Computerized Design and Analysis of Face-Milled, Uniform Tooth Height Spiral Bevel Gear Drives

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COMPUTERIZED DESIGN AND ANALYSIS OF FACE-MILLED, UNIFORM TOOTH HEIGHT SPIRAL BEVEL GEAR DRIVES

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ABSTRACT

Face-milled spiral bevel gears with uniform tooth height are considered. An approach is proposed for the design of low-noise and localized bearing contact of such gears. The approach is based on the mismatch of contacting surfaces and permits two types of bearing contact either directed longitudinally or across the surface to be obtained. A Tooth Contact Analysis (TCA) computer program was developed. This analysis was used to determine the influence of misalignment on meshing and contact of the spiral bevel gears. A numerical example that illustrates the developed theory is provided.

1 INTRODUCTION

Two models for spiral bevel gears with uniform tooth height were proposed by Litvin et al [6]. The generation of tooth surfaces of such gears is based on application: (i) of two cones that are in tangency along their common generatrix (model 1), and (ii) a cone and a surface of revolution that are in tangency along a common circle (model 2). The pinion and the gear are face-milled by head-cutters whose blades by rotation form the generating surfaces.

The generating surfaces provide conjugate pinion-gear tooth surfaces with a localized bearing contact that is formed by a set of instantaneous contact ellipses. The path of contact is directed across the surfaces in model 1 (fig. 1), and in the longitudinal direction in model 2 (fig. 2). The transmission errors are zero but only for aligned gear drives.

It is well known that misalignment of a gear drive causes the shift of the bearing contact and transmission errors. The transmission errors are one of the main sources of vibration. Therefore, the direct application of the models discussed above for generating surfaces is undesirable.

It was discovered that misalignment of a gear drive causes an almost linear but discontinuous transmission function. However, such functions can be absorbed by a predesigned parabolic function of transmission errors. The interaction of the parabolic function and a linear function results in a parabolic function with the same parabola coefficient [9]. Based on this consideration, it becomes necessary to modify the process for generation discussed above to obtain a predesigned parabolic function of transmission errors. It was proposed in work [10] to obtain the desired parabolic function of transmission errors by executing proper nonlinear relations between the motions of the cradle and the gear (or the pinion) being generated. This approach requires the application of the CNC machines.

The purpose of this paper is to propose modifications of generating surfaces that will obtain: (i) a localized bearing contact that may be directed in the longitudinal direction or across the surface, and (ii) a predesigned parabolic function. These goals that will be proven later are obtained by the proper mismatch of the ideal generating surfaces shown in figs.1 and 2. The mismatch of surfaces is achieved by application of modified generating surfaces shown in fig. 3. The modified generating surfaces are in point contact instead of tangency along a line that the ideal generating surfaces have. The desired parabolic function of transmission errors, the orientation of the path of contact, and the magnitude of the major axis of the contact ellipses are obtained by the proper determination of the curvature and the mean radius of the surface of revolution of the generating tool.

The meshing and contact of the tooth surfaces was simulated by the TCA computer program developed by the authors. Numerical examples for the illustration of the proposed approach are considered.
2 METHOD FOR GENERATION OF CONJUGATE PINION-GEAR TOOTH SURFACES

Gear Generation:

The head-cutter for gear generation is provided with inner and outer straight-line blade (fig. 4), that form two cones while the blades are rotated about the Zc-axis of the head cutter. These cones will generate the convex and concave sides of the space of the gear, respectively.

We apply coordinate systems Sc, S, Sm that are rigidly connected to the cradle of the generating machine, the gear and the cutting machine, respectively (figs. 5 and 6). The cradle with coordinate system Sc performs rotation about the Zc-axis, and ψc is the current angle of rotation of the cradle (We take i = 2 in the designations of fig. 5). Coordinate system S is rigidly connected to the head-cutter that is mounted on the cradle. The installation of the head-cutter is determined with angle ϒa and Sc = [OcOia] (fig. 5(b)). The gear in the process for generation performs rotation about the Za-axis of the auxiliary fixed coordinate system Sa that is rigidly connected to the Sm coordinate system (fig. 6). The installation of Sa with respect to Sm is determined with angle ϒa, where ϒa is the angle of the gear pitch cone. The current angle of gear rotation is ψg (fig. 6). Angles ψc and ψg are related as

\[
\frac{\psi_c}{\psi_g} = \frac{\omega_c}{\omega_g} = \sin \gamma_a
\]

(1)

The observation of this equation guarantees that the Xc-axis is the instantaneous axis of rotation of the gear in its relative motion with respect to the cradle.

