The aim of the research was to further our understanding of the natural sources of sound near the ocean surface, which are known to be due mainly to oscillating bubbles and bubble clouds. A theory has been given for the damping of bubble oscillations by nonlinear coupling between different modes of oscillation of a spherical bubble. Some experimental confirmation was found by later workers. A simple statistical model has been proposed for the initial bubble sizes from breaking waves, which also has received experimental support. A direct method of calculating wave-generated ripples has been proposed, which accounts quantitatively for observations. It has been shown that such capillary waves can be important sources of vorticity, and hence of turbulence, near the crests of short gravity waves, and can lead to a certain a type of wave breaking. Other investigations concern the dynamics of capillary-gravity waves of solitary type on deep water, the emission of sound by the controlled release of bubbles from an underwater nozzle, and the formation of inward-going jets in bubble splitting or bubble collapse. Such jets are shown to be dynamically similar to those occurring in standing surface waves, or waves breaking against a harbor wall. Several other aspects of wave breaking have also been investigated.
ONR Grant N00014-91-J-1582:

Acoustical Emission from Bubbles

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FINAL REPORT
INTRODUCTION

This Contract was a sequel to Contract LJI-R-90-544: “Underwater Sound Generation at Low Wind Speeds,” and was based on the following Research Proposals:

<table>
<thead>
<tr>
<th>Date</th>
<th>Title</th>
<th>Period</th>
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<tbody>
<tr>
<td>1 Feb. 91</td>
<td>Acoustical Emission from Bubbles</td>
<td>1 Mar. - 28 Feb. 92</td>
</tr>
<tr>
<td>7 Jan. 91</td>
<td>Dynamics of Bubbles and Bubble Clouds</td>
<td>1 Mar. - 28 Feb. 94</td>
</tr>
<tr>
<td>17 Jan. 93</td>
<td>Dynamics of Bubbles and Bubble Clouds</td>
<td>1 Oct. 93 - 30 Sep. 95</td>
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Since 1 Oct. 93 the research has been closely related to that done under Grant N00014-94-1-0008, "Dynamics of Short Surface Waves".

The following Progress Reports have been submitted previously:

<table>
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<th>Date</th>
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<tr>
<td>7 Feb. 92</td>
<td>1 Mar. 91 - 31 Dec. 91</td>
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<tr>
<td>28 Jan. 93</td>
<td>1 Jan. 92 - 31 Dec. 92</td>
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<tr>
<td>14 Apr. 94</td>
<td>1 Jan. 93 - 31 Dec. 93</td>
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<td>20 Oct. 94</td>
<td>1 Oct. 93 - 30 Sep. 94</td>
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Research for the periods 1 Oct. 94 to 30 Sep. 95 and 1 Oct. 95 to 30 Sep. 96 was included in Progress Reports for ONR Grant N00014-94-1-0008.

The present report covers the acoustically-relevant work done in the entire period 1 Mar. 91 to 31 Dec. 96. Research specifically on wave breaking and surface turbulence, which is also related to ocean acoustics but less directly, is being reported in Progress Reports for ONR Grant N00014-94-1-0008.

SCIENTIFIC GOALS

The primary aim of the research was to understand the natural sources of sound in the ocean, the most important of which are the spontaneous emission of sound by bubbles when generated by breaking waves and drops of rain. (Previous work on this subject by the author is cited in References (1) to (6)). This led to an investigation of the damping of bubble oscillations by nonlinear modal interactions (ref. (7)), and to a simple theory for the statistical distribution of bubble sizes in breaking waves (ref. (27)).

At all wind speeds, including light winds less than 6 m/s, capillary waves are prevalent on the sea surface. Nonlinear capillary waves become rounded at the crests and steep in the troughs, where they can trap bubbles of air. Hence they must be considered as sources of sound. Capillary waves are particularly common on the forward face of short, steep gravity
waves of length less than 1m, being generated by surface tension and other forces near the wave crest. One of the aims of the research has been to calculate the steepness of such parasitic capillaries and to consider their effects. It turns out that they can be important sources of vorticity and horizontal current shear, which in turn can become unstable and initiate a type of spilling breaker.

Some aspects of bubble collapse, which involve bubble splitting and the formation of inward-pointing jets of water, were also investigated. We note that this has possible implications for the little understood phenomenon of sonoluminescence.

**APPROACH**

The methods used have been mainly mathematical, with close attention to observations, and a preference for simple analytical models rather than extensive numerical computation. The theory has been based mainly on the equations for an almost incompressible, time-dependent flow in a uniform fluid with a free surface, these equations being highly nonlinear in general.

