
by

Paul J. Kopp
# MAJOR CARDEROCK DIVISION TECHNICAL COMPONENTS

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This report documents the mathematical model of a combined maneuvering, stationkeeping, and seakeeping simulation computer program for two different classes of mine hunting vessels. The model features a mix of physical based models for the propulsion systems and control surfaces and hydrodynamic coefficient based hull model. The approach follows the concept of the modular maneuvering model. A quasi-steady assumption is utilized for the seakeeping motion effects. This allows the calm water maneuvering motions to be calculated separately, and the six degrees of freedom linear responses in waves to be superimposed. The effects of wind, current, and second order mean drift forces are included.
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<th>Description</th>
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<tr>
<td>$A_p$</td>
<td>plane area of propeller race over rudder</td>
</tr>
<tr>
<td>$A_r$</td>
<td>planeform area of rudder</td>
</tr>
<tr>
<td>$A_T$</td>
<td>cross sectional area of thruster opening</td>
</tr>
<tr>
<td>$C_D$</td>
<td>drag coefficient</td>
</tr>
<tr>
<td>$C_{D_{rudder}}$</td>
<td>rudder drag coefficient, coplanar to rudder centerline plane</td>
</tr>
<tr>
<td>$C_{Dx}$</td>
<td>non-dimensional wave drift force in direction of ship x axis (surge)</td>
</tr>
<tr>
<td>$C_{Dy}$</td>
<td>non-dimensional wave drift force in direction of ship y axis (sway)</td>
</tr>
<tr>
<td>$C_{Dz}$</td>
<td>non-dimensional wave drift moment about ship z axis (yaw)</td>
</tr>
<tr>
<td>$C_L$</td>
<td>lift coefficient</td>
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<tr>
<td>$C_{L_{rudder}}$</td>
<td>rudder lift coefficient, normal to rudder centerline plane</td>
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<tr>
<td>$C_{x_{wind}}$</td>
<td>wind drag force coefficient of superstructure in ship x axis direction (surge)</td>
</tr>
<tr>
<td>$C_{x_{z_{wind}}}$</td>
<td>wind drag moment coefficient of superstructure in ship x axis direction</td>
</tr>
<tr>
<td>$C_{y_{wind}}$</td>
<td>wind drag force coefficient of superstructure in ship y axis direction (sway)</td>
</tr>
<tr>
<td>$C_{y_{z_{wind}}}$</td>
<td>wind drag moment coefficient of superstructure in ship y axis direction</td>
</tr>
<tr>
<td>$D$</td>
<td>propeller diameter</td>
</tr>
<tr>
<td>$D_{orbit}$</td>
<td>vertical axis propeller orbit diameter</td>
</tr>
<tr>
<td>$F_x$</td>
<td>force in ship x axis direction (surge)</td>
</tr>
<tr>
<td>$F_y$</td>
<td>force in ship y axis direction (sway)</td>
</tr>
<tr>
<td>$I_{xx}$</td>
<td>roll mass moment of inertia</td>
</tr>
<tr>
<td>$I_{zz}$</td>
<td>yaw mass moment of inertia</td>
</tr>
<tr>
<td>$J$</td>
<td>propeller advance coefficient</td>
</tr>
<tr>
<td>$J_{shaft}$</td>
<td>propeller shaft system rotational moment of inertia</td>
</tr>
<tr>
<td>$K_D$</td>
<td>vertical axis propeller open water torque coefficient</td>
</tr>
<tr>
<td>$K_Q$</td>
<td>open water propeller torque coefficient</td>
</tr>
<tr>
<td>$K_g$</td>
<td>vertical axis propeller open water thrust coefficient</td>
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<td>$K_T$</td>
<td>open water propeller thrust coefficient</td>
</tr>
<tr>
<td>$L$</td>
<td>ship length</td>
</tr>
<tr>
<td>$L_{blade}$</td>
<td>vertical axis propeller blade length</td>
</tr>
<tr>
<td>$L_{cg}$</td>
<td>turning radius to ship center of gravity</td>
</tr>
<tr>
<td>$L_{CL}$</td>
<td>turning radius to a point on the ship centerline</td>
</tr>
<tr>
<td>$L_{pt}$</td>
<td>turning radius to a point off the ship centerline</td>
</tr>
<tr>
<td>$L_{rod}$</td>
<td>turning radius to rudder</td>
</tr>
<tr>
<td>$M_x$</td>
<td>moment about ship x axis (roll)</td>
</tr>
<tr>
<td>$M_z$</td>
<td>moment about ship z axis (yaw)</td>
</tr>
</tbody>
</table>
m  ship mass
N  revolutions per minute
n  revolutions per second
P/D  propeller pitch/diameter ratio
P_{e}  effective horsepower (EHP)
Q  torque
Q_{e}  torque provided by the engine
Q_{prop}  torque absorbed by the propeller
R_{A}  transfer function amplitude
R_{i}  resistance force
r  yaw rate
S_{c}(\omega)  wave spectra
T  thrust
T_{0}  wave modal period
T'_{0}  non-dimensional wave modal period
T_{s}  ship draft
t_{rise}  second order throttle control system rise time
U  general external force vector
U_{ps}  vertical axis propeller blade peripheral speed
u  surge velocity, in ship coordinate system
u_{c}  surge velocity relative to current, in ship coordinate system
u_{w}  surge velocity relative to wind, in ship coordinate system
V  velocity magnitude
V_{a}  speed of advance
V_{c}  ship velocity relative to current
V_{current}  velocity of current
V_{rad}  velocity at rudder
\bar{V}_{rad}  effective velocity over rudder
V_{wind}  velocity of wind
v  sway velocity, in ship coordinate system
v_{c}  sway velocity relative to current, in ship coordinate system
v_{w}  sway velocity relative to wind, in ship coordinate system
X  general state variable vector
X', Y'  earth fixed coordinate system axes
X_{pos}, Y_{pos}  coordinates of ship center of gravity in earth fixed coordinate system
$x, y$  
ship coordinate system axes

$x', y'$  
translating coordinate system axes

$x_{pr}$  
separation distance between propeller and rudder

$x_{rudder}$  
rudder distance from center of gravity (positive forward)

$x_{motor}$  
thruster distance from center of gravity (positive forward)

$y_{pt}$  
distance of a point off the centerline (positive to starboard)

$y_{sep}$  
separation distance between a pair of propellers or rudders

$z_{prop}$  
vertical distance between propeller force vector and ship center of gravity (positive above the cg)

$z_{rudder}$  
vertical distance between rudder force vector and ship center of gravity (positive above the cg)

