the bottlenecks are identified, their impact on the system is fully defined using the weighted utility functions for attributes. A mathematical programming model is used to determine the incremental plan which maximizes the 'utility' of the network. A flow model analysis of the expanded network may identify new bottlenecks for the short term characterization, beginning an iterative process of expanding the network, analyzing the flow model to recognize bottlenecks that may have shifted, identifying potential component expansions, and finding the optimal plan which maximizes the accomplishment of network objectives.

Scope / Limitations

This effort focuses on the expansion of a deployed communications network to meet a forecasted demand on capacity. It will not address the scheduling of network resources, but is based on developing a network topology that will optimally satisfy demand.

The network objectives and attributes which are used in the objectives hierarchy are specific not only to the C4 system chosen, but also to the decision maker used. They may not be representative of all C4 systems and all decision maker's preferences.

Thesis Overview

This chapter presents a background and basic introduction of the problem to be addressed. Chapter II provides a more detailed background on relevant topics, such as capacity expansion problems, value-focused thinking, multiattribute utility theory, and network algorithms. Chapter III gives the methodology used to obtain and quantify network objectives, identify bottleneck components and optimally expand the communication network. Chapter IV provides an analysis of the results from a network scenario. Chapter V makes conclusions and recommendations for future extensions of this research.

II. Literature Review

Networks

C4 networks can be broken into two parts: the user subnetwork, which provides network access through terminals or consoles, and the communication subnetwork consisting of network nodes, transmission links and signal converter equipment, which provides signal conversion and transmission/reception of data [Ahuja, 1982: 13]. The topology of a network includes the connectivity between nodes and the capacity of the transmission links. This effort focuses on expanding the existing topology of a communication subnetwork.

A network topology can be determined through network optimization techniques, such as constrained minimum spanning tree algorithms, capacity assignment models, link assignment algorithms, link capacity assignment models, and route assignment/flow control methods [Ahuja, 1982: 135; Gavish, 1992: 115-128]. Network algorithms take advantage of the special structure of the network linear program to produce an optimal solution much more quickly, with less storage required, and with virtually no round-off error in comparison with general linear programming codes [Woolsey and Swanson, 1975: 100]. A network topology that cannot meet the demands of users due to the capacity constraints on its components must be expanded to increase network throughput. The bottleneck components are those whose capacity is fully utilized; these should be the focus of any network expansion. Bottleneck components can be identified through several analysis methods. These include real-time network monitoring to identify link

utilization or traffic demands, simulation modeling, or mathematical programming options such as the minimum cost-maximum flow algorithm. Real time monitoring and simulation, if sufficient data are available, can be used to incorporate the stochastic nature of message traffic flow in a communication network [Frank & Frisch: 1971]. This is particularly critical in commercial communication networks. While demand for capacity on military network is stochastic, it is assumed in this study that the projected demand is known and the upgrade is designed to support that demand. This projected demand may be peak demand or average demand, as required by the specific operational environment and considering that the planned capacity of the C4 system must meet the needs of a wartime system [Joint Pub 6-02, 1996: II.11]. It should also be noted that priority messages will be given preference in a military network. Given this justification for a deterministic approach, a minimum cost-maximum flow algorithm will be used to identify bottlenecks in this study. Of course, if the stochastic elements of communication density are important in a particular setting, a simulation or real time analysis could be used to identify the bottlenecks without an adverse effect on the proposed methodology.

Minimum Cost - Maximum Flow Problem

The minimum cost-maximum flow network problem consists of finding a minimum cost (or maximum value) flow from supply nodes to demand nodes in a capacitated, directed graph, defined by $G = (X, A)$ [Evans and Minieka, 1992: 446], where X represents the set of nodes and A the set of arcs. In a minimum cost circulation network, all nodes exhibit conservation of flow. To achieve this, a super node is added which has arcs flowing out to the supply nodes and arcs flowing in from the demand

nodes. The capacity of arcs to supply nodes is bounded from above by the actual supply of the node. Likewise, the capacity of the arcs from demand nodes is bounded from below by the actual demand of the node. The corresponding linear programming model takes the following form [Hartley, 1976]:

$$min \ \Sigma_{(i,j) \in A} \ c_{ij} x_{ij} \tag{1}$$

$$s.t. \ \Sigma_k \ x_{ki} - \Sigma_k \ x_{ik} = 0, \text{ for all } i \in X \tag{2}$$

$$x_{ij} \geq l_{ij}, \text{ for all } (i,j) \in A \tag{3}$$

$$x_{ij} \leq h_{ij}, \text{ for all } (i,j) \in A \tag{4}$$

The cost coefficients, c_{ij}, represent the cost of flowing one unit from i to j. The actual flow from i to j is represented by x_{ij}. Thus, (1) gives the objective of minimizing the cost (or maximizing the value) of actual flow. Constraints of type (2) are the conservation of flow constraints; i.e., what flows into the node must flow out. The capacity of each arc is limited by its lower bound, l_{ij}, and its upper bound, h_{ij} as seen in constraints (3) and (4) respectively.

The out-of-kilter algorithm solves this problem using the primal-dual theory of linear programming [Ford and Fulkerson, 1962: 164]. The process also identifies the bottleneck components in the network. The out-of-kilter algorithm operates in such a way as to maintain a circulation in the network while rerouting flows to minimize the sum of cost times flow and satisfy capacity restrictions on each arc [Woolsey and Swanson, 1975: 102].

An optimal solution of the minimum cost-maximum flow problem identifies the bottleneck components using the reduced costs for each arc. These reduced costs have a physical interpretation which can be easily seen by constructing the dual of the minimum cost-maximum flow problem. To form the dual, define dual variables q_i for constraints (2), v_{ij} for constraints (3) and u_{ij} for constraints (4). The dual formulation is as follows [Hartley, 1976: 408].

$$max \; \sum_{(i,j) \in A} l_{ij} v_{ij} - \sum_{(i,j) \in A} h_{ij} u_{ij} \qquad (5)$$

$$s.t. \quad q_j - q_i + v_{ij} - u_{ij} \leq c_{ij}, \; for \; all \; (i,j) \in A \qquad (6)$$

$$v_{ij}, \; u_{ij} \geq 0, \; for \; all \; (i,j) \in A \qquad (7)$$

$$q_i \; unrestricted \; in \; sign \qquad (8)$$

Each dual variable q_i imputes a value of a unit flow at that node. From primal-dual theory, a dual variable is greater than or equal to zero in an optimal solution if the primal constraint is binding. Therefore, only one of the variables v_{ij} and u_{ij} will be greater than zero, since only one of the primal constraints (3) or (4) can be binding. By rewriting (6) as

$$v_{ij} - u_{ij} \leq c_{ij} + q_i - q_j \qquad (9)$$

and letting

$$C_{ij} = c_{ij} + q_i - q_j \qquad (10)$$

then C_{ij} is interpreted as the value of flow at node i plus the cost of moving from node i to node j along (i,j) minus the value of flow at node j. This is the reduced cost of the arc (i,j). If $C_{ij} < 0$, then the value of flow at node j is greater than the value of flow at node i

plus the cost of moving from node i to node j. The flow on this arc is then equal to its upper bound. Therefore, those arcs in an optimal solution whose reduced costs, C_{ij}, are less than zero, are the bottleneck arcs. There is value in increasing the flow from i to j, but the arc capacity has been exhausted [Woolsey and Swanson, 1975: 100-106]. The network must be expanded to increase throughput.

Capacity Expansion Problems

Planning for capacity expansion is of vital importance in many industrial sectors including electrical power systems, water resource systems, transportation and communication networks [Luss, 1982: 907] and as such, has received a great deal of attention in developing mathematical models. Capacity expansion problems (CEPs) have two properties which drive model development. First, capacity expansion costs exhibit substantial economies-of-scale. That is, the average cost per capacity unit decreases with expansion size [Luss, 1982: 907]. This introduces an economic tradeoff between the savings of larger expansion sizes versus the cost of unused capacity. The second property is that time is important. There is a continuing, possibly changing, need for the facilities, and the added equipment will provide service over many time periods [Freidenfelds, 1981: 5]. This requires a specific discount rate of money applied over time. The expansion of network capacity involves real capital investment decisions; the efficient commitment of that capital depends on making the best decisions in individual capacity expansion projects [Freidenfelds, 1981: 3]. Logically, most mathematical models are designed to find optimal expansion sizes, times and locations in order to meet forecasted demands while minimizing the discounted costs associated with the expansion process.

Most corporations in the communications sector have a fairly long planning horizon for their networks and as such plan for several expansions depending on forecasted demand.

The minimum cost objective of CEP models can encompass costs for expansions, shortages depending on the timing of the upgrade, congestion when there is insufficient spare capacity, holding costs for excess capacity and, of course, operating costs [Luss, 1982: 913]. Constraints range from physical restrictions such as conservation of flow and capacity constraints to budget limitations and acceptable policies on excess capacity, capacity shortages, or grade of service requirements by users [Luss, 1982: 913; Veroy and Zwass, 1987: 53]. Formulations deal with network uncertainties by using parameter estimates such as demand growth, message arrival rates or traffic intensity within the constraints. All have network model representations, and most model the networks as store-and-forward in which nodes have memory capacity to store messages until they can be transmitted. Frank and Frisch [Frank and Frisch, 1971] devote a chapter to this type of network. The developments typically involve message queues or stochastic message flows. Solution techniques include dynamic programming, mixed integer and linear programming, and a combination of these and network optimization algorithms [Lee and Luss, 1987; Parrish, Cox, Kuehner and Qiu, 1992; Veroy and Zwass, 1987; Zwass and Veroy, 1988]. The models find the optimal sizes, times and locations by scheduling resources, and specifying flow routes for point-to-point demands.

Cederbaum and Paz [3] select as an objective in a multicommodity flow network optimizing the grade of service -- that is, to satisfy all demands as fully as possible. They introduced branch weights based on a concept of the topological importance of a branch

within a capacitated network. A forecast of most probable traffic distribution is used to schedule channels. Their solution approach was a shortest path in a weighted network.

Freidenfelds' book [Freidenfelds, 1981: 5] covers capacity expansion models from many different public sectors. He focuses on capacity expansion decisions as capital investments and uses discounted present value as the decision criteria to evaluate expansion plans. The transmission network problem is to determine the optimal network expansion (install new links or expand old) with different types of capacity *and* the optimal routing (scheduling) of network traffic to meet various point-to-point demands. Solution approaches are driven to heuristic methods due to the never ending possibility of complications when solving both problems simultaneously [Freidenfelds, 1981: 281].

The military C4 network planner has different decision criteria for network expansions. Due to the crucial nature of information requirements, the critical priority of network performance often far exceeds expansion costs. When building or expanding a C4 system, Joint military doctrine provides the following seven principles and other relevant factors to be considered [JP 6-0; JP 6-02]. In order to be useful to the warfighter, these systems must be:

1. Interoperable. Interoperability is the condition achieved among C4 systems or items of C4 equipment when information or services can be exchanged directly and satisfactorily between them and their users;
2. Flexible. Systems must have the ability to meet changing situations and diversified operations with a minimum of disruption or delay;
3. Responsive. C4 systems must respond instantaneously to the warriors' demands for information;
4. Mobile. Warriors at all levels must have C4 systems that are as mobile as the forces, elements or organizations they support without degrade of information quality or flow;
5. Disciplined. C4 systems and associated resources may be limited -- this calls for a minimum of essential information critical to decision making and mission execution;

6. Survivable. The degree of survivability for C4 systems supporting the function of command and control (C2) should be commensurate with the survival potential of the associated command centers and weapon systems; and

7. Sustainable. C4 systems must provide continuous support during any type and length of joint operation.

Network planners must be fully aware of the mission the system is to support. They should clearly understand the capabilities and limitations of all potentially available strategic, operational and tactical C4 systems and equipment [JP 6-02: I.3]. Other key factors to be considered in the design include the mission of the system, standardization, connectivity, spectrum management, and information protection.

The application of military objectives to network decisions is seen in Hale's development of a decision analysis model for the Australian Defense Force to evaluate communication systems [Hale, 1995]. Hale's decision criteria were cost, and system effectiveness, as developed from communication system objectives similar to those cited above. A two-way analysis of system effectiveness versus cost portrayed the tradeoff between the criteria.

Value-Focused Thinking

The criteria of system effectiveness used by Hale [Hale, 1995] were quantified using value functions and measurable attributes, as discussed by Keeney [Keeney, 1992]. The concept focuses on first articulating and understanding your values, then using them to create the decision alternatives which might achieve them, and finally, evaluating how well alternatives achieve them.

Values of decision makers are made explicit with objectives [Keeney, 1992: 33]. These objectives qualitatively capture the values of concern in the decision to be made. Objectives are structured in the fundamental objectives hierarchy for quantitative modeling. The overall fundamental objective identifies the purpose for investigating the decision situation. Lower level objectives define a part of the higher level objective above it. These lower level objectives must be mutually exclusive and collectively exhaustive [Keeney, 1992: 78]. Each lower level objective is further broken down until reaching a level at which attributes can be defined to indicate the degree to which the objective is met. Attributes must be measurable in order to construct a value model that quantifies multiple objectives.

A value model provides a method to quantify the relationships among all the objectives of the decision. It assigns a number to each consequence which specifies a level for each attribute. If there are no uncertainties in the consequences of an alternative, the model is a measurable value function; with uncertainty, the model should be a utility function [Keeney, 1992: 132]. The value function is derived so that the expected utility of each alternative is an indication of its achievement of objectives. The concepts and procedures for constructing measurable value functions and utility functions are analogous [Keeney, 1992: 132]. Therefore, the words 'utility function' will be used to represent both utility and value functions. This will generalize the discussion so that uncertainty can be handled.

Multiattribute Utility Theory

The concept of 'utility' introduces the capability of comparing the consequences of varied levels of a set of attributes which have different units of measure with a common measure. A utility function for a decision with multiple objectives incorporates utility functions for each measurable attribute from the fundamental objectives hierarchy and attribute weights indicating relative importance of each attribute to the overall objective. In this way, incommensurate units can be combined into a single measure of effectiveness, utility. Defining the utility functions and weights for the attributes involves value judgments or preferences of the decision maker. There are many methods for eliciting these preferences. These are discussed later in this chapter.

The form of the multiattribute utility function depends on the independence conditions which characterize the interaction of the attributes. A set of attributes with no interactions can be modeled with a more simple form. The additive utility function is an exceptionally useful and easy way to model preferences [Clemen, 1996: 553]. An additive utility function, with the set of attributes $X = \{X_1, \ldots, X_n\}$, $n \geq 2$, has the form

$$U(x_1, \ldots, x_n) = \Sigma_{i=1 \; to \; n} \; k_i u_i(x_i) \qquad (11)$$

with

$$\Sigma_{i=1 \; to \; n} \; k_i = 1 \qquad (12)$$

where x_i is the level of attribute X_i, u_i is the utility function of attribute X_i, and k_i is the weight of attribute X_i. The additive utility function exists if and only if the attributes are additive independent [Keeney, 1992: 139]. Attributes are additive independent if the

preference order for specified lotteries does not depend on the joint probability

distributions of these lotteries, but depends only on their marginal probability

distributions [Keeney, 1992: 134]. In other words, changes in lotteries in one attribute do

not affect preferences for lotteries in other attributes [Clemen, 1996: 584]. If there is no

uncertainty, the additive utility function exists if and only if the attributes possess

mutually preferential independence [Kirkwood, 1997: 238]. This concept is fully defined

in the following section. Clemen gives a justification for using the additive utility

function in light of von Winterfeldt and Edwards' discussion that additive independence

usually does not hold:

> "Many multiattribute decisions that we make involve little or no uncertainty, and
> evidence has shown that the additive model is reasonable for most situations
> under conditions of certainty. And in extremely complicated situations with many
> attributes, the additive model may be a useful rough-cut approximation."
> [Clemen, 1996: 585]

This effort considers the consequences of alternatives to be certain. Therefore, mutual

preferential independence of attributes should be shown.

Mutual Preferential Independence

Given a partition of the set of attributes X into sets Y and Z, Y is preferentially

independent of Z if the rank ordering of alternatives that have common levels for all

attributes in Z does not depend on these common levels [Kirkwood, 1997: 238]. That is,

the preference ranking of attributes in Y does not change for different levels of attributes

in Z. A set of attributes displays mutual preferential independence (MPI) if Y is

preferentially independent of Z for every partition $\{Y, Z\}$ of $\{X_1, X_2, \ldots X_n\}$ [Kirkwood,

1997: 238]. The Theorem of Pairwise Preferential Independence can be employed to

reduce the number of partitions which need to be examined. It states that given a partition $\{Y_k, Z_k\}$ with $Y_k = \{X_1, X_k\}$, $k = 2, 3, \ldots, n$ and Z_k containing all the attributes not in Y_k, then the set of attributes X will have MPI if Y_k is preferentially independent of Z_k for $k = 2, 3, \ldots, n$. Thus, it is sufficient to consider only pairs of attributes, $\{X_1, X_j\}$, $j = 2, 3, \ldots, n$ to establish MPI [Kirkwood, 1997: 239]. If a decision maker has established a good value hierarchy with mutually exclusive and collectively exhaustive objectives, MPI is reasonable to assume, but should not be taken for granted [Clemen, 1996: 579]. Therefore, assuming certain consequences and establishing mutual preferential independence of attributes, the value or objective model takes the form of the additive utility function. To realize the model, two elements demonstrating the decision maker's preferences must be elicited: the individual attribute utility functions and the attribute weights.

Defining Preferences: Attribute Utility Functions

Once the decision maker has completely defined the value hierarchy, the next step is to elicit his/her preferences for the attributes in the form of utility functions and weights. Generally, utility functions range from zero to one; however, they can be scaled to any range since they demonstrate a positive affine transformation [Kirkwood, 1997: 245]. A range of possible levels must be defined for each attribute, assigning the best utility score (i.e., one) to the most preferred level and the worst utility score (zero) to the least preferred level. There are a number of assessment methods for defining the utility function between the two extremes, including lottery assessments and scaling procedures [Keeney and Raiffa, 1976: 94, 261; Logical Decisions, 1995: 245-256]. Lottery methods

are developed for uncertain consequences; whereas, scaling procedures pertain to certain consequences. This effort focuses on techniques for additive value functions, specifically the midvalue splitting technique.

A specific scaling procedure, the midvalue splitting technique, finds the following attribute (x_i) levels over the value function: $v_i(x_i) = 0$, $v_i(x_i) = 1$, $v_i(x_i) = .5$, $v_i(x_i) = .25$, and $v_i(x_i) = .75$. The function can be adequately approximated with a curve through these five points [Kirkwood, 1997: 239-240]. An informal method of assessing the value functions is to have the decision maker draw them directly. Logical Decisions for Windows (LDW), a decision analysis software package incorporating MAUT techniques, provides both capabilities [Logical Decisions, 1995]. From a graphical representation of the value function, the decision maker can specify mid-preference levels for any utility range and split it into two subranges to further define the curve [Logical Decisions, 1995: 38-41]. This allows for construction of non-linear value functions. LDW also computes an estimate of the resulting value function for evaluation purposes.

Defining Preferences: Attribute Weights

To complete the additive model, each attribute's relative importance must be elicited from the decision maker. Assessment methods for attribute weights include direct assessment, pricing out, swing weighting, and rankings among others [Clemen, 1996: 546-552; Logical Decisions, 1995: 266-278]. The theory behind pricing out is to determine the marginal rate of substitution between one particular attribute and any other. This method lends itself to monetary tradeoffs, attributes which are commonly bought or sold, and linear utility functions [Clemen, 1996: 547].

In swing weighting, the decision maker numerically specifies the value of swinging each attribute from its least preferred to its most preferred level [Kirkwood, 1997: 240]. These weights are sensitive to the range of values for an attribute. If a range is changed, a new scaling constant must be found that will match the rescaled value function for that attribute [Kirkwood, 1997: 240].

Barron and Barrett present a case for rank-based methods of determining weights, considering the detailed and 'perhaps falsely precise' weight elicitation of weights from the decision maker [Barron and Barrett, 1996: 1515]. They cite as further reason the increased confidence level of a decision maker in specifying a ranking of the importance of the attribute ranges rather than assigning precise weights. Specifically, they recommend the use of rank-order centroid (ROC) weights. In general, for a ranking of $w_1 \leq w_2 \leq \ldots \leq w_n$, the centroid weight for the ith most important attribute is [Barron and Barrett, 1996: 1517]:

$$w_i(ROC) = (1/n) * \Sigma_{j=1 \text{ to } n} 1/j, \quad \text{for } i = 1, \ldots, n \qquad (13)$$

LDW uses this calculation scheme in its "Smarter" method of weight assessment [Logical Decisions, 1995: 267-268]. Tied weights and zero weights are allowed. This method provides a usable, efficacious set of weights in a case where little more than attribute rankings are certain, as evidenced in Barron and Barrett's simulation study [Barron and Barrett, 1996: 1515-1523].

Incorporating weights and utility functions completes an additive model. This model represents the total utility of attributes as a function of the alternatives. For each alternative, the model generates a specific utility value. When choosing an alternative,

the one with the highest utility is the most desirable. Cost is usually a limiting factor in selection. To investigate a number of different alternative combinations with different costs, a 'knapsack problem' formulation is one modeling option.

Knapsack Problems

The knapsack problem derives its name from the problem faced by a camper filling his/her knapsack. There are n objects to choose from, having weights w_i and values v_i ; however, the total weight of the knapsack must not exceed W. This weight is less than the sum of all the weights. The problem is to find the most valuable combination of objects to pack (see constraint (14) below) which meet weight constraints (see (15) below) [Evans and Minieka, 1992: 79]. The problem can be formulated using binary variables x_i for $i = 1, \ldots, n$ with $x_i = 1$ if object i is selected and $x_i = 0$ otherwise. The formulation for this 0-1 knapsack problem is [Evans and Minieka, 1992: 80]:

$$max \ \Sigma_i \ v_i x_i \qquad (14)$$

$$s.t. \ \Sigma_i \ w_i x_i \ \leq \ W \qquad (15)$$

$$x_i \ \in \ \{0,1\} \qquad (16)$$

This is the general 0-1 knapsack model. In using this formulation to find the optimal combination of alternatives described by utility functions, the objective is to maximize utility. The value, v_i, is the utility of each alternative; it contributes to the objective function only if the alternative, x_i, is chosen. The weight constraint (15) may

describe numerous different limitations. For a C4 network, these may be budget limitations, military standards and specifications, the availability of the equipment, or the amount of time until delivery.

Summary

The theory presented in this chapter establishes the framework for the methodology used in this effort. There are three key techniques that should be noted: 1) the minimum cost-maximum flow algorithm identifies the bottleneck components in a communication network; 2) value-focused thinking provides a mapping of the objectives of a C4I network expansion into a quantitative model for evaluating the system; and 3) a simple knapsack formulation selects the most valuable combination of alternatives for upgrading the network. These techniques are explored further in Chapter 3.

III. Methodology

This chapter specifies the techniques developed to provide an integrated methodology for the component evaluation and the expansion of an existing C4 network. This requires the use of three separate models: the network flow model, which identifies bottlenecks and provides the focus for generating expansion plans; the value model, which quantifies the objectives of the decision to make expansion plan comparisons and system evaluations; and the expansion model, which selects the components from the feasible set of options to optimally expand the network. Figure 3-1 demonstrates the four steps of the methodology, the models used, and the outputs generated. Before constructing the models, the general background of the modeling environment is defined.
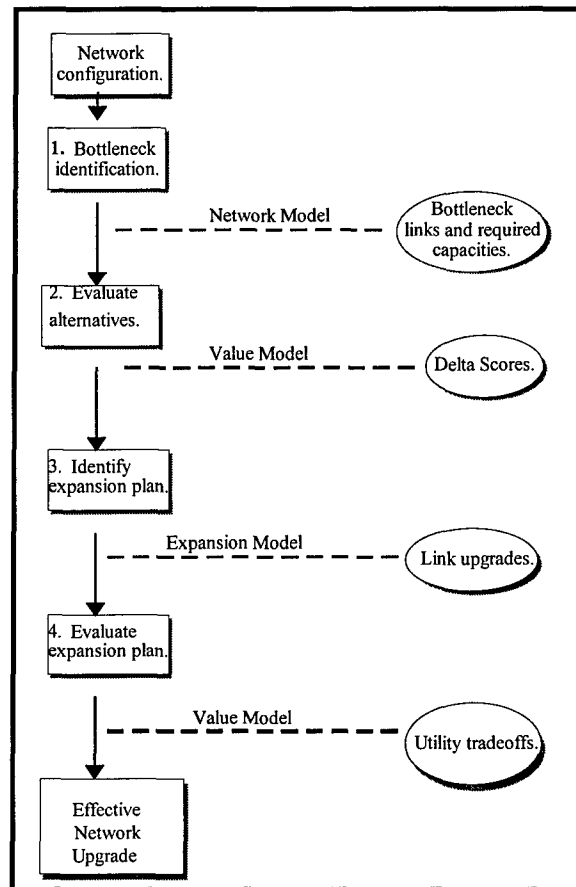
Figure 3-1. Flow Chart of Methodology

The Situation

To perform this analysis, specific network information is required. This includes

the network description, the current topology, its mission and capabilities, and the

problem it has meeting user demands. To illustrate this methodology, an example

scenario is used. The network is supporting a forward deployed joint task force with

extensive requirements for intelligence data. A notional network configuration is adapted

from the U.S. Army's area communications system [Dept. of the Army, 1995] (see

Figure 3-2). There is an existing structure or connectivity within the system as well as a

specified flow of information, unequivocally analogous to the military chain of

command. With the ramping build-up of forces in the area, the network has become

stressed and the demands for intelligence at the lowest levels (Brigade level shown) are

not being satisfied before the information "expires." Thus, the link capacities between

Theater and Brigade levels must be increased.
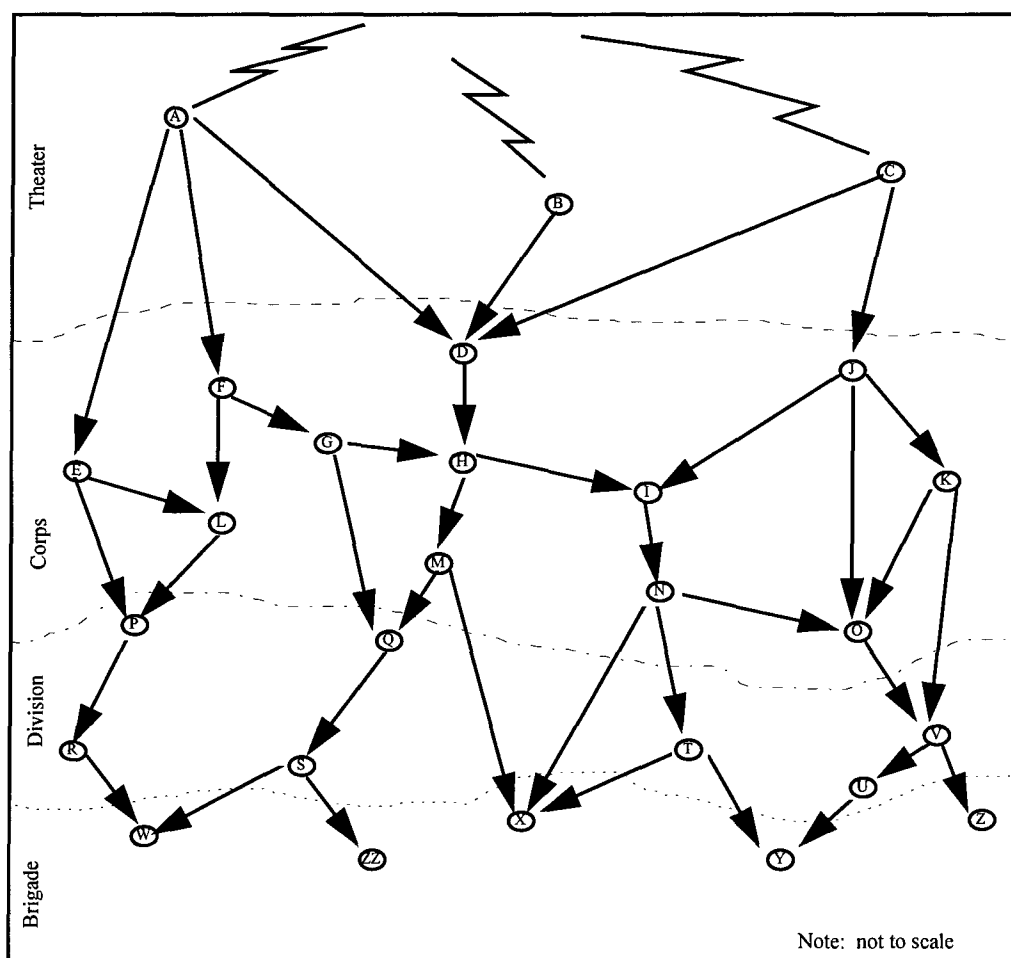
Figure 3-2. Notional Deployed C4I Network.

In a deployed scenario, there is usually a transformation from initial hub and

spokes to a full mesh network topology which increases the robustness and reliability of

the network. This metamorphosis results from the phasing of joint operations, to include

deployment, employment and sustainment [Joint Pub 6-02, 1996: III-1]. The objective of

the deployment phase is to maintain the continuous flow of information between commanders. Capacity is minimal, and the system may be severely degraded when disturbed. The employment phase produces the automated flow and processing of information, establishing numerous alternate routes to increase the robustness of the network. The focus of the sustainment phase is to support and improve the automated flow and processing of information to commanders. Overall capacity is increased to obtain transparent information transfer. Systems are adjusted to meet changing mission requirements and user demands or complaints [Joint Pub 6-02, 1996: III-2]. The model used here focuses on the sustainment phase of the network.

The principle transmission media in the notional network is the atmosphere or empty space, utilizing RF (radio frequency) and satellite communications. Each link represents a separate component, including but not limited to a receiver/transmitter, antenna, and a personal computer for control and display. A conceptual listing of the network links and their current capabilities has been place in Appendix A. An explanation of the capabilities listing is given in Appendix B, the value hierarchy. It is assumed that an estimate of increased network demands at the Brigade level has been assessed.

Network Model

Given a C4 network topology that must be expanded to meet the actual or predicted demands of deployed units or increased usage, the problem is to identify those arcs with no remaining capacity, the *bottleneck arcs,* by solving the minimum cost-

maximum flow network algorithm. This bottleneck identification step is highlighted in

Figure 3-3.
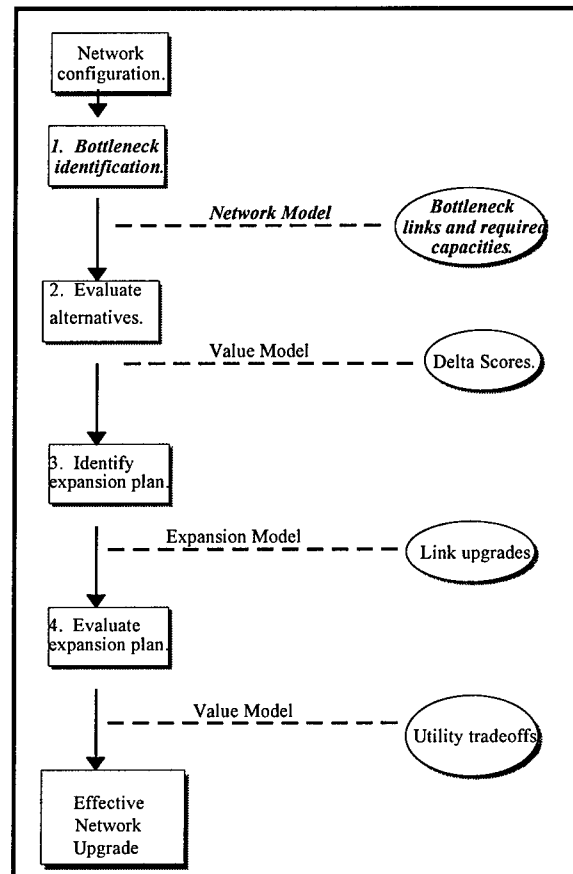
Figure 3-3. Methodology Flow Chart - Step 1.

Configuration. The network topology includes nodes and directed arcs,

representing transmit/receive equipment and transmission links respectively. A

circulation network with all nodes exhibiting conservation of flow is formed as stated in

Chapter II by adding a super node. In this case, two separate nodes are added, a *supply*

node and a *demand* node, which are connected by an arc over which flows the throughput

of the network (see Figure 3-4). The capacity of the arcs from the *supply* node to origin

nodes (theater level) has an upper bound of the actual supply of the origin node in Kilobits (Kb). Similarly, the capacity of the arcs from destination nodes to the *demand* node has a lower bound of the actual demand of the destination node in Kb. The capacity of real network arcs is given by the data rate of the transmission media in Kilobits per second (Kbps). This is a simplifying assumption that negates the need to schedule network resources and simulate exact message flow through a network. Implicit in the formulation is the 'per second' use of capacity on an arc; however, this will not affect the efficiency of the model in identifying bottlenecks.

Feasibility. If the current network configuration cannot meet demand, the problem is mathematically infeasible and the only information obtained is the nodes at which conservation of flow does not hold. Therefore, the problem must be kept feasible to recognize the bottleneck links. This can be accomplished by adding more arcs to the network. These feasibility arcs can flow directly from the supply node to the destination nodes to meet this excess demand. Figure 3-4 shows the use of an excess node, *ex*, with a link from the supply which clearly indicates the total amount of flow not feasibly supported by the network. While redundant in a strictly mathematical programming sense, these additional arcs ease the identification of capacity shortfalls at specific end nodes. In order to only draw flow which cannot be supported by the current network structure, the cost on this path is set prohibitively high, greater than the number of links in the longest path from the supply node to any demand node.
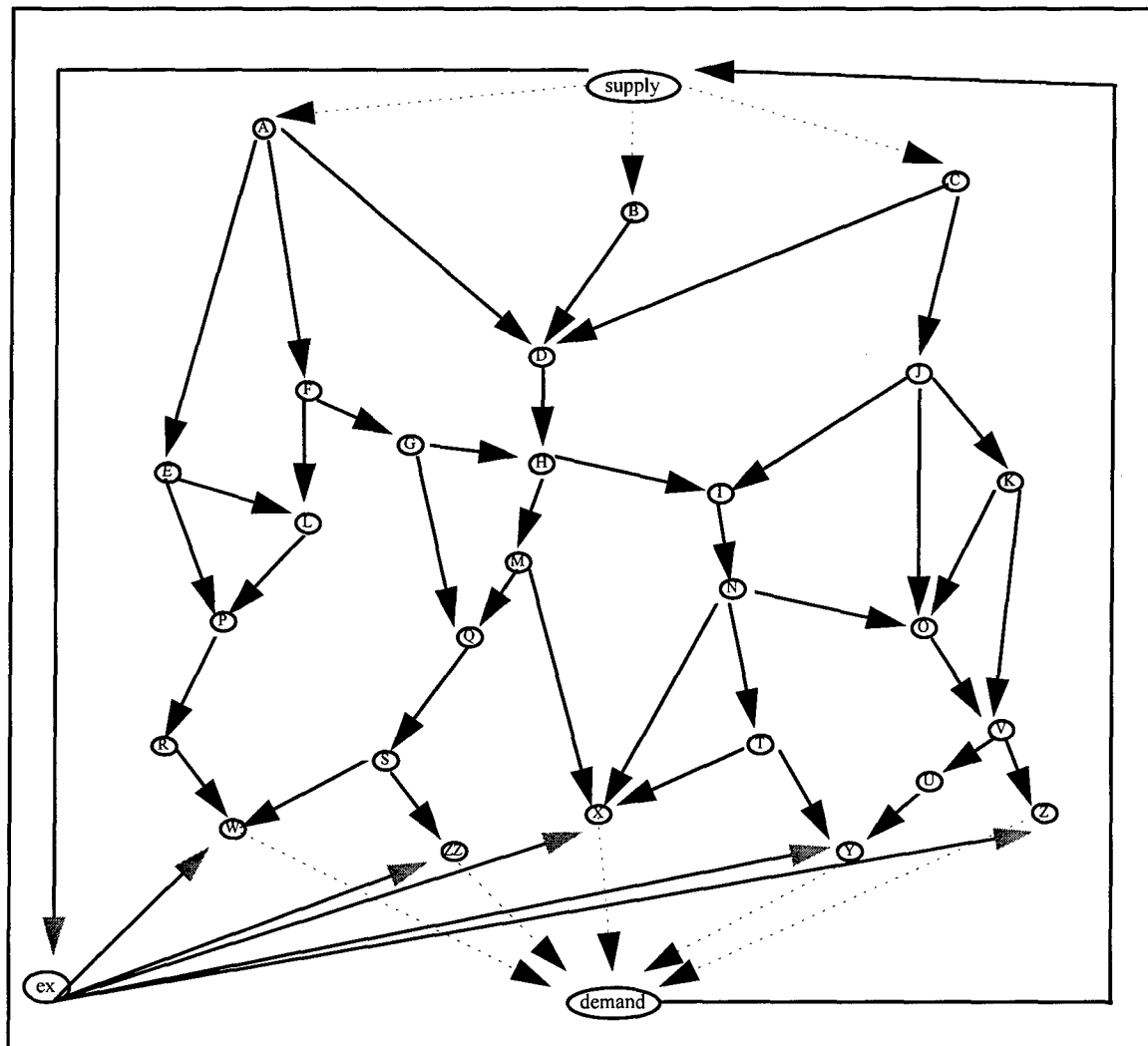
Figure 3-4. Network Configuration for Analysis Purposes

Model Input and Output. The optimal solution of the minimum cost-maximum

flow problem can be found using any number of algorithms and commercially available

codes. Netsolve [Netsolve, 1992], an interactive software package for network analysis,

was used in this study. This requires the same inputs as the linear program formulation

outlined in Chapter II. Given a graph defined by $G = (X, A)$ with X the set of nodes and

A the set of arcs, arc cost coefficients c_{ij}, and arc capacity bounds l_{ij} and h_{ij}, the

formulation is:

$$min \; \Sigma_{(i,j) \in A} \; c_{ij} x_{ij} \qquad\qquad (1)$$

$$s.t. \;\; \Sigma_k \, x_{ki} - \Sigma_k \, x_{ik} = 0, \, for \, all \; i \in X \qquad\qquad (2)$$

$$x_{ij} \geq l_{ij}, \, for \, all \; (i,j) \in A \qquad\qquad (3)$$

$$x_{ij} \leq h_{ij}, \, for \, all \; (i,j) \in A \qquad\qquad (4)$$

In this study, the cost coefficients equal one for all arcs representing transmission media. This allows the model to utilize the least number of links to satisfy demands. The arcs used to generate supply and demand in the network have a cost coefficient of zero, as these arcs are not associated with the depletion of any resources. For the notional network used, the arc from *supply* to *ex* has a cost of ten, since the longest path through the network consists of nine arcs having a cost of one. The value of the objective function is the sum of flow over all actual network arcs plus the "cost" of using the excess flow path to maintain feasibility. The arcs with flow values at their upper capacity bound and negative reduced costs are the *bottlenecks*. The more negative reduced costs indicate the greater the value of the existing arc capacity and the expansion of that capacity. Due to the analysis configuration, negative reduced costs for links will be present in the solution until the actual network components can satisfy demand *and* there are no arcs restricting a potential shorter path within the network. Bottleneck arcs need to be expanded to increase the throughput of the network.

