Acoustic Radiation Efficiency Models of a Simple Gearbox

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ACOUSTIC RADIATION EFFICIENCY MODELS OF A SIMPLE GEARBOX

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ABSTRACT

Acoustic intensity measurements were conducted on a simple spur gear transmission in a welded steel housing. The radiation efficiency of the housing was computed from the intensity data for the first three harmonics of mesh frequency. Finite element and boundary element methods (FEM/BEM) were used jointly to model acoustics and dynamics of the top plate of the housing. For a simply supported elastic plate, reasonable agreement was achieved between experimental radiation efficiencies and those predicted with FEM/BEM. However, predictions of the housing characteristics were only partially successful. Four simple analytical models were examined to judge their ability to predict the radiation efficiency. These models do not simulate the modal characteristics of a gearbox; therefore their predictions yield only general trends. Discrepancies are believed to be related to inaccurate modeling of the excitation of the structure as well as to interactions between modes of vibration.

INTRODUCTION

Much effort has been expended during the design process in recent years to predict the noise of machinery in order to reduce it. The mesh excitation of gear transmissions has been analyzed, and a method has been developed to model the transmission of vibration through bearings into the housing (Lim, 1989). The remaining step is to predict the characteristics of noise radiation from a vibrating structure. Seybert discusses a method for predicting radiation by a boundary element analysis of a vibrating surface (Seybert et al., 1994). Several studies have examined the sound radiation characteristics of single flat plates (panels) under various boundary conditions. A general review is given in Lim and Singh (1989). Other work includes Wallace (1972), Beranek (1971), Fahy (1985), Guyader (1994), and Cremer et al. (1973). Others have tried to correlate gearbox-radiated noise with structural excitation (Oswald et al., 1992; Seybert et al., 1991; Kato et al., 1994; Sabot and Perret-Liaudet, 1994; VanRoosmalen, 1994; Lim, 1989; and Heath and Bossler, 1993). Many other studies are summarized in Lim and Singh (1989). Since acoustic radiation efficiency is the link between structural vibrations and the radiated sound power, it should be studied when attempts are being made to predict noise. This paper looks at several acoustic radiation efficiency models and compares them to experimental results. Identifying the usefulness of these models in housing design is the main focus of this paper.

APPARATUS

The gear noise test rig at NASA Lewis Research Center is equipped to accommodate spur and helical gears. Several types of measurements, including sound pressure, acoustic intensity, and housing acceleration, can be made on this rig (Oswald et al., 1992). The test rig consists of a single-mesh gear pair powered by a 150-kW (200 hp) variable-speed electric motor. An eddy-current dynamometer loads the output shaft. The gearbox can operate at speeds up to 6000 rpm.

For this study, the gears were identical 28-tooth spur gears with a 6.35-mm face width; they were manufactured to AGMA class-15 accuracy. The gears have a linear profile modification (tip relief) of 11 mm (0.00045 in.) that extends 90 percent of the distance from the tip to the high point of single tooth contact. For these gears, the transmission error is minimized at a load of 45 N-m (400 in-lbf). The housing is made of 6.35-mm-thick steel plates. The four corners of the base plate are bolted to a foundation that is assumed to be rigid. The sides of the box are welded to each other and to the base plate; the top plate is bolted down, but can be removed to change test gears. The whole rig is within a room whose floors, walls, and ceiling are covered with acoustic absorbing

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material. This provides a semi-anechoic test chamber that above 500 Hz attenuates reflected sound by at least 20 dB.

MEASUREMENTS

The sound power radiated from the gearbox was measured by using an acoustic intensity probe consisting of a pair of phase-matched microphones mounted face-to-face and spaced 6 mm apart. The probe was positioned with the aid of a computer-controlled robot (Oswald et al., 1992). The robot was commanded to move the probe to 20 locations, each approximately 60 mm above the top of the housing. The 20 intensity spectra were averaged and multiplied by the area of the top of the housing to yield the radiated sound power at that operating condition.

During the course of the experiments, we noticed that the amplitudes of the sidebands were significant relative to the levels at mesh frequency \( f_m \) and the harmonics of \( f_m \). This was particularly true at the higher speeds. Figure 1 shows the sound power spectrum for speed \( \Omega = 6000 \text{ rpm} \) and torque \( T = 68 \text{ N-m} \). Here, the mesh frequency is 2800 Hz, and the shaft frequency is 100 Hz. In the spectrum some of the sidebands have significant amplitudes relative to the mesh frequency peak.

We decided that a characterization of the sound power should include three pairs of sidebands in the computation.