$z_{thruster}$  
vertical distance between thruster force vector and ship center of gravity (positive above the cg)

$z_{wave}$  
vertical distance between wave force vector and ship center of gravity (positive above the cg)

$z_{windage}$  
vertical distance between wind force center of pressure and ship center of gravity (positive above the cg)

$\Phi$  
blending function for ship speed

$\Psi$  
blending function for ship heading to the waves

$\alpha$  
inflow angle to propeller or rudder

$\beta$  
drift angle

$\beta_c$  
drift angle relative to current

$\beta_{CL}$  
drift angle at a point on the ship centerline

$\beta_{pt}$  
drift angle of a point off the ship centerline

$\beta_{prop}$  
drift angle at propeller

$\beta_{rudder}$  
drift angle at rudder

$\chi_{gear}$  
gear ratio

$\delta$  
rudder or steering angle

$\epsilon$  
flow straightening angle

$\epsilon_p$  
transfer function phase angle

$\phi$  
roll angle

$\gamma$  
random phase angle

$\eta_{shaft}$  
gine/shaft system efficiency

$\eta_1, \ldots, \eta_6$  
surge sway, heave, yaw, roll, and pitch in seakeeping coordinate system

$\Phi_{current}$  
compass angle of direction of current flow

$\Phi_{waves}$  
compass angle of direction of predominate waves

$\Phi_{wind}$  
compass angle of direction of wind

$\lambda$  
vertical axis propeller advance coefficient

$\rho$  
water density

$\rho_{air}$  
air density
\( \tau \)  
engine throttle setting (normalized between 0 and 1)

\( \tau_c \)  
commanded engine throttle setting (normalized between 0 and 1)

\( \omega_c \)  
encounter frequency

\( \omega_s \)  
ship natural roll frequency

\( \omega_{s\text{throt}} \)  
natural frequency of second order throttle control system

\( \zeta_r \)  
roll damping coefficient

\( \zeta_{s\text{throt}} \)  
throttle control system damping coefficient

\( \psi \)  
yaw angle

\( \zeta \)  
wave height

\( (\zeta)_{1/3} \)  
significant wave height

1-t  
thrust deduction factor

1-w\(_t\)  
thrust wake fraction
ABSTRACT

This report documents the mathematical model of a combined maneuvering, stationkeeping, and seakeeping simulation computer program for two different classes of mine hunting vessels. The model features a mix of physical based models for the propulsion systems and control surfaces and hydrodynamic coefficient based hull model. The approach follows the concept of the modular maneuvering model. A quasi-steady assumption is utilized for the seakeeping motion effects. This allows the calm water maneuvering motions to be calculated separately, and the six degrees of freedom linear responses in waves to be superimposed. The effects of wind, current, and second order mean drift forces are included.

ADMINISTRATIVE INFORMATION

This work was funded by the Coastal Systems Station, Panama City, Florida, under Work Request WR511991 and is identified at CDNSWC by Job Order Number 1-1561-432.

INTRODUCTION

The Carderock Division of the Naval Surface Warfare Center (CDNSWC) was originally tasked in 1992 by the Coastal Systems Station (CSS), Dahlgren Division, Naval Surface Warfare Center, in Panama City, Florida to develop a simulator model for ship maneuvering and stationkeeping, including seakeeping effects. That simulator was to be incorporated into a team training simulator for the MCM-1 (Avenger) class of mine hunter vessels. Training scenarios would cover all aspects of the MCM mission; mine sweeping, neutralization, and transiting, possibly in the presence of wind, waves, and/or a current. The result of this effort was called Version 1 of the MCMSIM computer program, as documented in [1].

In 1995, CSS tasked CDNSWC to expand the original simulator model to allow simulations of the MHC-51 (Osprey) class of mine hunters. The primary modification was to be the change of the propulsion system from controllable reversible pitch propellers to twin vertical axis propellers. Other changes and modifications include the addition of roll to the maneuvering equations of motion, a physically based engine model, real time graphical display of simulation results, and the option of automatic execution of standard maneuvers (turning circle, spiral, and zig-zag). The result of the modifications to the computer program (and underlying model) is now called MANSIM Version 2. This document is meant to replace reference [1] in its entirety.

Originally, the basic physical requirements for the combined maneuvering simulators were to operate in real time on an Intel 80386/80387 personal computer operating at 25 MHz, and be written in the FORTRAN programming language using no machine specific modules. For the development of MANSIM Version 2, an Intel 80486 DX 50 based computer was used with a 32bit FORTRAN compiler running on the Windows NT operating system. On this system, a minimum of five to ten times real time performance has been obtained (depending on
program execution options). The eventual platform for the simulator program is a Silicon Graphics model 4440 workstation operating at 40MHz. The source code will be translated by CSS into the ADA programming language for final inclusion into the team trainer system. In the final form, the simulation will be running at a 16Hz sample rate with approximately 20 to 30 milliseconds allowed for each time step.

For the original work, two different starting points were evaluated. First, one of the existing maneuvering programs could have been used and extensively modified to support the zero speed special case required for stationkeeping [2, 3, 4]. Second, a new program could have been written from scratch. Either option needed to be coupled with a wave induced ship motion calculation methodology. It was determined that because of the requirement of intrinsic support for zero speed as well as backing and sidewise motions, a new program should be written. This also allowed the new program to be written in a streamlined, minimal form for maximum computational speed.

The development of the original simulator model represented the first opportunity at CDNSWC to develop a maneuvering model based on the modular maneuvering model concept. The modular model concept differs from the traditional approach in several ways. The traditional approach is to use hydrodynamic coefficients derived from a Taylor series expansion of the forces and moments acting on a vessel [5]. The modular modeling approach treats the components of the forces and moments separately. For example, the forces due to the rudder are composed of the basic forces from the airfoil in a flow, the interaction of the hull on the rudder (expressed as flow blockage and straightening effects), and the interaction of the propeller on the rudder (through flow acceleration and straightening due to the propeller race). The forces on the hull would likewise be comprised of the bare hull forces and the interactions of the rudder and propeller on the hull. Implicit in the modular model concept is the use of physical based models for each of the components [6].

The modular maneuvering model offers several advantages over the coefficient based model. Since each component effecting maneuvering performance is separate in the modular model, changes can be isolated. This includes changes to program source code and model as well as changes to the ship design. For example, the effect of a different size or shape rudder is isolated to the rudder model and the interactions on other components. Hence, the hull force model or propeller force model remain unchanged. This is particularly useful in preliminary design where different concepts are investigated. In later phases of design, model test data may be available for a baseline design. The design can then be modified with different propellers or rudders without necessarily invalidating the previous model test work.