Shifting Bottlenecks. Capacity expansions on network links usually occur in discrete increments. For the system examined here, capacity levels for available

components include 2.4, 9.6, 16, and 32 Kbps. Because the minimum cost-maximum flow algorithm demonstrates a phenomena known as *shifting bottlenecks*, the expansion of all bottleneck components identified in one iteration will do one of four things:

1) increase network throughput and identify new bottlenecks;

2) increase network throughput and identify no new bottleneck components;

3) give no change in network throughput but identify new bottlenecks; or

4) give no change in network throughput and identify no new bottlenecks (but lower the network flow cost).

Depending on the purpose of the expansion, a stopping point for iterations of the algorithm may be having no bottlenecks remaining (results 2 and 4) or increasing network throughput until completely satisfying the demands of end users (results 1 and 2).

Expansion Methods. There are two methods considered for upgrading the capacities of the bottleneck links. One consists of first performing a nodal analysis on every bottleneck found during an iteration of the flow algorithm to determine the potential amount of flow into the start node and out of the end node. This information is then used with discrimination to select a new link capacity that eliminates it as a bottleneck. Not all links are upgraded. The full knowledge of the network is used to determine which links, if upgraded, will satisfy network demand. Iterations of the algorithm continue until the network can meet the system demands. Once the network demand is satisfied and no further bottlenecks are identified, a flow analysis gives an indication of which upgraded links are actually utilized to meet the projected demand. The expansion plans for the network then consist of only those upgraded links whose

increased capacity is used in the last iteration of the algorithm. This method resembles the long-term planning used by many growth-oriented companies. It produces a 'macro' upgrade for the system.

The second method consists of increasing each identified bottleneck myopically by one discrete jump at each iteration, re-running the algorithm with those increased capacities, and upgrading only those components whose augmented capacity is used to increase network throughput. This is analogous to a short range planning cycle, in which immediate return is the primary or sole concern. This "greedy," incremental method generates a set of expansion plans at each iteration and stops when demand is fully satisfied.

Expansion Plans. There are many options for relieving a bottleneck link, such as parallel arcs, expanded arc capacity, or alternate routes. Alternate routes may include current nodes or new node locations. The present phase of the network guides the options for expansion. This network, which is in the sustainment phase of deployment, has ample alternate routes and good connectivity. The main focus of expansion is increasing the capacity of existing links. Expansion plans for each method consist of a set of components (one for each bottleneck) that meet the capacity constraints of the identified bottlenecks. Any combination of these alternatives may be feasible. All components must meet military standards and specifications. Potential components are screened before inclusion for meeting other link-specific requirements. These may include component interoperability, encryption capability, frequency spectrum (interference and allocations), and transmission range. Only range is directly utilized as a screening factor here. All conceptual alternatives possess encryption capability.

Interoperability and spectrum management are assumed. A complete set of available

components and their features is listed in Appendix C. With a feasible set of alternatives,

the impact of each combination of alternatives on the system must be evaluated using the

value model.

Value Model

The value model is utilized to quantify network objectives and evaluate

components using a common measure. Figure 3-5 highlights this step. From the

identified bottlenecks and required capacities, this model is used to evaluate all possible

alternatives for each bottleneck.
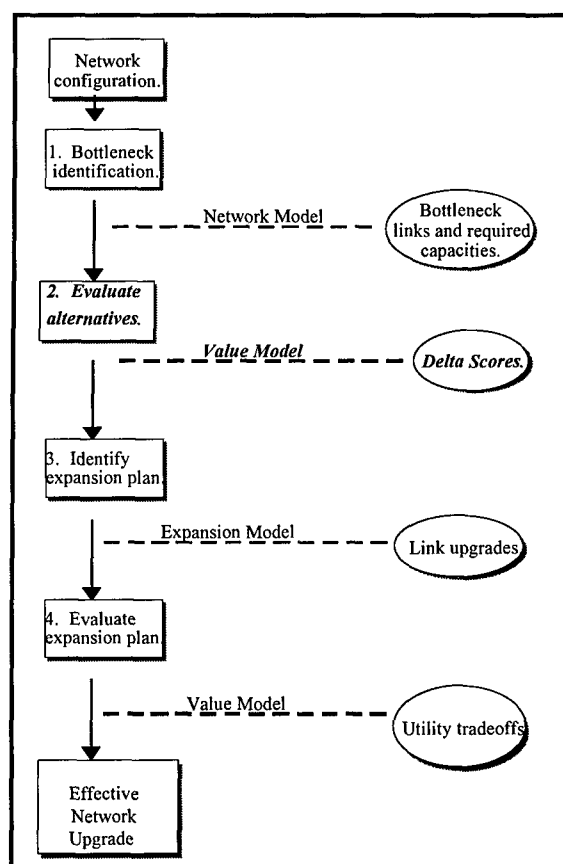
Figure 3-5. Flow Chart of Methodology - Step 2.

The network values or objectives can be drawn from literature, the network

mission, and the expertise of the decision maker. The value hierarchy must be tailored to

the network and the specific decision at hand. The overall fundamental objective

identifies the purpose for investigating the decision situation. The overall objective of this

network expansion is system effectiveness. A group of sub-objectives further define the

objective above it. In this case, objectives for constructing and expanding a Joint C4

network were initially drawn from the Joint Publications as cited in Chapter II. These

were then tailored by the decision makers for the specific problem of expanding the

network to support the demands of a deployed task force. In this illustrative example,

two decision makers were used to develop this notional evaluation. Both hold PhDs in

engineering disciplines. Collectively, they have over twenty years of experience in

communications and computer systems, from planning, building, repairing, and

evaluating C3 (command, control and communications) systems for deployments, to

teaching graduate level courses in radar, communications, and information warfare. Input

was also obtained from four Army Signal Corps personnel to identify the operational

aspects and considerations of the notional network. The initial value hierarchy as drawn

from the literature for the expansion of a deployed C4 network is pictured in Figure 3-6.

The final hierarchy was a modification of Figure 3-6 and is displayed in Chapter 4 and in

Appendix B, where details of the subobjectives and attributes are outlined.

C4 Network Expansion Hierarchy
(first cut)
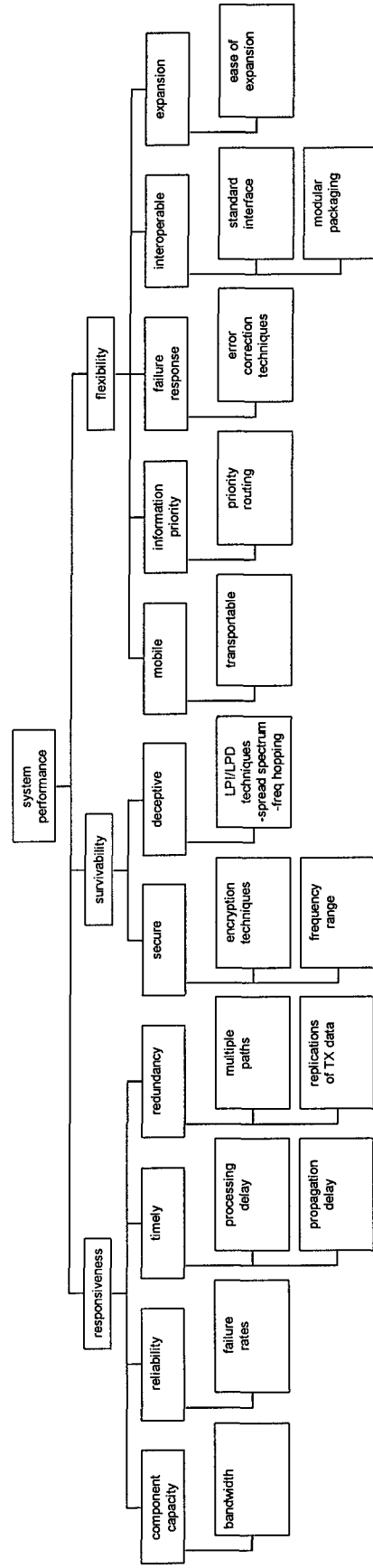
Figure 3-6. Initial Value Hierarchy

Multiattribute Utility Function. This effort assumes that the consequences regarding attributes for alternatives are certain. Therefore, in order to determine the form of the multiattribute utility function, independence conditions must be established. The additive utility function is relatively easy to assess, especially under conditions of certainty. To use this function, mutual preferential independence (MPI) should be shown. By the Theorem of Pairwise Preferential Independence, MPI can be demonstrated by examining each partition of the set X of n attributes, into the pairs $\{X_1, X_j\}$, $j = 2, 3, \ldots,$ n. If the preference ranking of outcomes of the attribute X_1 does not change for different levels of the attribute X_j for all j, then the set X displays mutual preferential independence.

MPI can be established by interviewing the decision makers. If MPI cannot be established, the value hierarchy may be structured to achieve mutually exclusive and collectively exhaustive subobjectives and measures. MPI was assumed as a reasonable approximation [Clemen, 1995: 579] for this effort since the structure of the hierarchy was conceived based on the attributes being mutually exclusive and collectively exhaustive. Therefore, an additive utility function was used. For a set of attributes $X = \{X_1, \ldots, X_n\}$

$$U(x_1, \ldots, x_n) = \sum_{i=1 \text{ to } n} k_i u_i(x_i) \tag{5}$$

with

$$\sum_{i=1 \text{ to } n} k_i = 1 \tag{6}$$

where x_i is the level of attribute X_i, u_i is the utility function of attribute X_i, and k_i is the weight of attribute X_i. These elements were elicited from the decision makers.

Assessment of Preferences. Aside from their operational/technical communications experience, the decision makers also had a recent exposure to the use of

value functions and weights in an additive model. They were entirely comfortable with performing a direct assessment of both weights and functions. Logical Decisions for Windows (LDW) aided in deriving the individual utility functions using a combination of direct assessment and the graphical split-range technique discussed in Chapter 2 [Logical Decisions, 1995]. Weights were based on proportional comparisons between attributes and objectives. For example, the subgoal responsiveness may be viewed as twice as important as flexibility, and about the same in importance as survivability. This would result in a weight of .4 for both responsiveness and survivability and a weight of .2 for flexibility.

Component Evaluation. The resulting value hierarchy, utility functions, weights and attribute descriptions are located in Appendix B. Appendix B also contains the background for each attribute area, how each system component was evaluated, and how the impact of that component on the system was captured. In general, each system component receives a score for an attribute. The system score for that attribute is based on an average score taken over all the components in the system. The utility function relates that average score to a common measure. Using a straight average for these components indicates that there are no components whose scores should be more important due to their location, utilization, or impact on network throughput. If such differences exist, a weighted average could be used to model this. However, the notional network analyzed here assumes equal importance for all network links.

The system effectiveness objective has the form of an additive utility function. It is the weighted sum of all the individual attribute utility functions. The model is validated by using test cases of the best and worst ratings on attributes, giving the overall

scores of one and zero respectively. A specific set of system components corresponds to a particular system effectiveness utility. Any change in the system produces a change in the overall utility. The baseline system utility establishes a benchmark for comparison of potential component upgrades. In order to evaluate the impact of upgrade alternatives for a set of bottleneck links, every possible combination of alternatives has to be scored. This may not appear to be overly taxing; however, consider the following scenario with eight bottleneck components to be upgraded. After pre-screening and capacity considerations, one link has 2 alternatives, four links have 3 alternatives, and 3 links have four alternatives. This translates into 10,368 different combinations or expansion plans to evaluate in order to identify the optimal combination. However, the general structure of the utility functions allows an efficacious approximation of the impact of each component at each bottleneck location.

Given the baseline system score, one bottleneck link, A-B for example, and n links in the system, the current component on link A-B contributes ($1/n$) times its component score to the system score on each attribute. A component, if substituted for an element on link A-B, would have an added contribution to the system (over the currently installed configuration of the link) that can be approximated as the difference in system scores from the baseline system and the evaluation of the system with the new component on link A-B. To illustrate this, given a baseline system score of 650 and a score of 668 for the system with a specific new component on link A-B, the delta score for the component on link A-B would be 18. The use of approximations gains a significant decrease in the number of evaluations accomplished. For this example, the evaluation of 26 upgraded systems and the baseline system is required, rather than 10,368

evaluations of all possible combinations. Note that the averaging method of component scores does not apply to all attributes. However, the approximation does not severely damage this proof-of-concept model. These differences in utility, or delta scores, for each potential alternative are used in the expansion model to select the best (based on the approximations just discussed) combination of components subject to any further constraints in the system.

The expansion plans resulting from both the global and the incremental methods of capacity expansion are evaluated for this effort. The global method produces just one set of bottlenecks and potential upgrades. It attempts to locate a configuration which optimizes the global value of the network. However, the incremental method has one set of bottlenecks and potential component upgrades for each bottleneck analysis iteration. This short term, "greedy" approach mimics the behavior of an incremental upgrade to the network which is based on the "next best" configuration, rather than a final, overall configuration. Within each set, each feasible component is evaluated as described above, resulting in the delta scores to be used in the expansion model. The incremental delta scores for each iteration are based on the system state after the previous iteration was completed. In other words, each previous iteration becomes the new baseline system for comparison.

Expansion Model

Given the delta scores for each upgrade component, the problem is to maximize the 'utility' of the network upgrade while alleviating the bottleneck arcs by choosing the best possible component expansion. Figure 3-7 shows that the output of this step is the

link upgrades for all bottlenecks. A zero-one knapsack problem tailored to the network selects optimal expansion components.
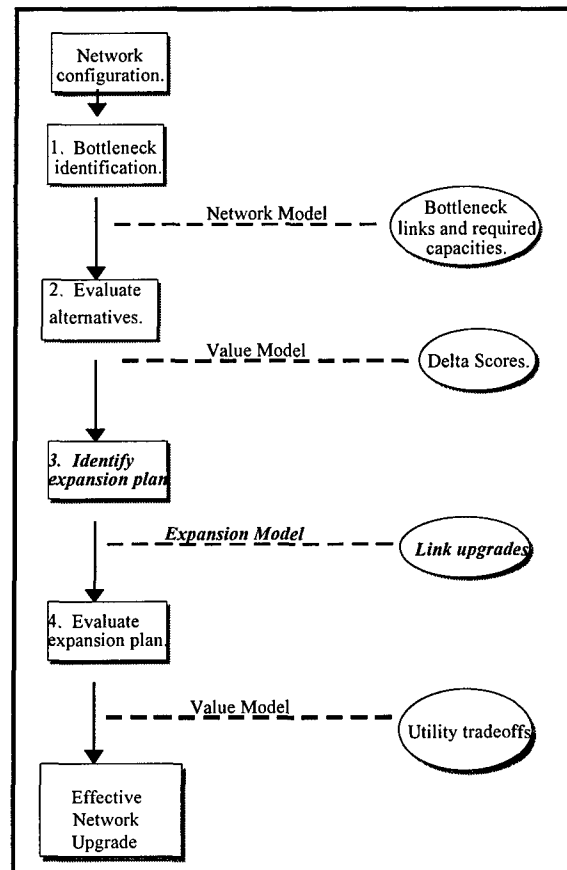
Figure 3-7. Flow Chart of Methodology - Step 3.

Knapsack Formulation. The delta scores from the value model become coefficients, v_{ij}, in the objective function of the formulation, where i represents the upgrade component and j represents the bottleneck location. Binary variables, x_{ij}, act as switches for selecting/not selecting each component. The first constraint (8) is the traditional knapsack "weight" problem, in the form of a budget. The next constraint type (9) limits the selection to one component per bottleneck. This is the additional constraint

used in a multiple choice knapsack problem (MCKP) where the item set (all upgrade alternatives) is partitioned into subsets (alternatives for each bottleneck) and at most one item per subset is selected [Martello & Toth, 1990: 77].   There may be other restrictions as to which components and/or locations are feasible, depending on military and commercial asset availability, frequency availability, geographic operational area, and the transitory state of node locations. While the MCKP structure may be lost, other integer and linear programming approaches could be utilized for these constraints. However, only the two constraints outlined above are used here for illustrative purposes. The complete formulation is:

$$max \ \Sigma_{i,j} \ v_{ij} x_{ij} \tag{7}$$

$$s.t. \qquad \Sigma_{i,j} \ c_{ij} x_{ij} \ \leq \ B \tag{8}$$

$$\Sigma_i \ x_{ij} \ \leq \ 1, \qquad \forall \ j \tag{9}$$

$$x_{ij} \ \in \ \{0,1\} \tag{10}$$

B is the budget amount, and c_{ij} are the costs of components at each bottleneck location. This model indicates the combination of upgrade components which maximizes the value of the items selected subject to cost considerations. A model with the budget constraint (8) relaxed is also run. This allows the analysis to determine the 'best' configuration if cost is not a constraint.

System Evaluation

Once these expansion plans are selected, the network is expanded. The resulting systems are evaluated using the value model. This is step four of the methodology (see

Figure 3-8). These system utilities and the corresponding costs provide insight into the

tradeoff of system effectiveness versus cost. The increase in utility which has been

realized is the value of upgrading the system based on the decision makers' objectives.

There is an increase in system throughput associated with the upgraded network. This

increase corresponds to an increase in the amount of information received. Therefore, the

increase in system utility represents the value of the increase in information.
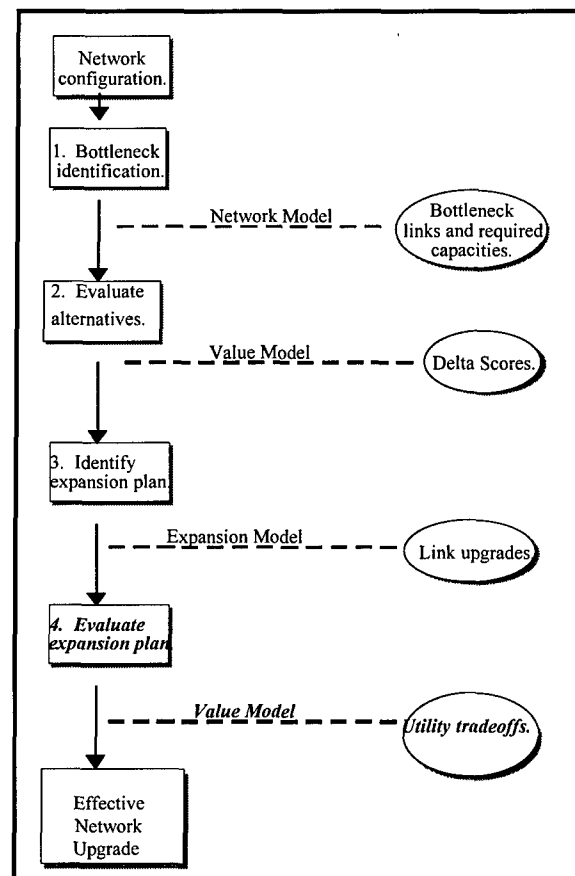
Figure 3-8. Flow Chart of Methodology - Step 4.

Summary

The "optimally" upgraded systems have been selected using the delta scores of component impact. These are not guaranteed to be optimal, having been based on the approximation of one component upgrades. To identify the optimal system upgrade, all combinations of component alternatives would have to be evaluated on the system level with the value model. Due to the prohibitive combinatorial nature of such an undertaking, such an effort is not performed for this study. This methodology does provide a technique to systematically evaluate a network upgrade with a realistic number of component evaluations.

System Changes. New demands introduced to the system require a full iteration of identifying bottlenecks, evaluating expansion plans, and selecting the optimal expansion with cost considerations. Changes in the network due to the current deployment phase may require a new or adjusted value hierarchy. For the illustrative example used in this study, it is assumed that the weights and utility functions for attributes remain constant despite changes in the deployment scenario. The needs of the actual operational setting will dictate the level of modeling required.

IV. Results and Analysis

This chapter presents the results of an analysis based on the notional scenario and detailed methodology developed in the previous chapters. This includes the results from the network model, the value model for component and system evaluations, and the expansion model for both macro and incremental expansion methods.

Network Model Results

The network model was used to identify the bottleneck links within the network. The minimum cost-maximum flow algorithm was run using the software code Netsolve [Netsolve, 1992]. A listing of the model input and the results is provided in Appendix D. The model identifies links whose capacities are at their upper bounds, and their reduced edge costs are negative. This means that there is value in expanding the link to potentially provide greater throughput to the network. These are the bottleneck links. For the macro expansion method, bottlenecks are identified at each iteration (see Table 4-1). The final flow model indicates the upgrades actually required to satisfy projected demands. For the incremental expansion method, bottleneck links are expanded at each iteration only if the expansion results in increased throughput of the network (see Table 4-2). Upgrades are required at each iteration. Figure 4-1 depicts the network scenario.

The following tables list the identified bottlenecks and the capacity upgrades required for each method. One system wide upgrade effort for the macro method yields 100% demand satisfaction; whereas, three separate upgrades are required for the

incremental method to reach the same level of throughput. The network flow costs are

due to the flow across network links and routing flow on the excess paths when the

network could not satisfy 100% of the demand. Note that the macro method continues

relieving bottlenecks after the demand is satisfied; whereas the incremental method stops

as soon as 100% demand satisfaction is attained. Due to the costs on network links, the

expansion of bottlenecks identified after meeting demand should lower the cost of the

network flow. These bottlenecks are on potentially more efficient (shorter) paths for

satisfying demand.
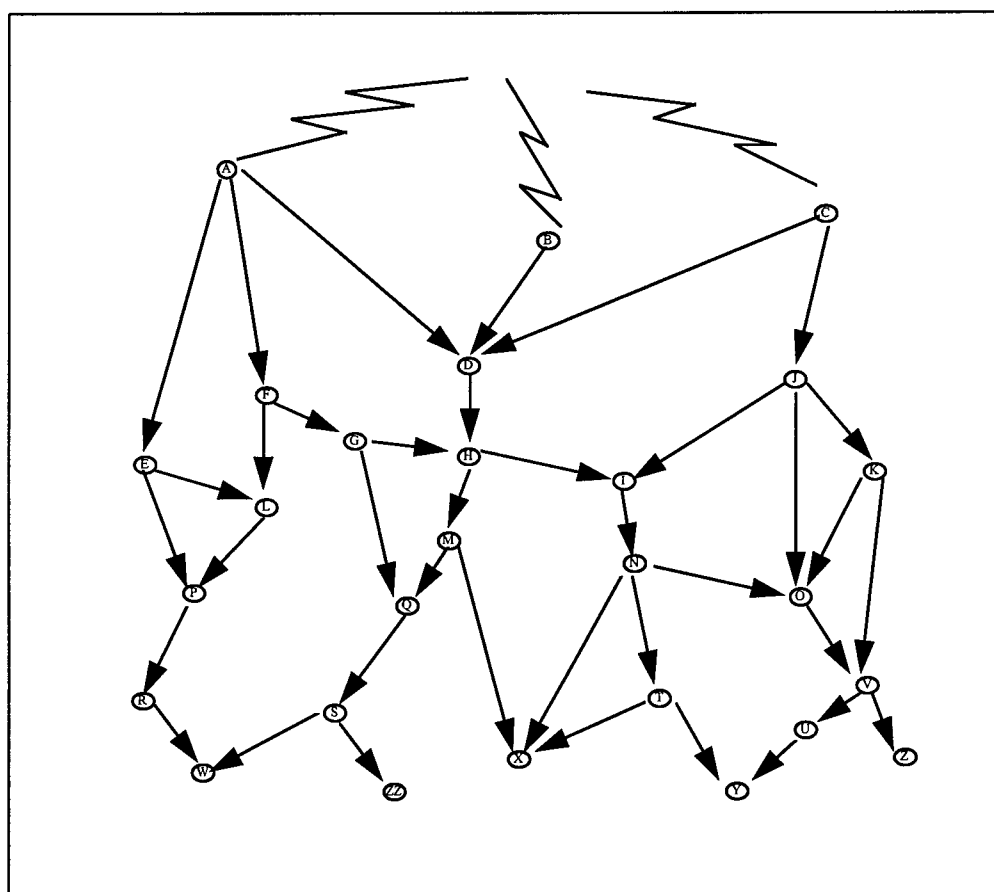
Figure 4-1. Notional Deployed C4I Network

Not all the bottlenecks identified in the macro method are upgraded. Those not expanded are listed in italics (see Table 4-1). This discriminant method of selection is acceptable because the iterations continue until all bottlenecks are removed. Any true bottlenecks hindering more efficient flow will be identified again in a later iteration.

Table 4-1. Macro Method Expansion Requirements

Iteration 1				
bottleneck link	current capacity (Kbps)	new capacity (Kbps)		
A-F	9.60	32.00		
C-J	9.60	32.00		
D-H	9.60	32.00		
E-L	2.40	*2.40*		
E-P	2.40	9.60		
G-Q	2.40	9.60		
I-N	9.60	32.00		
M-X	2.40	9.60		
P-R	9.60	16.00		
cost:	330.60	251.40		
% of demand satisfied:	65.88	89.41		
Iteration 2				
bottleneck link	current capacity (Kbps)	new capacity (Kbps)		
K-V	2.40	*2.40*		
O-V	9.60	*9.60*		
Q-S	9.60	*9.60*		
R-W	9.60	16.00		
T-Y	2.40	9.60		
cost:		224.40		
% of demand satisfied:		100.00		
Iteration 3				
bottleneck link	current capacity (Kbps)	new capacity (Kbps)		
E-P	9.60	16.00		
cost:		224.40		
% of demand satisfied:		100.00		
Iteration 4			Results	
bottleneck link	current capacity (Kbps)	new capacity (Kbps)	links to be upgraded	capacity required (Kbps)
A-E	9.60	16.00	A-E	16.00
			C-J	32.00
			E-P	16.00
			G-Q	9.60
			M-X	9.60
			P-R	16.00
			R-W	16.00
			T-Y	9.60
cost:		220.00		
% of demand satisfied:		100.00		

There are eight links whose capacity must be expanded for the macro method. These are identified in the deployed network shown below as the continuous lines. For example, link A-F has a current capacity of 9.6 Kbps and must be expanded to 32 Kbps to meet system flow requirements.
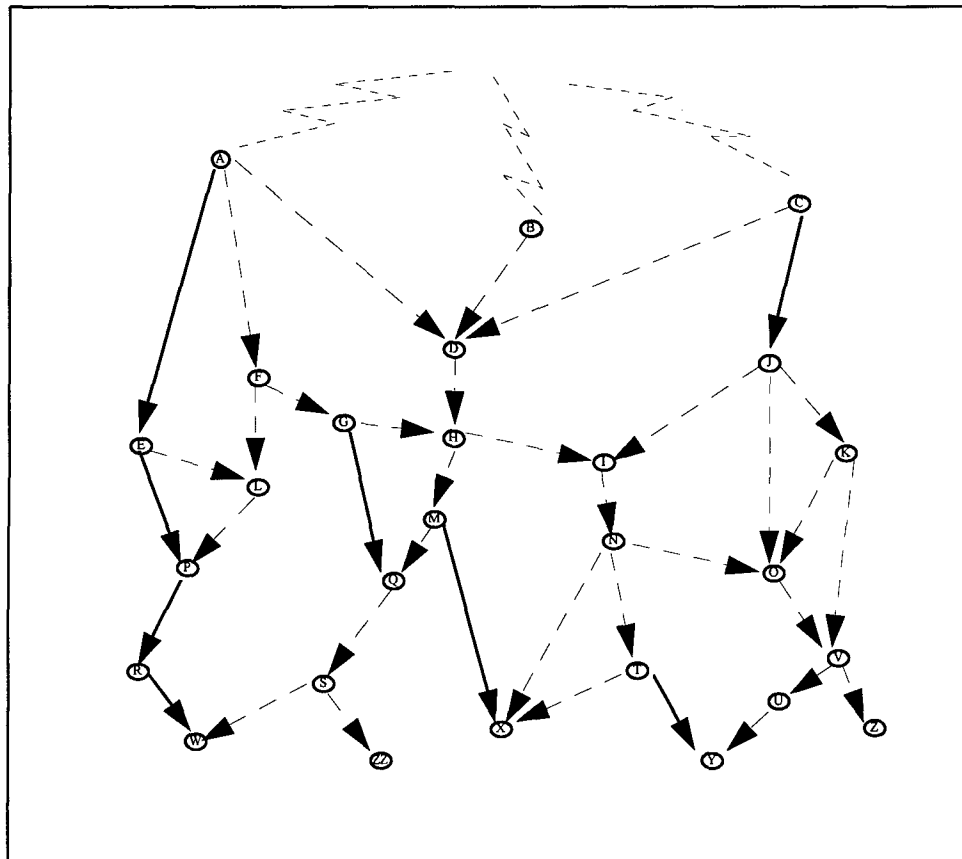
Figure 4-2. Network with Macro Expansion Links Identified

Table 4-2. Incremental Method Expansion Requirements

| Iteration 1 | | | | Result | |
bottleneck link	current capacity (Kbps)	new capacity (Kbps)	upgrade required for increased throughput	links to be upgraded	capacity required (Kbps)
A-F	9.60	16.00	N	C-J	16.00
C-J	9.60	16.00	Y	D-H	16.00
D-H	9.60	16.00	Y	E-P	9.60
E-L	2.40	9.60	N	G-Q	9.60
E-P	2.40	9.60	Y	M-X	9.60
G-Q	2.40	9.60	Y		
I-N	9.60	16.00	N		
M-X	2.40	9.60	Y		
P-R	9.60	16.00	N		
cost:	330.60				252.20
% of demand satisfied:	65.88				89.41

| Iteration 2 | | | | Result | |
bottleneck link	current capacity (Kbps)	new capacity (Kbps)	upgrade required for increased throughput	links to be upgraded	capacity required (Kbps)
C-J	16.00	32.00	Y	C-J	32.00
H-M	9.60	16.00	Y	H-M	16.00
J-O	9.60	16.00	N	K-V	9.60
K-V	2.40	9.60	Y	Q-S	16.00
O-V	9.60	16.00	N	T-Y	9.60
Q-S	9.60	16.00	Y		
R-W	9.60	16.00	N		
T-Y	2.40	9.60	Y		
cost:					228.80
% of demand satisfied:					99.22

| Iteration 3 | | | | Result | |
bottleneck link	current capacity (Kbps)	new capacity (Kbps)	upgrade required for increased throughput	links to be upgraded	capacity required (Kbps)
G-Q	9.60	16.00	N	M-Q	9.60
M-Q	2.40	9.60	Y		
R-W	9.60	16.00	N		
cost:					227.20
% of demand satisfied:					100.00

The two methods provide a total of four sets of bottlenecks for evaluation. Each bottleneck within a set has several different component alternatives. The value model was used to evaluate all the alternatives.

Value Model Results

The value model consists of the multiattribute utility function structured from the

decision makers' value hierarchy. The final value hierarchy is shown below. The local

weights (within each subgroup) are included for each group. The three major areas

defining system effectiveness are service, survivability, and flexibility. These three

sectors are further divided into subobjectives and finally actual attributes or measures.

There are twenty-one individual attribute utility functions. A complete explanation of the

objectives and attributes is presented in Appendix B.
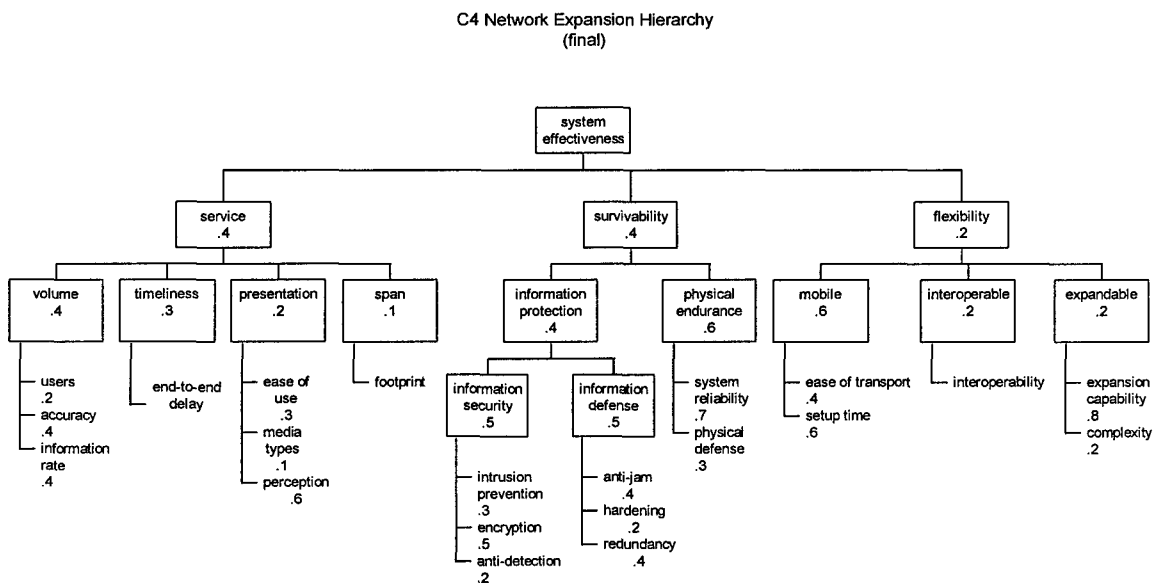
Figure 4-3. Value Hierarchy for C4 Network Expansion

The value model was used in two ways; it provided an evaluation of a complete

system, such as the baseline and the upgraded systems, and an evaluation of the impact an

individual upgrade component had on the system. The baseline system served as the

benchmark comparison for the systems obtained with the macro method and the first iteration of the incremental method. Further iterations of the incremental method used the previous iteration result as a comparison point for the increased values. The baseline system utility was evaluated using the twenty-one features of all thirty-eight links. Each of the attributes contributed a portion of their system score based on the established weights. For example, the attribute setup time has a global weight of 0.072. This is obtained by multiplying the weights at each level as one descends from system effectiveness to setup time (.2*.6*.6). In general, for a set of twenty-one attributes (X = $\{X_1, \ldots, X_{21}\}$), the system model is

$$U(x_1, \ldots, x_{21}) = \sum_{i=1 \text{ to } 21} k_i u_i(x_i)$$

where x_i is the level of attribute X_i, u_i is the system utility function of attribute X_i, and k_i is the global weight of attribute X_i. The system setup time was evaluated as the average setup time over all network links. For an average setup time of one hour, the corresponding utility is 500 (based on a scale of 0-1000). This translates into a contribution of 36 toward the system effectiveness utility. Figure 4-4 shows the global weights for each of the subobjectives and the attributes.

```
                              ┌──────────────┐
                              │   system     │
                              │ effectiveness│
                              └──────────────┘
        ┌─────────────────────────┼─────────────────────────┐
   ┌─────────┐               ┌──────────────┐          ┌───────────┐
   │ service │               │ survivability│          │flexibility│
   │   .4    │               │     .4       │          │    .2     │
   └─────────┘               └──────────────┘          └───────────┘
```

| volume .16 | timeliness .12 | presentation .08 | span .04 | | information protection .16 | physical endurance .24 | | mobile .12 | interoperable .04 | expandable .04 |

— users .032
— accuracy .064
— information rate .064

— end-to-end delay

— ease of use .024
— media types .008
— perception .048

— footprint

information security .08 | information defense .08

— system reliability .168
— physical defense .072

— ease of transport .048
— setup time .072

— interoperability

— expansion capability .032
— complexity .008

— intrusion prevention .024
— encryption .04
— anti-detection .016

— anti-jam .032
— hardening .016
— redundancy .032

Figure 4-4. Value Hierarchy with Global Weights

The global weights range from 0.168 to 0.008 (or 16.8% to .8%). Obviously the system utility will be more sensitive to changes in those attributes with larger weights. There are eight attributes with weights above .04, or 4%. These include (in descending order of weights) reliability, end-to-end delay, setup time, physical defense, accuracy, information rate, ease of transport, and perception. Of these, perception and physical defense are held at a constant value throughout the evaluations. It is assumed that all components have the same capability for presenting information, and that the physical defense at a station will not change due to a change in components. It is noted that there are forty-three parallel paths within the network. True to design, this translates into a very reliable network. Therefore, even though reliability has the largest weight, the reliability of the system will probably not be a determining factor in system or component evaluations.