**RESULTS**

A. **Damping of bubble oscillations.**

In Longuet-Higgins (1991) an explanation has been given for the anomalously high damping, sometimes non-monotonic, which is frequently observed in the decay of acoustical pulses from newly-formed bubbles. The explanation is that the sound-producing, radially symmetric, mode of oscillation can interact nonlinearly with one of the shape oscillations of the bubble. Near resonance, there is an exchange of energy from the symmetric (or "breathing") mode to the shape oscillation, the sign of the energy exchange depending on the relative phase of the two modes. When energy is put into the shape oscillation it is dissipated by viscosity, but emits only relatively small amounts of sound. Therefore less energy is subsequently returned to the breathing mode than was initially transferred. Meanwhile the breathing mode itself decays, mainly through radiation and heat transfer. The whole process may undergo one or more cycles before the oscillations are finally damped out. The theory in ref. (7) made use of the coupled equations derived in ref. (6) with the important addition of damping terms. Comparison with the experimental data of Medwin and Beaky (1989) on the acoustical pulses from near-surface bubbles in a wind-wave channel showed remarkably good agreement.

An extension of the theory to shape oscillations of large amplitude also accounted for the occurrence of "spin-off" bubbles, evidenced by sporadic high-frequency pulses of sound. The assumption was that at a certain amplitude a shape oscillation would "break", producing
spin-off. The timing and phase of the observed pulses agreed with the theory.

Some experimental confirmation of the theory was subsequently obtained by Mao (1993) who in controlled experiments found a family of decay curves closely resembling those predicted; see Crum (1995).

B. Capillary-gravity waves

In Longuet-Higgins (1992b) a new method proposed by Ruvinsky et al. (1991) for calculating steep gravity-capillary waves was given a physical interpretation. The theory included the first-order damping effects of parasitic capillaries on the Stokes waves. However, it did not include second-order effects such as the generation of a mean vorticity and shear by the capillaries. A nonlinear theory for the capillary waves alone was then given in Longuet-Higgins (1992c), which showed moreover that the mean vorticity appeared capable of producing a surprisingly strong shear beneath the crests of short gravity waves, as reported by Ebuchi et al. (1987). It was subsequently shown (Longuet-Higgins 1994) that the initial shearing current produced in this way was unstable and could cause the wave crest to become turbulent and then collapse, as was observed by Duncan et al. (1994). Thus the wave could break, in a sense, without overturning of the free surface.

An analytic theory for the calculation of parasitic capillary waves (wave-generated ripples) on the forward face of gravity waves was given in Longuet-Higgins (1995). This accounted quantitatively for previous observations by Cox (1958), Ebuchi et al. (1987) and Perlin et al. (1993). In it the ripples were treated as linear perturbations of the gravity waves. It predicted that the ripple steepness, considered as a function of the steepness $ak$ of the gravity waves, would be greatest at a certain value of $ak$, for gravity waves of a given wavelength. However in a subsequent paper (Longuet-Higgins 1996a) in which the nonlinearity of the ripples was included, it was shown that the ripple steepness sometimes increased monotonically with $ak$. A physical interpretation of this result in terms of the properties of steep capillary-gravity waves (as exemplified by the deep-water solitary wave) was also given.

Longuet-Higgins (1993) gave a physical discussion of capillary-gravity waves of solitary type on deep water, pointing out that they can be considered as special cases of "envelope solitons" in which the speed of the wave envelope (the group-velocity) happens to be equal to the speed of the carrier wave (the phase-speed). Both speeds are decreasing functions of the wave amplitude, which explains qualitatively (Longuet-Higgins 1996b) how stationary nonlinear ripples can exist near the crest of a gravity wave even when linear ripples could not.

C. Sound from bubbles at an underwater nozzle.

In Longuet-Higgins et al. (1991) the author made a theoretical and experimental study of the slow release of bubbles from an underwater nozzle, and the associated acoustical pulses. This was the first such study for large bubbles (diameter > 1 mm). The relation
between the bubble size and nozzle diameter was found for nozzle diameters up to 6 mm, the stable limit. The process of detachment was also studied, and it was shown that at break-off the theoretical half-angle in the conical "neck" is 45°. This is followed by the formation of a high-speed jet going into the bubble along the axis of symmetry.