By using the modular maneuvering model concept, both the MCMSIM and MANSIM program codes have been written in a highly structured manner with clear divisions between force and moment contributors. This facilitates better internal documentation of the source code and allows easier maintenance and future expansion of capabilities.

This report documents the combined maneuvering, stationkeeping, and seakeeping simulation mathematical models used in the computer program. A separate programmers manual documents the computer program source code as implemented for the MCM and MHC class vessels.
COMBINED MANEUVERING, STATIONKEEPING, AND SEAKEEPING

In developing a time domain model of ship maneuvering in waves, there are several problems to be considered. There are frequency dependent coefficients in the equations of motion, unsteady ship speed and heading, effects of the past history of a body's motion (memory effects), and first and second order wave induced oscillatory exciting forces. Previous attempts to model ship maneuvering in waves have generally focused on the specific problems of course keeping ability, or capsizing and broaching. Few complete models that can represent general maneuvering in waves have been developed, with even fewer implementations of those models [7]. In general, such models are computationally expensive, and not well suited to preliminary design applications or real time, man-in-the-loop simulations.

The approach taken here is to de-couple the relatively slow maneuvering motions from the relatively faster seakeeping motions. This is accomplished by calculating the calm water maneuvering motions and adding the linear seakeeping responses. The advantage of this method is that the seakeeping motions can be pre-computed outside of the maneuvering simulation. Figure 1 shows the conceptual model that was developed as a framework for the development of the components of the model.

It is important to note that when estimating or experimentally determining the hydrodynamic characteristics of each component of the ship (hull, rudders, props, etc.), it is assumed that the body is moving with sufficient speed and has sufficient surface roughness to insure that the flow is fully turbulent. However, the simulation model does have the capability to operate at very low speeds where the flow could be laminar. In these cases, appropriate empirical corrections have been implemented to provide proper low speed behavior.

COORDINATE SYSTEMS

Within this simulator model, a multitude of coordinate systems and frames of reference are employed. The standard coordinate systems for ship motions and maneuvering are different from each other by a 180 degree rotation about the ship's x axis. The bottom of Figure 2 shows the orientation of both coordinate systems [5]. In this work, the standard maneuvering coordinate system will be used unless explicitly stated. Figure 2 shows the translating ship coordinate system (x',y'), the earth fixed coordinate system (X',Y'), and the ship reference coordinate system (x,y). The positive sense of the rudder and drift angles and propeller and bow thruster thrust are also shown. The origin of the ship coordinate systems are at the ship's center of gravity. Current and wind speed and direction and predominate wave direction are specified in earth fixed coordinate system using compass heading angles.
MANEUVERING EQUATIONS OF MOTION

The general, nonlinear equations of motion for a maneuvering ship in the horizontal plane, with roll, are given in [5],

\[ \begin{align*}
    m(\dot{u} - v \dot{\psi}) &= \sum F_x \\
    m(\dot{v} + u \dot{\psi}) &= \sum F_y \\
    I_{zz} \ddot{\psi} &= \sum M_z \\
    I_{xx} \ddot{\phi} + I_{xx} 2 \omega_n \xi \dot{\phi} + I_{xx} \omega_n^2 \sin \phi &= \sum M_x
\end{align*} \] (1)

The cross product terms on the left side of the x and y force equations are the result of those equations being written in the ship coordinate system. Note that the notation for the moment equations has changed from [1]. Rewriting equation (1) as a set of first order differential equations,

\[ \begin{align*}
    \dot{u} &= \frac{1}{m} (v r + \sum F_x) \\
    \dot{v} &= \frac{1}{m} (-ur + \sum F_y) \\
    \dot{r} &= \frac{\sum M_z}{I_{zz}} \\
    \dot{\phi} &= -(2 \omega_n \xi \dot{\phi} + \omega_n^2 \sin \phi) + \frac{\sum M_x}{I_{xx}}
\end{align*} \] (2)

where \( \dot{\psi} \) has been replaced by \( r \). The roll equation is still shown as a second order differential equation but is implemented as a set of two coupled first order differential equations. Relating the ship coordinate system back to the earth fixed coordinate system yields another set of first order differential equations,

\[ \begin{align*}
    \dot{X}_{pos} &= u \sin \psi + v \cos \psi \\
    \dot{Y}_{pos} &= u \cos \psi - v \sin \psi
\end{align*} \] (3)

Equations (2) and (3), when taken together, form a set of seven, first order nonlinear state equations*. The general form is given by

\[ \dot{X} = A(X) + B(U, X) \] (4)

where \( X \) is the state variable vector, \( U \) is the external force vector, and \( A \) and \( B \) are matrices representing the characteristics of the system. Solution of the time evolution of these equations is performed using either a first order Euler time step,

* the second order equation for roll in equation (2) can be written as a set of two first order equations.
\[ X^{n+1} = X^n + [A(X^n) + B(U^n, X^n)]\Delta t \]  

(5a)

or a fourth order Runge-Kutta time step,

\[ k_1 = [A(X^n) + B(U^n, X^n)]\Delta t \]

\[ k_2 = [A(X^n + (k_1/2)) + B(U^n, X^n + (k_1/2))]\Delta t \]

\[ k_3 = [A(X^n + (k_2/2)) + B(U^n, X^n + (k_2/2))]\Delta t \]

\[ k_4 = [A(X^n + k_3) + B(U^n, X^n + k_3)]\Delta t \]

\[ X^{n+1} = X^n + \frac{1}{6} [k_1 + 2k_2 + 2k_3 + k_4] \]  

(5b)

The first order Euler method is generally stable (for dynamically stable ships) and will not propagate errors if the time step size is relatively small compared to the speed of ship maneuvering motions. However, due to the presence of the restoring force term in the roll equation, the first order integration is numerically unstable if used. Therefore, the higher order method is required when roll is included. If roll is not included, the first order method may be used and the expense of the extra derivative evaluations avoided.