The baseline score was also used in obtaining the delta scores for each potential

upgrade component. As described in Chapter 3, the delta score is the difference in utility

between the baseline system score and the system score realized with each potential

expansion component replacing its corresponding bottleneck. These scores provide an

approximation of the impact of a replacement component on the system. The evaluation

of components is examined first. System evaluations were used to analyze the results of

the knapsack expansion model. These results follow the expansion model.

Value Model Results for Expansion Components

The system scores have been scaled to indicate a utility between 0 and 1000,

primarily to highlight the delta score effects. In some cases, the impact on the generally

used averages for system attribute scores of replacing one component in a network of

thirty-eight components was minimal. However, this is not totally unexpected. Short of

a fundamental shift in technology and its related benefits, one would expect a marginal

overall system increase for the addition of a single component.

Components are described by twenty-two features, corresponding to the twenty-

one attributes of the value model and a cost. Cost is not considered for the evaluation of

component impact on the network. A complete listing of all component alternatives is

presented in Appendix C. The thirty-eight system links are compiled in Appendix A.

Appendix A is the spreadsheet used for system calculations. The baseline system features

for each of the twenty-one attributes are presented.

For each of the four sets of network bottlenecks, the system and delta scores were

calculated for all expansion alternatives. The baseline system score is included for the

calculation. Many of the delta scores were the same. These correspond to a specific component, such as the 16A, replacing a specific component, such as the 9.6B. The bottleneck location did not affect these delta scores, which approximate the system marginal change.

Table 4-3. Component Delta Scores - Macro Method

upgrade link	component alternative	system score	delta score
baseline system	------	598.67	------
A-E	16A	608.78	10.11
	16B	609.16	10.49
C-J	32A	632.00	33.33
	32B	620.78	22.11
	32C	620.96	22.29
E-P	16A	617.40	18.73
	16B	617.16	18.49
	16C	617.81	19.14
	16D	622.53	23.86
G-Q	9.6A	607.79	9.12
	9.6B	609.96	11.29
	9.6C	606.02	7.35
M-X	9.6A	607.79	9.12
	9.6B	609.96	11.29
	9.6C	606.02	7.35
P-R	16A	610.93	12.26
	16B	611.31	12.64
	16C	611.13	12.46
	16D	615.87	17.20
R-W	16A	610.93	12.26
	16B	611.31	12.64
	16C	611.13	12.46
	16D	615.87	17.20
T-Y	9.6A	607.79	9.12
	9.6B	609.96	11.29
	9.6C	606.02	7.35

Table 4-4. Component Delta Scores - Incremental Iterations

Iteration 1			
upgrade link	component alternative	system score	delta score
baseline system	------	598.67	------
C-J	16A	608.78	10.11
	16B	609.16	10.49
D-H	16A	610.93	12.26
	16B	611.32	12.65
	16C	611.13	12.46
	16D	615.87	17.20
E-P	9.6A	607.79	9.12
	9.6B	609.96	11.29
	9.6C	606.02	7.35
G-Q	9.6A	607.79	9.12
	9.6B	609.96	11.29
	9.6C	606.02	7.35
M-X	9.6A	607.79	9.12
	9.6B	609.96	11.29
	9.6C	606.02	7.35
Iteration 2			
upgrade link	component alternative	system score	delta score
baseline system	------	598.67	------
C-J	32A	632.00	33.33
	32B	620.78	22.11
	32C	620.96	22.29
H-M	16A	610.93	12.26
	16B	611.32	12.65
	16C	611.13	12.46
	16D	615.87	17.20
K-V	9.6A	607.79	9.12
	9.6B	609.96	11.29
	9.6C	606.02	7.35
Q-S	16A	610.93	12.26
	16B	611.31	12.64
	16C	611.13	12.46
	16D	615.87	17.20
T-Y	9.6A	607.79	9.12
	9.6B	609.96	11.29
	9.6C	606.02	7.35
Iteration 3			
upgrade link	component alternative	system score	delta score
baseline system	------	598.67	------
M-Q	9.6A	607.79	9.12
	9.6B	609.96	11.29
	9.6C	606.02	7.35

Expansion Model

This knapsack formulation utilizes the delta scores from the value model to find an effective combination of upgrade components to relieve the identified bottlenecks. It does not ensure the optimal combination because the delta scores are an approximation of component impact on the system. The macro bottleneck set required a single basic model. This model was solved with and without budget constraints, as well as a relaxed model with the cost constraint, but without the binary variable constraint. This was accomplished only to directly identify the budget shortfall necessary to obtain the optimal system. The same information could be obtained by paramaterizing the right hand side value of the budget constraint in the model. The two models analyzed for the macro method are the constrained (GC) and the unconstrained (GU). The budget used for both methods was $220,000. The incremental method had three separate sets of bottlenecks, one from each iteration. The first model (I1C) was run with the budget constraint, and the residual funds were applied as the second model's (I2C) budget constraint. This second model was also run without the budget constraint (I2U). With no budget remaining and only one bottleneck to upgrade, the third iteration solution was not modeled, but observed. These are identified as I3C, which selects the least expensive option due to the depleted resources, and I3U, which has no constraint on cost and is therefore able to consider the best improvement for selection. Given the relatively manageable size of the illustrative example, no specialized code was required to solve these models. The student version of LINDO Optimization Software was used to determine the solution. The formulations and output are listed in Appendix E. The following tables present the results of the expansion models.

Table 4-5. Macro Method Expansion Results

GC model			
bottleneck link	component upgrade	cost ($)	delta score
A-E	16B	30200.00	10.49
C-J	32A	35000.00	33.33
E-P	16C	26500.00	19.14
G-Q	9.6B	21500.00	11.29
M-X	9.6A	20000.00	9.12
P-R	16D	32750.00	17.20
R-W	16D	32750.00	17.20
T-Y	9.6A	20000.00	9.12
total		218700.00	126.89
GU model			
bottleneck link	component upgrade	cost ($)	delta score
A-E	16B	30200.00	10.49
C-J	32A	35000.00	33.33
E-P	16D	32750.00	23.86
G-Q	9.6B	21500.00	11.29
M-X	9.6B	21500.00	11.29
P-R	16D	32750.00	17.20
R-W	16D	32750.00	17.20
T-Y	9.6B	21500.00	11.29
total		227950.00	135.95

The relaxed macro model (not shown in table) selected the same upgrade components as the unconstrained model. All corresponding relaxed binary variables had a value of one, with the exception of A-E. This variable had a value of 0.737. This means that with the proposed budget, the "optimal" selection of component upgrades was only approximately $7,943 over-budget {(1-.737)*(cost of best link option)}.

The difference in cost in the two macro models with and without budget constraints was $9250. This does not seem like a substantial distinction; however, further comment is deferred until system utilities are evaluated.

Table 4-6. Incremental Method Expansion Results

I1C model			
bottleneck link	component upgrade	cost ($)	delta score
C-J	16B	30200.00	10.49
D-H	16D	32750.00	17.20
E-P	9.6B	21500.00	11.29
G-Q	9.6B	21500.00	11.29
M-X	9.6B	21500.00	11.29
total		127450.00	61.56

I2C model			
bottleneck link	component upgrade	cost ($)	delta score
C-J	32A	35000.00	33.33
H-M	-----	-----	-----
K-V	9.6B	21500.00	11.29
Q-S	16D	32750.00	17.20
T-Y	-----	-----	-----
total		89250.00	61.82

I3C model			
bottleneck link	component upgrade	cost ($)	delta score
M-Q	9.6A	20000.00	9.12

I2U model			
bottleneck link	component upgrade	cost ($)	delta score
C-J	32A	35000.00	33.33
H-M	16D	32750.00	17.20
K-V	9.6B	21500.00	11.29
Q-S	16D	32750.00	17.20
T-Y	9.6B	21500.00	11.29
total		143500.00	90.31

I3U model			
bottleneck link	component upgrade	cost ($)	delta score
M-Q	9.6B	21500.00	11.29

The total delta score listed for each model in Tables 4-5 and 4-6 has no meaning in terms of the additional utility of the expanded system. The delta scores were used as an indicator of direction of preference, acting as an approximation of the impact each component would have on the network. This is further explored in the next section.

Value Model Results for System Expansions

The value model was used to evaluate the seven system expansions resulting from the expansion model. Figure 4-5 shows the contribution breakdown of each system on fourteen of the twenty-one attributes and the overall system utility. The best and worst cases were validation models. The grayscale bars are arranged based on the weight of the attribute, with the highest weight on the left. The key corresponds to the attributes along the bar from left to right, line by line. Each system is represented by the same acronym used in the expansion models.

System	Utility
best	1000.00
I3U	685
I2U	681
GU	678
GC	671
I3C	664
I2C	662
I1C	647
baseline	599
worst	0.00

- reliability
- physical defense
- ease of transport
- encryption
- anti-jam
- end-to-end delay
- accuracy
- perception
- footprint
- redundancy
- setup time
- information rate
- interoperability
- expansion capability
- Other
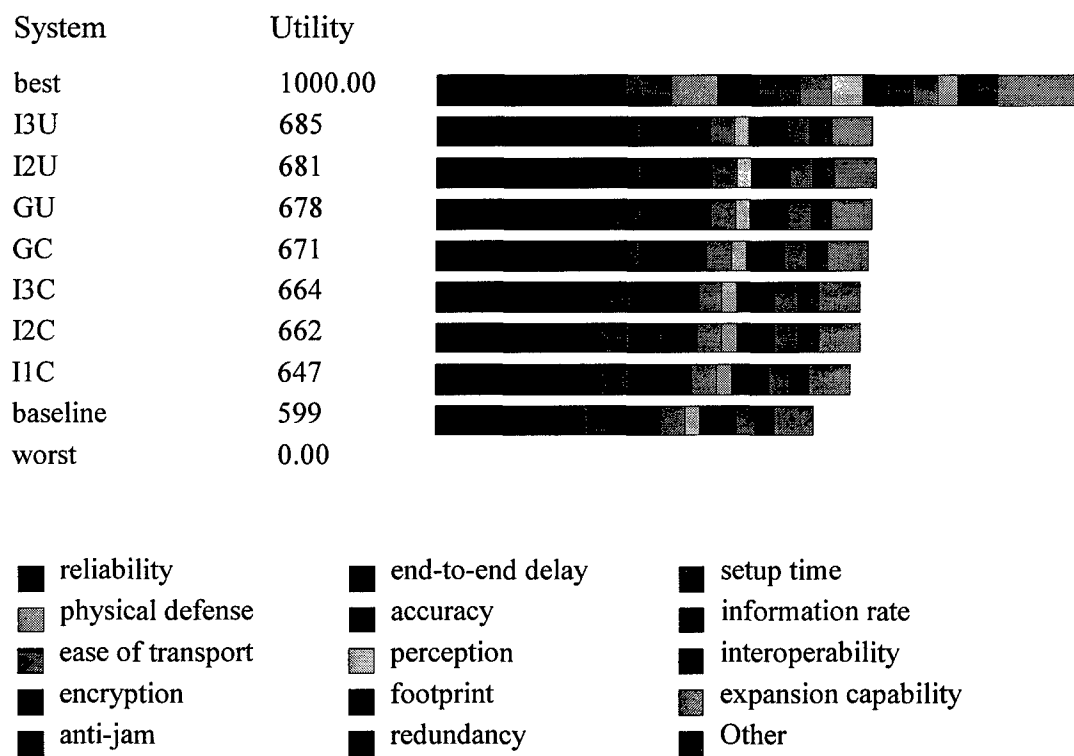
Figure 4-5. System Scores By Attribute Contribution

From Figure 4-5, it is clear that reliability did not change noticeably among the

system upgrades. End-to-end delay appears to have the most variation among all

systems. Its weight and the differences among iterations justifies this result.

The unconstrained incremental systems demonstrated the highest system utilities,

followed by the macro systems, and finally the constrained incremental systems. The

difference between the highest and lowest expanded system utility is 38. As identified

previously in this chapter, the attributes that have the greatest impact on the scores (with

over 4% contribution to the overall utility) and are modeled here are end-to-end delay,

setup time, accuracy, information rate, and ease of transport. Figures 4-6 through 4-9

demonstrate the differences between and within the macro and the incremental system

scores based on the top ten attributes contributing to their differences. The five high-

impact attributes are shown in italics.
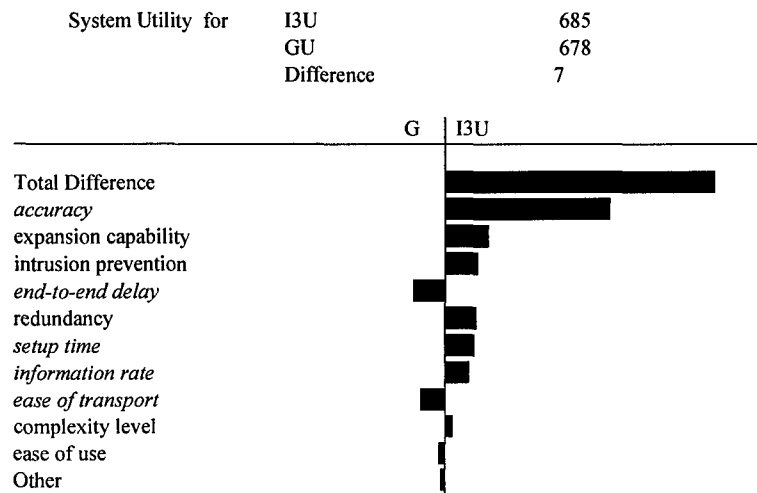
Figure 4-6. GU versus I3U Attribute Comparison

The incremental and macro methods have different purposes. When comparing the two, it is best to keep in mind that the macro method results in one upgrade with 100% demand satisfaction and no bottlenecks; whereas the incremental (3rd iteration) is the result of three consecutive upgrades to reach 100% demand satisfaction, and has bottlenecks remaining. Note that the utility scores do not address either of these issues. I3U dominates GU in three of the five high-impact attributes, with accuracy carrying over half the point spread between the two systems. GU upgraded eight links; whereas, I3U upgraded eleven.

System Utility for	GC	671
	I3C	664
	Difference	7

I3C | GC

Total Difference
end-to-end delay
accuracy
expansion capabilit
setup time
intrusion preventio
redundancy
information rate
ease of use
ease of transport
complexity level
Other
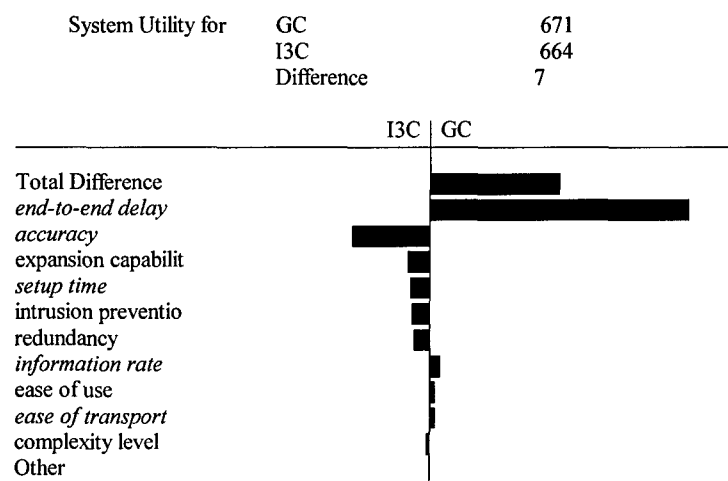
Figure 4-7. GC versus I3C Attribute Comparison

GC's dominance in end-to-end delay overshadows I3C's attribute scores for accuracy and setup time, among others. GC upgraded eight links; whereas, I3C upgraded nine links.

System Utility for	GU	678
	GC	671
	Difference	7

```
                              GC | GU
_____
Total Difference
accuracy
setup time
redundancy
end-to-end delay
intrusion preventio
expansion capabilit
ease of use
ease of transport
interoperability
information rate
Other
```
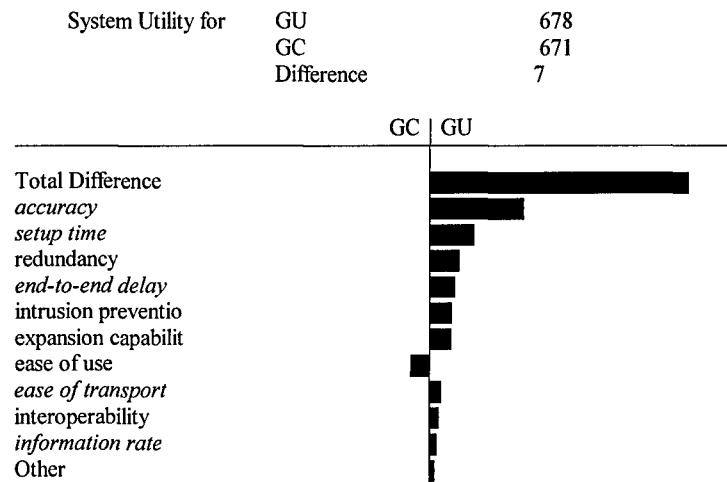
Figure 4-8. GC versus GU Attribute Comparison

GU dominates in all five high-impact attributes. This graphic illustrates substantially more than the straight utility scores. For the cost difference, GU performs better in the highest weighted attribute areas.

System Utility for	I3U	685
	I3C	664
	Difference	21

```
                              I3C | I3U
_____
Total Difference
end-to-end delay
accuracy
information rate
setup time
redundancy
intrusion preventio
expansion capabilit
ease of use
interoperability
complexity level
```
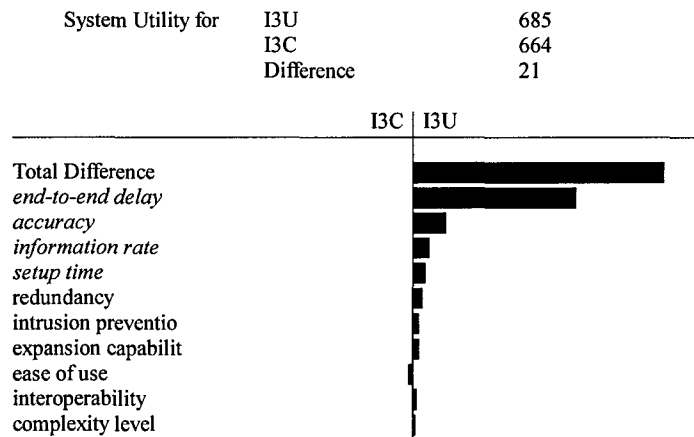
Figure 4-9. I3U versus I3C Attribute Comparison

This graphic resembles the macro comparison of constrained versus unconstrained. The unconstrained system dominates in four of the high-impact attributes. The fifth attribute, ease of transport, showed no significant difference. I3C upgraded nine components; whereas, I3U upgraded eleven links.

Delta Scores versus System Utility. The next portion of the analysis concerns the use of delta scores for selecting components within an expansion plan. As stated earlier, the delta scores were used as an approximation of the impact each component would have on the network. The total delta score listed for each model in the expansion results has no meaning in terms of the additional utility of the expanded system. The use of delta scores cannot guarantee an optimal expansion plan, but should give an effective comparison of component impacts. In the table below, system utility is compared to the baseline plus total delta scores for each expansion plan. The baseline for incremental iterations is the previous iteration's delta+baseline. A ranking of the systems is also shown based on the two measures. It should be recalled, however, that the objectives and funding resources of the models varied when comparing between systems.

Table 4-7. Delta Scores and System Utility

system	delta score	delta + baseline*	system utility	delta + baseline ranking	system utility ranking
I3U	11.29	761.83	685.08	1	1
I2U	90.31	750.54	681.46	2	2
GU	135.95	734.62	678.12	3	3
GC	126.89	725.56	671.25	5	4
I3C	9.12	731.17	664.26	4	5
I2C	61.82	722.05	661.58	6	6
I1C	61.56	660.23	646.86	7	7

* indicates the use of appropriate baseline

Table 4-7 shows that the use of delta scores gives an over-estimated component contribution to the upgraded network. The resulting system utilities were substantially lower than the delta+baseline. The reason for this large difference is not intuitively obvious. Most utility functions demonstrate decreasing marginal returns for increased levels of attributes. Therefore, in general, the delta+baseline should be an over-estimation. However, an investigation into the differences among attribute scores for systems and component evaluations revealed that while all other categories generally improved for the system evaluations, the goal of mobility showed a steady decline for adding more components to the network. The initial network was composed of the most transportable, easily set up components. As the components increase their technological capabilities, there is a resulting decline in both ease of transport and setup time. Mobility, with a global weight of 12%, did not decrease significantly with the replacement of one component. However, the delta scores for each of these replacements are added to find the total delta for the system. The combined effect of two or more replacement components more severely impacts the mobility of the network; thus, delta scores are not additive by nature. The rankings obtained for delta scores compared to system evaluations, however, are reasonably consistent.

System Tradeoffs. The table below lists the actual new system utilities, costs and demand satisfaction statistics for each of the seven models. Following this are three graphs which investigate the pairwise interaction of these factors.

Table 4-8. System Results for Both Methods

system	% demand satisfied	utility	upgrade cost ($)	upgrade total cost ($)
baseline	65.88	599	------	------
GC	100	671	218,700	218,700
GU	100	678	227,950	227,950
I1C	89.41	647	127,450	127,450
I2C	99.22	662	89,250	216,700
I3C	100	664	21,500	238,200
I2U	99.22	681	143,500	270,950
I3U	100	685	21,500	292,450

Both macro system utilities dominate the constrained incremental utilities. When comparing 100% demand satisfaction levels, the macro systems dominate all others in (total) cost, but its best utility score is beaten by the incremental unconstrained utility score by about 7. These are different approaches, and while they merit comparison, their purposes are distinct. Meeting demand is a high priority for the macro approach; whereas, the incremental method concentrates on getting the most return for a lower cost (per iteration).

The incremental iterations 2 and 3, constrained and unconstrained, meet the same demand levels at vastly different costs and utilities. This is a result of the many paths available in the network. Even though the constrained iteration 2 does not upgrade two recommended links, the ones it does select are enough to increase the throughput.

Constrained iterations 2 and 3 cost approximately $54,000 less than their unconstrained

counterparts at a tradeoff of about 20 points in utility.
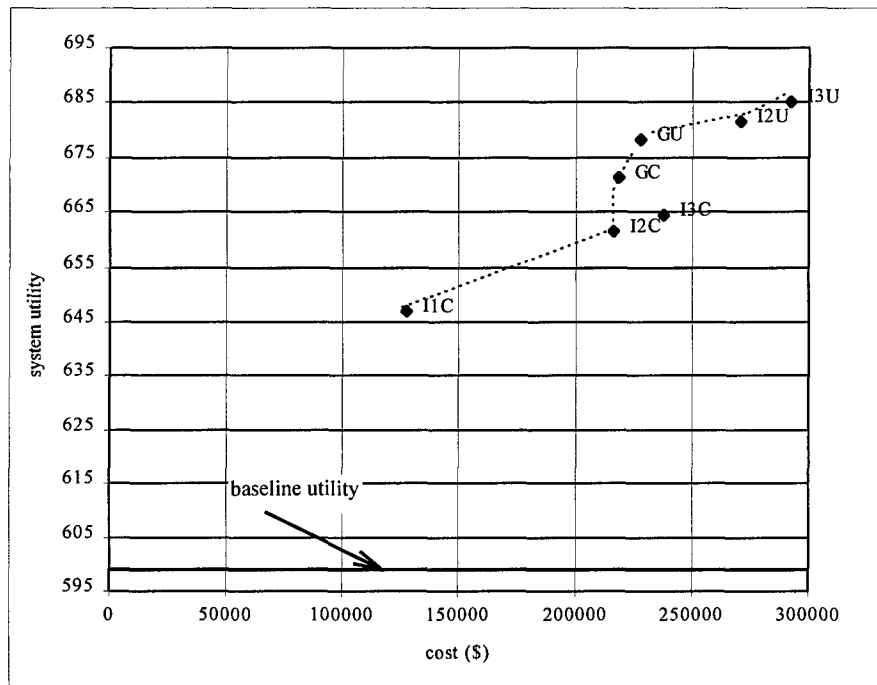
Figure 4-10. Cost versus Utility

Figure 4-10 above shows the tradeoffs in utility for the less expensive upgrades.

The dotted line represents the efficient frontier. Of the systems evaluated, those below

and to the right of the frontier are dominated. Those lying along the frontier demonstrate

a utility/cost tradeoff at each point. All upgrades appear to give a substantial increase in

utility over the baseline system. The incremental unconstrained upgrades should give a

higher utility but at a higher cost than the incremental constrained and the macro method

because they usually involve upgrading more network links. This is true in this case.

Using the slope as a comparison, moving from the macro method constrained to the

unconstrained gives a proportionally larger jump in utility for cost than when moving from either incremental iteration constrained to unconstrained. These moves appear to cost more for less utility gains. For example, the macro method move costs approximately $580 per utility point; whereas, the iteration 3 move costs approximately $1030 per utility point. The macro method appears to be more cost efficient for utility gains.
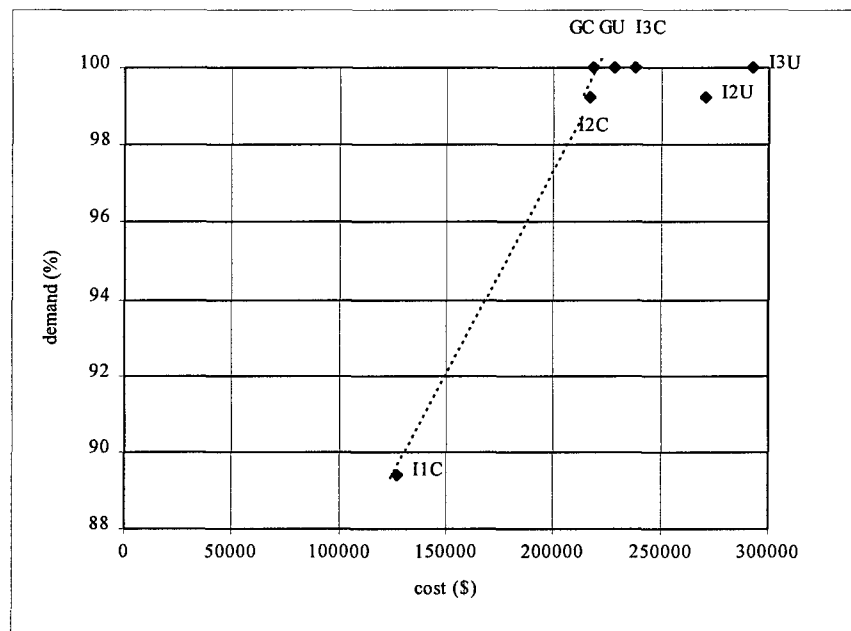
Figure 4-11. Cost versus Demand

Figure 4-11 depicts the tradeoff between cost and demand. Those systems falling below and to the right of the efficient frontier are dominated. The macro systems dominate both iteration 3 incremental systems in cost at 100% demand satisfaction levels. I3U presents a higher utility as a tradeoff; however, I3C gives the lowest utility score for the 100% demand level. It is dominated in demand and utility by less expensive system

expansions. At 99.22%, incremental iteration 2 constrained dominates its unconstrained

counterpart in cost with a tradeoff of about 20 points in utility.
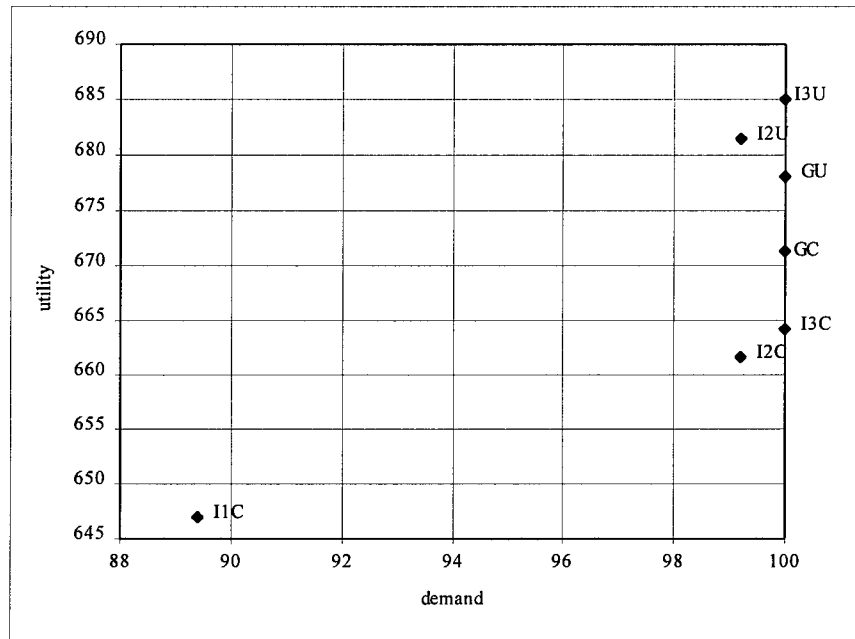
Figure 4-12. Demand versus Utility

Figure 4-12 depicts the tradeoff between utility and demand. Incremental

iteration 3 unconstrained dominates all other systems in utility and demand, but costs at

least $21,000 more than the second most expensive system. The unconstrained systems

in general do the best in both categories of utility and demand, but are the more expensive

systems. The differences in constrained and unconstrained utilities for iterations 2 and 3

are approximately the same, with the same difference in cost. Iterations 2 and 3 differ by

only one component addition to the upgrade.

Any of these pairwise graphs can be utilized to visually screen expansion plans

for dominant systems using the efficient frontier, or perhaps meeting minimum standards.

There is no one dominant system in all three categories. The macro and incremental methods have specific purposes, which are made obvious with these graphs. Macro methods concentrate on demand satisfaction and elimination of bottlenecks; whereas, incremental methods focus on generating improvements in steps.

There are tradeoffs for every method and each system within the method. The most demonstrative is cost versus utility, which every decision maker would need to consider. The attribute breakdown provides more information for the decision.

Summary

The four step methodology used provided an approximation of the impact of upgrade components on system effectiveness. This guided the determination of an efficient set of expansion alternatives. Within the resulting macro method systems, the tradeoff between cost and utility becomes clear with the attribute comparison. The unconstrained model not only has a higher utility, but the attributes in which it dominates the constrained model are those with the highest global weights. The constrained/unconstrained incremental iterations demonstrate a similar comparison. Of the five identified high-impact attributes, end-to-end delay, setup time, and accuracy seem to drive the scores of the seven examined models.

The tradeoffs among utility, cost and demand are also made clear with the pairwise graphs. By outlining a specific priority, such as 100% demand satisfaction, dominated systems can be removed from consideration. System screening can also be accomplished by establishing acceptable ranges for each of these factors. The

information gained with this analysis provides lucid insight into the system expansion

question.

V. Conclusions and Recommendations

This chapter provides conclusions from the analysis completed and makes recommendations for further study of the topic area.

Overview

This effort accomplished the goal of providing a quantitative method that incorporates the objectives of any C4 system into an expansion model. The expansion plan was not guaranteed to be *the* overall system optimal due to the use of approximations for component impacts, but it does provide an excellent 'first cut' analysis of the system expansion. More importantly, a systematic methodology, which incorporates expert opinion and operational necessities, is provided to evaluate communications network upgrades. The approach produced graphical displays that provide a great deal of insight into the tradeoffs which could be made and the driving factors behind the system scores.

Research Results

The research results for the example network scenario and components were used to illustrate this proof-of-concept model. The primary contribution of this research is the detailed methodology which can be applied to any system or decision. Any attempt to utilize this model should recommence with a realistic scenario, an actual C4 system, and

real components. The value structure should reflect the realities of the actual operational system and its decision makers.

The bottleneck identification method provides insight into the bottleneck links of a network for which demand levels have been assessed. The value model presents a thorough examination of the important factors involved in this decision based on the knowledge, experience and objectives of the decision makers. The method contributes a means to evaluate any communications network at the system level. Although an optimal expansion set is not identified, the capability is presented. The insights into system scores, costs and demand are invaluable to a decision maker.

Due to the increased throughput of the network, the increase in system utility corresponds to the "value" of the additional information received by the users. This is a novel concept that may prove useful to any C4 systems analyst.

Limitations of The Study

The most significant limitation of this study is the inability to produce an optimal expansion set of components. The number of possible combinations of alternatives that would have to be evaluated is prohibitive. While not providing an overall optimal, the use of delta scores based on a single component's impact on the system appears to be a reasonable approximation.

Communications network traffic was handled deterministically. However, the assumption of known demand levels does not hinder the evaluation of expansion plans.

Recommendations for Future Research

As mentioned above, the optimal expansion plan was not identified here. Due to the massive amount of evaluations required, this accomplishment was neither effective or efficient. Future research should focus on screening techniques to reduce the evaluations while providing a more effective approximation of the optimal solution.

Most of the attributes from the value hierarchy are best evaluated with real-time monitoring. Research into the area of stochastic message traffic could expand the bottleneck analysis and enhance the attribute description with real data. Other areas within the decision analysis realm include dealing with the uncertainty of attribute consequences and verifying the assumption of mutual preferential independence.

Many of the other assumptions and notional aspects of the network deserve further investigation. For example, the restrictions on the expansion model in the areas of spectrum management, geographical hindrances, and asset availability were not considered. The sustainment phase of deployment was the focus for this effort; however, all stages of deployment could benefit from this type of analysis. New techniques could be explored to handle different phases of deployment. These and other realities, such as the use of actual components in a real scenario, warrant further research.

Conclusions

The approach developed in this thesis effort can serve as a method for developing communications network upgrades. By combining both expert opinion and operational necessities, the value model provides a benchmark and metric for the analysis. The use of mathematical programming to both identify links for consideration for expansion and

to make enlightened choices for that expansion are sound applications of operations

research. The outputs of the model, while not optimal in the classic sense, provide the

decision maker with an improved communication structure in a "traceable" format.