The mean square velocity was obtained from the accelerometers located on the gearbox housing (we assumed sinusoidal response and integrated). Three housing locations were chosen for the spatial averaging (see Fig. 2). The velocity spectra contained significant sideband activity, just as the sound power spectra did.

Sound intensity measurements were taken only on the top plate rather than over the surface of an enclosing volume. This limitation was dictated by the difficulty and danger of taking measurements next to a rotating shaft. Limiting measurements to the top plate reduced the unwanted effects of noise from couplings adjacent to the gearbox.

One justification for limiting the measurements was that the top plate was less stiff than the sides and, thus, could vibrate more than the others, especially at low frequencies. Although the top plate is more flexible, how much this affects the vibratory power flow through the housing is not clear. The peak velocities of the bearing cap were an order of magnitude lower than those of the side or top plates. This is as expected since the spur gears generate no thrust force that would cause significant motion along the axis of the shaft. In addition, the bearing mounting plates are very stiff relative to the housing. The peak velocities of the side plate were about the same order of magnitude as those of the top plate. This suggests that a significant amount of power may be flowing through the side plates and could be radiated as sound. Future experiments should attempt to examine sound radiation from the side plates by using more accelerometer locations.

The acoustic radiation efficiency of a structure such as a gearbox may be defined as the measured sound power radiated from the structure divided by the sound power radiated by a piston in an infinite baffle. The area of the piston is equal to the surface area of the structure, and the vibration velocity of the piston equals that of the structure. The radiation efficiency may be computed for each frequency of interest and it may exceed one.

The experimental radiation efficiency \( \sigma^*_{rad} \) of the gearbox housing under operating conditions was estimated from the sound power and velocity by the following (Cremer et al., 1973):

\[
\sigma^*_{rad}(f, T, \Omega) = \frac{W^* (f, T, \Omega)}{\rho c S \langle v^2 \rangle_{n1}(f, T, \Omega)^*}
\]

where \( W^* \) is the radiated sound power in watts, \( \rho c \) is the acoustic impedance of the surrounding medium in pascals, \( S \) is the radiating area of the housing in meters squared, and \( \langle v^2 \rangle_{n1} \) is the spatially and temporally averaged mean square velocity of the housing in meters squared per second squared. In Eq. (1), the * indicates that these are in situ quantities, which depend on the measurement and operating conditions, including harmonic and sideband frequencies \( f_n \), speed \( \Omega \), and torque load \( T \).

![Intensity probe](image1)

**Figure 1.**—Typical sound power spectrum (6000 rpm, torque = 68 N-m).

![Instrumentation](image2)

**Figure 2.**—Instrumentation.
RADIATION EFFICIENCY RESULTS

The radiation efficiency was computed from Eq. (1). The acoustic impedance of air is 415 rayls at 25°C and 1 atm. The surface area S of the top and four sides of the housing is 0.41 m². No acceleration measurements were taken on the bottom plate because it is not easily accessible. Therefore, the area of the bottom plate was not included in the computed surface area. The bottom plate is somewhat thicker than the sides and has stiffeners, so we expect it is not a good sound radiator. The radiation efficiency computed from these parameters is plotted in Fig. 3. In general, the radiation efficiency tends to increase with operating speed for each harmonic and approaches unity as the speed becomes large enough so that the acoustic wavelength is no longer large relative to the dimensions of the gearbox. Several exceptions to the trend can be observed. Most notably, the 1st harmonic, at Ω = 1000 rpm (f_m = 467 Hz), had a radiation efficiency nearly 10 times as large as at Ω = 1500 rpm (f_m = 700 Hz). This variation in the radiation efficiency across the speed range is most probably the result of the interaction (coupling) between vibration modes of the housing. At least five housing modes are present below 900 Hz alone, as reported by Lim (1989), so the possibility of multimode excitation is likely. Each of these modes should have different modal radiation efficiencies, so an examination of the radiation efficiency on a modal basis would be useful. This will be left for a future study.

The three traces in Fig. 3 partially overlap. This allows us to compare the radiation efficiency for different harmonics at the same frequency. For example, at 2800 Hz, the mesh frequency (1st harmonic) occurs at 6000 rpm; twice the mesh frequency (2nd harmonic), at 3000 rpm; and three times the mesh frequency (3rd harmonic), at 2000 rpm. In many cases (except at the lowest frequencies) the overlapping curves agree reasonably well. This suggests that data from higher harmonics taken at fairly low speeds may be used to predict the radiation efficiency for low harmonics at higher speeds. In other words, perhaps we can predict the radiation efficiency for speeds beyond our operating range by examining data from higher harmonics within the operating range. This would be useful for future high-power-density rotocraft transmissions.