**EXTERNAL FORCES**

If we disregard the seakeeping aspects of the problem for now, the total external forces and moments acting on a maneuvering ship are represented by \( \sum F_x, \sum F_y, \sum M_z, \text{ and } \sum M_x \). These summations can be expanded into components for the hull, rudders, propellers, thrusters, wind, and waves [5].

\[ \sum F_x = F_{x_{\text{hull}}} + F_{x_{\text{rudder}}} + F_{x_{\text{prop}}} + F_{x_{\text{thrust}}} + F_{x_{\text{wind}}} + F_{x_{\text{waves}}} \]

\[ \sum F_y = F_{y_{\text{hull}}} + F_{y_{\text{rudder}}} + F_{y_{\text{prop}}} + F_{y_{\text{thrust}}} + F_{y_{\text{wind}}} + F_{y_{\text{waves}}} \]

\[ \sum M_z = M_{z_{\text{hull}}} + M_{z_{\text{rudder}}} + M_{z_{\text{prop}}} + M_{z_{\text{thrust}}} + M_{z_{\text{wind}}} + M_{z_{\text{waves}}} \]

\[ \sum M_x = M_{x_{\text{hull}}} + M_{x_{\text{rudder}}} + M_{x_{\text{prop}}} + M_{x_{\text{thrust}}} + M_{x_{\text{wind}}} + M_{x_{\text{waves}}} \]  

(6)

**CURRENT EFFECTS**

Hull forces are due the rotation and translation of the vessel through the water. In order to include the effects of a current, the relative velocity between the ship and the current must be calculated.

\[ u_c = V \cos \beta + V_{\text{current}} \cos(\varphi_{\text{current}} - \psi) \]

\[ v_c = -V \sin \beta - V_{\text{current}} \cos(\varphi_{\text{current}} - \psi) \]  

(7)

where the ship velocity and drift angle are
\[ V = \sqrt{\dot{X}_{\text{pos}}^2 + \dot{Y}_{\text{pos}}^2} \]

\[ \beta = \psi - \arctan \frac{\dot{X}_{\text{pos}}}{\dot{Y}_{\text{pos}}} \]

and the drift angle is considered zero if both the \(X_{\text{pos}}\) and \(Y_{\text{pos}}\) velocities are zero. The velocity and drift of the ship relative to the current is therefore

\[ V_c = \sqrt{u_c^2 + v_c^2} \]

\[ \beta_c = \arctan \frac{-v_c}{u_c} \]  

(9)

Using the relative velocity through the water requires that the surge and sway equations in equation (2) be rewritten as

\[ \dot{u}_c = \frac{1}{m} (v_c r + \sum F_x) \]

\[ \dot{v}_c = \frac{1}{m} (-u_c r + \sum F_y) \]  

(10).

Equation (3) must then be corrected for the current and written as

\[ \dot{X}_{\text{pos}} = \hat{u} \sin \psi + \hat{v} \cos \psi \]

\[ \dot{Y}_{\text{pos}} = \hat{u} \cos \psi - \hat{v} \sin \psi \]  

(11)

where

\[ \hat{u} = u_c - V_{\text{current}} \cos(\psi - \phi_{\text{current}}) \]

\[ \hat{v} = v_c + V_{\text{current}} \sin(\psi - \phi_{\text{current}}) \]  

(12).

HULL FORCE MODEL

As stated in the introduction, the traditional approach to modeling hull forces has been to assume the linearity of forces and expand them with a Taylor series expansion. This is still the most straightforward approach to developing hull forces as a function of surge, sway, and yaw motions. Here we will not include the effects of rudder, props, or bow thrusters in the expansion.

If the hull forces are considered to be functions of surge, sway, and yaw motion only, we may write
\[ F_{x_{null}} = \frac{\partial F_x}{\partial \theta} v + \frac{\partial F_x}{\partial \dot{\theta}} \dot{v} + \frac{\partial F_x}{\partial \phi} r + \frac{\partial F_x}{\partial \dot{\phi}} \dot{r} + \frac{\partial F_x}{\partial \chi} \chi + \frac{\partial F_x}{\partial \dot{\chi}} \dot{\chi} \ldots \]

\[ F_{y_{null}} = \frac{\partial F_y}{\partial \theta} v + \frac{\partial F_y}{\partial \dot{\theta}} \dot{v} + \frac{\partial F_y}{\partial \phi} r + \frac{\partial F_y}{\partial \dot{\phi}} \dot{r} + \frac{\partial F_y}{\partial \chi} \chi + \frac{\partial F_y}{\partial \dot{\chi}} \dot{\chi} \ldots \]

\[ M_{z_{null}} = \frac{\partial M_z}{\partial \theta} v + \frac{\partial M_z}{\partial \dot{\theta}} \dot{v} + \frac{\partial M_z}{\partial \phi} r + \frac{\partial M_z}{\partial \dot{\phi}} \dot{r} + \frac{\partial M_z}{\partial \chi} \chi + \frac{\partial M_z}{\partial \dot{\chi}} \dot{\chi} \ldots \]

\[ M_{x_{null}} = \frac{\partial M_x}{\partial \theta} v + \frac{\partial M_x}{\partial \dot{\theta}} \dot{v} + \frac{\partial M_x}{\partial \phi} r + \frac{\partial M_x}{\partial \dot{\phi}} \dot{r} + \frac{\partial M_x}{\partial \chi} \chi + \frac{\partial M_x}{\partial \dot{\chi}} \dot{\chi} \ldots \] (13)

where any potential higher order and cross coupling terms have been omitted. The derivatives, also known as hydrodynamic or maneuvering coefficients, are usually written in a shorthand notation such that \( \frac{\partial F_i}{\partial \theta} = X_i \),

\( \frac{\partial F_i}{\partial \dot{\theta}} = Y_i \), \( \frac{\partial M_i}{\partial \theta} = N_i \), and \( \frac{\partial M_i}{\partial \dot{\theta}} = K_i \), for example. The acceleration terms in the equations basically represent low frequency added mass effects [5].

Hydrodynamic coefficients can be obtained from captive model tests, estimation, or system identification from full scale trials or free running model experiments. Each of these methods has its own merits and drawbacks. A discussion of these is beyond the scope of this document and the reader is referred to the references [5, 8, 9, 10, 11].

The normal practice is to express the hydrodynamic coefficients in non-dimensional form, for example

\[ Y_i = \frac{Y_i}{\frac{1}{2} \rho L^2 V_c^2} \] (14)

Here, the velocity in the denominator is the relative velocity through the water. When a non-dimensional hydrodynamic coefficient is used to calculate the non-dimensional force or moment, it implies that the associated variable (v in the example case of equation (14)) to be multiplied is also in its non-dimensional form. Hence, the dimensional force is obtained from summing the products of the non-dimensional coefficients and associated non-dimensional variables, and then multiplying by the factor \( \frac{1}{2} \rho L^2 V_c^2 \) (L^3 for moments)

Within the series expansion for the x force, a term has been added to account for the hull resistance. The effective power (EHP) curve is represented as a polynomial curve from which the hull resistance is obtained,

\[ P_E(V) = V^3 [c_1 V + c_2 V^2 + c_3 V^3] \]

\[ R_T = \frac{550 P_E}{V} \] (15)

The polynomial coefficients are obtained from curve fits of upright straight line (zero drift angle) resistance tests [12]. The ship speed applied to equation (15) in the program implementation is the x component of the relative speed through the water (\( w_c \)). In the special case of zero ship speed, the resistance is zero.
When running the simulator program for low ship speeds, it was observed that the behavior of the model was incorrect. If the vessel was allowed to decelerate from some nominal forward speed, the model did not show the vessel coming to rest within a reasonable time. To correct this behavior, an empirical correction has been made for ship speeds of 5 knots and below. This correction takes the form of a multiplying factor to the hull resistance \( R_f \) in equation (15). Table 1 lists the correction factors used in the model.