Appendix A. Baseline System Links

This appendix contains the complete description of the baseline system network links. They are described in terms of the measurable attributes in the value hierarchy. This hierarchy structure, is presented in Appendix B with comprehensive a background of each attribute and the system level measurement explanation. The network links have been fabricated for this proof-of-concept model. This spread sheet also served as the system score template. For upgrades, new link component attributes were inserted and the system attribute level recorded. The attribute levels for all systems examined are presented in Appendix G.

link-system scores

link component ID	A-D	A-E	A-F	B-D	C-D	C-J	D-H	E-L	E-P
users	1	1	1	1	1	1	1	1	1
Pe	-7	-7	-7	-7	-7	-7	-5	-5	-5
Ri (info rate)	1200	1200	1200	1200	1200	1200	1200	300	300
delay	0	0	0	0	0	0	0	0	0
ease of use	0.5	0.5	0.5	0.5	0.5	0.5	0.75	0.5	0.5
media types	2	2	2	2	2	2	2	2	2
perception	60	60	60	60	60	60	60	60	60
range	400km	400km	400km	400km	400km	400km	40km	40km	40km
intrusion prevention	1	1	1	1	1	1	0	0	0
encrypt	0	0	0	0	0	0	0	0	0
anti-detection	4	4	4	4	4	4	4	4	4
anti-jam	4	4	4	4	4	4	4	4	4
hardening	2	2	2	2	2	2	2	2	2
redundancy	2	2	2	2	2	2	1	1	1
reliability	0.93	0.93	0.93	0.93	0.93	0.93	0.75	0.8	0.8
physical defense	0	0	0	0	0	0	0	0	0
ease of transport	0.75	0.75	0.75	0.75	0.75	0.75	0.75	1	1
setup time	0.416666667	0.416666667	0.416666667	0.416666667	0.416666667	0.416666667	0.9166667	0.333333333	0.333333333
interoperability	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
expansion capability	1.666666667	1.666666667	1.666666667	1.666666667	1.666666667	1.666666667	1	1	1
complexity	0.5	0.5	0.5	0.5	0.5	0.5	0.25	0.25	0.25

link-system scores

link	F-G	F-L	G-H	G-Q	H-I	H-M	I-N	J-I	J-K	J-O	K-O
component ID	9.6A	9.6A	2.4A	2.4A	9.6A	9.6A	9.6A	9.6A	9.6A	9.6A	2.4A
users	1	1	1	1	1	1	1	1	1	1	1
Pe	-5	-5	-5	-5	-5	-5	-5	-5	-5	-5	-5
Ri (info rate)	1200	1200	300	300	1200	1200	1200	1200	1200	1200	300
delay	0	0	0	0	0	0	0	0	0	0	0
ease of use	0.75	0.75	0.5	0.5	0.75	0.75	0.75	0.75	0.75	0.75	0.5
media types	2	2	2	2	2	2	2	2	2	2	2
perception	60	60	60	60	60	60	60	60	60	60	60
range	40km	40km	40km	40km	40km	40km	40km	40km	40km	40km	40km
intrusion prevention	0	0	0	0	0	0	0	0	0	0	0
encrypt	0	0	0	0	0	0	0	0	0	0	0
anti-detection	4	4	4	4	4	4	4	4	4	4	4
anti-jam	4	4	4	4	4	4	4	4	4	4	4
hardening	2	2	2	2	2	2	2	2	2	2	2
redundancy	1	1	1	1	1	1	1	1	1	1	1
reliability	0.75	0.75	0.8	0.8	0.75	0.75	0.75	0.75	0.75	0.75	0.8
physical defense	0	0	0	0	0	0	0	0	0	0	0
ease of transport	0.75	0.75	1	1	0.75	0.75	0.75	0.75	0.75	0.75	1
setup time	0.9166667	0.9166667	0.333333333	0.333333333	0.9166667	0.9166667	0.9166667	0.9166667	0.9166667	0.9166667	0.333333333
interoperability	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
expansion capability	1	1	1	1	1	1	1	1	1	1	1
complexity	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25

link-system scores

link component ID	K-V	L-P	M-Q	M-X	N-O	N-T	N-X	O-V	P-R	Q-S	R-W	S-W
component ID	2.4A	9.6A	2.4A	2.4A	9.6A	9.6A	9.6A	9.6A	9.6A	9.6A	9.6A	9.6A
users	1	1	1	1	1	1	1	1	1	1	1	1
Pe	-5	-5	-5	-5	-5	-5	-5	-5	-5	-5	-5	-5
Ri (info rate)	300	1200	300	300	1200	1200	1200	1200	1200	1200	1200	1200
delay	0	0	0	0	0	0	0	0	0	0	0	0
ease of use	0.5	0.75	0.5	0.5	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75
media types	2	2	2	2	2	2	2	2	2	2	2	2
perception	60	60	60	60	60	60	60	60	60	60	60	60
range	40km	40km	40km	40km	40km	40km	40km	40km	40km	40km	40km	40km
intrusion prevention	0	0	0	0	0	0	0	0	0	0	0	0
encrypt	0	0	0	0	0	0	0	0	0	0	0	0
anti-detection	4	4	4	4	4	4	4	4	4	4	4	4
anti-jam	4	4	4	4	4	4	4	4	4	4	4	4
hardening	2	2	2	2	2	2	2	2	2	2	2	2
redundancy	1	1	1	1	1	1	1	1	1	1	1	1
reliability	0.8	0.75	0.8	0.8	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75
physical defense	0	0	0	0	0	0	0	0	0	0	0	0
ease of transport	1	0.75	1	1	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75
setup time	0.333333333	0.9166667	0.333333333	0.333333333	0.9166667	0.9166667	0.9166667	0.9166667	0.9166667	0.9166667	0.9166667	0.9166667
interoperability	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
expansion capability	1	1	1	1	1	1	1	1	1	1	1	1
complexity	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25

link-system scores

link	S-ZZ	T-X	T-Y	U-Y	V-U	V-Z	SYSTEM	attribute level
component ID	9.6A	2.4A	2.4A	9.6A	9.6A	9.6A	SYSTEM	attribute level
users	1	1	1	1	1	1	38	100
Pe	-5	-5	-5	-5	-5	-5	-5.315789474	-5.315789474
Ri (info rate)	1200	300	300	1200	1200	1200	963.1578947	100
delay	0	0	0	0	0	0		100
ease of use	0.75	0.5	0.5	0.75	0.75	0.75	0.644736842	0.644736842
media types	2	2	2	2	2	2	2	2
perception	60	60	60	60	60	60	60	60
range	40km	40km	40km	40km	40km	40km		90
intrusion prevention	0	0	0	0	0	0	0.157894737	0.157894737
encrypt	0	0	0	0	0	0		100
anti-detection	4	4	4	4	4	4	4	4
anti-jam	4	4	4	4	4	4	4	4
hardening	2	2	2	2	2	2	2	2
redundancy	1	1	1	1	1	1	1.157894737	1.157894737
reliability	0.75	0.8	0.8	0.75	0.75	0.75	0.999997789	99.99977889
physical defense	0	0	0	0	0	0		0
ease of transport	0.75	1	1	0.75	0.75	0.75	0.815789474	0.815789474
setup time	0.9166667	0.333333333	0.333333333	0.9166667	0.9166667	0.9166667	0.684210526	0.684210526
interoperability	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
expansion capability	1	1	1	1	1	1	1.105263158	1.105263158
complexity	0.25	0.25	0.25	0.25	0.25	0.25	0.289473684	0.289473684

Appendix B. Value Hierarchy

The value hierarchy below was constructed to evaluate the incremental changes in a deployed C4 digital network to adapt to demands at the lowest levels within the deployed region. The purpose of the hierarchy is to aid in evaluating the impact of network changes on system effectiveness. Decimal numbers listed in the hierarchy are the local weights established for each subgoal group, demonstrating relative importance within the hierarchy. The weights within each subgoal group sum to one.

C4 Network Expansion Hierarchy
(final)

```
                              system
                           effectiveness
        ┌──────────────────────┼──────────────────────┐
     service              survivability            flexibility
       .4                     .4                       .2
   ┌────┬────┬────┐        ┌──────┴──────┐       ┌──────┬──────┐
 volume time- presen- span  information  physical  mobile inter-  expand-
  .4    liness tation  .1   protection   endurance  .6   operable able
        .3     .2             .4           .6             .2      .2

 — users  end-to-end  — ease of  — footprint
   .2     — delay        use
 — accuracy              .3
   .4                  — media
 — information           types
   rate                  .1
   .4                  — perception
                         .6
```

information protection .4:
- information security .5
  - intrusion prevention .3
  - encryption .5
  - anti-detection .2
- information defense .5
  - anti-jam .4
  - hardening .2
  - redundancy .4

physical endurance .6:
- system reliability .7
- physical defense .3

mobile .6:
- ease of transport .4
- setup time .6

interoperable .2:
- interoperability

expandable .2:
- expansion capability .8
- complexity .2

The overall goal for the C4 network expansion is enhanced system effectiveness. This is broken into three subgoals defined as follows:

1. *Service*. The ability of the system to act as an 'information pump' for the user under normal conditions.

2. *Survivability*. The ability of the system to perform under duress or stressed conditions.

3. *Flexibility*. The ability of the system to adapt to changes in network topology, demands, etc.

1. Aspects of *service* include *volume, timeliness, presentation* and *span*. The warfighter needs to receive the required information accurately, quickly, presorted, in a format that's understandable and usable. The attributes within these four aspects quantify the warfighters' needs into required system features.

- *Volume* attributes include *users, accuracy* and *information rate*.

  - *Users* is a measure of the number of persons that can manipulate, transmit or receive information through the system. The magnitude of the increase/decrease is relative to the current size of the system. Therefore, a percentage measure is used that can be incorporated for any system size. The current number of users supported (100%) scores a .5 on utility. If the number of resulting users is 80% of the current number, the utility is .1. Likewise, 120% of the number of current users scores a .8 on utility. For this effort, the number of users corresponds to the total number of receiver/transmitters within the system.
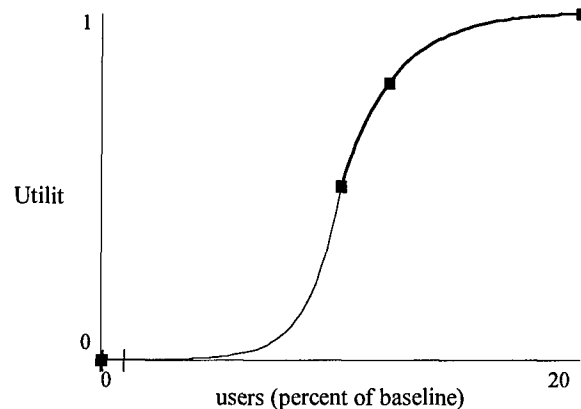
- *Accuracy* measures the ability of the system to perform when it is corrupted by noise. Channel noise can result from natural or man-made sources, such as lightning or high-voltage transmission lines, near the transmission medium. For an analog system, one useful performance measure is the output signal-to-noise ratio; whereas, in a digital system, a measure of performance is the probability of bit error (P_e) of the received signal. The probability of bit error varies for

different binary signaling schemes. Typically one error in 10^6 bits is desirable. This translates into a P_e of 10^{-6}. To capture the system accuracy and the impact of a new component on system accuracy, the average P_e over all network links is used. The use of a straight average implies that all links have equal topological importance. The P_e on low utilization links and redundant paths has the same impact on the system as critical paths and links with high utilization. In order to account for the importance of these critical links, a weighted average may be used. However, defining the topological weights for a scenario is beyond the scope of this effort. Therefore, all links are considered equally important. The utility function for accuracy scores the average P_e over all links. The x axis for the function is the log of the average P_e, which shows increasing orders of magnitude of P_e. Notationally, the system average is

$$\overline{P}_e = \sum_{i=1\,to\,L} w_i * \log(P_{ei})$$

where L is the number of links, w_i is the topological weight, and P_{ei} is the probability of bit error of the ith link. The P_e for network links must be estimated or measured directly. An actual measurement is preferred; however, it is not usually available. An estimate can be obtained from graphs of P_e versus E_b/N_o (the value of received bit energy per noise power spectral density measured in decibels (dB)) by knowing or assuming a particular modulation scheme and E_b/N_o [Couch, 1990; Sklar, 1988].
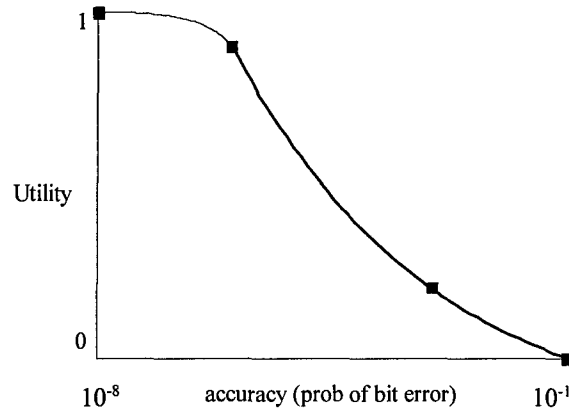
- *Information rate* captures the overall ability of the system to move information. It is essentially the effective throughput to a system. In a digital system, the information sent when the *j*th message is transmitted is defined mathematically as

$$I_j = \log_2 (1/P_j)$$

where P_j is the probability of transmitting the *j*th message. Thus, the information content is high if there is a low probability of it occurring. The average information measure (H), called entropy, is defined as

$$H = \Sigma_{j=1 \text{ to } m} P_j * I_j \qquad \text{bits}$$

for m messages. The information rate then is given by

$$R = H/T \qquad \text{bps}$$

with T as the time required to generate a message. Assuming messages with equal probability of occurring, H becomes a constant. For a particular coding scheme, (such as ASCII with 7 bits to describe each character) T is defined as $(N+n)*\tau$, where n is the number of bits used for each character, N accounts for error coding redundancy bits and overhead, and τ is the bit interval, or 1/(bit rate). The information rate becomes (H/(N+n)) times the bit rate. Thus,

$$R \approx 1/T = (1/(N+n))*\text{bit rate}$$

The information rate of each link in the system (R_i) can be evaluated as a fraction of its corresponding bit rate [Couch, 1995]. The system information rate is defined as the *average* information rate over all L links. This measure assumes equal topological importance of all links. Due to critical paths and specific

demands, this may not be representative of the true system. To introduce topological importance to specific links, a weighted average is shown; however, this effort considers the special case where all weights are equal.

$$\bar{R} = \sum_{i=1 \text{ to } L} w_i * R_i$$

The utility function for information rate is defined as a percentage of the current system. The current system information rate has a utility of .5. A 20% increase in the average information rate corresponds to a utility of .9. Concurrently, a 20% decrease corresponds to a utility of .1. This measure doesn't account for the marginal value obtained beyond doubling the current average information rate of the system.

Utilit

0   information rate (percent of baseline)   200
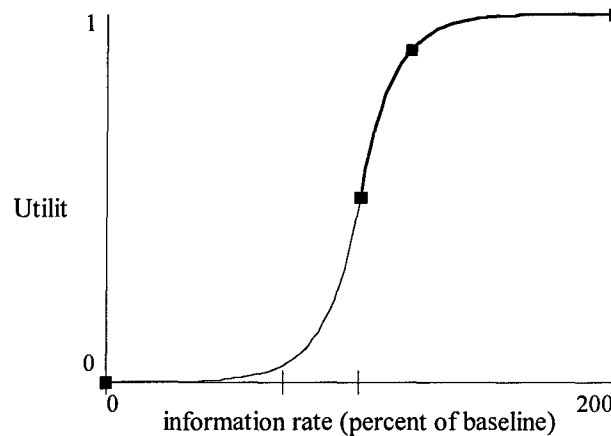
- *Timeliness* has one measure, *end-to-end delay*.
  - *End-to-end delay* is defined as the amount of time it takes to send a packet of information from the source to the destination. Link delay consists of processing, queueing, transmission and propagation components. Processing delay is the time from receiving the packet at the head node of the link to assigning it to an outgoing link queue for transmission. Queueing delay is the time from assigning the packet to a queue for transmission to the start of transmission. Transmission delay is the time to transmit all bits of the packet. Propagation delay is the time between transmitting the last bit of the packet at the head node of the link and receiving the last bit of the packet at the tail node. This is proportional to the

physical distance between the transmitter and receiver [Bertsekas and Gallagher, 1992: 149-150]. A value for end-to-end delay can be found through direct measure of an operational system or a simulation. Since this value is dependent on the system size, the utility function is based on a percentage of the baseline system. The baseline delay (100%) scores a utility of .5. A decrease in the baseline delay translates to a higher utility and vice versa. For this effort, end-to-end delay is evaluated based on a comparison of the relevant parameters on the current link and the characteristics of the proposed new component which may impact the system end-to-end delay.
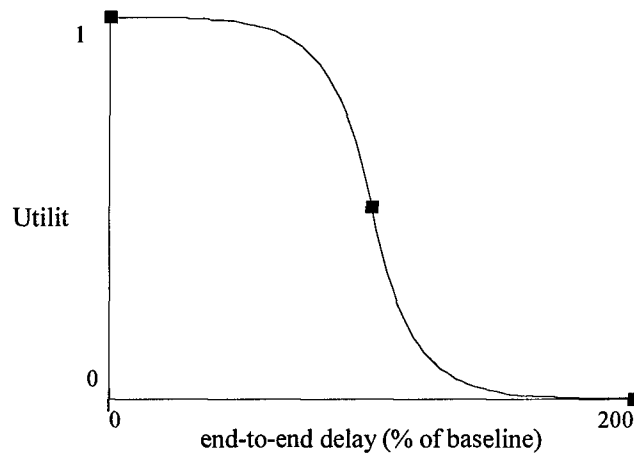
- *Presentation* is quantified through three measures: *ease of use, media types* and *perception.*

  - *Ease of use* evaluates the amount of difficulty involved in using the equipment, the use of priority routing and sorting (intelligent processing). A scoring function is used with utility values for each level as shown below. Levels are assigned based on a subjective judgment of difficulty in using each component. The system ease of use is an average of the ease of use scores for all the components. Thus the system utility may fall between two of the identified levels. This is possible because the utility function for ease of use is linear (see below). The system average assumes equal importance of all nodes.

Component scores for ease of use

Level	Utility
trivial	1.0
easy	0.75
medium	0.5
difficult	0.25
very difficult	0

Utilit

ease of use (average)
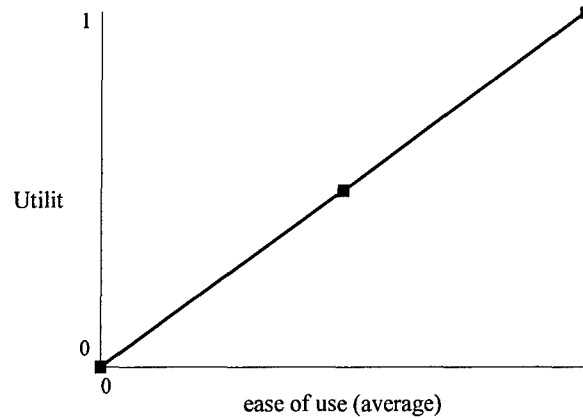
- The measure *media types* assesses the number of different transmission media types a system supports. This is taken as an average of the number of media types each piece of equipment within the system supports. This rewards the system for having components that support different media types, regardless of whether or not the system as a whole currently supports different media types. Similarly, the method does not consider which combinations of media types each component supports. The five types considered here are voice, data (text), imagery, video, and 3-D video/sound (holography). Each type rates a utility of .2.

Utilit (y-axis), media types (number) (x-axis), with values 0 and 1 marked
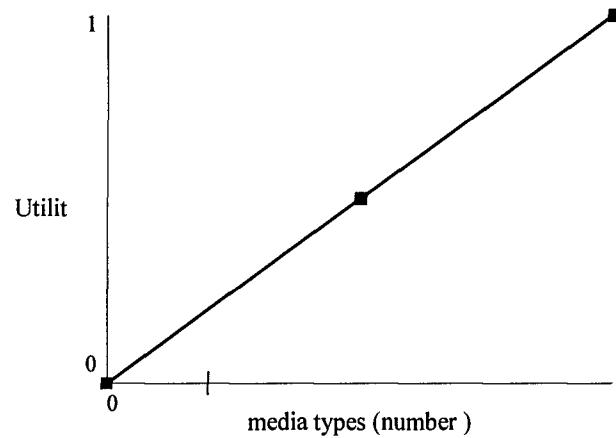
- *Perception* is evaluated as the percentage of the presented data (audio or visual) that is actually absorbed and useable by the user. This is a result of the quality and arrangement of presented information. It is a subjective evaluation by the user. It is evaluated notionally for this effort. The system score on perception is based on the average perception score for all links. This assumes equal weighting of all nodes.

Utilit (y-axis), perception (%) (x-axis), with values 0 and 10 marked
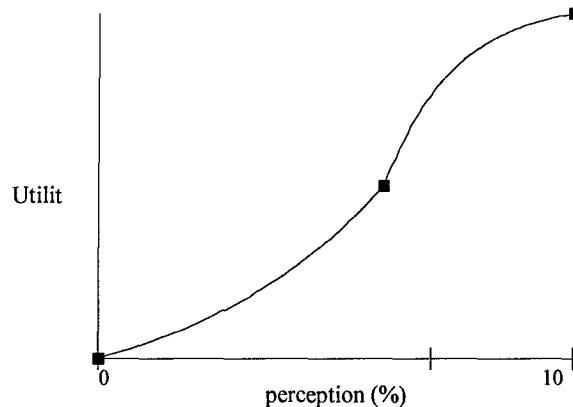
- *Span* involves the range or reach of the equipment being used. It is quantified with one measure, *footprint*.
  - The measure *footprint* is actually the percentage of the area of operations that is covered given the ranges and locations of the equipment currently being used.

The system score with a proposed new component reflects the range the new component introduces. Covering 50% of the area has a utility of .3.
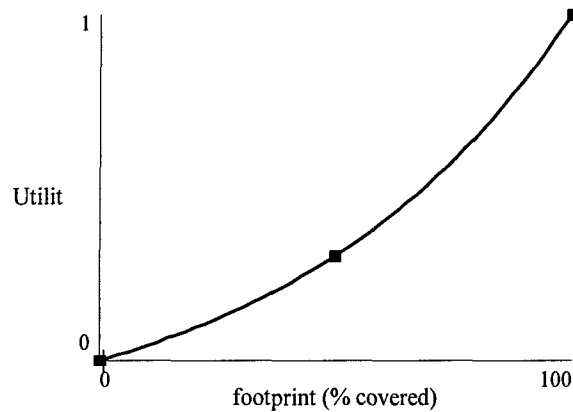
2. *Survivability* involves the survivability of the *information* (protecting the signal and the internal system) and survivability of the *physical* system (protection against component failure and physical damage).

- *Information protection* consists of two areas: 1) minimizing the unintended interception of information and the correct interpretation if intercepted (*information security*); and 2) optimizing the intended reception of information (*information defense*).

    - *Information security* consists of three individual measures: *intrusion prevention, encryption, and anti-detection.*

        - *Intrusion prevention* measures the number of techniques being employed within the system on average to prevent system intrusion. The five techniques identified here include password/authentication, firewall, decoy, biometrics, and intelligent agents. Passwords protect the system from unauthorized entry by requiring individual identification codes. A firewall is a mechanism that compares the address of the transmitting machine to an approved list. Access is denied if the address is not on the list. A decoy is a more sophisticated technique that allows addresses not on the approved list entry into a phony system to track their actions. Biometrics involve using individual identification such as finger prints,

retina scans, typing style and speed, etc. Intelligent agents are programs designed to identify users employing out-of-the-ordinary commands, perhaps at the system level. The utility function for this measure shows large gains in utility for employing a technique, and marginally decreasing gains for further techniques. The system score is simply the average number of techniques used at each node. This assumes equal importance of intrusion prevention techniques at all nodes. The scenario, however, may place more importance on using techniques for locations closest to enemy lines. This could be modeled using a weighted average.

intrusion preventio (# of techniques)

Selected Point -- Level   Utility
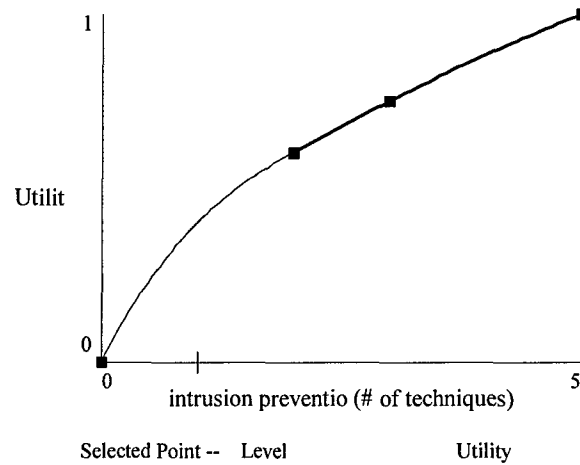
- *Encryption* provides a method of encoding the signal to prevent unauthorized decoding [Sklar, 1988]. The measure is designed to indicate the percent of data that is transmitted encrypted. This will vary depending on the utilization (U_i) and capacity (C_i in bits per second) of each link. For the set of links, S, and a subset of encrypted links, E, this measure can be calculated using

$$\% \text{ secure} = \left[\sum\nolimits_{\forall i \in E} C_i * U_i \right] / \left[\sum\nolimits_{\forall j \in S} C_j * U_j \right]$$

This effort does not consider the utilization of links. All components considered for this scenario possess encryption capability.

Utilit

0   100

encryption (% of data)
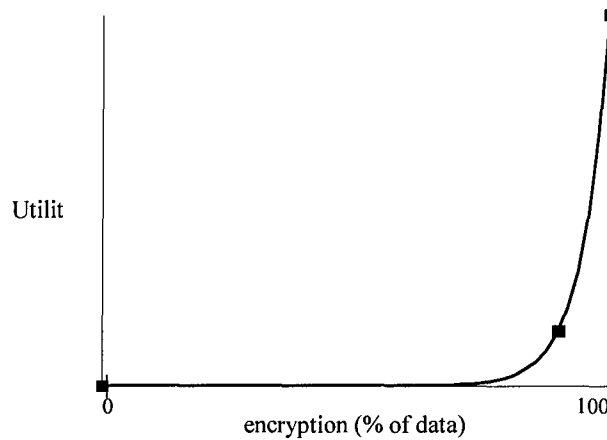
- *Anti-detection* evaluates the impact various techniques have in preventing signal detection (low probability of intercept/detection or LPI/LPD). The main objective of LPI communications is to reduce the intercept range with respect to the communicaiton range [Ghordlo, 1-3: 1996]. The performance metrics for LPI communication systems are the LPI quality factors. Each of the five techniques identified for this study have their own quality factor, which when combined, form the LPI quality factor. The five techniques include modulation, antenna, interference suppression, terrain and atmosphere. These items can be exploited to minimize signal detection. For antennas, covertness can be improved by using directional antennas with high gain in the direction of the intended receiver and small side lobes or nulls in the interceptor direction. In interference suppression, covertness can be improved by increasing the ability of the receiver to distinguish the signal from interference relative to the ability of the interceptor to do the same. Different modulation techniques can improve covertness by decreasing the required power spectral density (PSD) of the receiver to receive the signal and extract information from it or by increasing the required PSD of the interceptor to do the same. The atmosphere and terrain can be exploited by minimizing the atmospheric loss (in decibels/kilometer) of the signal on the

communication path while simultaneously maximizing the loss on the interception path.

The impact of using these techniques is a function of either receiver, transmitter and interceptor gains, signal to noise ratios, or ranges. They result in a decibel (dB) gain in the LPI quality factor. The reader is encouraged to refer to [Ghordlo: 1996] or another source for a complete characterization of each technique. This effort does not consider the interceptor's location and communication parameters to be known. Therefore, a simplification of these measures is used. For each LPI technique employed on a transmission link, there is an assumed 2dB gain in the LPI quality factor. Each link is characterized by a dB gain for LPI techniques. The system anti-detection score is defined as the average dB gain achieved by all links. This assumes that the use of anti-detection techniques is of equal importance for each location within the deployed region. A weighted average can be used if this is not the case.
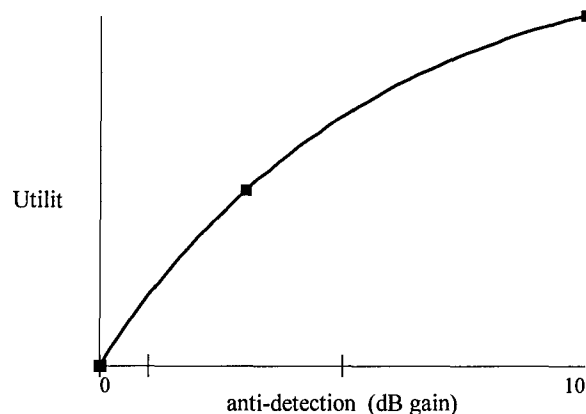
- *Information defense* consists of numerous measures, three of which are considered here. These include *anti-jam, hardening,* and *redundancy.*

  - *Anti-jam* evaluates the dB gain obtained by using techniques employed to reduce the effects of jamming. The five techniques identified for this study include modulation, antenna, interference suppression, atmosphere, and terrain. These techniques, which were used offensively for low

probability of intercept (LPI), can be used defensively for anti-jam purposes [Ghordlo: 1996]. For each anti-jam technique employed on a transmission link, there is an assumed 2dB gain for the system. The system anti-jam score is defined as the average dB gain achieved by all links. This assumes that the use of anti-jam techniques is of equal importance for each location within the deployed region. A weighted average can be used if this is not the case.

Utilit

0   anti-jam (dB gain)   10

Selected Point --   Level   Utility
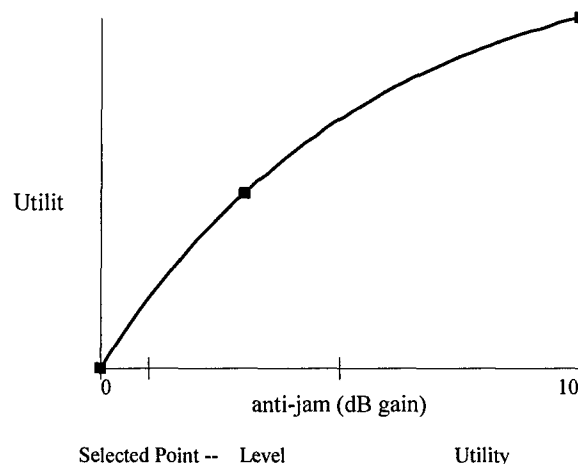
- *Hardening* rewards the utilization of three types of information hardening. These are electromagnetic pulse (EMP)hardening, error codes, and shadowing. EMP hardening is the protection against electromagnetic pulse. Methods include shielding, filtering, pulse rejection, non-programmable ROM storage, etc. Error coding refers to the structured sequences used for the detection and correction of errors. Shadowing is buffering multiple copies of data in case one is corrupted. The system hardening score is defined as the average number of techniques used over all the area links. This assumes that using hardening techniques is of equal importance at each location within the network. The utility function shows marginally decreasing returns for increasing the types of hardening used.

Utilit

0   hardening (# of types)   3
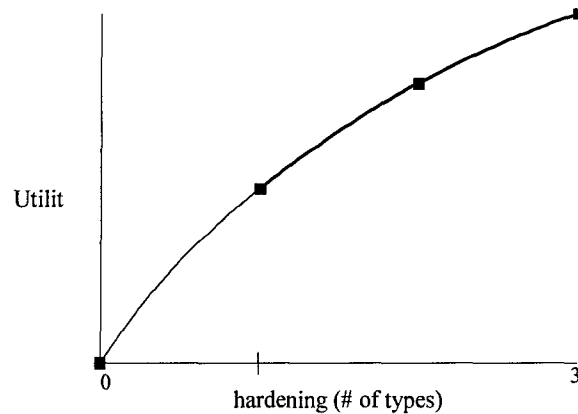
- *Redundancy* rewards the number of techniques, or diversity schemes used to provide redundancy in the network. The five diversity schemes cited in this study are frequency, spatial, polarization, time, and code. Frequency diversity involves using multiple frequencies to transmit data. Spatial diversity is usually implemented using two or more antennas at different heights/orientations to transmit/receive. Polarization diversity is achieved by using multiple polarizations for transmission. Time diversity is simply transmitting at different times to defeat any signal degradation factors. Code diversity is using different codes to transmit data to defeat 'smart coded jammers' or interference from other coded signals. The system redundancy score is an average of the number of redundancy techniques used over all communication links. This assumes that the use of redundancy techniques is of equal importance at each location within the deployed region. The utility curve shows a marginally decreasing gain for more than two diversity schemes.

Utilit

0

0   redundancy (# of techniques)   5

Selected Point -- Level: Utility
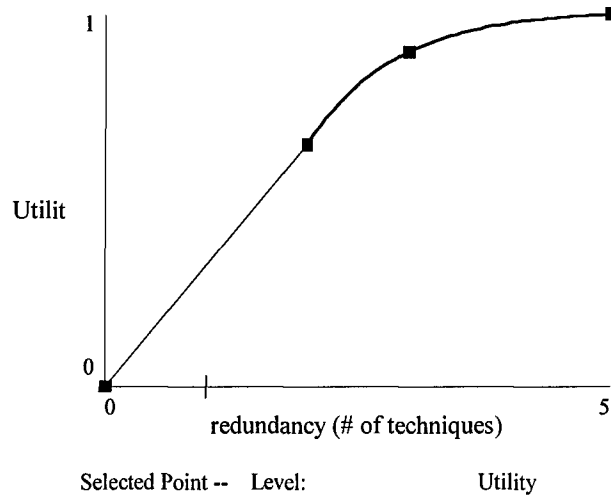
- *Physical endurance* includes two separate measures, *system reliability* and *physical defense.*

  - *System reliability* is defined using the network structure and each component or subsystem's availability. Availability (A) is the percentage of time that the system is available for tasking. It is expressed as

  $$A = (MTBF) / (MTBF + MTTR)$$

  where MTBF is mean time between failures and MTTR is mean time to repair a failure. From component availability, reliability can be calculated using the parallel and series relationships within the network. For n systems in parallel,

  $$R = 1 - (\Pi_{i=1 \text{ to } n} (1-A_i))$$

  and for n systems in series,

  $$R = \Pi_{i=1 \text{ to } n} (A_i)$$

  [Sharma, 1990: 42-44] These calculations do not consider the reliability of components on critical or high utilization links to be any more important than other network links. Appendix F lists the network reliability calculations for this effort.

Utilit

0   reliability (%)   10
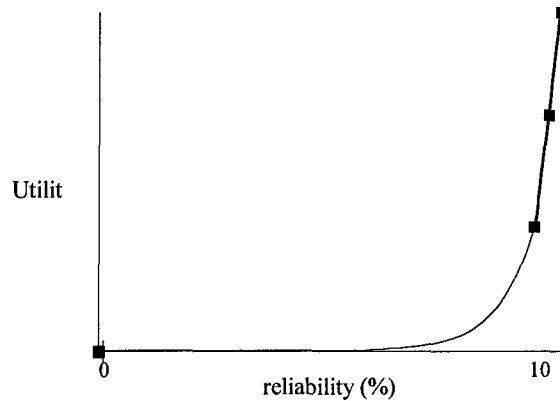
- *Physical defense* evaluates the number of techniques used to defend the components. There are four used in this study. These self-explanatory techniques include locked door, guard, bunker, and camouflage. The system score for physical defense is simply an average of the number of techniques used over all network links. This assumes that using physical defense techniques is of equal importance at each node location. The utility plot shows marginally decreasing gains for increasing the number of techniques.

1

Utilit

0

0   physical (# of techniques)   4
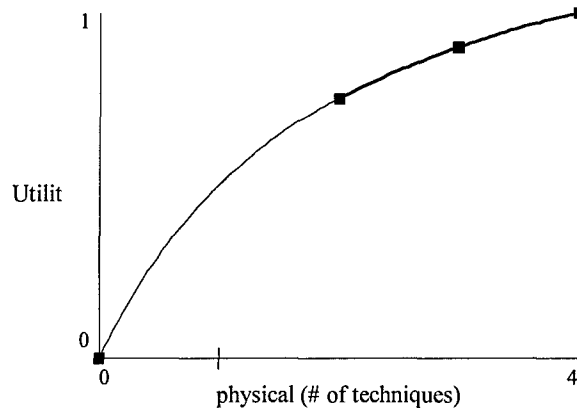
3. *Flexibility* is decomposed into three subgoals, *mobile, interoperable,* and *expandable.*

- *Mobile* is defined with two measures, *ease of transport* and *setup time.* Both are important within the deployed scenario.

- *Ease of transport* is simply a scoring of different equipment types as to how easily transportable they are. The components generally fall into categories such as manpacks, vehicle mounted, fixed transportable, and fixed permanent. These correspond to the component scores below. The system score is the average of all the component scores. As such, it will probably fall between two of the identified levels. The linear utility function is also shown below.

Component scores for ease of transport

Level	Utility
manpack	1
vehicle mount	0.75
fixed transportable	0.25
fixed permanent	0

Utilit — ease of transport (average)

- *Setup time* varies depending on the equipment and transmission options. The system score for setup time is based on the average setup time of all equipment in the network. The impact of a new component on setup time is found by calculating a new average. The plot shows a much larger utility for times less than one hour. Points used: 1hr = .5 utility.

Utilit — setup time (hours) — 0, 5

- *Interoperable* is characterized with one measure, *interoperability.*

  - Component *interoperability* is captured with descriptive labels, indicating the magnitude of interoperability. This is a subjective , relative measure. It should capture how interoperable the new component is with the system and also if the new component adds interoperability with outside systems that the network didn't currently have. The 5 levels progress from very low at a utility of 0, through low, medium, high and very high with a utility of 1. System interoperability is simply an average of all component scores.