D. Distribution of bubble sizes

The statistical distribution of bubble sizes in the upper layers of the ocean is important for understanding acoustic propagation in these layers; it has been the subject of many observational studies since Johnson and Cooke (1979). Below a certain depth in the water, the size distribution may be determined physically by a balance between the rise velocity of the individual bubbles, on the one hand, and turbulent diffusion on the other (see Wu, 1988; Baldy 1993). The question considered in Longuet-Higgins (1992d) was this: What would one expect for the initial distribution of bubble sizes from a breaking wave, that is immediately following on the entrainment and disintegration of a given volume of air?

In the proposed model, the formation of the bubbles was compared to the breakup of a cubical block by three sets of planes, each set parallel to a face of the cube, each plane being inserted independently of the others. When the total number \( N \) of planes is large, this gives a certain limiting distribution of bubble sizes, depending only on the initial block size and on \( N \), which can be related to the available energy. The normalised distribution has a certain standard deviation and skewness. When compared with observations obtained acoustically (from bubble pulses), in mountain streams (Leighton and Walton (1987), the ocean (Updegraff 1989) and in the laboratory (Medwin and Daniel 1990) a fair measure of agreement was found. This in spite of the crudity of the model and the absence of a specific physical mechanism of bubble splitting.

Quite recently the model has been used by Clegg and Auffret (1996), with some success, for simulating the bubble size distributions derived from ambient noise measurements.

E. Jetting in bubble collapse

The phenomenon of high-speed, inward-going jets in a collapsing cavity, which was observed in bubble pinch-off, is of interest also in the phenomenon of cavitation damage to propellers and other underwater surfaces. No exact analytical solution for the flow in such a situation is known. However the author (1993) has discovered a family of analytic solutions for standing surface waves which generalise the well-known Dirichlet hyperbola (see Longuet-Higgins 1972) and which do approximately satisfy all the necessary conditions. In this family of solutions the velocity and acceleration become large like \((t - t_j)^{1/3}\) and \((t - t_j)^{-4/3}\) respectively near a critical instant \(t = t_j\), the displacement remaining finite. This has been called an "inertial shock". In Longuet-Higgins (1994) the author has found analogous solutions for a cylindrical cavity. Such solutions are given by very simple expressions, in semi-Lagrangian coordinates; in fact by two circular harmonics, whose coefficients satisfy a system of coupled, nonlinear, ordinary differential equations, making the time-integration
relatively straightforward. The solutions also display inward-going jets, with inertial shocks at which the velocities vary as \((t - t_j)^{2/3}\).

For a study of three-dimensional cavity collapse, the author has collaborated with Dr. H. Oguz of Johns Hopkins University who had developed a numerical code based on a boundary-integral technique such as was originally used for breaking surface waves (Longuet-Higgins and Cokelet 1976). In a first paper (Longuet-Higgins and Oguz 1995) it was discovered that a flow with very simple initial conditions, namely a near-spherical cavity enclosing a moving source, can give rise either to jet formation or to bubble pinch-off, depending on the parameters of the initial flow. Moreover at the critical parameters separating jet formation from bubble pinch-off there forms a "critical jet", in which the fluid velocity can become arbitrarily large, behaving in fact like \((t - t_j)^{a}\), where \(a\) is a positive constant. It was verified that the flow near \(t = t_j\) is indeed self-similar, though it was not determined whether \(a\) is a constant.

In a second paper (Longuet-Higgins and Oguz 1996a) it has been demonstrated, by considering other examples of asymmetric initial condition, that \(a\) is not an absolute constant but depends in a predictable way on the flow near the axis of symmetry. It was also shown that the so-called "flip-through" found in ocean waves, when a steep wave impacts a harbor wall and throws up a jet of water, is another example of such a critical jet. The velocity varies as an inverse power of \((t - t_j)\) in this case also.

F. Single-bubble sonoluminescence

The presence of inward-going jets in a collapsing cavity may have a bearing on another acoustical phenomenon, that of sonoluminescence. A. Prosperetti (1996) has proposed that in single-bubble sonoluminescence the very brief flash of light given off near the instant of minimum bubble radius may be due to the high-speed impact of the jet with the opposite wall of the bubble, giving rise to a kind of fracture in the liquid phase. This would account for the extreme brevity (\(~10^{-11}\)s) of the flash and the observed asymmetry in the emitted pattern of light. In parallel work (Longuet-Higgins 1996b) the author has suggested that the impact should give rise to vortices in the surrounding fluid. Hence an examination of the fluid flow in the neighborhood of the bubble should provide evidence for or against the theory.
REFERENCES


Published papers


Presentations (outside U.C.S.D.)