**Rudder Force Model**

In the case of the MHC, the rudders are replaced by the omni directional thrust provided by the vertical axis propellers. However, for the MCM, the rudder is the primary device used to maintain directional control. In order to determine the forces acting on the rudder, the actual velocity and direction of the flow at the rudder must be determined from the general motion of the ship. Figure 3 shows a schematic representation of a ship in a turn. If the velocity components of the ship center of gravity (\( c_g \)) and the yaw rate are known then the drift angle, turning radius, and inflow angle at the rudder (or any arbitrary point along the vessel centerline) is calculated from

\[
L_{c_g} = \frac{V_c}{r}
\]

\[
L_{\text{rad}} = \text{sign}(r) \cdot \sqrt{(x_{\text{rad}} \cos \beta_c)^2 + (L_{c_g} - x_{\text{rad}} \sin \beta_c)^2}
\]

\[
\beta_{\text{rad}} = \beta - \text{sign}(r) \cdot \text{atan} \left( \frac{x_{\text{rad}} \cos \beta_c}{L_{c_g} - x_{\text{rad}} \sin \beta_c} \right)
\]

\[
V_{\text{rad}} = |r L_{\text{rad}}|
\]

\[
\alpha = \beta_{\text{rad}} + \delta - \epsilon
\]

(16)

In the case of \( r=0 \) and \( V \neq 0 \), then the turning radius is infinite and \( \beta_{\text{rad}} = \beta_c \) and \( V_{\text{rad}} = V_c \). The rudder angle is given by the commanded rudder angle up to the point that a set maximum rudder rate or the maximum allowed angle is achieved.

When the point in question is not on the ship centerline, an additional correction is required. If the turning radius and drift angle of a point on the centerline is \( L_{\text{CL}} \) and \( \beta_{\text{CL}} \), then the turning radius and drift angle of a point off the centerline is

\[
L_{\text{pt}} = \text{sign}(L_{\text{CL}}) \cdot \sqrt{(y_{\text{pt}} \sin \beta_{\text{CL}})^2 + (L_{\text{CL}} - y_{\text{pt}} \cos \beta_{\text{CL}})^2}
\]

\[
\beta_{\text{pt}} = \beta_{\text{CL}} + \text{sign}(L_{\text{CL}}) \cdot \text{atan} \left( \frac{y_{\text{pt}} \sin \beta_{\text{CL}}}{L_{\text{CL}} - y_{\text{pt}} \cos \beta_{\text{CL}}} \right)
\]

(17)
The correction, $\varepsilon$ in equation (16), is a flow straightening coefficient which represents the blockage of the hull to the flow. It has been seen that the flow straightening is primarily dependent on the geometric drift angle at the rudder, $\beta_{\text{rud}}$. A relation derived from the data presented in [13] is employed,

$$
\varepsilon = c_0 + c_1 \beta_{\text{rud}} + c_2 \beta_{\text{rud}}^3 + c_3 \beta_{\text{rud}}^5
$$

where it is assumed that $\varepsilon = 0$ if $|\beta_{\text{rud}}| \geq 90$ degrees.

There is an additional interaction effect is on the effective flow velocity when the propeller race is directed over the rudder [14],

$$
\overline{v}_{\text{rud}}^2 = \frac{A_p}{A_r} \left[ (1 - w_T) v_{\text{rud}} \cos \beta_{\text{rud}} + k v_{\infty} \right] + \frac{A_r - A_p}{A_r} (1 - w_T)^2 v_{\text{rud}}^2 \cos^2 \beta_{\text{rud}}
$$

where

$$
v_{\infty} = - (1 - w_T) v_{\text{rud}} \cos \beta_{\text{rud}} + \sqrt{ (1 - w_T)^2 v_{\text{rud}}^2 \cos^2 \beta_{\text{rud}} + \frac{8}{\pi} K_T n^2 D^2 }
$$

$$
k = 0.5 + 0.26527 \tanh (1.2775 \xi) + 0.17533 \tanh (2.555 \xi)
$$

$$
\xi = 2x_{p-r} / D
$$

Some portion of the propeller race will be directed over the rudder when: $u_c > 0$ and $p/d > 0$, and possibly $u_c < 0$ and $p/d > 0$.

The forces acting on the rudder are given by the lift and drag coefficients. As the rudder will operate in a diversity of flow conditions, lift and drag characteristics are needed for forward, reverse, and sideways flow [15]. Like the hull hydrodynamic coefficients, rudder lift and drag coefficients are generally determined in a fully turbulent flow condition. These coefficients are used at all times in the simulator model for both port and starboard rudders, even for low speed operation.

Lift and drag are defined respectively as parallel and normal to the flow. In order to obtain the forces acting on the hull, a transformation is performed from the flow reference frame to the rudder reference frame and finally to the moving ship’s coordinate system. This is done using

$$
C_{L_{\text{rud}}} = C_L \cos \alpha + C_D \sin \alpha
$$

$$
C_{D_{\text{rud}}} = C_D \cos \alpha - C_L \sin \alpha
$$

and
\[ F_x = \left( C_{1_{\text{rad}}} \cos \delta - C_{D_{\text{rad}}} \sin \delta \right) \left( \frac{1}{2} \rho V_{\text{rad}}^2 A_{r} \right) \]
\[ F_y = \left( -C_{D_{\text{rad}}} \cos \delta - C_{1_{\text{rad}}} \sin \delta \right) \left( \frac{1}{2} \rho V_{\text{rad}}^2 A_{r} \right) \]
\[ M_z = F_y x_{\text{rad}} + \frac{y_{\text{sep}}}{2} F_x \]
\[ M_x = F_y z_{\text{rad}} \]

(22).

For the yaw moment, the positive sign is taken for the port rudder and the negative for the starboard rudder. Though the Rudders on the MCM operate together, each will experience a different inflow geometry and potentially different propeller races. Therefore, the forces and moments should be calculated for each rudder separately and summed together. The machinery model for the steering gear imposes limits on the extremes of position and the maximum rate of change of position.