Pinion Generation:

The head-cutters for pinion generation are provided with separate blades that will generate the convex and concave sides of the space of the pinion, respectively (fig. 7). The pinion generating tool is installed on the cradle similarly to the installation of the gear generating cone (We take i = 1 in the designations of fig. 5). An auxiliary fixed coordinate system Sa is rigidly connected to the Sm coordinate system (fig. 8). The installation of Sa with respect to Sm is determined with angle γ1 of the pinion pitch cone. An imaginary process for the pinion generation for the purpose of simplification of the TCA program is considered. The installation of coordinate system Sa with respect to Sm is determined in the real process of cutting by the angle γ1 that is measured clockwise, opposite to the direction shown in fig. 8. The pinion performs rotation about the Za-axis and ψ1 is the current angle of rotation. The angles of rotation of the pinion and the cradle are related as

\[
\frac{\psi_1}{\psi_c} = \frac{\omega_1}{\omega_c} = \sin \gamma_1
\]

(2)

Axis Xc in accordance to equation (2) is the instantaneous axis of rotation of the pinion in its relative motion with respect to the cradle.

3 DERIVATION OF GEAR TOOTH SURFACE

We consider that the gear head-cutter surface is represented in S1 by vector function r1(ςg, θg), where ςg and θg are the surface parameters. A family of tool surfaces is generated in gear coordinate system S2 while the cradle and the mounted tool and the gear perform the rotational motions that are shown in figs. 5 and 6. The family of surfaces is represented in S2 by the matrix equation

\[
r_2(ς_g, θ_g, ψ_2) = M_{2i}(ψ_2)M_{sm}(ς_g)M_{c2i}(ψ_c)M_{c2i1}r_{11}(ς_g, θ_g)
= M_{2i1}r_{11}(ς_g, θ_g)
\]

(3)

The product of matrices M2i1 is based on the coordinate transformations from S1 to S2 (figs. 5 and 6). The derivation of vector function r1(ςg, θg) is represented in Appendix 1.

The envelope to the family of surfaces r1(ςg, θg, ψ2) is determined with equation (3) and the equation of meshing is [9]

\[
N_c \cdot v_{c2} = f_2(ς_g, θ_g, ψ_2) = 0
\]

(4)

where Nc is the normal to the generating surface, and vc2 is the relative velocity of the tool with respect to the gear. Vectors in equation (4) are represented in coordinate system Sc.

An alternative approach for the derivation of the equation of meshing is based on the consideration that the normal to the generating surface at a point of tangency of the contacting surfaces passes through the instantaneous axis of rotation.

Equations (3) and (4) represent the gear tooth surface by three related parameters. Taking into account that these equations are linear with respect to ςg, we may eliminate ςg and represent the gear tooth surface by two independent parameters, θg and ψ2.

4 DERIVATION OF PINION TOOTH SURFACE

The derivations are similar to those that have been described in section 3. The family of generating surfaces is represented by the matrix equation

\[
r_1(λ_p, θ_p, ψ_1) = M_{1i}(ψ_1)M_{sm}(ς_p)M_{c1i}(ψ_c)M_{c1i1}r_{11}(λ_p, θ_p)
= M_{1i1}r_{11}(λ_p, θ_p)
\]

(5)

Here, r1(λ_p, θ_p) is the vector function that represents the generating surface ( Appendix 2 ), where λ_p and θ_p are the surface parameters.

The equation of meshing is

\[
N_{c1} \cdot v_{c1} = f_1(λ_p, θ_p, ψ_1) = 0
\]

(6)
Equations (5) and (6) represent the pinion tooth surface by three related parameters. After elimination of parameter \( \lambda_p \), we may represent the pinion tooth surface by two independent parameters, \( \phi_1 \) and \( \psi_1 \).