Component scores for interoperability

Level	Utility	
very high	1	████████████████████
high	0.75	███████████████
medium	0.5	██████████
low	0.25	█████
very low	0	

Utilit

0

interoperability (average)
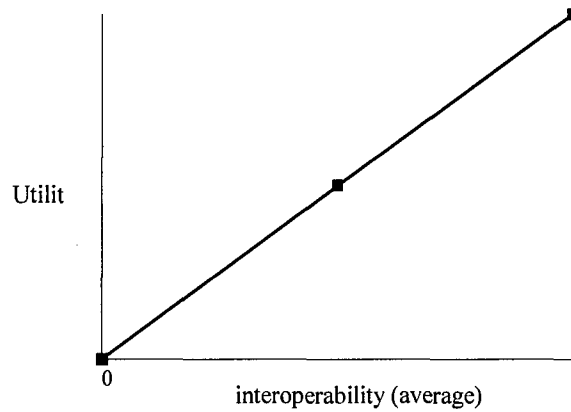
- *Expandable* is captured with two measures, *expansion capability* and *complexity*.
  - *Expansion capability* measures the magnitude of a possible expansion based on the system's current capabilities. It deals strictly with capacity expansion. No expansion, or a multiple of 1 times the current system, receives a utility of 0. A doubling capability has a utility of .5. Five times has a utility of .9 and 10 times has a utility of 1. The system expansion score is based on the average rating of all network links.

Utilit

expansion potential (multiple of baseline)
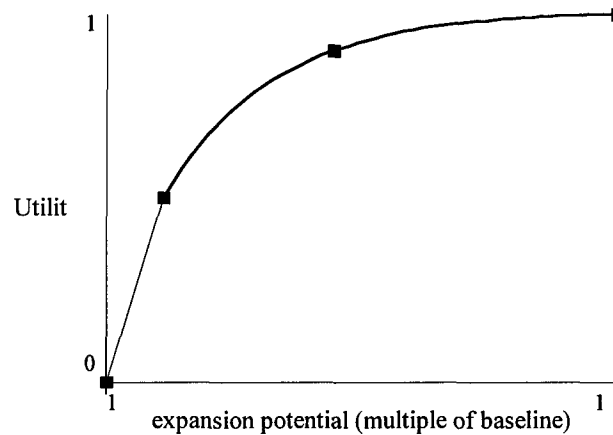
- *Complexity* captures the modularity of a system, and how easily it can be upgraded. This may be for expansion purposes, or for adding new features or compatibility components. It is a measure of the level of difficulty of changing or adapting the system to changes in needs. The system score is an average of the complexity ratings for each of the network members. Component ratings are somewhat subjective, based on the user's interpretation and experience.

Component scores for complexity level

Label	Utility	
trivial	1	
easy	0.75	
medium	0.5	
difficult	0.25	
very difficult	0	

Utilit

0

complexity level (average)
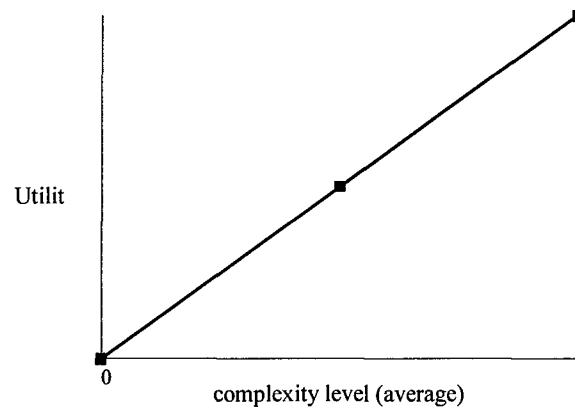
Appendix C. Component Alternatives for System Upgrade

This appendix contains the complete description of the components which are being considered to expand the baseline network. They are described in terms of the measurable attributes in the value hierarchy. This hierarchy structure, is presented in Appendix B with comprehensive a background of each attribute and the system level measurement explanation. This appendix also lists the cost of each component. All components have been fabricated for this proof-of-concept model.

component scores and costs

	2.4kbps components		9.6kbps components					
component ID	2.4A		9.6A		9.6B		9.6C	
	features	score	features	score	features	score	features	score
users	1	1	1	1	1	1	1	1
Pe	0.00001	-5.00	0.00001	-5.00	0.0000001	-7	0.000001	-6
Ri (info rate)	300	300	1200	1200	1200	1200	960	960
delay								
ease of use	medium	0.5	easy	0.75	medium	0.5	easy	0.75
media types	2	2	2	2	2	2	2	2
perception	60	60	60	60	60	60	60	60
range	40km	40km	40km	40km	400km	400km	40km	40km
intrusion prevention	0	0	0	0	1	1	0	0
encrypt	y		y		y		y	
anti-detection	2 techs	4	2 techs	4	2 techs	4	3 techs	6
anti-jam	2 techs	4	2 techs	4	2 techs	4	3 techs	6
hardening	2 types	2	2 types	2	2 types	2	2 types	2
redundancy	1 tech	1	1 tech	1	2 techs	2	1 tech	1
reliability	0.8	0.8	0.75	0.75	0.93	0.93	0.9	0.9
physical defense								
ease of transport	manpack	1	veh mount	0.75	veh mount	0.75	manpack	1
setup time	20 min	0.333333333	55 min	0.916666667	25 min	0.416666667	15 min	0.25
interoperability	medium	0.5	medium	0.5	medium	0.5	medium	0.5
expansion capability	none	1	none	1	to 16kbps	1.666666667	none	1
complexity	difficult	0.25	difficult	0.25	medium	0.5	difficult	0.25
cost	18000		20000		21500		26000	

component scores and costs

component ID	16kbps components							
	16A		16B		16C		16D	
	features	score	features	score	features	score	features	score
users	1	1	1	1	1	1	1	1
Pe	0.000001	-6	0.00001	-5	0.00001	-5	0.000001	-6
Ri (info rate)	1600	1600	2000	2000	2000	2000	2285.714286	2285.714286
delay								
ease of use	difficult	0.25	medium	0.5	easy	0.75	medium	0.5
media types	3	3	2	2	2	2	2	2
perception	60	60	60	60	60	60	60	60
range	area	area	400km	400km	40km	40km	40km	40km
intrusion prevention	2	2	1	1	0	0	0	0
encrypt	y		y		y		y	
anti-detection	4 techs	8	3 techs	6	2 techs	4	2 techs	4
anti-jam	4 techs	8	2 techs	4	2 techs	4	2 techs	4
hardening	2 types	2	2 types	2	2 types	2	2 types	2
redundancy	2 techs	2	2 techs	2	1 tech	1	2 techs	2
reliability	0.85	0.85	0.95	0.95	0.9	0.9	0.92	0.92
physical defense								
ease of transport	fixed trans	0.25	veh mount	0.75	veh mount	0.75	manpack	1
setup time	2.5 hours	2.5	1 hour	1	40 min	0.666666667	10 min	0.166666667
interoperability	low	0.25	medium	0.5	medium	0.5	high	0.75
expansion capability	to 32kbps	2	none	1	none	1	none	1
complexity	easy	0.75	difficult	0.25	medium	0.5	easy	0.75
cost	31500		30200		26500		32750	

component scores and costs

component ID	32kbps components					
	32A		32B		32C	
	features	score	features	score	features	score
users	1	1	1	1	1	1
Pe	0.00001	-5	0.000001	-6	0.000001	-6
Ri (info rate)	4000	4000	3200	3200	3200	3200
delay						
ease of use	medium	0.5	medium	0.5	easy	0.75
media types	2	2	3	3	2	2
perception	60	60	60	60	60	60
range	area	area	400km	400km	400km	400km
intrusion prevention	2	2	2	2	1	1
encrypt	y		y		y	
anti-detection	4 techs	8	3 techs	6	2 techs	4
anti-jam	4 techs	8	2 techs	4	2 techs	4
hardening	2 types	2	2 types	2	2 types	2
redundancy	3 techs	3	2 techs	2	2 techs	2
reliability	0.82	0.82	0.93	0.93	0.9	0.9
physical defense						
ease of transport	fixed trans	0.25	veh mount	0.75	manpack	1
setup time	2 hours	2	40 min	0.666666667	25 min	0.416666667
interoperability	high	0.75	medium	0.5	medium	0.5
expansion capability	to 64kbps	2	none	1	none	1
complexity	difficult	0.25	medium	0.5	difficult	0.25
cost	35000		31600		31500	

Appendix D. Netsolve code for bottleneck identification.

Macro Method for Bottleneck Identification

file: thesis

FROM	TO	COST	LOWER	UPPER
A	D	1.00	0.00	9.60
A	E	1.00	0.00	9.60
A	F	1.00	0.00	9.60
B	D	1.00	0.00	9.60
C	D	1.00	0.00	9.60
C	J	1.00	0.00	9.60
D	H	1.00	0.00	9.60
DE	SU	0.00	0.00	999999.00
E	L	1.00	0.00	2.40
E	P	1.00	0.00	2.40
EX	W	0.00	0.00	999999.00
EX	X	0.00	0.00	999999.00
EX	Y	0.00	0.00	999999.00
EX	Z	0.00	0.00	999999.00
EX	ZZ	0.00	0.00	999999.00
F	G	1.00	0.00	9.60
F	L	1.00	0.00	9.60
G	H	1.00	0.00	2.40
G	Q	1.00	0.00	2.40
H	I	1.00	0.00	9.60
H	M	1.00	0.00	9.60
I	N	1.00	0.00	9.60
J	I	1.00	0.00	9.60
J	K	1.00	0.00	9.60
J	O	1.00	0.00	9.60
K	O	1.00	0.00	2.40
K	V	1.00	0.00	2.40
L	P	1.00	0.00	9.60
M	Q	1.00	0.00	2.40
M	X	1.00	0.00	2.40
N	O	1.00	0.00	9.60
N	T	1.00	0.00	9.60
N	X	1.00	0.00	9.60
O	V	1.00	0.00	9.60
P	R	1.00	0.00	9.60
Q	S	1.00	0.00	9.60
R	W	1.00	0.00	9.60
S	W	1.00	0.00	9.60
S	ZZ	1.00	0.00	9.60
SU	A	0.00	0.00	999999.00
SU	B	0.00	0.00	999999.00
SU	C	0.00	0.00	999999.00
SU	EX	10.00	0.00	999999.00
T	X	1.00	0.00	2.40
T	Y	1.00	0.00	2.40
U	Y	1.00	0.00	9.60
V	U	1.00	0.00	9.60
V	Z	1.00	0.00	9.60
W	DE	0.00	14.00	999999.00
X	DE	0.00	12.00	999999.00
Y	DE	0.00	8.00	999999.00
Z	DE	0.00	9.00	999999.00
ZZ	DE	0.00	8.00	999999.00

MINIMUM COST FLOW PROBLEM: MINIMUM COST IS   330.60

FROM	TO	LOWER	FLOW	UPPER	COST
SU	A	0.00	24.00	999999.00	0.00
SU	C	0.00	9.60	999999.00	0.00
A	D	0.00	9.60	9.60	1.00
W	DE	14.00	14.00	999999.00	0.00
X	DE	12.00	12.00	999999.00	0.00
Y	DE	8.00	8.00	999999.00	0.00
Z	DE	9.00	9.00	999999.00	0.00
ZZ	DE	8.00	8.00	999999.00	0.00
A	E	0.00	4.80	9.60	1.00
SU	EX	0.00	17.40	999999.00	10.00
A	F	0.00	9.60	9.60	1.00
F	G	0.00	4.80	9.60	1.00
D	H	0.00	9.60	9.60	1.00
G	H	0.00	2.40	2.40	1.00
H	I	0.00	9.00	9.60	1.00
J	I	0.00	0.60	9.60	1.00
C	J	0.00	9.60	9.60	1.00
J	K	0.00	2.40	9.60	1.00
E	L	0.00	2.40	2.40	1.00
F	L	0.00	4.80	9.60	1.00
H	M	0.00	3.00	9.60	1.00
I	N	0.00	9.60	9.60	1.00
J	O	0.00	6.60	9.60	1.00
E	P	0.00	2.40	2.40	1.00
L	P	0.00	7.20	9.60	1.00
G	Q	0.00	2.40	2.40	1.00
M	Q	0.00	0.60	2.40	1.00
P	R	0.00	9.60	9.60	1.00
Q	S	0.00	3.00	9.60	1.00
DE	SU	0.00	51.00	999999.00	0.00
K	V	0.00	2.40	2.40	1.00
O	V	0.00	6.60	9.60	1.00
EX	W	0.00	4.40	999999.00	0.00
R	W	0.00	9.60	9.60	1.00
M	X	0.00	2.40	2.40	1.00
N	X	0.00	9.60	9.60	1.00
EX	Y	0.00	8.00	999999.00	0.00
V	Z	0.00	9.00	9.60	1.00
EX	ZZ	0.00	5.00	999999.00	0.00
S	ZZ	0.00	3.00	9.60	1.00

FROM	TO	EDGE STATE	REDUCED COST	LOWER	COST RANGE CURRENT	UPPER
A	D	BASIC	0.00	-999999.00	1.00	1.00
A	E	BASIC	0.00	-999999.00	1.00	4.00
A	F	UPPER	-3.00	-999999.00	1.00	4.00
B	D	LOWER	0.00	1.00	1.00	999999.00
C	D	LOWER	0.00	1.00	1.00	999999.00
C	J	UPPER	-5.00	-999999.00	1.00	6.00
D	H	UPPER	-4.00	-999999.00	1.00	5.00
DE	SU	BASIC	0.00	-9.00	0.00	999999.00
E	L	UPPER	-3.00	-999999.00	1.00	4.00
E	P	UPPER	-4.00	-999999.00	1.00	5.00
EX	W	BASIC	0.00	-2.00	0.00	0.00
EX	X	BASIC	0.00	-1.00	0.00	999999.00
EX	Y	BASIC	0.00	-10.00	0.00	0.00
EX	Z	LOWER	1.00	-1.00	0.00	999999.00
EX	ZZ	BASIC	0.00	0.00	0.00	1.00
F	G	BASIC	0.00	-1.00	1.00	4.00
F	L	BASIC	0.00	-2.00	1.00	3.00
G	H	BASIC	0.00	-1.00	1.00	4.00
G	Q	UPPER	-2.00	-999999.00	1.00	3.00
H	I	BASIC	0.00	1.00	1.00	2.00
H	M	BASIC	0.00	0.00	1.00	1.00
I	N	UPPER	-1.00	-999999.00	1.00	2.00
J	I	BASIC	0.00	0.00	1.00	1.00
J	K	BASIC	0.00	0.00	1.00	1.00
J	O	BASIC	0.00	1.00	1.00	2.00
K	O	LOWER	1.00	0.00	1.00	999999.00
K	V	UPPER	0.00	-999999.00	1.00	1.00
L	P	BASIC	0.00	-3.00	1.00	3.00
M	Q	BASIC	0.00	0.00	1.00	1.00
M	X	UPPER	-2.00	-999999.00	1.00	3.00
N	O	LOWER	3.00	-2.00	1.00	999999.00
N	T	BASIC	0.00	0.00	1.00	999999.00
N	X	BASIC	0.00	-999999.00	1.00	2.00
O	V	BASIC	0.00	1.00	1.00	2.00
P	R	UPPER	-2.00	-999999.00	1.00	3.00
Q	S	BASIC	0.00	0.00	1.00	1.00
R	W	BASIC	0.00	-999999.00	1.00	3.00
S	W	LOWER	0.00	1.00	1.00	999999.00
S	ZZ	BASIC	0.00	0.00	1.00	1.00
SU	A	BASIC	0.00	-999999.00	0.00	0.00
SU	B	BASIC	0.00	0.00	0.00	999999.00
SU	C	BASIC	0.00	0.00	0.00	5.00
SU	EX	BASIC	0.00	7.00	10.00	999999.00
T	X	LOWER	1.00	0.00	1.00	999999.00
T	Y	LOWER	1.00	0.00	1.00	999999.00
U	Y	LOWER	0.00	1.00	1.00	999999.00
V	U	BASIC	0.00	1.00	1.00	999999.00
V	Z	BASIC	0.00	-8.00	1.00	2.00
W	DE	LOWER	10.00	-10.00	0.00	999999.00
X	DE	LOWER	10.00	-10.00	0.00	999999.00
Y	DE	LOWER	10.00	-10.00	0.00	999999.00
Z	DE	LOWER	9.00	-9.00	0.00	999999.00
ZZ	DE	LOWER	10.00	-10.00	0.00	999999.00

```
file: thes21
   FROM      TO              COST       LOWER        UPPER
   ----      --          --------    --------     --------
     A       D              1.00        0.00         9.60
     A       E              1.00        0.00         9.60
     A       F              1.00        0.00        32.00
     B       D              1.00        0.00         9.60
     C       D              1.00        0.00         9.60
     C       J              1.00        0.00        32.00
     D       H              1.00        0.00        32.00
    DE       SU             0.00        0.00    999999.00
     E       L              1.00        0.00         2.40
     E       P              1.00        0.00         9.60
    EX       W              0.00        0.00    999999.00
    EX       X              0.00        0.00    999999.00
    EX       Y              0.00        0.00    999999.00
    EX       Z              0.00        0.00    999999.00
    EX       ZZ             0.00        0.00    999999.00
     F       G              1.00        0.00         9.60
     F       L              1.00        0.00         9.60
     G       H              1.00        0.00         2.40
     G       Q              1.00        0.00         9.60
     H       I              1.00        0.00         9.60
     H       M              1.00        0.00         9.60
     I       N              1.00        0.00        32.00
     J       I              1.00        0.00         9.60
     J       K              1.00        0.00         9.60
     J       O              1.00        0.00         9.60
     K       O              1.00        0.00         2.40
     K       V              1.00        0.00         2.40
     L       P              1.00        0.00         9.60
     M       Q              1.00        0.00         2.40
     M       X              1.00        0.00         9.60
     N       O              1.00        0.00         9.60
     N       T              1.00        0.00         9.60
     N       X              1.00        0.00         9.60
     O       V              1.00        0.00         9.60
     P       R              1.00        0.00        16.00
     Q       S              1.00        0.00         9.60
     R       W              1.00        0.00         9.60
     S       W              1.00        0.00         9.60
     S       ZZ             1.00        0.00         9.60
    SU       A              0.00        0.00    999999.00
    SU       B              0.00        0.00    999999.00
    SU       C              0.00        0.00    999999.00
    SU       EX            10.00        0.00    999999.00
     T       X              1.00        0.00         2.40
     T       Y              1.00        0.00         2.40
     U       Y              1.00        0.00         9.60
     V       U              1.00        0.00         9.60
     V       Z              1.00        0.00         9.60
     W       DE             0.00       14.00    999999.00
     X       DE             0.00       12.00    999999.00
     Y       DE             0.00        8.00    999999.00
     Z       DE             0.00        9.00    999999.00
    ZZ       DE             0.00        8.00    999999.00
```

MINIMUM COST FLOW PROBLEM: MINIMUM COST IS 251.40

FROM	TO	LOWER	FLOW	UPPER	COST
SU	A	0.00	28.80	999999.00	0.00
SU	C	0.00	16.80	999999.00	0.00
A	D	0.00	9.60	9.60	1.00
W	DE	14.00	14.00	999999.00	0.00
X	DE	12.00	12.00	999999.00	0.00
Y	DE	8.00	8.00	999999.00	0.00
Z	DE	9.00	9.00	999999.00	0.00
ZZ	DE	8.00	8.00	999999.00	0.00
A	E	0.00	9.60	9.60	1.00
SU	EX	0.00	5.40	999999.00	10.00
A	F	0.00	9.60	32.00	1.00
F	G	0.00	9.60	9.60	1.00
D	H	0.00	9.60	32.00	1.00
J	I	0.00	4.80	9.60	1.00
C	J	0.00	16.80	32.00	1.00
J	K	0.00	2.40	9.60	1.00
H	M	0.00	9.60	9.60	1.00
I	N	0.00	4.80	32.00	1.00
J	O	0.00	9.60	9.60	1.00
E	P	0.00	9.60	9.60	1.00
G	Q	0.00	9.60	9.60	1.00
P	R	0.00	9.60	16.00	1.00
Q	S	0.00	9.60	9.60	1.00
DE	SU	0.00	51.00	999999.00	0.00
N	T	0.00	2.40	9.60	1.00
V	U	0.00	3.00	9.60	1.00
K	V	0.00	2.40	2.40	1.00
O	V	0.00	9.60	9.60	1.00
EX	W	0.00	2.80	999999.00	0.00
R	W	0.00	9.60	9.60	1.00
S	W	0.00	1.60	9.60	1.00
M	X	0.00	9.60	9.60	1.00
N	X	0.00	2.40	9.60	1.00
EX	Y	0.00	2.60	999999.00	0.00
T	Y	0.00	2.40	2.40	1.00
U	Y	0.00	3.00	9.60	1.00
V	Z	0.00	9.00	9.60	1.00
S	ZZ	0.00	8.00	9.60	1.00

FROM	TO	EDGE STATE	REDUCED COST	LOWER	COST RANGE CURRENT	UPPER
A	D	UPPER	0.00	-999999.00	1.00	1.00
A	E	BASIC	0.00	1.00	1.00	2.00
A	F	BASIC	0.00	0.00	1.00	1.00
B	D	BASIC	0.00	1.00	1.00	1.00
C	D	LOWER	0.00	1.00	1.00	999999.00
C	J	BASIC	0.00	1.00	1.00	2.00
D	H	BASIC	0.00	0.00	1.00	1.00
DE	SU	BASIC	0.00	-4.00	0.00	999999.00
E	L	LOWER	0.00	1.00	1.00	999999.00
E	P	BASIC	0.00	-999999.00	1.00	2.00
EX	W	BASIC	0.00	-5.00	0.00	0.00
EX	X	LOWER	6.00	-6.00	0.00	999999.00
EX	Y	BASIC	0.00	-5.00	0.00	1.00
EX	Z	LOWER	1.00	-1.00	0.00	999999.00
EX	ZZ	LOWER	0.00	0.00	0.00	999999.00
F	G	BASIC	0.00	0.00	1.00	2.00
F	L	BASIC	0.00	0.00	1.00	1.00
G	H	LOWER	1.00	0.00	1.00	999999.00
G	Q	BASIC	0.00	-999999.00	1.00	2.00
H	I	LOWER	1.00	0.00	1.00	999999.00
H	M	UPPER	0.00	-999999.00	1.00	1.00
I	N	BASIC	0.00	1.00	1.00	6.00
J	I	BASIC	0.00	1.00	1.00	2.00
J	K	BASIC	0.00	0.00	1.00	6.00
J	O	BASIC	0.00	-999999.00	1.00	2.00
K	O	LOWER	1.00	0.00	1.00	999999.00
K	V	UPPER	-5.00	-999999.00	1.00	6.00
L	P	LOWER	1.00	0.00	1.00	999999.00
M	Q	LOWER	1.00	0.00	1.00	999999.00
M	X	BASIC	0.00	-999999.00	1.00	1.00
N	O	LOWER	2.00	-1.00	1.00	999999.00
N	T	BASIC	0.00	0.00	1.00	6.00
N	X	BASIC	0.00	1.00	1.00	2.00
O	V	UPPER	-5.00	-999999.00	1.00	6.00
P	R	BASIC	0.00	-999999.00	1.00	7.00
Q	S	UPPER	-5.00	-999999.00	1.00	6.00
R	W	UPPER	-6.00	-999999.00	1.00	7.00
S	W	BASIC	0.00	1.00	1.00	6.00
S	ZZ	BASIC	0.00	-9.00	1.00	1.00
SU	A	BASIC	0.00	-1.00	0.00	0.00
SU	B	BASIC	0.00	0.00	0.00	0.00
SU	C	BASIC	0.00	0.00	0.00	1.00
SU	EX	BASIC	0.00	5.00	10.00	999999.00
T	X	LOWER	1.00	0.00	1.00	999999.00
T	Y	UPPER	-5.00	-999999.00	1.00	6.00
U	Y	BASIC	0.00	0.00	1.00	6.00
V	U	BASIC	0.00	0.00	1.00	6.00
V	Z	BASIC	0.00	-8.00	1.00	2.00
W	DE	LOWER	10.00	-10.00	0.00	999999.00
X	DE	LOWER	4.00	-4.00	0.00	999999.00
Y	DE	LOWER	10.00	-10.00	0.00	999999.00
Z	DE	LOWER	9.00	-9.00	0.00	999999.00
ZZ	DE	LOWER	10.00	-10.00	0.00	999999.00

file: thes22

FROM	TO	COST	LOWER	UPPER
A	D	1.00	0.00	9.60
A	E	1.00	0.00	9.60
A	F	1.00	0.00	32.00
B	D	1.00	0.00	9.60
C	D	1.00	0.00	9.60
C	J	1.00	0.00	32.00
D	H	1.00	0.00	32.00
DE	SU	0.00	0.00	999999.00
E	L	1.00	0.00	2.40
E	P	1.00	0.00	9.60
EX	W	0.00	0.00	999999.00
EX	X	0.00	0.00	999999.00
EX	Y	0.00	0.00	999999.00
EX	Z	0.00	0.00	999999.00
EX	ZZ	0.00	0.00	999999.00
F	G	1.00	0.00	9.60
F	L	1.00	0.00	9.60
G	H	1.00	0.00	2.40
G	Q	1.00	0.00	9.60
H	I	1.00	0.00	9.60
H	M	1.00	0.00	9.60
I	N	1.00	0.00	32.00
J	I	1.00	0.00	9.60
J	K	1.00	0.00	9.60
J	O	1.00	0.00	9.60
K	O	1.00	0.00	2.40
K	V	1.00	0.00	2.40
L	P	1.00	0.00	9.60
M	Q	1.00	0.00	2.40
M	X	1.00	0.00	9.60
N	O	1.00	0.00	9.60
N	T	1.00	0.00	9.60
N	X	1.00	0.00	9.60
O	V	1.00	0.00	9.60
P	R	1.00	0.00	16.00
Q	S	1.00	0.00	9.60
R	W	1.00	0.00	16.00
S	W	1.00	0.00	9.60
S	ZZ	1.00	0.00	9.60
SU	A	0.00	0.00	999999.00
SU	B	0.00	0.00	999999.00
SU	C	0.00	0.00	999999.00
SU	EX	10.00	0.00	999999.00
T	X	1.00	0.00	2.40
T	Y	1.00	0.00	9.60
U	Y	1.00	0.00	9.60
V	U	1.00	0.00	9.60
V	Z	1.00	0.00	9.60
W	DE	0.00	14.00	999999.00
X	DE	0.00	12.00	999999.00
Y	DE	0.00	8.00	999999.00
Z	DE	0.00	9.00	999999.00
ZZ	DE	0.00	8.00	999999.00

MINIMUM COST FLOW PROBLEM: MINIMUM COST IS   224.40

FROM	TO	LOWER	FLOW	UPPER	COST
SU	A	0.00	31.60	999999.00	0.00
SU	C	0.00	19.40	999999.00	0.00
A	D	0.00	9.60	9.60	1.00
W	DE	14.00	14.00	999999.00	0.00
X	DE	12.00	12.00	999999.00	0.00
Y	DE	8.00	8.00	999999.00	0.00
Z	DE	9.00	9.00	999999.00	0.00
ZZ	DE	8.00	8.00	999999.00	0.00
A	E	0.00	9.60	9.60	1.00
A	F	0.00	12.40	32.00	1.00
F	G	0.00	8.00	9.60	1.00
D	H	0.00	9.60	32.00	1.00
J	I	0.00	9.60	9.60	1.00
C	J	0.00	19.40	32.00	1.00
J	K	0.00	2.40	9.60	1.00
F	L	0.00	4.40	9.60	1.00
H	M	0.00	9.60	9.60	1.00
I	N	0.00	9.60	32.00	1.00
J	O	0.00	7.40	9.60	1.00
E	P	0.00	9.60	9.60	1.00
L	P	0.00	4.40	9.60	1.00
G	Q	0.00	8.00	9.60	1.00
P	R	0.00	14.00	16.00	1.00
Q	S	0.00	8.00	9.60	1.00
DE	SU	0.00	51.00	999999.00	0.00
N	T	0.00	7.20	9.60	1.00
V	U	0.00	0.80	9.60	1.00
K	V	0.00	2.40	2.40	1.00
O	V	0.00	7.40	9.60	1.00
R	W	0.00	14.00	16.00	1.00
M	X	0.00	9.60	9.60	1.00
N	X	0.00	2.40	9.60	1.00
T	Y	0.00	7.20	9.60	1.00
U	Y	0.00	0.80	9.60	1.00
V	Z	0.00	9.00	9.60	1.00
S	ZZ	0.00	8.00	9.60	1.00

FROM	TO	EDGE STATE	REDUCED COST	LOWER	COST RANGE CURRENT	UPPER
A	D	UPPER	0.00	-999999.00	1.00	1.00
A	E	BASIC	0.00	1.00	1.00	2.00
A	F	BASIC	0.00	0.00	1.00	1.00
B	D	BASIC	0.00	1.00	1.00	1.00
C	D	LOWER	0.00	1.00	1.00	999999.00
C	J	BASIC	0.00	1.00	1.00	2.00
D	H	BASIC	0.00	0.00	1.00	1.00
DE	SU	BASIC	0.00	-4.00	0.00	999999.00
E	L	LOWER	0.00	1.00	1.00	999999.00
E	P	UPPER	-1.00	-999999.00	1.00	2.00
EX	W	LOWER	5.00	-5.00	0.00	999999.00
EX	X	LOWER	6.00	-6.00	0.00	999999.00
EX	Y	LOWER	5.00	-5.00	0.00	999999.00
EX	Z	LOWER	6.00	-6.00	0.00	999999.00
EX	ZZ	LOWER	5.00	-5.00	0.00	999999.00
F	G	BASIC	0.00	1.00	1.00	2.00
F	L	BASIC	0.00	0.00	1.00	1.00
G	H	LOWER	1.00	0.00	1.00	999999.00
G	Q	BASIC	0.00	1.00	1.00	2.00
H	I	LOWER	1.00	0.00	1.00	999999.00
H	M	UPPER	0.00	-999999.00	1.00	1.00
I	N	BASIC	0.00	0.00	1.00	1.00
J	I	UPPER	0.00	-999999.00	1.00	1.00
J	K	BASIC	0.00	0.00	1.00	1.00
J	O	BASIC	0.00	1.00	1.00	2.00
K	O	LOWER	1.00	0.00	1.00	999999.00
K	V	UPPER	0.00	-999999.00	1.00	1.00
L	P	BASIC	0.00	0.00	1.00	1.00
M	Q	LOWER	1.00	0.00	1.00	999999.00
M	X	BASIC	0.00	-999999.00	1.00	1.00
N	O	LOWER	2.00	-1.00	1.00	999999.00
N	T	BASIC	0.00	0.00	1.00	1.00
N	X	BASIC	0.00	1.00	1.00	2.00
O	V	BASIC	0.00	1.00	1.00	2.00
P	R	BASIC	0.00	-4.00	1.00	1.00
Q	S	BASIC	0.00	1.00	1.00	6.00
R	W	BASIC	0.00	-4.00	1.00	1.00
S	W	LOWER	0.00	1.00	1.00	999999.00
S	ZZ	BASIC	0.00	-4.00	1.00	6.00
SU	A	BASIC	0.00	-1.00	0.00	0.00
SU	B	BASIC	0.00	0.00	0.00	0.00
SU	C	BASIC	0.00	0.00	0.00	1.00
SU	EX	BASIC	0.00	5.00	10.00	999999.00
T	X	LOWER	1.00	0.00	1.00	999999.00
T	Y	BASIC	0.00	0.00	1.00	1.00
U	Y	BASIC	0.00	1.00	1.00	2.00
V	U	BASIC	0.00	1.00	1.00	2.00
V	Z	BASIC	0.00	-3.00	1.00	7.00
W	DE	LOWER	5.00	-5.00	0.00	999999.00
X	DE	LOWER	4.00	-4.00	0.00	999999.00
Y	DE	LOWER	5.00	-5.00	0.00	999999.00
Z	DE	LOWER	4.00	-4.00	0.00	999999.00
ZZ	DE	LOWER	5.00	-5.00	0.00	999999.00

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file: thes23
   FROM     TO          COST        LOWER        UPPER
   ----     --        --------    --------     --------
    A        D          1.00        0.00          9.60
    A        E          1.00        0.00          9.60
    A        F          1.00        0.00         32.00
    B        D          1.00        0.00          9.60
    C        D          1.00        0.00          9.60
    C        J          1.00        0.00         32.00
    D        H          1.00        0.00         32.00
   DE        SU         0.00        0.00     999999.00
    E        L          1.00        0.00          2.40
    E        P          1.00        0.00         16.00
   EX        W          0.00        0.00     999999.00
   EX        X          0.00        0.00     999999.00
   EX        Y          0.00        0.00     999999.00
   EX        Z          0.00        0.00     999999.00
   EX        ZZ         0.00        0.00     999999.00
    F        G          1.00        0.00          9.60
    F        L          1.00        0.00          9.60
    G        H          1.00        0.00          2.40
    G        Q          1.00        0.00          9.60
    H        I          1.00        0.00          9.60
    H        M          1.00        0.00          9.60
    I        N          1.00        0.00         32.00
    J        I          1.00        0.00          9.60
    J        K          1.00        0.00          9.60
    J        O          1.00        0.00          9.60
    K        O          1.00        0.00          2.40
    K        V          1.00        0.00          2.40
    L        P          1.00        0.00          9.60
    M        Q          1.00        0.00          2.40
    M        X          1.00        0.00          9.60
    N        O          1.00        0.00          9.60
    N        T          1.00        0.00          9.60
    N        X          1.00        0.00          9.60
    O        V          1.00        0.00          9.60
    P        R          1.00        0.00         16.00
    Q        S          1.00        0.00          9.60
    R        W          1.00        0.00         16.00
    S        W          1.00        0.00          9.60
    S        ZZ         1.00        0.00          9.60
   SU        A          0.00        0.00     999999.00
   SU        B          0.00        0.00     999999.00
   SU        C          0.00        0.00     999999.00
   SU        EX        10.00        0.00     999999.00
    T        X          1.00        0.00          2.40
    T        Y          1.00        0.00          9.60
    U        Y          1.00        0.00          9.60
    V        U          1.00        0.00          9.60
    V        Z          1.00        0.00          9.60
    W        DE         0.00       14.00     999999.00
    X        DE         0.00       12.00     999999.00
    Y        DE         0.00        8.00     999999.00
    Z        DE         0.00        9.00     999999.00
   ZZ        DE         0.00        8.00     999999.00
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MINIMUM COST FLOW PROBLEM: MINIMUM COST IS   224.40

FROM	TO	LOWER	FLOW	UPPER	COST
SU	A	0.00	31.60	999999.00	0.00
SU	C	0.00	19.40	999999.00	0.00
A	D	0.00	9.60	9.60	1.00
W	DE	14.00	14.00	999999.00	0.00
X	DE	12.00	12.00	999999.00	0.00
Y	DE	8.00	8.00	999999.00	0.00
Z	DE	9.00	9.00	999999.00	0.00
ZZ	DE	8.00	8.00	999999.00	0.00
A	E	0.00	9.60	9.60	1.00
A	F	0.00	12.40	32.00	1.00
F	G	0.00	8.00	9.60	1.00
D	H	0.00	9.60	32.00	1.00
J	I	0.00	9.60	9.60	1.00
C	J	0.00	19.40	32.00	1.00
J	K	0.00	2.40	9.60	1.00
F	L	0.00	4.40	9.60	1.00
H	M	0.00	9.60	9.60	1.00
I	N	0.00	9.60	32.00	1.00
J	O	0.00	7.40	9.60	1.00
E	P	0.00	9.60	16.00	1.00
L	P	0.00	4.40	9.60	1.00
G	Q	0.00	8.00	9.60	1.00
P	R	0.00	14.00	16.00	1.00
Q	S	0.00	8.00	9.60	1.00
DE	SU	0.00	51.00	999999.00	0.00
N	T	0.00	7.20	9.60	1.00
V	U	0.00	0.80	9.60	1.00
K	V	0.00	2.40	2.40	1.00
O	V	0.00	7.40	9.60	1.00
R	W	0.00	14.00	16.00	1.00
M	X	0.00	9.60	9.60	1.00
N	X	0.00	2.40	9.60	1.00
T	Y	0.00	7.20	9.60	1.00
U	Y	0.00	0.80	9.60	1.00
V	Z	0.00	9.00	9.60	1.00
S	ZZ	0.00	8.00	9.60	1.00