Figure 3.—Radiation efficiency for first three harmonics over speed range of 1000 to 6000 rpm and torque of 68 N-m.

Figure 4.—Summary of analytical acoustic radiation models (a = radius, h = height, d = diameter).

Ideal acoustic source models such as the monopole (pulsating sphere of radius a) and the dipole (acoustic doublet—two simple sources of equal strength, separated by distance a, vibrating at the same frequency but 180° out of phase with each other) yield fast estimations of the housing radiation efficiency, but the utility of these models is difficult to justify on the basis of any apparent housing geometry. An equivalent (same mass and estimated modal density as gearbox housing) plate model that relies on an approximation of the modal density of the housing appears to be a better estimator. (Unfortunately, the modal densities of transmission housings may not be known.) An equivalent cylinder model based on the geometry of the housing, where plate thickness h are equal and cylinder diameter d equals the length of the gear housing, may also be a good estimator (Guyader, 1994). Figure 4 compares some of the best models studied, along with the experimental radiation efficiency values. Obviously, none of these models capture the variation that is present in the experimental radiation efficiency curves.

Figure 4.—Summary of analytical acoustic radiation models (a = radius, h = height, d = diameter).

DYNAMIC AND ACOUSTIC MODELS OF THE TOP PLATE

The test gearbox is a rectangular housing made from steel plates welded together and a top plate bolted on. For a simple analysis, a finite element method (FEM) model was generated for only the top plate. The top plate is 286 by 362 by 6.35 mm. Shell elements were used to generate a model with 120 elements and 143 nodes. The boundary conditions of the plate were modeled so that the nodes at the four corners
had all degrees of freedom set to zero (clamped condition), and the nodes along the edges alternated between being completely free and being clamped. This scheme simulated the bolted connection between the top plate and the top flange of the housing, which is depicted in Fig. 5. With this model, the first four natural frequencies were determined. They are shown in Table 1 along with the previous experimental measurements (Oswald et al., 1992). The modal index is the number of antinodes along the width and length of the plate. The discrepancy between the FEM and measured results is less than 9 percent.

<table>
<thead>
<tr>
<th>Modal index</th>
<th>Frequency, Hz</th>
<th>Measured*</th>
<th>Error, percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1,1)</td>
<td>508</td>
<td>511</td>
<td>0.6</td>
</tr>
<tr>
<td>(1,2)</td>
<td>898</td>
<td>975</td>
<td>8.6</td>
</tr>
<tr>
<td>(2,1)</td>
<td>1175</td>
<td>1273</td>
<td>8.3</td>
</tr>
<tr>
<td>(2,2)</td>
<td>1530</td>
<td>1631</td>
<td>6.6</td>
</tr>
</tbody>
</table>

*Oswald et al. (1992).

The FEM and measured mode shapes (not shown) were also similar. This indicates that the model reasonably simulates the response of the gearbox top in the frequency range studied. Using this plate model, we applied three arbitrary loading conditions to conduct forced response and acoustic radiation studies. These loading conditions were not meant to represent actual loads seen by the test gearbox; rather, they were meant to be a starting point for comparing the effects of different loads on the acoustic efficiency of the plate and the housing. The different loads were (i) a 1-N force normal to the plate, applied at a point on the edge as shown in Fig. 6(a); (ii) a 1-N-m moment about the y-axis of Fig. 6(b), applied at the same node as for case (i); and (iii) several 1-N forces normal to the plate and equal in phase, applied along the plate edge at the free nodes as shown in Fig. 6(c). For a linear system, the predicted radiation efficiency is not affected by the magnitude of the applied force. To simulate the range of speeds in the experiment, the loads were applied at frequencies of 400 to 2900 Hz in steps of 100 Hz.

The forced vibration response predicted by the FEM model was used as the input to a boundary element method (BEM) model. The BEM model calculated the acoustic response of the plate, including the radiation efficiency. Figure 7 shows the radiation efficiencies predicted for the three loading cases by the combined BEM/FEM model. An
2. A boundary element model can predict the acoustic response of a vibrating structure if the vibration characteristics are known. A combined finite element/boundary element model may be used to predict the operating noise characteristics.

3. Ideal acoustic models (such as a monopole, dipole, flat plate, or cylinder) do not adequately simulate the modal vibration behavior of a gearbox. These ideal models cannot predict the variation in acoustic response due to vibration modes. Therefore, ideal models predict only general trends.

REFERENCES


**Title and Subtitle**

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**Supplementary Notes**

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**Subject Terms**

Acoustic intensity; Gears; Gearbox; Radiation efficiency

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