4 Feb. 91   Univ. of California, Santa Barbara. Seminar: "Nonlinear dynamics of bubble oscillations."

9 Apr. 91   ONR Conf. on Air-Sea Acoustics: Paper: "Lagrangian distortion of bubble clouds."

1-2 Apr 91  Univ. of Central Florida. Two lectures "Surface sources of underwater sound in the ocean."
            "Nonlinear dynamics of bubble oscillations."

3 May 91    A.S.A. Meeting, Baltimore. Two papers: "The release of air bubbles from an underwater nozzle."
            "Dynamics of air bubbles entrapped by capillary waves" (with H.N. Oguz).

31 May 91   ONR Symp. on Nonlinear Waves, APL, John Hopkins Univ. "Breaking capillary-gravity waves."

17 Jul. 91   IUTAM Symp. on Breaking Waves, Sydney, Australia. Invited lecture: "Capillary rollers and bores."

23 Oct. 91   Texas A. & M. Univ., College Station, TX. Seminar: "Jets, breakers and standing waves."

22 Nov. 91   Univ. of Colorado, Boulder, Colorado. Seminar: "Capillary rollers."

12 May 92   ASA Meeting, Salt Lake City. Contributed paper: "Distortion of bubble clouds by orbital motions in surface waves."

20 May 92   ONR Workshop on Air-Sea Acoustics, Sidney, B.C. Contributed paper: "The shattering of air cavities in a liquid."


19 Feb. 93   Univ. of Maryland, College Park, MD. Seminar: "Inertial shocks in surface waves."

24 Feb. 93   Univ. of California, Santa Barbara. Seminar: "Inertial shocks in surface waves."
<table>
<thead>
<tr>
<th>Date</th>
<th>Event</th>
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<tr>
<td>11 Mar. 93</td>
<td>San Diego State Univ. Seminar:</td>
<td>&quot;Ocean waves, bubbles and underwater sound.&quot;</td>
</tr>
<tr>
<td>22 Aug. 94</td>
<td>Int. Symp. on Waves — Physical and Numerical Modelling, Univ. of British Columbia, Vancouver, B.C.</td>
<td>Keynote address: &quot;The initiation of spilling breakers.&quot;</td>
</tr>
<tr>
<td>23 Feb. 95</td>
<td>ONR Workshop on Bubbly Flows, La Jolla.</td>
<td>Contributed paper: &quot;Critical microjets in collapsing cavities.&quot;</td>
</tr>
<tr>
<td>28 Feb. 95</td>
<td>ONR Workshop on Free-Surface Turbulence, Pasadena.</td>
<td>Contributed paper: &quot;Surface manifestations of turbulent flow.&quot;</td>
</tr>
<tr>
<td>28 Mar. 95</td>
<td>Univ. of California, Santa Barbara. Seminar:</td>
<td>Capillary waves generated by surface waves and turbulent currents.&quot;</td>
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4 Oct. 95  4th Workshop on Capillary Waves and Remote-Sensing Phenomena, La Jolla. Contributed paper: “Nonlinear development of a crest instability.”

12 Oct. 95  Lamont-Doherty Earth Observatory, Palisades, N.Y. Seminar: “Breaking waves and their importance for air-sea interaction.”

15 Nov. 95  ASME Meeting, San Francisco. Contributed paper: “On the disintegration of the jet in a plunging breaker.”


24 Jan. 96  Univ. of California, Berkeley. Seminar: “Recent studies of wave breaking.”


Seminars given at U.C.S.D. include the following:

April - May  Three seminars in SIO postgraduate course, Surface Wave Phenomena:
92  1. "Laboratory measurements of the modulation of short-wave slopes by long  
surface waves."
2. "A stochastic model of sea surface roughness."
3. "Capillary rollers and bores."

11-13 May    Two lectures in SIO graduate course: Advanced Acoustics: "Natural sources  
93 of underwater sound.

Oct. - Dec.    Six seminars in SIO postgraduate course, Microstructure and Dynamics of the  
96 Sea Surface:
1. "Types of wave breaking."
2. "Nonlinear theory of gravity waves."
3. "Crest instabilities."
4. "Parasitic capillary waves."
5. "Capillary-gravity waves of solitary type."
6. "Viscosity and turbulence in surface waves."

Also occasional seminars at U.C.S.D. in Physics Department, AMES and INLS.

Number of graduate students: 0

Committees and Panel Service

2. Member, ONR Review Committee for University Research Initiative at U. Michigan,  
1993-6.