FROM	TO	EDGE STATE	REDUCED COST	LOWER	COST RANGE CURRENT	UPPER
A	D	UPPER	0.00	-999999.00	1.00	1.00
A	E	UPPER	-1.00	-999999.00	1.00	2.00
A	F	BASIC	0.00	0.00	1.00	2.00
B	D	BASIC	0.00	1.00	1.00	1.00
C	D	LOWER	0.00	1.00	1.00	999999.00
C	J	BASIC	0.00	1.00	1.00	2.00
D	H	BASIC	0.00	0.00	1.00	1.00
DE	SU	BASIC	0.00	-4.00	0.00	999999.00
E	L	LOWER	1.00	0.00	1.00	999999.00
E	P	BASIC	0.00	-999999.00	1.00	2.00
EX	W	LOWER	5.00	-5.00	0.00	999999.00
EX	X	LOWER	6.00	-6.00	0.00	999999.00
EX	Y	LOWER	5.00	-5.00	0.00	999999.00
EX	Z	LOWER	6.00	-6.00	0.00	999999.00
EX	ZZ	LOWER	5.00	-5.00	0.00	999999.00
F	G	BASIC	0.00	1.00	1.00	2.00
F	L	BASIC	0.00	0.00	1.00	1.00
G	H	LOWER	1.00	0.00	1.00	999999.00
G	Q	BASIC	0.00	1.00	1.00	2.00
H	I	LOWER	1.00	0.00	1.00	999999.00
H	M	UPPER	0.00	-999999.00	1.00	1.00
I	N	BASIC	0.00	0.00	1.00	1.00
J	I	UPPER	0.00	-999999.00	1.00	1.00
J	K	BASIC	0.00	0.00	1.00	1.00
J	O	BASIC	0.00	1.00	1.00	2.00
K	O	LOWER	1.00	0.00	1.00	999999.00
K	V	UPPER	0.00	-999999.00	1.00	1.00
L	P	BASIC	0.00	0.00	1.00	1.00
M	Q	LOWER	1.00	0.00	1.00	999999.00
M	X	BASIC	0.00	-999999.00	1.00	1.00
N	O	LOWER	2.00	-1.00	1.00	999999.00
N	T	BASIC	0.00	0.00	1.00	1.00
N	X	BASIC	0.00	1.00	1.00	2.00
O	V	BASIC	0.00	1.00	1.00	2.00
P	R	BASIC	0.00	-4.00	1.00	1.00
Q	S	BASIC	0.00	1.00	1.00	6.00
R	W	BASIC	0.00	-4.00	1.00	1.00
S	W	LOWER	0.00	1.00	1.00	999999.00
S	ZZ	BASIC	0.00	-4.00	1.00	6.00
SU	A	BASIC	0.00	-1.00	0.00	0.00
SU	B	BASIC	0.00	0.00	0.00	0.00
SU	C	BASIC	0.00	0.00	0.00	1.00
SU	EX	BASIC	0.00	5.00	10.00	999999.00
T	X	LOWER	1.00	0.00	1.00	999999.00
T	Y	BASIC	0.00	0.00	1.00	1.00
U	Y	BASIC	0.00	1.00	1.00	2.00
V	U	BASIC	0.00	1.00	1.00	2.00
V	Z	BASIC	0.00	-3.00	1.00	7.00
W	DE	LOWER	5.00	-5.00	0.00	999999.00
X	DE	LOWER	4.00	-4.00	0.00	999999.00
Y	DE	LOWER	5.00	-5.00	0.00	999999.00
Z	DE	LOWER	4.00	-4.00	0.00	999999.00
ZZ	DE	LOWER	5.00	-5.00	0.00	999999.00

file: thes24

FROM	TO	COST	LOWER	UPPER
A	D	1.00	0.00	9.60
A	E	1.00	0.00	16.00
A	F	1.00	0.00	32.00
B	D	1.00	0.00	9.60
C	D	1.00	0.00	9.60
C	J	1.00	0.00	32.00
D	H	1.00	0.00	32.00
DE	SU	0.00	0.00	999999.00
E	L	1.00	0.00	2.40
E	P	1.00	0.00	16.00
EX	W	0.00	0.00	999999.00
EX	X	0.00	0.00	999999.00
EX	Y	0.00	0.00	999999.00
EX	Z	0.00	0.00	999999.00
EX	ZZ	0.00	0.00	999999.00
F	G	1.00	0.00	9.60
F	L	1.00	0.00	9.60
G	H	1.00	0.00	2.40
G	Q	1.00	0.00	9.60
H	I	1.00	0.00	9.60
H	M	1.00	0.00	9.60
I	N	1.00	0.00	32.00
J	I	1.00	0.00	9.60
J	K	1.00	0.00	9.60
J	O	1.00	0.00	9.60
K	O	1.00	0.00	2.40
K	V	1.00	0.00	2.40
L	P	1.00	0.00	9.60
M	Q	1.00	0.00	2.40
M	X	1.00	0.00	9.60
N	O	1.00	0.00	9.60
N	T	1.00	0.00	9.60
N	X	1.00	0.00	9.60
O	V	1.00	0.00	9.60
P	R	1.00	0.00	16.00
Q	S	1.00	0.00	9.60
R	W	1.00	0.00	16.00
S	W	1.00	0.00	9.60
S	ZZ	1.00	0.00	9.60
SU	A	0.00	0.00	999999.00
SU	B	0.00	0.00	999999.00
SU	C	0.00	0.00	999999.00
SU	EX	10.00	0.00	999999.00
T	X	1.00	0.00	2.40
T	Y	1.00	0.00	9.60
U	Y	1.00	0.00	9.60
V	U	1.00	0.00	9.60
V	Z	1.00	0.00	9.60
W	DE	0.00	14.00	999999.00
X	DE	0.00	12.00	999999.00
Y	DE	0.00	8.00	999999.00
Z	DE	0.00	9.00	999999.00
ZZ	DE	0.00	8.00	999999.00

MINIMUM COST FLOW PROBLEM: MINIMUM COST IS   220.00

FROM	TO	LOWER	FLOW	UPPER	COST
SU	A	0.00	31.60	999999.00	0.00
SU	C	0.00	19.40	999999.00	0.00
A	D	0.00	9.60	9.60	1.00
W	DE	14.00	14.00	999999.00	0.00
X	DE	12.00	12.00	999999.00	0.00
Y	DE	8.00	8.00	999999.00	0.00
Z	DE	9.00	9.00	999999.00	0.00
ZZ	DE	8.00	8.00	999999.00	0.00
A	E	0.00	14.00	16.00	1.00
A	F	0.00	8.00	32.00	1.00
F	G	0.00	8.00	9.60	1.00
D	H	0.00	9.60	32.00	1.00
J	I	0.00	9.60	9.60	1.00
C	J	0.00	19.40	32.00	1.00
J	K	0.00	2.40	9.60	1.00
H	M	0.00	9.60	9.60	1.00
I	N	0.00	9.60	32.00	1.00
J	O	0.00	7.40	9.60	1.00
E	P	0.00	14.00	16.00	1.00
G	Q	0.00	8.00	9.60	1.00
P	R	0.00	14.00	16.00	1.00
Q	S	0.00	8.00	9.60	1.00
DE	SU	0.00	51.00	999999.00	0.00
N	T	0.00	7.20	9.60	1.00
V	U	0.00	0.80	9.60	1.00
K	V	0.00	2.40	2.40	1.00
O	V	0.00	7.40	9.60	1.00
R	W	0.00	14.00	16.00	1.00
M	X	0.00	9.60	9.60	1.00
N	X	0.00	2.40	9.60	1.00
T	Y	0.00	7.20	9.60	1.00
U	Y	0.00	0.80	9.60	1.00
V	Z	0.00	9.00	9.60	1.00
S	ZZ	0.00	8.00	9.60	1.00

FROM	TO	EDGE STATE	REDUCED COST	COST RANGE LOWER	CURRENT	UPPER
A	D	UPPER	0.00	-999999.00	1.00	1.00
A	E	BASIC	0.00	1.00	1.00	2.00
A	F	BASIC	0.00	0.00	1.00	1.00
B	D	BASIC	0.00	1.00	1.00	999999.00
C	D	LOWER	0.00	1.00	1.00	999999.00
C	J	BASIC	0.00	1.00	1.00	2.00
D	H	BASIC	0.00	0.00	1.00	1.00
DE	SU	BASIC	0.00	-4.00	0.00	999999.00
E	L	LOWER	0.00	1.00	1.00	999999.00
E	P	BASIC	0.00	-3.00	1.00	2.00
EX	W	LOWER	6.00	-6.00	0.00	999999.00
EX	X	LOWER	6.00	-6.00	0.00	999999.00
EX	Y	LOWER	5.00	-5.00	0.00	999999.00
EX	Z	LOWER	6.00	-6.00	0.00	999999.00
EX	ZZ	LOWER	5.00	-5.00	0.00	999999.00
F	G	BASIC	0.00	0.00	1.00	2.00
F	L	BASIC	0.00	0.00	1.00	1.00
G	H	LOWER	1.00	0.00	1.00	999999.00
G	Q	BASIC	0.00	0.00	1.00	2.00
H	I	LOWER	1.00	0.00	1.00	999999.00
H	M	UPPER	0.00	-999999.00	1.00	1.00
I	N	BASIC	0.00	0.00	1.00	1.00
J	I	UPPER	0.00	-999999.00	1.00	1.00
J	K	BASIC	0.00	0.00	1.00	1.00
J	O	BASIC	0.00	1.00	1.00	2.00
K	O	LOWER	1.00	0.00	1.00	999999.00
K	V	UPPER	0.00	-999999.00	1.00	1.00
L	P	LOWER	1.00	0.00	1.00	999999.00
M	Q	LOWER	1.00	0.00	1.00	999999.00
M	X	BASIC	0.00	-999999.00	1.00	1.00
N	O	LOWER	2.00	-1.00	1.00	999999.00
N	T	BASIC	0.00	0.00	1.00	1.00
N	X	BASIC	0.00	1.00	1.00	2.00
O	V	BASIC	0.00	1.00	1.00	2.00
P	R	BASIC	0.00	-3.00	1.00	2.00
Q	S	BASIC	0.00	0.00	1.00	6.00
R	W	BASIC	0.00	-3.00	1.00	2.00
S	W	LOWER	1.00	0.00	1.00	999999.00
S	ZZ	BASIC	0.00	-4.00	1.00	6.00
SU	A	BASIC	0.00	-1.00	0.00	0.00
SU	B	BASIC	0.00	0.00	0.00	0.00
SU	C	BASIC	0.00	0.00	0.00	1.00
SU	EX	BASIC	0.00	5.00	10.00	999999.00
T	X	LOWER	1.00	0.00	1.00	999999.00
T	Y	BASIC	0.00	0.00	1.00	1.00
U	Y	BASIC	0.00	1.00	1.00	2.00
V	U	BASIC	0.00	1.00	1.00	2.00
V	Z	BASIC	0.00	-3.00	1.00	7.00
W	DE	LOWER	4.00	-4.00	0.00	999999.00
X	DE	LOWER	4.00	-4.00	0.00	999999.00
Y	DE	LOWER	5.00	-5.00	0.00	999999.00
Z	DE	LOWER	4.00	-4.00	0.00	999999.00
ZZ	DE	LOWER	5.00	-5.00	0.00	999999.00

Incremental method for bottleneck identification

file: thesis   (baseline system)

FROM	TO	COST	LOWER	UPPER
A	D	1.00	0.00	9.60
A	E	1.00	0.00	9.60
A	F	1.00	0.00	9.60
B	D	1.00	0.00	9.60
C	D	1.00	0.00	9.60
C	J	1.00	0.00	9.60
D	H	1.00	0.00	9.60
DE	SU	0.00	0.00	999999.00
E	L	1.00	0.00	2.40
E	P	1.00	0.00	2.40
EX	W	0.00	0.00	999999.00
EX	X	0.00	0.00	999999.00
EX	Y	0.00	0.00	999999.00
EX	Z	0.00	0.00	999999.00
EX	ZZ	0.00	0.00	999999.00
F	G	1.00	0.00	9.60
F	L	1.00	0.00	9.60
G	H	1.00	0.00	2.40
G	Q	1.00	0.00	2.40
H	I	1.00	0.00	9.60
H	M	1.00	0.00	9.60
I	N	1.00	0.00	9.60
J	I	1.00	0.00	9.60
J	K	1.00	0.00	9.60
J	O	1.00	0.00	9.60
K	O	1.00	0.00	2.40
K	V	1.00	0.00	2.40
L	P	1.00	0.00	9.60
M	Q	1.00	0.00	2.40
M	X	1.00	0.00	2.40
N	O	1.00	0.00	9.60
N	T	1.00	0.00	9.60
N	X	1.00	0.00	9.60
O	V	1.00	0.00	9.60
P	R	1.00	0.00	9.60
Q	S	1.00	0.00	9.60
R	W	1.00	0.00	9.60
S	W	1.00	0.00	9.60
S	ZZ	1.00	0.00	9.60
SU	A	0.00	0.00	999999.00
SU	B	0.00	0.00	999999.00
SU	C	0.00	0.00	999999.00
SU	EX	10.00	0.00	999999.00
T	X	1.00	0.00	2.40
T	Y	1.00	0.00	2.40
U	Y	1.00	0.00	9.60
V	U	1.00	0.00	9.60
V	Z	1.00	0.00	9.60
W	DE	0.00	14.00	999999.00
X	DE	0.00	12.00	999999.00
Y	DE	0.00	8.00	999999.00
Z	DE	0.00	9.00	999999.00
ZZ	DE	0.00	8.00	999999.00

MINIMUM COST FLOW PROBLEM: MINIMUM COST IS   330.60

FROM	TO	LOWER	FLOW	UPPER	COST
SU	A	0.00	24.00	999999.00	0.00
SU	C	0.00	9.60	999999.00	0.00
A	D	0.00	9.60	9.60	1.00
W	DE	14.00	14.00	999999.00	0.00
X	DE	12.00	12.00	999999.00	0.00
Y	DE	8.00	8.00	999999.00	0.00
Z	DE	9.00	9.00	999999.00	0.00
ZZ	DE	8.00	8.00	999999.00	0.00
A	E	0.00	4.80	9.60	1.00
SU	EX	0.00	17.40	999999.00	10.00
A	F	0.00	9.60	9.60	1.00
F	G	0.00	4.80	9.60	1.00
D	H	0.00	9.60	9.60	1.00
G	H	0.00	2.40	2.40	1.00
H	I	0.00	9.00	9.60	1.00
J	I	0.00	0.60	9.60	1.00
C	J	0.00	9.60	9.60	1.00
J	K	0.00	2.40	9.60	1.00
E	L	0.00	2.40	2.40	1.00
F	L	0.00	4.80	9.60	1.00
H	M	0.00	3.00	9.60	1.00
I	N	0.00	9.60	9.60	1.00
J	O	0.00	6.60	9.60	1.00
E	P	0.00	2.40	2.40	1.00
L	P	0.00	7.20	9.60	1.00
G	Q	0.00	2.40	2.40	1.00
M	Q	0.00	0.60	2.40	1.00
P	R	0.00	9.60	9.60	1.00
Q	S	0.00	3.00	9.60	1.00
DE	SU	0.00	51.00	999999.00	0.00
K	V	0.00	2.40	2.40	1.00
O	V	0.00	6.60	9.60	1.00
EX	W	0.00	4.40	999999.00	0.00
R	W	0.00	9.60	9.60	1.00
M	X	0.00	2.40	2.40	1.00
N	X	0.00	9.60	9.60	1.00
EX	Y	0.00	8.00	999999.00	0.00
V	Z	0.00	9.00	9.60	1.00
EX	ZZ	0.00	5.00	999999.00	0.00
S	ZZ	0.00	3.00	9.60	1.00

FROM	TO	EDGE STATE	REDUCED COST	COST RANGE LOWER	CURRENT	UPPER
A	D	BASIC	0.00	-999999.00	1.00	1.00
A	E	BASIC	0.00	-999999.00	1.00	4.00
A	F	UPPER	-3.00	-999999.00	1.00	4.00
B	D	LOWER	0.00	1.00	1.00	999999.00
C	D	LOWER	0.00	1.00	1.00	999999.00
C	J	UPPER	-5.00	-999999.00	1.00	6.00
D	H	UPPER	-4.00	-999999.00	1.00	5.00
DE	SU	BASIC	0.00	-9.00	0.00	999999.00
E	L	UPPER	-3.00	-999999.00	1.00	4.00
E	P	UPPER	-4.00	-999999.00	1.00	5.00
EX	W	BASIC	0.00	-2.00	0.00	0.00
EX	X	BASIC	0.00	-1.00	0.00	999999.00
EX	Y	BASIC	0.00	-10.00	0.00	0.00
EX	Z	LOWER	1.00	-1.00	0.00	999999.00
EX	ZZ	BASIC	0.00	0.00	0.00	1.00
F	G	BASIC	0.00	-1.00	1.00	4.00
F	L	BASIC	0.00	-2.00	1.00	3.00
G	H	BASIC	0.00	-1.00	1.00	4.00
G	Q	UPPER	-2.00	-999999.00	1.00	3.00
H	I	BASIC	0.00	1.00	1.00	2.00
H	M	BASIC	0.00	0.00	1.00	1.00
I	N	UPPER	-1.00	-999999.00	1.00	2.00
J	I	BASIC	0.00	0.00	1.00	1.00
J	K	BASIC	0.00	0.00	1.00	1.00
J	O	BASIC	0.00	1.00	1.00	2.00
K	O	LOWER	1.00	0.00	1.00	999999.00
K	V	UPPER	0.00	-999999.00	1.00	1.00
L	P	BASIC	0.00	-3.00	1.00	3.00
M	Q	BASIC	0.00	0.00	1.00	1.00
M	X	UPPER	-2.00	-999999.00	1.00	3.00
N	O	LOWER	3.00	-2.00	1.00	999999.00
N	T	BASIC	0.00	0.00	1.00	999999.00
N	X	BASIC	0.00	-999999.00	1.00	2.00
O	V	BASIC	0.00	1.00	1.00	2.00
P	R	UPPER	-2.00	-999999.00	1.00	3.00
Q	S	BASIC	0.00	0.00	1.00	1.00
R	W	BASIC	0.00	-999999.00	1.00	3.00
S	W	LOWER	0.00	1.00	1.00	999999.00
S	ZZ	BASIC	0.00	0.00	1.00	1.00
SU	A	BASIC	0.00	-999999.00	0.00	0.00
SU	B	BASIC	0.00	0.00	0.00	999999.00
SU	C	BASIC	0.00	0.00	0.00	5.00
SU	EX	BASIC	0.00	7.00	10.00	999999.00
T	X	LOWER	1.00	0.00	1.00	999999.00
T	Y	LOWER	1.00	0.00	1.00	999999.00
U	Y	LOWER	0.00	1.00	1.00	999999.00
V	U	BASIC	0.00	1.00	1.00	999999.00
V	Z	BASIC	0.00	-8.00	1.00	2.00
W	DE	LOWER	10.00	-10.00	0.00	999999.00
X	DE	LOWER	10.00	-10.00	0.00	999999.00
Y	DE	LOWER	10.00	-10.00	0.00	999999.00
Z	DE	LOWER	9.00	-9.00	0.00	999999.00
ZZ	DE	LOWER	10.00	-10.00	0.00	999999.00

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file:   what0   (all bottlenecks expanded one discrete increment)
   FROM     TO            COST        LOWER         UPPER
   ----     --         --------     --------      --------
     A       D            1.00         0.00          9.60
     A       E            1.00         0.00          9.60
     A       F            1.00         0.00         16.00
     B       D            1.00         0.00          9.60
     C       D            1.00         0.00          9.60
     C       J            1.00         0.00         16.00
     D       H            1.00         0.00         16.00
    DE      SU            0.00         0.00     999999.00
     E       L            1.00         0.00          9.60
     E       P            1.00         0.00          9.60
    EX       W            0.00         0.00     999999.00
    EX       X            0.00         0.00     999999.00
    EX       Y            0.00         0.00     999999.00
    EX       Z            0.00         0.00     999999.00
    EX      ZZ            0.00         0.00     999999.00
     F       G            1.00         0.00          9.60
     F       L            1.00         0.00          9.60
     G       H            1.00         0.00          2.40
     G       Q            1.00         0.00          9.60
     H       I            1.00         0.00          9.60
     H       M            1.00         0.00          9.60
     I       N            1.00         0.00         16.00
     J       I            1.00         0.00          9.60
     J       K            1.00         0.00          9.60
     J       O            1.00         0.00          9.60
     K       O            1.00         0.00          2.40
     K       V            1.00         0.00          2.40
     L       P            1.00         0.00          9.60
     M       Q            1.00         0.00          2.40
     M       X            1.00         0.00          9.60
     N       O            1.00         0.00          9.60
     N       T            1.00         0.00          9.60
     N       X            1.00         0.00          9.60
     O       V            1.00         0.00          9.60
     P       R            1.00         0.00         16.00
     Q       S            1.00         0.00          9.60
     R       W            1.00         0.00          9.60
     S       W            1.00         0.00          9.60
     S      ZZ            1.00         0.00          9.60
    SU       A            0.00         0.00     999999.00
    SU       B            0.00         0.00     999999.00
    SU       C            0.00         0.00     999999.00
    SU      EX           10.00         0.00     999999.00
     T       X            1.00         0.00          2.40
     T       Y            1.00         0.00          2.40
     U       Y            1.00         0.00          9.60
     V       U            1.00         0.00          9.60
     V       Z            1.00         0.00          9.60
     W      DE            0.00        14.00     999999.00
     X      DE            0.00        12.00     999999.00
     Y      DE            0.00         8.00     999999.00
     Z      DE            0.00         9.00     999999.00
    ZZ      DE            0.00         8.00     999999.00
```

MINIMUM COST FLOW PROBLEM: MINIMUM COST IS   252.20

FROM	TO	LOWER	FLOW	UPPER	COST
SU	A	0.00	28.80	999999.00	0.00
SU	B	0.00	0.80	999999.00	0.00
SU	C	0.00	16.00	999999.00	0.00
A	D	0.00	9.60	9.60	1.00
B	D	0.00	0.80	9.60	1.00
W	DE	14.00	14.00	999999.00	0.00
X	DE	12.00	12.00	999999.00	0.00
Y	DE	8.00	8.00	999999.00	0.00
Z	DE	9.00	9.00	999999.00	0.00
ZZ	DE	8.00	8.00	999999.00	0.00
A	E	0.00	9.60	9.60	1.00
SU	EX	0.00	5.40	999999.00	10.00
A	F	0.00	9.60	16.00	1.00
F	G	0.00	9.60	9.60	1.00
D	H	0.00	10.40	16.00	1.00
H	I	0.00	0.80	9.60	1.00
J	I	0.00	4.00	9.60	1.00
C	J	0.00	16.00	16.00	1.00
J	K	0.00	2.40	9.60	1.00
H	M	0.00	9.60	9.60	1.00
I	N	0.00	4.80	16.00	1.00
J	O	0.00	9.60	9.60	1.00
E	P	0.00	9.60	9.60	1.00
G	Q	0.00	9.60	9.60	1.00
P	R	0.00	9.60	16.00	1.00
Q	S	0.00	9.60	9.60	1.00
DE	SU	0.00	51.00	999999.00	0.00
N	T	0.00	2.40	9.60	1.00
V	U	0.00	3.00	9.60	1.00
K	V	0.00	2.40	2.40	1.00
O	V	0.00	9.60	9.60	1.00
EX	W	0.00	2.80	999999.00	0.00
R	W	0.00	9.60	9.60	1.00
S	W	0.00	1.60	9.60	1.00
M	X	0.00	9.60	9.60	1.00
N	X	0.00	2.40	9.60	1.00
EX	Y	0.00	2.60	999999.00	0.00
T	Y	0.00	2.40	2.40	1.00
U	Y	0.00	3.00	9.60	1.00
V	Z	0.00	9.00	9.60	1.00
S	ZZ	0.00	8.00	9.60	1.00

FROM	TO	EDGE STATE	REDUCED COST	LOWER	COST RANGE CURRENT	UPPER
A	D	UPPER	0.00	-999999.00	1.00	1.00
A	E	BASIC	0.00	1.00	1.00	2.00
A	F	BASIC	0.00	0.00	1.00	1.00
B	D	BASIC	0.00	1.00	1.00	1.00
C	D	LOWER	0.00	1.00	1.00	999999.00
C	J	UPPER	-1.00	-999999.00	1.00	2.00
D	H	BASIC	0.00	0.00	1.00	2.00
DE	SU	BASIC	0.00	-5.00	0.00	999999.00
E	L	LOWER	0.00	1.00	1.00	999999.00
E	P	BASIC	0.00	-999999.00	1.00	2.00
EX	W	BASIC	0.00	-5.00	0.00	0.00
EX	X	LOWER	5.00	-5.00	0.00	999999.00
EX	Y	BASIC	0.00	-3.00	0.00	1.00
EX	Z	LOWER	1.00	-1.00	0.00	999999.00
EX	ZZ	LOWER	0.00	0.00	0.00	999999.00
F	G	BASIC	0.00	0.00	1.00	3.00
F	L	BASIC	0.00	0.00	1.00	1.00
G	H	LOWER	1.00	0.00	1.00	999999.00
G	Q	BASIC	0.00	-999999.00	1.00	3.00
H	I	BASIC	0.00	0.00	1.00	4.00
H	M	UPPER	-1.00	-999999.00	1.00	2.00
I	N	BASIC	0.00	0.00	1.00	5.00
J	I	BASIC	0.00	0.00	1.00	2.00
J	K	BASIC	0.00	0.00	1.00	2.00
J	O	UPPER	-1.00	-999999.00	1.00	2.00
K	O	BASIC	0.00	0.00	1.00	2.00
K	V	UPPER	-4.00	-999999.00	1.00	5.00
L	P	LOWER	1.00	0.00	1.00	999999.00
M	Q	LOWER	2.00	-1.00	1.00	999999.00
M	X	BASIC	0.00	-999999.00	1.00	2.00
N	O	LOWER	1.00	0.00	1.00	999999.00
N	T	BASIC	0.00	0.00	1.00	5.00
N	X	BASIC	0.00	0.00	1.00	2.00
O	V	UPPER	-3.00	-999999.00	1.00	4.00
P	R	BASIC	0.00	-999999.00	1.00	7.00
Q	S	UPPER	-5.00	-999999.00	1.00	6.00
R	W	UPPER	-6.00	-999999.00	1.00	7.00
S	W	BASIC	0.00	1.00	1.00	6.00
S	ZZ	BASIC	0.00	-9.00	1.00	1.00
SU	A	BASIC	0.00	-1.00	0.00	0.00
SU	B	BASIC	0.00	0.00	0.00	0.00
SU	C	BASIC	0.00	0.00	0.00	1.00
SU	EX	BASIC	0.00	7.00	10.00	999999.00
T	X	LOWER	1.00	0.00	1.00	999999.00
T	Y	UPPER	-4.00	-999999.00	1.00	5.00
U	Y	BASIC	0.00	0.00	1.00	4.00
V	U	BASIC	0.00	0.00	1.00	4.00
V	Z	BASIC	0.00	-8.00	1.00	2.00
W	DE	LOWER	10.00	-10.00	0.00	999999.00
X	DE	LOWER	5.00	-5.00	0.00	999999.00
Y	DE	LOWER	10.00	-10.00	0.00	999999.00
Z	DE	LOWER	9.00	-9.00	0.00	999999.00
ZZ	DE	LOWER	10.00	-10.00	0.00	999999.00

Iteration 1

```
file:   incr1   (bottleneck upgrades if utilized in previous flow)
  FROM    TO         COST        LOWER        UPPER
  ----    --       --------     --------     --------
   A      D          1.00         0.00          9.60
   A      E          1.00         0.00          9.60
   A      F          1.00         0.00          9.60
   B      D          1.00         0.00          9.60
   C      D          1.00         0.00          9.60
   C      J          1.00         0.00         16.00
   D      H          1.00         0.00         16.00
   DE     SU         0.00         0.00     999999.00
   E      L          1.00         0.00          2.40
   E      P          1.00         0.00          9.60
   EX     W          0.00         0.00     999999.00
   EX     X          0.00         0.00     999999.00
   EX     Y          0.00         0.00     999999.00
   EX     Z          0.00         0.00     999999.00
   EX     ZZ         0.00         0.00     999999.00
   F      G          1.00         0.00          9.60
   F      L          1.00         0.00          9.60
   G      H          1.00         0.00          2.40
   G      Q          1.00         0.00          9.60
   H      I          1.00         0.00          9.60
   H      M          1.00         0.00          9.60
   I      N          1.00         0.00          9.60
   J      I          1.00         0.00          9.60
   J      K          1.00         0.00          9.60
   J      O          1.00         0.00          9.60
   K      O          1.00         0.00          2.40
   K      V          1.00         0.00          2.40
   L      P          1.00         0.00          9.60
   M      Q          1.00         0.00          2.40
   M      X          1.00         0.00          9.60
   N      O          1.00         0.00          9.60
   N      T          1.00         0.00          9.60
   N      X          1.00         0.00          9.60
   O      V          1.00         0.00          9.60
   P      R          1.00         0.00          9.60
   Q      S          1.00         0.00          9.60
   R      W          1.00         0.00          9.60
   S      W          1.00         0.00          9.60
   S      ZZ         1.00         0.00          9.60
   SU     A          0.00         0.00     999999.00
   SU     B          0.00         0.00     999999.00
   SU     C          0.00         0.00     999999.00
   SU     EX        10.00         0.00     999999.00
   T      X          1.00         0.00          2.40
   T      Y          1.00         0.00          2.40
   U      Y          1.00         0.00          9.60
   V      U          1.00         0.00          9.60
   V      Z          1.00         0.00          9.60
   W      DE         0.00        14.00     999999.00
   X      DE         0.00        12.00     999999.00
   Y      DE         0.00         8.00     999999.00
   Z      DE         0.00         9.00     999999.00
   ZZ     DE         0.00         8.00     999999.00
```

MINIMUM COST FLOW PROBLEM: MINIMUM COST IS   252.20

FROM	TO	LOWER	FLOW	UPPER	COST
SU	A	0.00	28.80	999999.00	0.00
SU	B	0.00	0.80	999999.00	0.00
SU	C	0.00	16.00	999999.00	0.00
A	D	0.00	9.60	9.60	1.00
B	D	0.00	0.80	9.60	1.00
W	DE	14.00	14.00	999999.00	0.00
X	DE	12.00	12.00	999999.00	0.00
Y	DE	8.00	8.00	999999.00	0.00
Z	DE	9.00	9.00	999999.00	0.00
ZZ	DE	8.00	8.00	999999.00	0.00
A	E	0.00	9.60	9.60	1.00
SU	EX	0.00	5.40	999999.00	10.00
A	F	0.00	9.60	9.60	1.00
F	G	0.00	9.60	9.60	1.00
D	H	0.00	10.40	16.00	1.00
H	I	0.00	0.80	9.60	1.00
J	I	0.00	4.00	9.60	1.00
C	J	0.00	16.00	16.00	1.00
J	K	0.00	2.40	9.60	1.00
H	M	0.00	9.60	9.60	1.00
I	N	0.00	4.80	9.60	1.00
J	O	0.00	9.60	9.60	1.00
E	P	0.00	9.60	9.60	1.00
G	Q	0.00	9.60	9.60	1.00
P	R	0.00	9.60	9.60	1.00
Q	S	0.00	9.60	9.60	1.00
DE	SU	0.00	51.00	999999.00	0.00
N	T	0.00	2.40	9.60	1.00
V	U	0.00	3.00	9.60	1.00
K	V	0.00	2.40	2.40	1.00
O	V	0.00	9.60	9.60	1.00
EX	W	0.00	2.80	999999.00	0.00
R	W	0.00	9.60	9.60	1.00
S	W	0.00	1.60	9.60	1.00
M	X	0.00	9.60	9.60	1.00
N	X	0.00	2.40	9.60	1.00
EX	Y	0.00	2.60	999999.00	0.00
T	Y	0.00	2.40	2.40	1.00
U	Y	0.00	3.00	9.60	1.00
V	Z	0.00	9.00	9.60	1.00
S	ZZ	0.00	8.00	9.60	1.00

FROM	TO	EDGE STATE	REDUCED COST	COST RANGE LOWER	COST RANGE CURRENT	COST RANGE UPPER
A	D	UPPER	0.00	-999999.00	1.00	1.00
A	E	BASIC	0.00	1.00	1.00	2.00
A	F	BASIC	0.00	0.00	1.00	1.00
B	D	BASIC	0.00	1.00	1.00	1.00
C	D	LOWER	0.00	1.00	1.00	999999.00
C	J	UPPER	-1.00	-999999.00	1.00	2.00
D	H	BASIC	0.00	0.00	1.00	2.00
DE	SU	BASIC	0.00	-5.00	0.00	999999.00
E	L	LOWER	0.00	1.00	1.00	999999.00
E	P	BASIC	0.00	-999999.00	1.00	2.00
EX	W	BASIC	0.00	-5.00	0.00	0.00
EX	X	LOWER	5.00	-5.00	0.00	999999.00
EX	Y	BASIC	0.00	-3.00	0.00	1.00
EX	Z	LOWER	1.00	-1.00	0.00	999999.00
EX	ZZ	LOWER	0.00	0.00	0.00	999999.00
F	G	BASIC	0.00	0.00	1.00	3.00
F	L	BASIC	0.00	0.00	1.00	1.00
G	H	LOWER	1.00	0.00	1.00	999999.00
G	Q	BASIC	0.00	-999999.00	1.00	3.00
H	I	BASIC	0.00	0.00	1.00	4.00
H	M	UPPER	-1.00	-999999.00	1.00	2.00
I	N	BASIC	0.00	0.00	1.00	5.00
J	I	BASIC	0.00	0.00	1.00	2.00
J	K	BASIC	0.00	0.00	1.00	2.00
J	O	UPPER	-1.00	-999999.00	1.00	2.00
K	O	BASIC	0.00	0.00	1.00	2.00
K	V	UPPER	-4.00	-999999.00	1.00	5.00
L	P	LOWER	1.00	0.00	1.00	999999.00
M	Q	LOWER	2.00	-1.00	1.00	999999.00
M	X	BASIC	0.00	-999999.00	1.00	2.00
N	O	LOWER	1.00	0.00	1.00	999999.00
N	T	BASIC	0.00	0.00	1.00	5.00
N	X	BASIC	0.00	0.00	1.00	2.00
O	V	UPPER	-3.00	-999999.00	1.00	4.00
P	R	BASIC	0.00	-999999.00	1.00	7.00
Q	S	UPPER	-5.00	-999999.00	1.00	6.00
R	W	UPPER	-6.00	-999999.00	1.00	7.00
S	W	BASIC	0.00	1.00	1.00	6.00
S	ZZ	BASIC	0.00	-9.00	1.00	1.00
SU	A	BASIC	0.00	-1.00	0.00	0.00
SU	B	BASIC	0.00	0.00	0.00	0.00
SU	C	BASIC	0.00	0.00	0.00	1.00
SU	EX	BASIC	0.00	7.00	10.00	999999.00
T	X	LOWER	1.00	0.00	1.00	999999.00
T	Y	UPPER	-4.00	-999999.00	1.00	5.00
U	Y	BASIC	0.00	0.00	1.00	4.00
V	U	BASIC	0.00	0.00	1.00	4.00
V	Z	BASIC	0.00	-8.00	1.00	2.00
W	DE	LOWER	10.00	-10.00	0.00	999999.00
X	DE	LOWER	5.00	-5.00	0.00	999999.00
Y	DE	LOWER	10.00	-10.00	0.00	999999.00
Z	DE	LOWER	9.00	-9.00	0.00	999999.00
ZZ	DE	LOWER	10.00	-10.00	0.00	999999.00