**PROPELLER FORCE MODEL**

The propellers are the primary means for the ship operator to control the speed of his vessel and for the MHC, they are also the only devices for providing directional control. For the MCM, the propellers can provide some directional control when the two propellers are operated individually at different speeds and/or pitch settings.

Since the MCM has controllable reversible pitch (CRP) propellers, the full four quadrant open water propeller performance at all pitch settings must be modeled. The open water propeller data is represented as curves of thrust and torque coefficients \( (K_T \text{ and } K_Q \text{ respectively}) \) over a range of negative and positive advance coefficient, \( J \) [16]. These are defined as
\[ J = \frac{V_a}{nD}, \quad K_T = \frac{T}{\frac{1}{2} \rho n^2 D^4}, \quad \text{and} \quad K_Q = \frac{Q}{\frac{1}{2} \rho n^2 D^5} \]

(23).

where \( V_a \) is the speed of advance through the water, \( n \) is the propeller speed, \( D \) is the propeller diameter, \( T \) is the propeller thrust, and \( Q \) is the propeller torque. For the purposes of this model, the speed of advance is
\[ V_a = V(1 - w_T) \]

(24).

where \((1 - w_T)\) is the thrust wake fraction and \( V \) is the x component of the velocity of a point at the propeller hub (refer to the discussion in the previous rudder force section). The thrust wake fraction represents a blockage effect of the hull on the flow to the propeller and is usually determined by powering experiments. In straight line motion (zero drift angle), the wake fraction is the measured value from experiments [12]. However, as the ship maneuvers and operates at non-zero drift angles, there will be less blockage by the hull on the flow to the propellers. For inflow angles less than 90 degrees, a cosine squared correction is used,
\[ (1 - w_T) = \sin^2 \beta_{\text{prop}} + (1 - w_T)_{(\beta_{\text{prop}} = 0)} \cos^2 \beta_{\text{prop}} \]

(25).

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and \((1-w_T)=1\) for angles greater than 90 degrees. Here is the drift angle of a point at the propeller hub. Since the propellers are to operated independently from each other, the velocity \(V\) in equation (24) and drift angle \(\beta_{prop}\) in equation (25) are calculated for both port and starboard propellers.

A polynomial curve fit for each \(K_T\) and \(K_Q\) curve is made for each pitch/diameter value and both positive and negative \(J\). Reference [17] gives twelfth order polynomial coefficients for the MCM design propeller. In order to obtain the operating values for either port or starboard propeller, the polynomials are evaluated at the appropriate value of advance coefficient. Linear interpolation is used between curves to the actual propeller pitch/diameter setting. The engine model dynamics and state equations (discussed later) determine the rpm to be used in calculating \(J\) for the port and starboard props.

Having the total thrust calculated from the port and starboard propeller \(K_T\) values, the thrust must be decreased by the thrust deduction factor. Thrust deduction is corrected for non-zero inflow angles in a similar manor to that used on the wake fraction in equation (25). Propeller thrust acts only along the longitudinal direction of the ship hull. No oblique flow effect are included except for the wake fraction and thrust deduction factor corrections. As a result, and by assumption, no side forces are exerted. For the MCM model, the forces and moments delivered by the propellers to the ship are given by,

\[
F_x = K_T (1-t) \left( \frac{1}{2} \rho n^2 D^4 \right) \\
F_y = 0 \\
M_z = \pm \frac{y_{sep}}{2} F_x \\
M_x = 0
\]

For the yaw moment, the positive sign is taken for the port propeller and the negative for the starboard propeller.

The MHC vertical axis propellers use a different model based on information provided by J.M. Voith GmbH*. Define the thrust and torque coefficients (\(K_S\) and \(K_D\) respectively) as,

---

* Unpublished data.
\[ U_{ps} = nD_{\text{orb}}, \pi \]

\[ \lambda = \frac{V_s}{U_{ps}} \]

\[ K_s = \frac{T}{\frac{1}{2} \rho D_{\text{orb}} L_{\text{blade}} U_{ps}^2} \]

\[ K_D = \frac{2Q}{\frac{1}{2} \rho D_{\text{orb}}^2 L_{\text{blade}} U_{ps}^2} \]

The value of \( U_{ps} \) is the orbital velocity of a propeller blade (tangential velocity) and \( \lambda \) is the ratio of the inflow velocity at the propeller to the blade orbital velocity. Note that for the data obtained, \( \lambda \) is limited in value to the range of 0 to 1. Open water characteristics of the vertical axis propellers are given as curves of \( K_s \) and \( K_D \) versus \( \lambda \) for various pitch settings. Polynomial curve fits are used to determine the values of \( K_s \) and \( K_D \) for given values of \( \lambda \) and pitch setting. In a manner similar to that used in the rudder model, the force produced by the propeller and imparted to the hull must be resolved into longitudinal and lateral forces. For the MHC, the forces and moments delivered by the propellers to the ship are given by,

\[ \alpha = \beta_{\text{prop}} + \delta \]

\[ F_{\text{prop-x}} = K_s \left( \frac{1}{2} \rho D_{\text{orb}} L_{\text{blade}} U_{ps}^2 (1-t) \cos \alpha \right) \]

\[ F_{\text{prop-y}} = K_s \left( \frac{1}{2} \rho D_{\text{orb}} L_{\text{blade}} U_{ps}^2 (1-t) \sin \alpha \right) \]

\[ F_x = F_{\text{prop-x}} \cos \beta_{\text{prop}} - F_{\text{prop-y}} \sin \beta_{\text{prop}} \]

\[ F_y = F_{\text{prop-x}} \cos \beta_{\text{prop}} + F_{\text{prop-y}} \sin \beta_{\text{prop}} \]

\[ M_z = F_y z_{\text{prop}} \pm \frac{y_{\text{sep}}}{2} F_x \]

\[ M_x = F_y z_{\text{prop}} \]

For the yaw moment, the positive sign is taken for the port propeller and the negative for the starboard propeller.

Here the forces are first resolved into xy components relative to the inflow velocity and then resolved into ship xy components. Note that a flow straightening correction is not included in the definition of \( \alpha \).