```
file:  what1        (all bottlenecks expanded one discrete increment)
    FROM    TO          COST       LOWER       UPPER
    ----    --        --------   --------    --------
      A      D           1.00       0.00        9.60
      A      E           1.00       0.00        9.60
      A      F           1.00       0.00        9.60
      B      D           1.00       0.00        9.60
      C      D           1.00       0.00        9.60
      C      J           1.00       0.00       32.00
      D      H           1.00       0.00       16.00
     DE     SU           0.00       0.00   999999.00
      E      L           1.00       0.00        2.40
      E      P           1.00       0.00        9.60
     EX      W           0.00       0.00   999999.00
     EX      X           0.00       0.00   999999.00
     EX      Y           0.00       0.00   999999.00
     EX      Z           0.00       0.00   999999.00
     EX     ZZ           0.00       0.00   999999.00
      F      G           1.00       0.00        9.60
      F      L           1.00       0.00        9.60
      G      H           1.00       0.00        2.40
      G      Q           1.00       0.00        9.60
      H      I           1.00       0.00        9.60
      H      M           1.00       0.00       16.00
      I      N           1.00       0.00        9.60
      J      I           1.00       0.00        9.60
      J      K           1.00       0.00        9.60
      J      O           1.00       0.00       16.00
      K      O           1.00       0.00        2.40
      K      V           1.00       0.00        9.60
      L      P           1.00       0.00        9.60
      M      Q           1.00       0.00        2.40
      M      X           1.00       0.00        9.60
      N      O           1.00       0.00        9.60
      N      T           1.00       0.00        9.60
      N      X           1.00       0.00        9.60
      O      V           1.00       0.00       16.00
      P      R           1.00       0.00        9.60
      Q      S           1.00       0.00       16.00
      R      W           1.00       0.00       16.00
      S      W           1.00       0.00        9.60
      S     ZZ           1.00       0.00        9.60
     SU      A           0.00       0.00   999999.00
     SU      B           0.00       0.00   999999.00
     SU      C           0.00       0.00   999999.00
     SU     EX          10.00       0.00   999999.00
      T      X           1.00       0.00        2.40
      T      Y           1.00       0.00        9.60
      U      Y           1.00       0.00        9.60
      V      U           1.00       0.00        9.60
      V      Z           1.00       0.00        9.60
      W     DE           0.00      14.00   999999.00
      X     DE           0.00      12.00   999999.00
      Y     DE           0.00       8.00   999999.00
      Z     DE           0.00       9.00   999999.00
     ZZ     DE           0.00       8.00   999999.00
```

MINIMUM COST FLOW PROBLEM: MINIMUM COST IS   228.80

FROM	TO	LOWER	FLOW	UPPER	COST
SU	A	0.00	28.80	999999.00	0.00
SU	B	0.00	2.40	999999.00	0.00
SU	C	0.00	19.40	999999.00	0.00
A	D	0.00	9.60	9.60	1.00
B	D	0.00	2.40	9.60	1.00
W	DE	14.00	14.00	999999.00	0.00
X	DE	12.00	12.00	999999.00	0.00
Y	DE	8.00	8.00	999999.00	0.00
Z	DE	9.00	9.00	999999.00	0.00
ZZ	DE	8.00	8.00	999999.00	0.00
A	E	0.00	9.60	9.60	1.00
SU	EX	0.00	0.40	999999.00	10.00
A	F	0.00	9.60	9.60	1.00
F	G	0.00	9.60	9.60	1.00
D	H	0.00	12.00	16.00	1.00
J	I	0.00	9.60	9.60	1.00
C	J	0.00	19.40	32.00	1.00
J	K	0.00	9.60	9.60	1.00
H	M	0.00	12.00	16.00	1.00
I	N	0.00	9.60	9.60	1.00
J	O	0.00	0.20	16.00	1.00
E	P	0.00	9.60	9.60	1.00
G	Q	0.00	9.60	9.60	1.00
M	Q	0.00	2.40	2.40	1.00
P	R	0.00	9.60	9.60	1.00
Q	S	0.00	12.00	16.00	1.00
DE	SU	0.00	51.00	999999.00	0.00
N	T	0.00	7.20	9.60	1.00
V	U	0.00	0.80	9.60	1.00
K	V	0.00	9.60	9.60	1.00
O	V	0.00	0.20	16.00	1.00
EX	W	0.00	0.40	999999.00	0.00
R	W	0.00	9.60	16.00	1.00
S	W	0.00	4.00	9.60	1.00
M	X	0.00	9.60	9.60	1.00
N	X	0.00	2.40	9.60	1.00
T	Y	0.00	7.20	9.60	1.00
U	Y	0.00	0.80	9.60	1.00
V	Z	0.00	9.00	9.60	1.00
S	ZZ	0.00	8.00	9.60	1.00

FROM	TO	EDGE STATE	REDUCED COST	LOWER	COST RANGE CURRENT	UPPER
A	D	UPPER	0.00	-999999.00	1.00	1.00
A	E	BASIC	0.00	1.00	1.00	2.00
A	F	BASIC	0.00	0.00	1.00	1.00
B	D	BASIC	0.00	1.00	1.00	1.00
C	D	LOWER	0.00	1.00	1.00	999999.00
C	J	BASIC	0.00	1.00	1.00	2.00
D	H	BASIC	0.00	0.00	1.00	1.00
DE	SU	BASIC	0.00	-4.00	0.00	999999.00
E	L	LOWER	0.00	1.00	1.00	999999.00
E	P	BASIC	0.00	-999999.00	1.00	2.00
EX	W	BASIC	0.00	-4.00	0.00	0.00
EX	X	LOWER	6.00	-6.00	0.00	999999.00
EX	Y	LOWER	5.00	-5.00	0.00	999999.00
EX	Z	LOWER	6.00	-6.00	0.00	999999.00
EX	ZZ	LOWER	0.00	0.00	0.00	999999.00
F	G	BASIC	0.00	0.00	1.00	6.00
F	L	BASIC	0.00	0.00	1.00	1.00
G	H	LOWER	1.00	0.00	1.00	999999.00
G	Q	UPPER	-5.00	-999999.00	1.00	6.00
H	I	LOWER	1.00	0.00	1.00	999999.00
H	M	BASIC	0.00	-999999.00	1.00	1.00
I	N	UPPER	0.00	-999999.00	1.00	1.00
J	I	BASIC	0.00	-999999.00	1.00	1.00
J	K	BASIC	0.00	0.00	1.00	1.00
J	O	BASIC	0.00	1.00	1.00	2.00
K	O	LOWER	1.00	0.00	1.00	999999.00
K	V	UPPER	0.00	-999999.00	1.00	1.00
L	P	LOWER	1.00	0.00	1.00	999999.00
M	Q	UPPER	-4.00	-999999.00	1.00	5.00
M	X	UPPER	0.00	-999999.00	1.00	1.00
N	O	LOWER	2.00	-1.00	1.00	999999.00
N	T	BASIC	0.00	0.00	1.00	1.00
N	X	BASIC	0.00	1.00	1.00	2.00
O	V	BASIC	0.00	1.00	1.00	6.00
P	R	UPPER	-6.00	-999999.00	1.00	7.00
Q	S	BASIC	0.00	-999999.00	1.00	5.00
R	W	BASIC	0.00	-999999.00	1.00	7.00
S	W	BASIC	0.00	1.00	1.00	5.00
S	ZZ	BASIC	0.00	-9.00	1.00	1.00
SU	A	BASIC	0.00	-1.00	0.00	0.00
SU	B	BASIC	0.00	0.00	0.00	0.00
SU	C	BASIC	0.00	0.00	0.00	1.00
SU	EX	BASIC	0.00	6.00	10.00	999999.00
T	X	LOWER	1.00	0.00	1.00	999999.00
T	Y	BASIC	0.00	-5.00	1.00	1.00
U	Y	BASIC	0.00	1.00	1.00	6.00
V	U	BASIC	0.00	1.00	1.00	6.00
V	Z	BASIC	0.00	-3.00	1.00	7.00
W	DE	LOWER	10.00	-10.00	0.00	999999.00
X	DE	LOWER	4.00	-4.00	0.00	999999.00
Y	DE	LOWER	5.00	-5.00	0.00	999999.00
Z	DE	LOWER	4.00	-4.00	0.00	999999.00
ZZ	DE	LOWER	10.00	-10.00	0.00	999999.00

Iteration 2

```
file:   incr2       (bottleneck upgrades if utilized in previous flow)
   FROM      TO            COST          LOWER            UPPER
   ----      --         --------       --------         --------
      A       D            1.00           0.00             9.60
      A       E            1.00           0.00             9.60
      A       F            1.00           0.00             9.60
      B       D            1.00           0.00             9.60
      C       D            1.00           0.00             9.60
      C       J            1.00           0.00            32.00
      D       H            1.00           0.00            16.00
     DE      SU            0.00           0.00        999999.00
      E       L            1.00           0.00             2.40
      E       P            1.00           0.00             9.60
     EX       W            0.00           0.00        999999.00
     EX       X            0.00           0.00        999999.00
     EX       Y            0.00           0.00        999999.00
     EX       Z            0.00           0.00        999999.00
     EX      ZZ            0.00           0.00        999999.00
      F       G            1.00           0.00             9.60
      F       L            1.00           0.00             9.60
      G       H            1.00           0.00             2.40
      G       Q            1.00           0.00             9.60
      H       I            1.00           0.00             9.60
      H       M            1.00           0.00            16.00
      I       N            1.00           0.00             9.60
      J       I            1.00           0.00             9.60
      J       K            1.00           0.00             9.60
      J       O            1.00           0.00             9.60
      K       O            1.00           0.00             2.40
      K       V            1.00           0.00             9.60
      L       P            1.00           0.00             9.60
      M       Q            1.00           0.00             2.40
      M       X            1.00           0.00             9.60
      N       O            1.00           0.00             9.60
      N       T            1.00           0.00             9.60
      N       X            1.00           0.00             9.60
      O       V            1.00           0.00             9.60
      P       R            1.00           0.00             9.60
      Q       S            1.00           0.00            16.00
      R       W            1.00           0.00             9.60
      S       W            1.00           0.00             9.60
      S      ZZ            1.00           0.00             9.60
     SU       A            0.00           0.00        999999.00
     SU       B            0.00           0.00        999999.00
     SU       C            0.00           0.00        999999.00
     SU      EX           10.00           0.00        999999.00
      T       X            1.00           0.00             2.40
      T       Y            1.00           0.00             9.60
      U       Y            1.00           0.00             9.60
      V       U            1.00           0.00             9.60
      V       Z            1.00           0.00             9.60
      W      DE            0.00          14.00        999999.00
      X      DE            0.00          12.00        999999.00
      Y      DE            0.00           8.00        999999.00
      Z      DE            0.00           9.00        999999.00
     ZZ      DE            0.00           8.00        999999.00
```

MINIMUM COST FLOW PROBLEM: MINIMUM COST IS   228.80

FROM	TO	LOWER	FLOW	UPPER	COST
SU	A	0.00	28.80	999999.00	0.00
SU	B	0.00	2.40	999999.00	0.00
SU	C	0.00	19.40	999999.00	0.00
A	D	0.00	9.60	9.60	1.00
B	D	0.00	2.40	9.60	1.00
W	DE	14.00	14.00	999999.00	0.00
X	DE	12.00	12.00	999999.00	0.00
Y	DE	8.00	8.00	999999.00	0.00
Z	DE	9.00	9.00	999999.00	0.00
ZZ	DE	8.00	8.00	999999.00	0.00
A	E	0.00	9.60	9.60	1.00
SU	EX	0.00	0.40	999999.00	10.00
A	F	0.00	9.60	9.60	1.00
F	G	0.00	9.60	9.60	1.00
D	H	0.00	12.00	16.00	1.00
J	I	0.00	9.60	9.60	1.00
C	J	0.00	19.40	32.00	1.00
J	K	0.00	9.60	9.60	1.00
H	M	0.00	12.00	16.00	1.00
I	N	0.00	9.60	9.60	1.00
J	O	0.00	0.20	9.60	1.00
E	P	0.00	9.60	9.60	1.00
G	Q	0.00	9.60	9.60	1.00
M	Q	0.00	2.40	2.40	1.00
P	R	0.00	9.60	9.60	1.00
Q	S	0.00	12.00	16.00	1.00
DE	SU	0.00	51.00	999999.00	0.00
N	T	0.00	7.20	9.60	1.00
V	U	0.00	0.80	9.60	1.00
K	V	0.00	9.60	9.60	1.00
O	V	0.00	0.20	9.60	1.00
EX	W	0.00	0.40	999999.00	0.00
R	W	0.00	9.60	9.60	1.00
S	W	0.00	4.00	9.60	1.00
M	X	0.00	9.60	9.60	1.00
N	X	0.00	2.40	9.60	1.00
T	Y	0.00	7.20	9.60	1.00
U	Y	0.00	0.80	9.60	1.00
V	Z	0.00	9.00	9.60	1.00
S	ZZ	0.00	8.00	9.60	1.00

FROM	TO	EDGE STATE	REDUCED COST	LOWER	COST RANGE CURRENT	UPPER
A	D	UPPER	0.00	-999999.00	1.00	1.00
A	E	BASIC	0.00	1.00	1.00	2.00
A	F	BASIC	0.00	0.00	1.00	1.00
B	D	BASIC	0.00	1.00	1.00	1.00
C	D	LOWER	0.00	1.00	1.00	999999.00
C	J	BASIC	0.00	1.00	1.00	2.00
D	H	BASIC	0.00	0.00	1.00	1.00
DE	SU	BASIC	0.00	-4.00	0.00	999999.00
E	L	LOWER	0.00	1.00	1.00	999999.00
E	P	BASIC	0.00	-999999.00	1.00	2.00
EX	W	BASIC	0.00	-4.00	0.00	0.00
EX	X	LOWER	6.00	-6.00	0.00	999999.00
EX	Y	LOWER	5.00	-5.00	0.00	999999.00
EX	Z	LOWER	6.00	-6.00	0.00	999999.00
EX	ZZ	LOWER	0.00	0.00	0.00	999999.00
F	G	BASIC	0.00	0.00	1.00	6.00
F	L	BASIC	0.00	0.00	1.00	1.00
G	H	LOWER	1.00	0.00	1.00	999999.00
G	Q	UPPER	-5.00	-999999.00	1.00	6.00
H	I	LOWER	1.00	0.00	1.00	999999.00
H	M	BASIC	0.00	-999999.00	1.00	1.00
I	N	UPPER	0.00	-999999.00	1.00	1.00
J	I	BASIC	0.00	-999999.00	1.00	1.00
J	K	BASIC	0.00	0.00	1.00	1.00
J	O	BASIC	0.00	1.00	1.00	2.00
K	O	LOWER	1.00	0.00	1.00	999999.00
K	V	UPPER	0.00	-999999.00	1.00	1.00
L	P	LOWER	1.00	0.00	1.00	999999.00
M	Q	UPPER	-4.00	-999999.00	1.00	5.00
M	X	UPPER	0.00	-999999.00	1.00	1.00
N	O	LOWER	2.00	-1.00	1.00	999999.00
N	T	BASIC	0.00	0.00	1.00	1.00
N	X	BASIC	0.00	1.00	1.00	2.00
O	V	BASIC	0.00	1.00	1.00	6.00
P	R	BASIC	0.00	-999999.00	1.00	7.00
Q	S	BASIC	0.00	-999999.00	1.00	5.00
R	W	UPPER	-6.00	-999999.00	1.00	7.00
S	W	BASIC	0.00	1.00	1.00	5.00
S	ZZ	BASIC	0.00	-9.00	1.00	1.00
SU	A	BASIC	0.00	-1.00	0.00	0.00
SU	B	BASIC	0.00	0.00	0.00	0.00
SU	C	BASIC	0.00	0.00	0.00	1.00
SU	EX	BASIC	0.00	6.00	10.00	999999.00
T	X	LOWER	1.00	0.00	1.00	999999.00
T	Y	BASIC	0.00	-5.00	1.00	1.00
U	Y	BASIC	0.00	1.00	1.00	6.00
V	U	BASIC	0.00	1.00	1.00	6.00
V	Z	BASIC	0.00	-3.00	1.00	7.00
W	DE	LOWER	10.00	-10.00	0.00	999999.00
X	DE	LOWER	4.00	-4.00	0.00	999999.00
Y	DE	LOWER	5.00	-5.00	0.00	999999.00
Z	DE	LOWER	4.00	-4.00	0.00	999999.00
ZZ	DE	LOWER	10.00	-10.00	0.00	999999.00

```
file: what2    (all bottlenecks expanded one discrete increment)
     FROM       TO            COST         LOWER         UPPER
     ----       --         ---------     ---------     ---------
      A          D            1.00          0.00          9.60
      A          E            1.00          0.00          9.60
      A          F            1.00          0.00          9.60
      B          D            1.00          0.00          9.60
      C          D            1.00          0.00          9.60
      C          J            1.00          0.00         32.00
      D          H            1.00          0.00         16.00
      DE         SU           0.00          0.00     999999.00
      E          L            1.00          0.00          2.40
      E          P            1.00          0.00          9.60
      EX         W            0.00          0.00     999999.00
      EX         X            0.00          0.00     999999.00
      EX         Y            0.00          0.00     999999.00
      EX         Z            0.00          0.00     999999.00
      EX         ZZ           0.00          0.00     999999.00
      F          G            1.00          0.00          9.60
      F          L            1.00          0.00          9.60
      G          H            1.00          0.00          2.40
      G          Q            1.00          0.00         16.00
      H          I            1.00          0.00          9.60
      H          M            1.00          0.00         16.00
      I          N            1.00          0.00          9.60
      J          I            1.00          0.00          9.60
      J          K            1.00          0.00          9.60
      J          O            1.00          0.00          9.60
      K          O            1.00          0.00          2.40
      K          V            1.00          0.00          9.60
      L          P            1.00          0.00          9.60
      M          Q            1.00          0.00          9.60
      M          X            1.00          0.00          9.60
      N          O            1.00          0.00          9.60
      N          T            1.00          0.00          9.60
      N          X            1.00          0.00          9.60
      O          V            1.00          0.00          9.60
      P          R            1.00          0.00          9.60
      Q          S            1.00          0.00         16.00
      R          W            1.00          0.00         16.00
      S          W            1.00          0.00          9.60
      S          ZZ           1.00          0.00          9.60
      SU         A            0.00          0.00     999999.00
      SU         B            0.00          0.00     999999.00
      SU         C            0.00          0.00     999999.00
      SU         EX          10.00          0.00     999999.00
      T          X            1.00          0.00          2.40
      T          Y            1.00          0.00          9.60
      U          Y            1.00          0.00          9.60
      V          U            1.00          0.00          9.60
      V          Z            1.00          0.00          9.60
      W          DE           0.00         14.00     999999.00
      X          DE           0.00         12.00     999999.00
      Y          DE           0.00          8.00     999999.00
      Z          DE           0.00          9.00     999999.00
      ZZ         DE           0.00          8.00     999999.00
```

MINIMUM COST FLOW PROBLEM: MINIMUM COST IS   227.20

FROM	TO	LOWER	FLOW	UPPER	COST
SU	A	0.00	28.80	999999.00	0.00
SU	B	0.00	2.80	999999.00	0.00
SU	C	0.00	19.40	999999.00	0.00
A	D	0.00	9.60	9.60	1.00
B	D	0.00	2.80	9.60	1.00
W	DE	14.00	14.00	999999.00	0.00
X	DE	12.00	12.00	999999.00	0.00
Y	DE	8.00	8.00	999999.00	0.00
Z	DE	9.00	9.00	999999.00	0.00
ZZ	DE	8.00	8.00	999999.00	0.00
A	E	0.00	9.60	9.60	1.00
A	F	0.00	9.60	9.60	1.00
F	G	0.00	9.60	9.60	1.00
D	H	0.00	12.40	16.00	1.00
J	I	0.00	9.60	9.60	1.00
C	J	0.00	19.40	32.00	1.00
J	K	0.00	9.60	9.60	1.00
H	M	0.00	12.40	16.00	1.00
I	N	0.00	9.60	9.60	1.00
J	O	0.00	0.20	9.60	1.00
E	P	0.00	9.60	9.60	1.00
G	Q	0.00	9.60	16.00	1.00
M	Q	0.00	2.80	9.60	1.00
P	R	0.00	9.60	9.60	1.00
Q	S	0.00	12.40	16.00	1.00
DE	SU	0.00	51.00	999999.00	0.00
N	T	0.00	7.20	9.60	1.00
V	U	0.00	0.80	9.60	1.00
K	V	0.00	9.60	9.60	1.00
O	V	0.00	0.20	9.60	1.00
R	W	0.00	9.60	16.00	1.00
S	W	0.00	4.40	9.60	1.00
M	X	0.00	9.60	9.60	1.00
N	X	0.00	2.40	9.60	1.00
T	Y	0.00	7.20	9.60	1.00
U	Y	0.00	0.80	9.60	1.00
V	Z	0.00	9.00	9.60	1.00
S	ZZ	0.00	8.00	9.60	1.00

FROM	TO	EDGE STATE	REDUCED COST	COST RANGE LOWER	COST RANGE CURRENT	COST RANGE UPPER
A	D	UPPER	0.00	-999999.00	1.00	1.00
A	E	BASIC	0.00	1.00	1.00	2.00
A	F	BASIC	0.00	0.00	1.00	1.00
B	D	BASIC	0.00	1.00	1.00	1.00
C	D	LOWER	0.00	1.00	1.00	999999.00
C	J	BASIC	0.00	1.00	1.00	2.00
D	H	BASIC	0.00	0.00	1.00	1.00
DE	SU	BASIC	0.00	-4.00	0.00	999999.00
E	L	LOWER	0.00	1.00	1.00	999999.00
E	P	BASIC	0.00	-999999.00	1.00	2.00
EX	W	LOWER	4.00	-4.00	0.00	999999.00
EX	X	LOWER	6.00	-6.00	0.00	999999.00
EX	Y	LOWER	5.00	-5.00	0.00	999999.00
EX	Z	LOWER	6.00	-6.00	0.00	999999.00
EX	ZZ	LOWER	4.00	-4.00	0.00	999999.00
F	G	UPPER	-1.00	-999999.00	1.00	2.00
F	L	BASIC	0.00	0.00	1.00	1.00
G	H	LOWER	2.00	-1.00	1.00	999999.00
G	Q	BASIC	0.00	-999999.00	1.00	2.00
H	I	LOWER	1.00	0.00	1.00	999999.00
H	M	BASIC	0.00	0.00	1.00	1.00
I	N	UPPER	0.00	-999999.00	1.00	1.00
J	I	BASIC	0.00	-999999.00	1.00	1.00
J	K	BASIC	0.00	0.00	1.00	1.00
J	O	BASIC	0.00	1.00	1.00	2.00
K	O	LOWER	1.00	0.00	1.00	999999.00
K	V	UPPER	0.00	-999999.00	1.00	1.00
L	P	LOWER	1.00	0.00	1.00	999999.00
M	Q	BASIC	0.00	0.00	1.00	5.00
M	X	UPPER	0.00	-999999.00	1.00	1.00
N	O	LOWER	2.00	-1.00	1.00	999999.00
N	T	BASIC	0.00	0.00	1.00	1.00
N	X	BASIC	0.00	1.00	1.00	2.00
O	V	BASIC	0.00	1.00	1.00	6.00
P	R	UPPER	-2.00	-999999.00	1.00	3.00
Q	S	BASIC	0.00	-1.00	1.00	5.00
R	W	BASIC	0.00	-999999.00	1.00	3.00
S	W	BASIC	0.00	-1.00	1.00	5.00
S	ZZ	BASIC	0.00	-5.00	1.00	5.00
SU	A	BASIC	0.00	-999999.00	0.00	0.00
SU	B	BASIC	0.00	0.00	0.00	0.00
SU	C	BASIC	0.00	0.00	0.00	1.00
SU	EX	BASIC	0.00	6.00	10.00	999999.00
T	X	LOWER	1.00	0.00	1.00	999999.00
T	Y	BASIC	0.00	-5.00	1.00	1.00
U	Y	BASIC	0.00	1.00	1.00	6.00
V	U	BASIC	0.00	1.00	1.00	6.00
V	Z	BASIC	0.00	-3.00	1.00	7.00
W	DE	LOWER	6.00	-6.00	0.00	999999.00
X	DE	LOWER	4.00	-4.00	0.00	999999.00
Y	DE	LOWER	5.00	-5.00	0.00	999999.00
Z	DE	LOWER	4.00	-4.00	0.00	999999.00
ZZ	DE	LOWER	6.00	-6.00	0.00	999999.00

Iteration 3

file: incr3   (bottleneck upgrades if utilized in previous flow)

FROM	TO	COST	LOWER	UPPER
A	D	1.00	0.00	9.60
A	E	1.00	0.00	9.60
A	F	1.00	0.00	9.60
B	D	1.00	0.00	9.60
C	D	1.00	0.00	9.60
C	J	1.00	0.00	32.00
D	H	1.00	0.00	16.00
DE	SU	0.00	0.00	999999.00
E	L	1.00	0.00	2.40
E	P	1.00	0.00	9.60
EX	W	0.00	0.00	999999.00
EX	X	0.00	0.00	999999.00
EX	Y	0.00	0.00	999999.00
EX	Z	0.00	0.00	999999.00
EX	ZZ	0.00	0.00	999999.00
F	G	1.00	0.00	9.60
F	L	1.00	0.00	9.60
G	H	1.00	0.00	2.40
G	Q	1.00	0.00	9.60
H	I	1.00	0.00	9.60
H	M	1.00	0.00	16.00
I	N	1.00	0.00	9.60
J	I	1.00	0.00	9.60
J	K	1.00	0.00	9.60
J	O	1.00	0.00	9.60
K	O	1.00	0.00	2.40
K	V	1.00	0.00	9.60
L	P	1.00	0.00	9.60
M	Q	1.00	0.00	9.60
M	X	1.00	0.00	9.60
N	O	1.00	0.00	9.60
N	T	1.00	0.00	9.60
N	X	1.00	0.00	9.60
O	V	1.00	0.00	9.60
P	R	1.00	0.00	9.60
Q	S	1.00	0.00	16.00
R	W	1.00	0.00	9.60
S	W	1.00	0.00	9.60
S	ZZ	1.00	0.00	9.60
SU	A	0.00	0.00	999999.00
SU	B	0.00	0.00	999999.00
SU	C	0.00	0.00	999999.00
SU	EX	10.00	0.00	999999.00
T	X	1.00	0.00	2.40
T	Y	1.00	0.00	9.60
U	Y	1.00	0.00	9.60
V	U	1.00	0.00	9.60
V	Z	1.00	0.00	9.60
W	DE	0.00	14.00	999999.00
X	DE	0.00	12.00	999999.00
Y	DE	0.00	8.00	999999.00
Z	DE	0.00	9.00	999999.00
ZZ	DE	0.00	8.00	999999.00

MINIMUM COST FLOW PROBLEM: MINIMUM COST IS   227.20

FROM	TO	LOWER	FLOW	UPPER	COST
SU	A	0.00	28.80	999999.00	0.00
SU	B	0.00	2.80	999999.00	0.00
SU	C	0.00	19.40	999999.00	0.00
A	D	0.00	9.60	9.60	1.00
B	D	0.00	2.80	9.60	1.00
W	DE	14.00	14.00	999999.00	0.00
X	DE	12.00	12.00	999999.00	0.00
Y	DE	8.00	8.00	999999.00	0.00
Z	DE	9.00	9.00	999999.00	0.00
ZZ	DE	8.00	8.00	999999.00	0.00
A	E	0.00	9.60	9.60	1.00
A	F	0.00	9.60	9.60	1.00
F	G	0.00	9.60	9.60	1.00
D	H	0.00	12.40	16.00	1.00
J	I	0.00	9.60	9.60	1.00
C	J	0.00	19.40	32.00	1.00
J	K	0.00	9.60	9.60	1.00
H	M	0.00	12.40	16.00	1.00
I	N	0.00	9.60	9.60	1.00
J	O	0.00	0.20	9.60	1.00
E	P	0.00	9.60	9.60	1.00
G	Q	0.00	9.60	9.60	1.00
M	Q	0.00	2.80	9.60	1.00
P	R	0.00	9.60	9.60	1.00
Q	S	0.00	12.40	16.00	1.00
DE	SU	0.00	51.00	999999.00	0.00
N	T	0.00	7.20	9.60	1.00
V	U	0.00	0.80	9.60	1.00
K	V	0.00	9.60	9.60	1.00
O	V	0.00	0.20	9.60	1.00
R	W	0.00	9.60	9.60	1.00
S	W	0.00	4.40	9.60	1.00
M	X	0.00	9.60	9.60	1.00
N	X	0.00	2.40	9.60	1.00
T	Y	0.00	7.20	9.60	1.00
U	Y	0.00	0.80	9.60	1.00
V	Z	0.00	9.00	9.60	1.00
S	ZZ	0.00	8.00	9.60	1.00

FROM	TO	EDGE STATE	REDUCED COST	COST RANGE LOWER	COST RANGE CURRENT	COST RANGE UPPER
A	D	UPPER	0.00	-999999.00	1.00	1.00
A	E	BASIC	0.00	1.00	1.00	2.00
A	F	BASIC	0.00	0.00	1.00	1.00
B	D	BASIC	0.00	1.00	1.00	1.00
C	D	LOWER	0.00	1.00	1.00	999999.00
C	J	BASIC	0.00	1.00	1.00	2.00
D	H	BASIC	0.00	0.00	1.00	1.00
DE	SU	BASIC	0.00	-4.00	0.00	999999.00
E	L	LOWER	0.00	1.00	1.00	999999.00
E	P	BASIC	0.00	-999999.00	1.00	2.00
EX	W	LOWER	4.00	-4.00	0.00	999999.00
EX	X	LOWER	6.00	-6.00	0.00	999999.00
EX	Y	LOWER	5.00	-5.00	0.00	999999.00
EX	Z	LOWER	6.00	-6.00	0.00	999999.00
EX	ZZ	LOWER	4.00	-4.00	0.00	999999.00
F	G	BASIC	0.00	0.00	1.00	2.00
F	L	BASIC	0.00	0.00	1.00	1.00
G	H	LOWER	1.00	0.00	1.00	999999.00
G	Q	UPPER	-1.00	-999999.00	1.00	2.00
H	I	LOWER	1.00	0.00	1.00	999999.00
H	M	BASIC	0.00	0.00	1.00	1.00
I	N	UPPER	0.00	-999999.00	1.00	1.00
J	I	BASIC	0.00	-999999.00	1.00	1.00
J	K	BASIC	0.00	0.00	1.00	1.00
J	O	BASIC	0.00	1.00	1.00	2.00
K	O	LOWER	1.00	0.00	1.00	999999.00
K	V	UPPER	0.00	-999999.00	1.00	1.00
L	P	LOWER	1.00	0.00	1.00	999999.00
M	Q	BASIC	0.00	0.00	1.00	5.00
M	X	UPPER	0.00	-999999.00	1.00	1.00
N	O	LOWER	2.00	-1.00	1.00	999999.00
N	T	BASIC	0.00	0.00	1.00	1.00
N	X	BASIC	0.00	1.00	1.00	2.00
O	V	BASIC	0.00	1.00	1.00	6.00
P	R	BASIC	0.00	-999999.00	1.00	3.00
Q	S	BASIC	0.00	-1.00	1.00	5.00
R	W	UPPER	-2.00	-999999.00	1.00	3.00
S	W	BASIC	0.00	-1.00	1.00	5.00
S	ZZ	BASIC	0.00	-5.00	1.00	5.00
SU	A	BASIC	0.00	-1.00	0.00	0.00
SU	B	BASIC	0.00	0.00	0.00	0.00
SU	C	BASIC	0.00	0.00	0.00	1.00
SU	EX	BASIC	0.00	6.00	10.00	999999.00
T	X	LOWER	1.00	0.00	1.00	999999.00
T	Y	BASIC	0.00	-5.00	1.00	1.00
U	Y	BASIC	0.00	1.00	1.00	6.00
V	U	BASIC	0.00	1.00	1.00	6.00
V	Z	BASIC	0.00	-3.00	1.00	7.00
W	DE	LOWER	6.00	-6.00	0.00	999999.00
X	DE	LOWER	4.00	-4.00	0.00	999999.00
Y	DE	LOWER	5.00	-5.00	0.00	999999.00
Z	DE	LOWER	4.00	-4.00	0.00	999999.00
ZZ	DE	LOWER	6.00	-6.00	0.00	999999.00

Iteration 2 with slack constraint on budget

file: incr2slk

FROM	TO	COST	LOWER	UPPER
A	D	1.00	0.00	9.60
A	E	1.00	0.00	9.60
A	F	1.00	0.00	9.60
B	D	1.00	0.00	9.60
C	D	1.00	0.00	9.60
C	J	1.00	0.00	32.00
D	H	1.00	0.00	16.00
DE	SU	0.00	0.00	999999.00
E	L	1.00	0.00	2.40
E	P	1.00	0.00	9.60
EX	W	0.00	0.00	999999.00
EX	X	0.00	0.00	999999.00
EX	Y	0.00	0.00	999999.00
EX	Z	0.00	0.00	999999.00
EX	ZZ	0.00	0.00	999999.00
F	G	1.00	0.00	9.60
F	L	1.00	0.00	9.60
G	H	1.00	0.00	2.40
G	Q	1.00	0.00	9.60
H	I	1.00	0.00	9.60
H	M	1.00	0.00	9.60
I	N	1.00	0.00	9.60
J	I	1.00	0.00	9.60
J	K	1.00	0.00	9.60
J	O	1.00	0.00	9.60
K	O	1.00	0.00	2.40
K	V	1.00	0.00	9.60
L	P	1.00	0.00	9.60
M	Q	1.00	0.00	2.40
M	X	1.00	0.00	9.60
N	O	1.00	0.00	9.60
N	T	1.00	0.00	9.60
N	X	1.00	0.00	9.60
O	V	1.00	0.00	9.60
P	R	1.00	0.00	9.60
Q	S	1.00	0.00	16.00
R	W	1.00	0.00	9.60
S	W	1.00	0.00	9.60
S	ZZ	1.00	0.00	9.60
SU	A	0.00	0.00	999999.00
SU	B	0.00	0.00	999999.00
SU	C	0.00	0.00	999999.00
SU	EX	10.00	0.00	999999.00
T	X	1.00	0.00	2.40
T	Y	1.00	0.00	2.40
U	Y	1.00	0.00	9.60
V	U	1.00	0.00	9.60
V	Z	1.00	0.00	9.60
W	DE	0.00	14.00	999999.00
X	DE	0.00	12.00	999999.00
Y	DE	0.00	8.00	999999.00
Z	DE	0.00	9.00	999999.00
ZZ	DE	0.00	8.00	999999.00

MINIMUM COST FLOW PROBLEM: MINIMUM COST IS   228.80

FROM	TO	LOWER	FLOW	UPPER	COST
SU	A	0.00	28.80	999999.00	0.00
SU	C	0.00	21.80	999999.00	0.00
A	D	0.00	9.60	9.60	1.00
W	DE	14.00	14.00	999999.00	0.00
X	DE	12.00	12.00	999999.00	0.00
Y	DE	8.00	8.00	999999.00	0.00
Z	DE	9.00	9.00	999999.00	0.00
ZZ	DE	8.00	8.00	999999.00	0.00
A	E	0.00	9.60	9.60	1.00
SU	EX	0.00	0.40	999999.00	10.00
A	F	0.00	9.60	9.60	1.00
F	G	0.00	9.60	9.60	1.00
D	H	0.00	9.60	16.00	1.00
J	I	0.00	7.20	9.60	1.00
C	J	0.00	21.80	32.00	1.00
J	K	0.00	9.60	9.60	1.00
H	M	0.00	9.60	9.60	1.00
I	N	0.00	7.20	9.60	1.00
J	O	0.00	5.00	9.60	1.00
E	P	0.00	9.60	9.60	1.00
G	Q	0.00	9.60	9.60	1.00
M	Q	0.00	2.40	2.40	1.00
P	R	0.00	9.60	9.60	1.00
Q	S	0.00	12.00	16.00	1.00
DE	SU	0.00	51.00	999999.00	0.00
N	T	0.00	2.40	9.60	1.00
V	U	0.00	5.60	9.60	1.00
K	V	0.00	9.60	9.60	1.00
O	V	0.00	5.00	9.60	1.00
EX	W	0.00	0.40	999999.00	0.00
R	W	0.00	9.60	9.60	1.00
S	W	0.00	4.00	9.60	1.00
M	X	0.00	7.20	9.60	1.00
N	X	0.00	4.80	9.60	1.00
T	Y	0.00	2.40	2.40	1.00
U	Y	0.00	5.60	9.60	1.00
V	Z	0.00	9.00	9.60	1.00
S	ZZ	0.00	8.00	9.60	1.00

FROM	TO	EDGE STATE	REDUCED COST	COST RANGE LOWER	CURRENT	UPPER
A	D	UPPER	0.00	-999999.00	1.00	1.00
A	E	BASIC	0.00	1.00	1.00	2.00
A	F	BASIC	0.00	0.00	1.00	1.00
B	D	BASIC	0.00	1.00	1.00	1.00
C	D	LOWER	0.00	1.00	1.00	999999.00
C	J	BASIC	0.00	1.00	1.00	2.00
D	H	BASIC	0.00	0.00	1.00	1.00
DE	SU	BASIC	0.00	-4.00	0.00	999999.00
E	L	LOWER	0.00	1.00	1.00	999999.00
E	P	BASIC	0.00	-999999.00	1.00	2.00
EX	W	BASIC	0.00	-4.00	0.00	0.00
EX	X	LOWER	6.00	-6.00	0.00	999999.00
EX	Y	LOWER	5.00	-5.00	0.00	999999.00
EX	Z	LOWER	6.00	-6.00	0.00	999999.00
EX	ZZ	LOWER	0.00	0.00	0.00	999999.00
F	G	BASIC	0.00	0.00	1.00	6.00
F	L	BASIC	0.00	0.00	1.00	1.00
G	H	LOWER	1.00	0.00	1.00	999999.00
G	Q	UPPER	-5.00	-999999.00	1.00	6.00
H	I	LOWER	1.00	0.00	1.00	999999.00
H	M	UPPER	0.00	-999999.00	1.00	1.00
I	N	BASIC	0.00	1.00	1.00	1.00
J	I	BASIC	0.00	1.00	1.00	1.00
J	K	BASIC	0.00	0.00	1.00	1.00
J	O	BASIC	0.00	1.00	1.00	2.00
K	O	LOWER	1.00	0.00	1.00	999999.00
K	V	UPPER	0.00	-999999.00	1.00	1.00
L	P	LOWER	1.00	0.00	1.00	999999.00
M	Q	UPPER	-4.00	-999999.00	1.00	5.00
M	X	BASIC	0.00	-3.00	1.00	1.00
N	O	LOWER	2.00	-1.00	1.00	999999.00
N	T	BASIC	0.00	0.00	1.00	1.00
N	X	BASIC	0.00	1.00	1.00	2.00
O	V	BASIC	0.00	1.00	1.00	6.00
P	R	BASIC	0.00	-999999.00	1.00	7.00
Q	S	BASIC	0.00	-999999.00	1.00	5.00
R	W	UPPER	-6.00	-999999.00	1.00	7.00
S	W	BASIC	0.00	1.00	1.00	5.00
S	ZZ	BASIC	0.00	-9.00	1.00	1.00
SU	A	BASIC	0.00	-1.00	0.00	0.00
SU	B	BASIC	0.00	0.00	0.00	0.00
SU	C	BASIC	0.00	0.00	0.00	1.00
SU	EX	BASIC	0.00	6.00	10.00	999999.00
T	X	LOWER	1.00	0.00	1.00	999999.00
T	Y	UPPER	0.00	-999999.00	1.00	1.00
U	Y	BASIC	0.00	1.00	1.00	6.00
V	U	BASIC	0.00	1.00	1.00	6.00
V	Z	BASIC	0.00	-3.00	1.00	7.00
W	DE	LOWER	10.00	-10.00	0.00	999999.00
X	DE	LOWER	4.00	-4.00	0.00	999999.00
Y	DE	LOWER	5.00	-5.00	0.00	999999.00
Z	DE	LOWER	4.00	-4.00	0.00	999999.00
ZZ	DE	LOWER	10.00	-10.00	0.00	999999.00

Iteration 3 using Iteration2 with slack constraint results

file: incr3slk

FROM	TO	COST	LOWER	UPPER
A	D	1.00	0.00	9.60
A	E	1.00	0.00	9.60
A	F	1.00	0.00	9.60
B	D	1.00	0.00	9.60
C	D	1.00	0.00	9.60
C	J	1.00	0.00	32.00
D	H	1.00	0.00	16.00
DE	SU	0.00	0.00	999999.00
E	L	1.00	0.00	2.40
E	P	1.00	0.00	9.60
EX	W	0.00	0.00	999999.00
EX	X	0.00	0.00	999999.00
EX	Y	0.00	0.00	999999.00
EX	Z	0.00	0.00	999999.00
EX	ZZ	0.00	0.00	999999.00
F	G	1.00	0.00	9.60
F	L	1.00	0.00	9.60
G	H	1.00	0.00	2.40
G	Q	1.00	0.00	16.00
H	I	1.00	0.00	9.60
H	M	1.00	0.00	9.60
I	N	1.00	0.00	9.60
J	I	1.00	0.00	9.60
J	K	1.00	0.00	9.60
J	O	1.00	0.00	9.60
K	O	1.00	0.00	2.40
K	V	1.00	0.00	9.60
L	P	1.00	0.00	9.60
M	Q	1.00	0.00	9.60
M	X	1.00	0.00	9.60
N	O	1.00	0.00	9.60
N	T	1.00	0.00	9.60
N	X	1.00	0.00	9.60
O	V	1.00	0.00	9.60
P	R	1.00	0.00	9.60
Q	S	1.00	0.00	16.00
R	W	1.00	0.00	16.00
S	W	1.00	0.00	9.60
S	ZZ	1.00	0.00	9.60
SU	A	0.00	0.00	999999.00
SU	B	0.00	0.00	999999.00
SU	C	0.00	0.00	999999.00
SU	EX	10.00	0.00	999999.00
T	X	1.00	0.00	2.40
T	Y	1.00	0.00	2.40
U	Y	1.00	0.00	9.60
V	U	1.00	0.00	9.60
V	Z	1.00	0.00	9.60
W	DE	0.00	14.00	999999.00
X	DE	0.00	12.00	999999.00
Y	DE	0.00	8.00	999999.00
Z	DE	0.00	9.00	999999.00
ZZ	DE	0.00	8.00	999999.00

MINIMUM COST FLOW PROBLEM: MINIMUM COST IS   227.20

FROM	TO	LOWER	FLOW	UPPER	COST
SU	A	0.00	28.80	999999.00	0.00
SU	C	0.00	22.20	999999.00	0.00
A	D	0.00	9.60	9.60	1.00
W	DE	14.00	14.00	999999.00	0.00
X	DE	12.00	12.00	999999.00	0.00
Y	DE	8.00	8.00	999999.00	0.00
Z	DE	9.00	9.00	999999.00	0.00
ZZ	DE	8.00	8.00	999999.00	0.00
A	E	0.00	9.60	9.60	1.00
A	F	0.00	9.60	9.60	1.00
F	G	0.00	9.60	9.60	1.00
D	H	0.00	9.60	16.00	1.00
J	I	0.00	7.60	9.60	1.00
C	J	0.00	22.20	32.00	1.00
J	K	0.00	9.60	9.60	1.00
H	M	0.00	9.60	9.60	1.00
I	N	0.00	7.60	9.60	1.00
J	O	0.00	5.00	9.60	1.00
E	P	0.00	9.60	9.60	1.00
G	Q	0.00	9.60	16.00	1.00
M	Q	0.00	2.80	9.60	1.00
P	R	0.00	9.60	9.60	1.00
Q	S	0.00	12.40	16.00	1.00
DE	SU	0.00	51.00	999999.00	0.00
N	T	0.00	2.40	9.60	1.00
V	U	0.00	5.60	9.60	1.00
K	V	0.00	9.60	9.60	1.00
O	V	0.00	5.00	9.60	1.00
R	W	0.00	9.60	16.00	1.00
S	W	0.00	4.40	9.60	1.00
M	X	0.00	6.80	9.60	1.00
N	X	0.00	5.20	9.60	1.00
T	Y	0.00	2.40	2.40	1.00
U	Y	0.00	5.60	9.60	1.00
V	Z	0.00	9.00	9.60	1.00
S	ZZ	0.00	8.00	9.60	1.00

FROM	TO	EDGE STATE	REDUCED COST	LOWER	COST RANGE CURRENT	UPPER
A	D	UPPER	0.00	-999999.00	1.00	1.00
A	E	BASIC	0.00	1.00	1.00	2.00
A	F	BASIC	0.00	0.00	1.00	1.00
B	D	BASIC	0.00	1.00	1.00	1.00
C	D	LOWER	0.00	1.00	1.00	999999.00
C	J	BASIC	0.00	1.00	1.00	2.00
D	H	BASIC	0.00	0.00	1.00	1.00
DE	SU	BASIC	0.00	-4.00	0.00	999999.00
E	L	LOWER	0.00	1.00	1.00	999999.00
E	P	BASIC	0.00	-999999.00	1.00	2.00
EX	W	LOWER	4.00	-4.00	0.00	999999.00
EX	X	LOWER	6.00	-6.00	0.00	999999.00
EX	Y	LOWER	5.00	-5.00	0.00	999999.00
EX	Z	LOWER	6.00	-6.00	0.00	999999.00
EX	ZZ	LOWER	4.00	-4.00	0.00	999999.00
F	G	UPPER	-1.00	-999999.00	1.00	2.00
F	L	BASIC	0.00	0.00	1.00	1.00
G	H	LOWER	2.00	-1.00	1.00	999999.00
G	Q	BASIC	0.00	-999999.00	1.00	2.00
H	I	LOWER	1.00	0.00	1.00	999999.00
H	M	UPPER	0.00	-999999.00	1.00	1.00
I	N	BASIC	0.00	1.00	1.00	1.00
J	I	BASIC	0.00	1.00	1.00	1.00
J	K	BASIC	0.00	0.00	1.00	1.00
J	O	BASIC	0.00	1.00	1.00	2.00
K	O	LOWER	1.00	0.00	1.00	999999.00
K	V	UPPER	0.00	-999999.00	1.00	1.00
L	P	LOWER	1.00	0.00	1.00	999999.00
M	Q	BASIC	0.00	0.00	1.00	5.00
M	X	BASIC	0.00	-3.00	1.00	1.00
N	O	LOWER	2.00	-1.00	1.00	999999.00
N	T	BASIC	0.00	0.00	1.00	1.00
N	X	BASIC	0.00	1.00	1.00	2.00
O	V	BASIC	0.00	1.00	1.00	6.00
P	R	UPPER	-2.00	-999999.00	1.00	3.00
Q	S	BASIC	0.00	-1.00	1.00	5.00
R	W	BASIC	0.00	-999999.00	1.00	3.00
S	W	BASIC	0.00	-1.00	1.00	5.00
S	ZZ	BASIC	0.00	-5.00	1.00	5.00
SU	A	BASIC	0.00	-999999.00	0.00	0.00
SU	B	BASIC	0.00	0.00	0.00	0.00
SU	C	BASIC	0.00	0.00	0.00	1.00
SU	EX	BASIC	0.00	6.00	10.00	999999.00
T	X	LOWER	1.00	0.00	1.00	999999.00
T	Y	UPPER	0.00	-999999.00	1.00	1.00
U	Y	BASIC	0.00	1.00	1.00	6.00
V	U	BASIC	0.00	1.00	1.00	6.00
V	Z	BASIC	0.00	-3.00	1.00	7.00
W	DE	LOWER	6.00	-6.00	0.00	999999.00
X	DE	LOWER	4.00	-4.00	0.00	999999.00
Y	DE	LOWER	5.00	-5.00	0.00	999999.00
Z	DE	LOWER	4.00	-4.00	0.00	999999.00
ZZ	DE	LOWER	6.00	-6.00	0.00	999999.00

Appendix E. Expansion Model Codes

```
GC Model     (cost constrained)
  MAX      10.11 AE16A + 10.49 AE16B + 33.33 CJ32A + 22.11 CJ32B
       + 22.29 CJ32C + 18.73 EP16A + 18.49 EP16B + 19.14 EP16C + 23.86 EP16D
       + 9.12 GQ96A + 11.29 GQ96B + 7.35 GQ96C + 9.12 MX96A + 11.29 MX96B
       + 7.35 MX96C + 12.26 PR16A + 12.64 PR16B + 12.46 PR16C + 17.2 PR16D
       + 12.26 RW16A + 12.64 RW16B + 12.46 RW16C + 17.2 RW16D + 9.12 TY96A
       + 11.29 TY96B + 7.35 TY96C
  SUBJECT TO
         2)    AE16A + AE16B <=    1
         3)    CJ32A + CJ32B + CJ32C <=    1
         4)    EP16A + EP16B + EP16C + EP16D <=    1
         5)    GQ96A + GQ96B + GQ96C <=    1
         6)    MX96A + MX96B + MX96C <=    1
         7)    PR16A + PR16B + PR16C + PR16D <=    1
         8)    RW16A + RW16B + RW16C + RW16D <=    1
         9)    TY96A + TY96B + TY96C <=    1
        10)    31500 AE16A + 30200 AE16B + 35000 CJ32A + 31600 CJ32B
       + 31500 CJ32C + 31500 EP16A + 30200 EP16B + 26500 EP16C + 32750 EP16D
       + 20000 GQ96A + 21500 GQ96B + 26000 GQ96C + 20000 MX96A + 21500 MX96B
       + 26000 MX96C + 31500 PR16A + 30200 PR16B + 26500 PR16C + 32750 PR16D
       + 31500 RW16A + 30200 RW16B + 26500 RW16C + 32750 RW16D + 20000 TY96A
       + 21500 TY96B + 26000 TY96C <=    220000
  END
  GIN      26
```

```
     OBJECTIVE FUNCTION VALUE
        1)       126.8900
VARIABLE          VALUE          REDUCED COST
  AE16A          .000000          -10.110000
  AE16B         1.000000          -10.490000
  CJ32A         1.000000          -33.330000
  CJ32B          .000000          -22.110000
  CJ32C          .000000          -22.290000
  EP16A          .000000          -18.730000
  EP16B          .000000          -18.490000
  EP16C         1.000000          -19.140000
  EP16D          .000000          -23.860000
  GQ96A          .000000           -9.120000
  GQ96B         1.000000          -11.290000
  GQ96C          .000000           -7.350000
  MX96A         1.000000           -9.120000
  MX96B          .000000          -11.290000
  MX96C          .000000           -7.350000
  PR16A          .000000          -12.260000
  PR16B          .000000          -12.640000
  PR16C          .000000          -12.460000
  PR16D         1.000000          -17.200000
  RW16A          .000000          -12.260000
  RW16B          .000000          -12.640000
  RW16C          .000000          -12.460000
  RW16D         1.000000          -17.200000
  TY96A         1.000000           -9.120000
  TY96B          .000000          -11.290000
  TY96C          .000000           -7.350000
     ROW     SLACK OR SURPLUS     DUAL PRICES
      2)          .000000           .000000
      3)          .000000           .000000
      4)          .000000           .000000
      5)          .000000           .000000
      6)          .000000           .000000
      7)          .000000           .000000
      8)          .000000           .000000
      9)          .000000           .000000
     10)      1300.000000           .000000
NO. ITERATIONS=      870
BRANCHES=  172 DETERM.=  1.000E    0
```

GU Model   **(unconstrained)**

```
MAX      10.11 AE16A + 10.49 AE16B + 33.33 CJ32A + 22.11 CJ32B
       + 22.29 CJ32C + 18.73 EP16A + 18.49 EP16B + 19.14 EP16C + 23.86 EP16D
       + 9.12 GQ96A + 11.29 GQ96B + 7.35 GQ96C + 9.12 MX96A + 11.29 MX96B
       + 7.35 MX96C + 12.26 PR16A + 12.64 PR16B + 12.46 PR16C + 17.2 PR16D
       + 12.26 RW16A + 12.64 RW16B + 12.46 RW16C + 17.2 RW16D + 9.12 TY96A
       + 11.29 TY96B + 7.35 TY96C
   SUBJECT TO
       2)    AE16A + AE16B <=   1
       3)    CJ32A + CJ32B + CJ32C <=   1
       4)    EP16A + EP16B + EP16C + EP16D <=   1
       5)    GQ96A + GQ96B + GQ96C <=   1
       6)    MX96A + MX96B + MX96C <=   1
       7)    PR16A + PR16B + PR16C + PR16D <=   1
       8)    RW16A + RW16B + RW16C + RW16D <=   1
       9)    TY96A + TY96B + TY96C <=   1
   END
   GIN    26
```

```
 OBJECTIVE FUNCTION VALUE
      1)      135.9500
      VARIABLE          VALUE          REDUCED COST
        AE16A          .000000         -10.110000
        AE16B         1.000000         -10.490000
        CJ32A         1.000000         -33.330000
        CJ32B          .000000         -22.110000
        CJ32C          .000000         -22.290000
        EP16A          .000000         -18.730000
        EP16B          .000000         -18.490000
        EP16C          .000000         -19.140000
        EP16D         1.000000         -23.860000
        GQ96A          .000000          -9.120000
        GQ96B         1.000000         -11.290000
        GQ96C          .000000          -7.350000
        MX96A          .000000          -9.120000
        MX96B         1.000000         -11.290000
        MX96C          .000000          -7.350000
        PR16A          .000000         -12.260000
        PR16B          .000000         -12.640000
        PR16C          .000000         -12.460000
        PR16D         1.000000         -17.200000
        RW16A          .000000         -12.260000
        RW16B          .000000         -12.640000
        RW16C          .000000         -12.460000
        RW16D         1.000000         -17.200000
        TY96A          .000000          -9.120000
        TY96B         1.000000         -11.290000
        TY96C          .000000          -7.350000
         ROW    SLACK OR SURPLUS      DUAL PRICES
         2)         .000000             .000000
         3)         .000000             .000000
         4)         .000000             .000000
         5)         .000000             .000000
         6)         .000000             .000000
         7)         .000000             .000000
         8)         .000000             .000000
         9)         .000000             .000000
```

```
 NO. ITERATIONS=         9
 BRANCHES=     0 DETERM.=  1.000E     0
```

relaxed GC Model   (relaxed binary constraints)

```
MAX     10.11 AE16A + 10.49 AE16B + 33.33 CJ32A + 22.11 CJ32B
      + 22.29 CJ32C + 18.73 EP16A + 18.49 EP16B + 19.14 EP16C + 23.86 EP16D
      + 9.12 GQ96A + 11.29 GQ96B + 7.35 GQ96C + 9.12 MX96A + 11.29 MX96B
      + 7.35 MX96C + 12.26 PR16A + 12.64 PR16B + 12.46 PR16C + 17.2 PR16D
      + 12.26 RW16A + 12.64 RW16B + 12.46 RW16C + 17.2 RW16D + 9.12 TY96A
      + 11.29 TY96B + 7.35 TY96C
SUBJECT TO
      2)    AE16A + AE16B <=    1
      3)    CJ32A + CJ32B + CJ32C <=    1
      4)    EP16A + EP16B + EP16C + EP16D <=    1
      5)    GQ96A + GQ96B + GQ96C <=    1
      6)    MX96A + MX96B + MX96C <=    1
      7)    PR16A + PR16B + PR16C + PR16D <=    1
      8)    RW16A + RW16B + RW16C + RW16D <=    1
      9)    TY96A + TY96B + TY96C <=    1
     10)    31500 AE16A + 30200 AE16B + 35000 CJ32A + 31600 CJ32B
      + 31500 CJ32C + 31500 EP16A + 30200 EP16B + 26500 EP16C + 32750 EP16D
      + 20000 GQ96A + 21500 GQ96B + 26000 GQ96C + 20000 MX96A + 21500 MX96B
      + 26000 MX96C + 31500 PR16A + 30200 PR16B + 26500 PR16C + 32750 PR16D
      + 31500 RW16A + 30200 RW16B + 26500 RW16C + 32750 RW16D + 20000 TY96A
      + 21500 TY96B + 26000 TY96C <=    220000
END
```

OBJECTIVE FUNCTION VALUE

       1)   133.1886

VARIABLE	VALUE	REDUCED COST
AE16A	.000000	.831556
AE16B	.736755	.000000
CJ32A	1.000000	.000000
CJ32B	.000000	10.039010
CJ32C	.000000	9.824272
EP16A	.000000	4.695812
EP16B	.000000	4.484256
EP16C	.000000	2.549057
EP16D	1.000000	.000000
GQ96A	.000000	1.648974
GQ96B	1.000000	.000000
GQ96C	.000000	5.503079
MX96A	.000000	1.648974
MX96B	1.000000	.000000
MX96C	.000000	5.503079
PR16A	.000000	4.505812
PR16B	.000000	3.674255
PR16C	.000000	2.569057
PR16D	1.000000	.000000
RW16A	.000000	4.505812
RW16B	.000000	3.674255
RW16C	.000000	2.569057
RW16D	1.000000	.000000
TY96A	.000000	1.648974
TY96B	1.000000	.000000
TY96C	.000000	5.503079

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	.263245	.000000
3)	.000000	21.172720
4)	.000000	12.484260
5)	.000000	3.821954
6)	.000000	3.821954
7)	.000000	5.824256
8)	.000000	5.824256
9)	.000000	3.821954
10)	.000000	.000347

NO. ITERATIONS=   9

RANGES IN WHICH THE BASIS IS UNCHANGED:

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
AE16A	10.110000	.831556	INFINITY
AE16B	10.490000	5.368512	.797238
CJ32A	33.330000	INFINITY	9.824273
CJ32B	22.110000	10.039010	INFINITY
CJ32C	22.290000	9.824273	INFINITY
EP16A	18.730000	4.695812	INFINITY
EP16B	18.490000	4.484256	INFINITY
EP16C	19.140000	2.549057	INFINITY
EP16D	23.860000	INFINITY	2.549057
GQ96A	9.120000	1.648974	INFINITY
GQ96B	11.290000	INFINITY	1.648974
GQ96C	7.350000	5.503079	INFINITY
MX96A	9.120000	1.648974	INFINITY
MX96B	11.290000	INFINITY	1.648974
MX96C	7.350000	5.503079	INFINITY
PR16A	12.260000	4.505812	INFINITY
PR16B	12.640000	3.674255	INFINITY
PR16C	12.460000	2.569057	INFINITY
PR16D	17.200000	INFINITY	2.569057
RW16A	12.260000	4.505812	INFINITY
RW16B	12.640000	3.674255	INFINITY
RW16C	12.460000	2.569057	INFINITY
RW16D	17.200000	INFINITY	2.569057
TY96A	9.120000	1.648974	INFINITY
TY96B	11.290000	INFINITY	1.648974
TY96C	7.350000	5.503079	INFINITY

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	1.000000	INFINITY	.263245
3	1.000000	.635714	.227143
4	1.000000	.679389	.242748
5	1.000000	1.034884	.369767
6	1.000000	1.034884	.369767
7	1.000000	.679389	.242748
8	1.000000	.679389	.242748
9	1.000000	1.034884	.369767
10	220000.000000	7950.001000	22250.000000

I1C Model   Iteration 1

```
MAX      10.11 CJ16A + 10.49 CJ16B + 12.26 DH16A + 12.65 DH16B
       + 12.46 DH16C + 17.2 DH16D + 9.12 EP96A + 11.29 EP96B + 7.35 EP96C
       + 9.12 GQ96A + 11.29 GQ96B + 7.35 GQ96C + 9.12 MX96A + 11.29 MX96B
       + 7.35 MX96C
SUBJECT TO
       2)    CJ16A + CJ16B <=    1
       3)    DH16A + DH16B + DH16C + DH16D <=    1
       4)    EP96A + EP96B + EP96C <=    1
       5)    GQ96A + GQ96B + GQ96C <=    1
       6)    MX96A + MX96B + MX96C <=    1
      10)    31500 CJ16A + 30200 CJ16B + 31500 DH16A + 30200 DH16B
       + 26500 DH16C + 32750 DH16D + 20000 EP96A + 21500 EP96B + 26000 EP96C
       + 20000 GQ96A + 21500 GQ96B + 26000 GQ96C + 20000 MX96A + 21500 MX96B
       + 26000 MX96C <=    220000
END
GIN      15
```

```
OBJECTIVE FUNCTION VALUE
       1)      61.56000
   VARIABLE           VALUE          REDUCED COST
     CJ16A          .000000          -10.110000
     CJ16B         1.000000          -10.490000
     DH16A          .000000          -12.260000
     DH16B          .000000          -12.650000
     DH16C          .000000          -12.460000
     DH16D         1.000000          -17.200000
     EP96A          .000000           -9.120000
     EP96B         1.000000          -11.290000
     EP96C          .000000           -7.350000
     GQ96A          .000000           -9.120000
     GQ96B         1.000000          -11.290000
     GQ96C          .000000           -7.350000
     MX96A          .000000           -9.120000
     MX96B         1.000000          -11.290000
     MX96C          .000000           -7.350000

        ROW     SLACK OR SURPLUS      DUAL PRICES
        2)             .000000           .000000
        3)             .000000           .000000
        4)             .000000           .000000
        5)             .000000           .000000
        6)             .000000           .000000
       10)        92550.000000           .000000

NO. ITERATIONS=          5
BRANCHES=     0 DETERM.=  1.000E    0
```

I2C Model   Iteration 2

```
MAX     33.33 CJ32A + 22.11 CJ32B + 22.29 CJ32C + 12.26 HM16A
      + 12.65 HM16B + 12.46 HM16C + 17.2 HM16D + 9.12 KV96A + 11.29 KV96B
      + 7.35 KV96C + 12.26 QS16A + 12.64 QS16B + 12.46 QS16C + 17.2 QS16D
      + 9.12 TY96A + 11.29 TY96B + 7.35 TY96C
SUBJECT TO
        2)   CJ32A + CJ32B + CJ32C <=   1
        3)   HM16A + HM16B + HM16C + HM16D <=   1
        4)   KV96A + KV96B + KV96C <=   1
        5)   QS16A + QS16B + QS16C + QS16D <=   1
        5)   TY96A + TY96B + TY96C <=   1
       10)   35000 CJ32A + 31600 CJ32B + 31500 CJ32C + 31500 HM16A
      + 30200 HM16B + 26500 HM16C + 32750 HM16D + 20000 KV96A + 21500 KV96B
      + 26000 KV96C + 31500 QS16A + 30200 QS16B + 26500 QS16C + 32750 QS16D
      + 20000 TY96A + 21500 TY96B + 26000 TY96C <=    92550
END
GIN     17
```

```
OBJECTIVE FUNCTION VALUE
        1)      61.82000
VARIABLE          VALUE            REDUCED COST
   CJ32A        1.000000           -33.330000
   CJ32B         .000000           -22.110000
   CJ32C         .000000           -22.290000
   HM16A         .000000           -12.260000
   HM16B         .000000           -12.650000
   HM16C         .000000           -12.460000
   HM16D         .000000           -17.200000
   KV96A         .000000            -9.120000
   KV96B        1.000000           -11.290000
   KV96C         .000000            -7.350000
   QS16A         .000000           -12.260000
   QS16B         .000000           -12.640000
   QS16C         .000000           -12.460000
   QS16D        1.000000           -17.200000
   TY96A         .000000            -9.120000
   TY96B         .000000           -11.290000
   TY96C         .000000            -7.350000

     ROW    SLACK OR SURPLUS     DUAL PRICES
     2)          .000000            .000000
     3)         1.000000            .000000
     4)          .000000            .000000
     5)          .000000            .000000
     5)         1.000000            .000000
    10)       3300.000000           .000000
```

```
NO. ITERATIONS=      160
BRANCHES=   24 DETERM.=  1.000E    0
```

I2U Model   (Iteration 2, no budget constraint)

```
MAX     33.33 CJ32A + 22.11 CJ32B + 22.29 CJ32C + 12.26 HM16A
      + 12.65 HM16B + 12.46 HM16C + 17.2 HM16D + 9.12 KV96A + 11.29 KV96B
      + 7.35 KV96C + 12.26 QS16A + 12.64 QS16B + 12.46 QS16C + 17.2 QS16D
      + 9.12 TY96A + 11.29 TY96B + 7.35 TY96C
   SUBJECT TO
        2)    CJ32A + CJ32B + CJ32C <=    1
        3)    HM16A + HM16B + HM16C + HM16D <=    1
        4)    KV96A + KV96B + KV96C <=    1
        5)    QS16A + QS16B + QS16C + QS16D <=    1
        5)    TY96A + TY96B + TY96C <=    1
   END
   GIN     17
```

OBJECTIVE FUNCTION VALUE

	1)	90.31001	
VARIABLE	VALUE	REDUCED COST	
CJ32A	1.000000	-33.330000	
CJ32B	.000000	-22.110000	
CJ32C	.000000	-22.290000	
HM16A	.000000	-12.260000	
HM16B	.000000	-12.650000	
HM16C	.000000	-12.460000	
HM16D	1.000000	-17.200000	
KV96A	.000000	-9.120000	
KV96B	1.000000	-11.290000	
KV96C	.000000	-7.350000	
QS16A	.000000	-12.260000	
QS16B	.000000	-12.640000	
QS16C	.000000	-12.460000	
QS16D	1.000000	-17.200000	
TY96A	.000000	-9.120000	
TY96B	1.000000	-11.290000	
TY96C	.000000	-7.350000	

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	.000000	.000000
3)	.000000	.000000
4)	.000000	.000000
5)	.000000	.000000
5)	.000000	.000000

```
NO. ITERATIONS=        6
BRANCHES=    0 DETERM.=  1.000E    0
```

Appendix F. Baseline System Reliability Calculation

This appendix contains the MathCad listing used for calculating all the system reliabilities. For systems other than the baseline, the upgraded components' reliability was changed accordingly. These system reliability levels were used in the system evaluations (see Appendix G).

The following calculations were used for network reliability. the first set of variables are the baseline link reliability values. The 'p*' variables are the paths through the network. Thus reliability is calculated as the parallel combination of series paths.

$bd := .93$ $el := .8$ $gh := .8$ $in := .75$ $ko := .8$ $mx := .8$ $no := .75$

$af := .93$ $dh := .75$ $fl := .75$ $hm := .75$ $jo := .75$ $mq := .8$ $nt := .75$

$ae := .93$ $cj := .93$ $fg := .75$ $hi := .75$ $jk := .75$ $lp := .75$ $nx := .75$

$ad := .93$ $cd := .93$ $ep := .8$ $gq := .8$ $ji := .75$ $kv := .8$ $ov := .75$

$pr := .75$ $qs := .75$ $rw := .75$ $sw := .75$ $szz := .75$ $tx := .8$ $ty := .8$

$uy := .75$ $vu := .75$ $vz := .75$

$p1 := ae \cdot ep \cdot pr \cdot rw$

$p2 := ae \cdot el \cdot lp \cdot pr \cdot rw$

$p3 := af \cdot fl \cdot lp \cdot pr \cdot rw$

$p4 := af \cdot fg \cdot gq \cdot qs \cdot sw$

$p5 := af \cdot fg \cdot gq \cdot qs \cdot szz$

$p6 := af \cdot fg \cdot gh \cdot hm \cdot mq \cdot qs \cdot sw$

$p7 := af \cdot fg \cdot gh \cdot hm \cdot mq \cdot qs \cdot szz$

$p8 := af \cdot fg \cdot gh \cdot hm \cdot mx$

$p9 := af \cdot fg \cdot gh \cdot hi \cdot in \cdot nx$

$p10 := af \cdot fg \cdot gh \cdot hi \cdot in \cdot nt \cdot tx$

$p11 := af \cdot fg \cdot gh \cdot hi \cdot in \cdot nt \cdot ty$

$p12 := af \cdot fg \cdot gh \cdot hi \cdot in \cdot no \cdot ov \cdot vu \cdot uy$

$p13 := af \cdot fg \cdot gh \cdot hi \cdot in \cdot no \cdot ov \cdot vz$

$p14 := ad \cdot dh \cdot hm \cdot mq \cdot qs \cdot sw$

$p15 := ad \cdot dh \cdot hm \cdot mq \cdot qs \cdot sw$

$p16 := ad \cdot dh \cdot hm \cdot mx$

$p17 := ad \cdot dh \cdot hi \cdot in \cdot nx$

$p18 := ad \cdot dh \cdot hi \cdot in \cdot nt \cdot tx$

$p19 := ad \cdot dh \cdot hi \cdot in \cdot nt \cdot ty$

$p20 := ad \cdot dh \cdot hi \cdot in \cdot no \cdot ov \cdot vu \cdot uy$

$p21 := ad \cdot dh \cdot hi \cdot in \cdot no \cdot ov \cdot vz$

$p22 := bd \cdot dh \cdot hm \cdot mq \cdot qs \cdot sw$

$p23 := bd \cdot dh \cdot hm \cdot mq \cdot qs \cdot szz$

$p24 := bd \cdot dh \cdot hm \cdot mx$

$p25 := bd \cdot dh \cdot hi \cdot in \cdot nx$

$p26 := bd \cdot dh \cdot hi \cdot in \cdot nt \cdot tx$

$p27 := bd \cdot dh \cdot hi \cdot in \cdot nt \cdot ty$

$p28 := bd \cdot dh \cdot hi \cdot in \cdot no \cdot ov \cdot vu \cdot uy$

$p29 := bd \cdot dh \cdot hi \cdot in \cdot no \cdot ov \cdot vz$

$p30 := cd \cdot dh \cdot hm \cdot mq \cdot qs \cdot sw$

$p31 := cd \cdot dh \cdot hm \cdot mq \cdot qs \cdot szz$

$p32 := cd \cdot dh \cdot hm \cdot mx$

$p33 := cd \cdot dh \cdot hi \cdot in \cdot nx$

$p34 := cd \cdot dh \cdot hi \cdot in \cdot nt \cdot tx$

$p35 := cd \cdot dh \cdot hi \cdot in \cdot nt \cdot ty$

$p36 := cd \cdot dh \cdot hi \cdot in \cdot no \cdot ov \cdot vu \cdot uy$

$p37 := cd \cdot dh \cdot hi \cdot in \cdot no \cdot ov \cdot vz$

$p38 := cj \cdot jo \cdot ov \cdot vu \cdot uy$

$p39 := cj \cdot jo \cdot ov \cdot vz$

$p40 := cj \cdot jk \cdot ko \cdot ov \cdot vu \cdot uy$

$p41 := cj \cdot jk \cdot ko \cdot ov \cdot vz$

$p42 := cj \cdot jk \cdot kv \cdot vu \cdot uy$

$p43 := cj \cdot jk \cdot kv \cdot vz$

$$p1t10 := (1 - p1) \cdot (1 - p2) \cdot (1 - p3) \cdot (1 - p4) \cdot (1 - p5) \cdot (1 - p6) \cdot (1 - p7) \cdot (1 - p8) \cdot (1 - p9) \cdot (1 - p10)$$

$$p11t20 := (1 - p11) \cdot (1 - p12) \cdot (1 - p13) \cdot (1 - p14) \cdot (1 - p15) \cdot (1 - p16) \cdot (1 - p17) \cdot (1 - p18) \cdot (1 - p19) \cdot (1 - p20)$$

$$p21t30 := (1 - p21) \cdot (1 - p22) \cdot (1 - p23) \cdot (1 - p24) \cdot (1 - p25) \cdot (1 - p26) \cdot (1 - p27) \cdot (1 - p28) \cdot (1 - p29) \cdot (1 - p30)$$

$$p31t40 := (1 - p31) \cdot (1 - p32) \cdot (1 - p33) \cdot (1 - p34) \cdot (1 - p35) \cdot (1 - p36) \cdot (1 - p37) \cdot (1 - p38) \cdot (1 - p39) \cdot (1 - p40)$$

$$p41t43 := (1 - p41) \cdot (1 - p42) \cdot (1 - p43)$$

$$R := 1 - (p1t10 \cdot p11t20 \cdot p21t30 \cdot p31t40 \cdot p41t43)$$

$$R = 0.99999778893133$$

Appendix G. System Attribute Levels

This appendix contains the input matrix used by Logical Decisions for Windows

for all systems examined. A system level score is presented for each of the attributes

found in the value hierarchy (see Appendix B).

Logical Decisions Levels for NEW ANALYSIS

	reliability	end-to-end delay	setup time	physical defense	accuracy	information rate	ease of transport	perception	interoperability	encryption	footprint	expansion capabilit	users
baseline	99.9998	100	0.684211	0	-5.31579	100	0.815789	60	0.5	100	90	1.1053	100
best	100	0	0	4	-8	200	1	100	1	100	100	10	200
GC	99.9999	84.3	0.743421	0	-5.31579	127.791	0.789474	60	0.519737	100	95	1.11404	100
GU	100	83.8	0.703947	0	-5.44737	128.571	0.796053	60	0.526316	100	95	1.14912	100
I1C	100	93	0.686403	0	-5.44737	112.529	0.802632	60	0.506579	100	90	1.14035	100
I2C	99.9999	92.8	0.695175	0	-5.52632	123.419	0.789474	60	0.519737	100	95	1.18421	100
I2U	100	85.4	0.677632	0	-5.60526	128.845	0.789474	60	0.526316	100	95	1.20175	100
I3C	99.9999	91.8	0.710526	0	-5.52632	125.878	0.782895	60	0.519737	100	95	1.18421	100
I3U	100	84.4	0.679824	0	-5.65789	131.304	0.78289	60	0.526316	100	95	1.2193	100
worst	0	200	5	0	-1	0	0	0	0	0	0	1	0

Logical Decisions Levels for NEW ANALYSIS

	anti-jam	redundancy	ease of use	intrusion preventio	hardening	anti-detection	complexity level	media types
baseline	4	1.1579	0.6447	0.1579	2	4	0.2895	2
best	10	5	1	5	3	10	1	5
GC	4.10526	1.26316	0.651316	0.210526	2	4.15789	0.315789	2
GU	4.10526	1.34211	0.631579	0.263158	2	4.15789	0.335526	2
I1C	4	1.26316	0.63816	0.23684	2	4.05263	0.315789	2
I2C	4.10526	1.34211	0.63158	0.289474	2	4.10526	0.335526	2
I2U	4.10526	1.39474	0.625	0.31579	2	4.10526	0.355263	2
I3C	4.10526	1.34211	0.638157	0.289474	2	4.10526	0.335526	2
I3U	4.10526	1.42105	0.625	0.34211	2	4.10526	0.361842	2
worst	0	0	0	0	0	0	0	0

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This effort focuses on expanding the bottleneck links of an existing C4I network to meet projected demands. The minimum cost-maximum flow algorithm is used to identify bottlenecks. A decision analysis approach enables the mapping of performance objectives for a network expansion into measureable attributes within the structure of a value hierarchy. A multi-attribute value function combines these measures into an overall system effectiveness measure. This allows for an evaluation of the impact of potential upgrade components on the system. A knapsack model imposes budgetary constraints on the selection of components. This methodology allows for the design of an optimized system upgrade based on system effectiveness, while offering an indication of the "value" of the increased information flow through the network.

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