**BOW THRUSTER FORCE MODEL**

The bow thruster is an auxiliary device for providing directional control. The MCM class of vessels use a bow thruster which has a sea chest in the keel of the vessel for a water inlet and a split so that the flow travels to either a port or starboard side outlet. For the purposes of this simulator, a relatively simple model will be used. Note that the MHC does not have a thruster of any kind.
The ship operator will issue a thruster command of between +100% and -100% thrust. A second order polynomial is used to transform the command setting to the full scale thrust. The data needed to perform the curve fit is usually provided by the thruster manufacture and should assume no forward speed. Forward speed degradation effects have been studied by Chislett and Bjorheden [18] and McCreight has performed curve fits of the published data [14]. The thruster forces can be determined from

\[ F_x = 0 \]

\[ F_y = T \left[ \exp(-13.3m^2) + 0.627396m - 0.385772m^2 + 0.124873m^3 \right] \]

\[ M_z = \left[ T + (1 - \frac{2m}{3}) (F_y - T) \right] x_{thruer} \]

\[ M_x = F_y z_{thruer} \]  

(29)

where

\[ m = \frac{V \cos \beta}{\sqrt{T/\rho A_T}} \]  

(30)

and T is the thruster thrust with no ship velocity. It is assumed that m=0 if T=0. For a ship moving aft, it is assumed that there is negligible degradation. This is represented here by m=0 for \( \beta_c = 180 \) degrees. The velocity and drift angle at the thruster should also be calculated for the ship in a turn using the method outlined in equation (16).

WIND FORCE MODEL

In a manner similar to the current and relative speed through the water, a transform between the wind speed and direction in the earth fixed reference system and the ship reference system is given by

\[ u_w = V \cos \beta + V_{wind} \cos(\varphi_{wind} - \psi) \]

\[ v_w = -V \sin \beta - V_{wind} \cos(\varphi_{wind} - \psi) \]

\[ V_w = \sqrt{u_w^2 + v_w^2} \]

\[ \beta_w = \tan^{-1} \frac{-v_w}{u_w} \]  

(31)

The sway force and moment coefficients are defined as the side force and yaw moment (non-dimensionalized) acting on the ship when the wind is blowing across the port beam. The surge force and moment coefficients are defined when the wind is blowing across the bow. The total force and moment contribution from the wind is a blended value of the two sets of wind force and moment coefficients [14].
\[ F_x = \frac{1}{2} \rho_{\text{air}} V_w^2 \left( \text{Area}_x C_{x,\text{wind}} \cos \beta_w \right) \]

\[ F_y = \frac{1}{2} \rho_{\text{air}} V_w^2 \left( \text{Area}_y C_{y,\text{wind}} \sin \beta_w \right) \]

\[ M_z = \frac{1}{2} \rho_{\text{air}} V_w^2 \left[ \left( \text{Area}_x C_{xz,\text{wind}} \cos 2\beta_w \right) + \left( \text{Area}_y C_{yz,\text{wind}} \sin 2\beta_w \right) \right] \]

\[ M_x = F_y z_{\text{windage}} \]  

(32).

WAVE DRIFT FORCE MODEL

The wave drift forces being addressed here are the mean, second order wave drift forces. This is an acceptable model for use in real time man-in-the-loop simulation where calculation of the full time varying second order forces would be burdensome. The mean wave drift forces are represented by

\[ F_x = \rho g B \xi_{1/3}^2 C_{D_x} \left( T_0, \phi_{\text{wave}} \right) \]

\[ F_y = \rho g B \xi_{1/3}^2 C_{D_y} \left( T_0, \phi_{\text{wave}} \right) \]

\[ M_z = \rho g B \xi_{1/3}^2 C_{D_z} \left( T_0, \phi_{\text{wave}} \right) \]

\[ M_x = F_y z_{\text{wave}} \]  

(33)

where

\[ T_0' = T_0 \sqrt{\frac{g}{T_x}} \]  

(34)

is the non-dimensional wave modal period [14].

McCreight and Jones have investigated the wave drift forces on an MCM model in uni-directional regular seas [19]. Since then, McCreight has developed an unpublished interpolation method for the original model test data. The non-dimensional wave drift force coefficients used the simulation computer program are determined by interpolation of the test data.

ENGINE MACHINERY MODEL

The MCM-1 is powered by four Waukesha LN 1616 DSIN diesel engines geared to two Transamerica-Delaval reduction gears. There are also twin light load electric motors that can be used. For this model, it is reasonable to assume that two diesel engines driving one shaft are represented by one single large engine driving its own shaft with no cross coupling between shafts. No separate model for the light load motors is included. Even though the MHC power plants are different than those on the MCM, the basic power plant dynamics represented by this model are sufficient for performing simulations for both vessels.

The original engine machinery model used in MCMSIM Ver. 1 was a constant torque model [20]. While that model worked, it did require solving for the roots of a twelfth order polynomial. Experience with the constant torque model implemented in MCMSIM Ver. 1 has shown that in transient conditions, the actual torque output of the
engine (computed from root solving) exhibited unrealistic behavior (signal spikes). This behavior led to the development and inclusion of a more realistic and physical based engine model.

Define the rate of change of the propeller shaft speed to be,

$$\dot{n} = \frac{Q_e - Q_{prop}}{J_{shaft}}$$  \hspace{1cm} (35)

where $Q_e$ is the torque produced by the engine for a given throttle setting, $Q_{prop}$ is the torque absorbed by the propeller, and $J_{shaft}$ is the rotational inertia of the propeller, shaft, gears, and engine crank shaft. This is a first order differential equation and is included in the total system of equations solved in equation (4). The torque absorbed by the propeller is computed from,

$$Q_{prop} = \begin{cases} K_0 \left( \frac{1}{2} \rho n^2 D^5 \right) & \text{[for MCM]} \\ \frac{K_D}{2} \left( \frac{1}{2} \rho D_{orbit} L_{blade} U_{ps}^2 \right) & \text{[for MHC]} \end{cases}$$  \hspace{1cm} (36)

depending on which vessel is being simulated.

The engine throttle setting is given as 0 to 100% where maximum engine torque is ordered for a 100% throttle setting. However, engine torque does not respond instantaneously to throttle setting. Therefore, a critically damped second order control system is used to model the response of the engine to throttle setting. If the actual engine throttle setting is $\tau$ and the commanded throttle setting is $\tau_c$, then the engine torque is computed from,

$$\ddot{\tau} = \tau_c - \omega_{ramp}^2 \tau - 2 \xi_{throt} \omega_{ramp} \dot{\tau}$$

$$Q_e = (X_{gear} \eta_{shaft}) (\tau Q_e \text{MAX})$$  \hspace{1cm} (37)

The second order control system is implemented as a pair of coupled first order differential equations and solved with the equations of motion. Losses in the system are accounted for by the gear ratio $X_{gear}$ and the efficiency of the system $\eta_{shaft}$ (includes losses due to friction). For the throttle control system to be critically damped, $\xi_{throt} = 1$. The system can then be defined in terms of a “rise time” which is the time that it takes for the throttle position to rise from 10% to 90% setting (starting from 0%) due to a step command of 0% to 100% [21],

$$\omega_{ramp} \equiv 3.5/t_{\text{rise}}$$  \hspace{1cm} (38)

**SEAKEEPING EFFECTS**

The six degree of freedom responses are governed by frequency domain transfer functions, calculated using a linear ship motions computer program [22]. Time histories for the six degrees of freedom are generated from the amplitude and phase information contained within the transfer functions,

$$\eta_i(t) = \sum_k \left[ (R_{A_i} \xi_k) \cos (\omega_{e_i} t + \gamma_{e_i} + \epsilon_{e_i}) \right]$$  \hspace{1cm} (39)
This gives the motion at time $t$ where the summation is over discrete frequency values, $R_n$ and $\varepsilon_p$ is the transfer function amplitude and phase at the $k^{th}$ wave frequency, $\omega_k$ is the frequency of encounter at the $k^{th}$ wave frequency, and $\xi_k$ is the wave height at the $k^{th}$ wave frequency defined by

$$\xi_k = \sqrt{2 \int_{\omega_k - \Delta \omega/2}^{\omega_k + \Delta \omega/2} S_\xi(\omega) \, d\omega} \tag{40}.$$ 

A uniformly distributed random phase angle, $\gamma_k$, is included. Equation (39) applies specifically to long crested seas. Short crested sea responses require an additional summation over wave direction [23].

The time history data files used are composed of 576 individual six degree of freedom time histories. They are for six ship speeds (0 to 10 knots in 2 knot increments), twenty four headings (0 to 345 degrees in 15 degree increments), and four significant wave height/modal period combinations; 1 foot and 7 seconds (Sea State 2), 3 feet and 7 seconds (Sea State 3), 6.2 feet and 9 seconds (Sea State 4), and 10.7 feet and 9 seconds (Sea State 5). Each of these individual time histories is 10 minutes in length at 4 samples per seconds with short crested seas assumed. All of the individual time histories are concatenated with the others in a particular order to create a single composite time history data file. Each composite time history data file therefore represents a 10 minute sample of ship motions for a particular sea state.

Access to the time history data by the maneuvering simulator is performed using a data table look-up with linear interpolation between heading and speed as needed. However, it should be understood that linear ship motion theory assumes constant mean heading and constant mean ship speed. Both the transfer functions and generated time histories are sensitive to speed and heading variations. Hence, an interpolated time history for a given mode of motion may not necessarily be the same as a time history derived from a transfer function at that identical speed and heading.

Interpolation between speed and heading is performed using linear blending functions. These functions are dependent only on speed or heading and can therefore be used for all modes of motion without recomputing. For example, the ship speed and relative heading to the waves are given at discrete values $V_1, \ldots, V_N$ and $\lambda_1, \ldots, \lambda_N$ respectively. The blending functions for ship speed ($\Phi$) and heading ($\Psi$) when interpolating to speed $V$ and heading $\lambda$ between the $i^{th}$ and the $i+1^{th}$ speed and $j^{th}$ and $j+1^{th}$ heading are then,

$$\Phi(i+1) = 1 - \frac{V - V_{i+1}}{V_{i+1} - V_i},$$

$$\Phi(i) = 1 - \Phi(i+1),$$

$$\Psi(j+1) = 1 - \frac{\lambda - \lambda_{j+1}}{\lambda_{j+1} - \lambda_j},$$

$$\Psi(j) = 1 - \Psi(j+1) \tag{41}.$$ 

Interpolating at time $t$ takes place between time history values at speeds $i$ and $i+1$ and heading $j$ and $j+1$,
\[ \eta(t, V, \lambda) = [\eta(t, V, \lambda, \lambda_{j+1}) \Psi(j+1) + \eta(t, V, \lambda) \Psi(j)] \Phi(i+1) + \\
[\eta(t, V, \lambda_{j+1}) \Psi(j+1) + \eta(t, V, \lambda) \Psi(j)] \Phi(i) \] (42).

**SUMMARY**

A description of the mathematical model used in developing a simulator for ship maneuvering, stationkeeping, and seakeeping has been presented. The model is based on the concept of a modular ship maneuvering model. This treats the ship hull, rudders, propellers, and bow thruster as individual components, modeled separately using methods appropriate for each component with interaction effects between components accounted for.

Two of the originally planned modifications to the model have been incorporated; the inclusion of ship roll to the maneuvering equation of motion and the ability to model vertical axes propellers. Within the computer program implementation of the model, these two modifications led to the complete reformulation of the structure of the generation of the state equations, the addition of a higher order integration method, and a rewrite of the engine dynamics model. The implementation of MANSIM Version 2 has also included a graphical presentation of the simulation as it advances in time.

More changes to the computer model and computer program code are planned. These changes could include; towing forces, shallow water effects, ship-ship interaction effects, ship-shore interaction effects, time varying second order wave drift forces, man in the loop control input, and automatic control systems for dynamic positioning. An important change for the computer program will be the replacement of the “hard wired” input data, which represent the vessel model, with a file based data representation. This will make the computer program capable of more general vessel simulations. Another important modification is to improve upon the method used for the superposition of seakeeping motions on the calm water maneuvering motions, possible adding the capability to solve the full maneuvering in waves problem.
Table 1. Low Speed Hull Resistance Correction Factors

<table>
<thead>
<tr>
<th>Ship Speed (knots)</th>
<th>Hull Resistance Correction Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_k \geq 5.0$</td>
<td>1.0</td>
</tr>
<tr>
<td>$5.0 &gt; V_k \geq 4.0$</td>
<td>1.2</td>
</tr>
<tr>
<td>$4.0 &gt; V_k \geq 3.0$</td>
<td>2.5</td>
</tr>
<tr>
<td>$3.0 &gt; V_k \geq 2.0$</td>
<td>5.0</td>
</tr>
<tr>
<td>$2.0 &gt; V_k \geq 1.0$</td>
<td>10.0</td>
</tr>
<tr>
<td>$1.0 &gt; V_k \geq 0.5$</td>
<td>50.0</td>
</tr>
<tr>
<td>$V_k &lt; 0.5$</td>
<td>100.0</td>
</tr>
</tbody>
</table>
Figure 1. Conceptual Model
Coordinate System and Sign Conventions

(Earth fixed coordinate system)

Figure 2. Coordinate Systems and Sign Conventions
Rudder Inflow Angle Geometry

Figure 3. Rudder Inflow Angle Geometry
REFERENCES