5 LOCAL SYNTHESIS

The ideas of local synthesis are based on the following considerations [9]:

1. The pinion and gear tooth surfaces are in tangency at the mean contact point \( M \) that is in the middle of the contacting surface.

2. The gear ratio is equal to the theoretical one.

3. Considering the principal curvatures of the contacting surfaces, we have to provide in the neighborhood of \( M \) the following transmission function (fig. 9)

\[
\phi_2(\phi_1) = \frac{N_1}{N_2} \phi_1 - \frac{1}{2} m_{21}' \phi_1^2
\]

where \( \frac{1}{2} m_{21}' \) is the parabola parameter of the predesigned parabolic function of transmission errors

\[
\Delta \phi_2(\phi_1) = -\frac{1}{2} m_{21}' \phi_1^2
\]

4. In addition it is necessary to provide the desired direction of the contact path.

All these goals can be achieved by the proper mismatch of the contacting surfaces of the pinion-gear tooth surfaces. The procedure of the local synthesis is as follows:

Step 1: We consider as given the surface of the head-cutter that generates the gear tooth surface. The head-cutter surface is a cone and is in line contact with the surface of the gear. One of such contact lines passes through the mean point \( M \) of tangency of the pinion and the tooth surfaces. Considering the surface of the gear head-cutter as known, we determine at point \( M \) the principal curvatures and directions of the gear head-cutter.

Step 2: Our next goal is to determine at \( M \) the principal curvatures \( k_f \) and \( k_g \) and directions of the gear tooth surface \( \Sigma_2 \). We apply for this purpose the equations that have been proposed in [9] and represent the direct relations between the principal curvatures and directions for two surfaces being in line contact.

Step 3: We consider at this step that gear and pinion tooth surfaces, \( \Sigma_2 \) and \( \Sigma_1 \), are in tangency at \( M \). As a reminder the mismatched gear and pinion tooth surfaces are in point contact at every instant.

Unit vectors \( e_f \) and \( e_g \) represent the known directions of the principal directions on surface \( \Sigma_2 \). The principal curvatures \( k_f \) and \( k_g \) on the gear principal directions are known. Our goal is to determine angle \( \sigma_{12} \) that is formed by vectors \( e_f \) and \( e_g \) (fig. 10) and the principal curvatures \( k_f \) and \( k_h \) of the pinion tooth surface at point \( M \). Unit vectors \( e_f \) and \( e_h \) represent the sought-for principal directions on the pinion tooth surface \( \Sigma_1 \).

Step 4: The three unknowns: \( k_f \), \( k_h \) and \( \sigma_{12} \) can be determined using the approach developed in [9]. We use for this purpose the following system of three linear equations [9].

\[
a_{11}v_f^{(1)} + a_{12}v_g^{(1)} = a_{13} \quad (i = 1, 2, 3)
\]

The augmented matrix formed by the coefficients \( a_{11}, a_{12}, a_{13} \) and \( a_{13} \) is a skew-symmetric one [9]. Here, \( v_f^{(1)} \) and \( v_g^{(1)} \) are the components of the velocity of the contact point that moves in the process of meshing over the pinion tooth surfaces \( \Sigma_1 \). Coefficients \( a_{11}, a_{12}, a_{13} \) are represented in terms of \( k_s, k_g, k_f, k_h, \sigma_{12} \) and the parameters of motion. Coefficient \( a_{13} \) contains the derivative

\[
m_{21}' = \frac{d}{d\phi_1}(m_{21}(\phi_1))
\]

where

\[
m_{21} = \frac{d\phi_2}{d\phi_1}
\]

Step 5: Equation system (9) represents a system of three linear equations in two unknowns: \( v_f^{(1)} \) and \( v_g^{(1)} \). Surfaces \( \Sigma_1 \) and \( \Sigma_2 \) are in point contact, the path of contact has a definite direction, and the solution of equation system (9) with respect to \( v_f^{(1)} \) and \( v_g^{(1)} \) must be unique. Therefore, the rank of the augmented matrix formed by \( a_{11}, a_{12}, a_{13} \) is equal to two. This yields that

\[
\begin{bmatrix}
a_{11} & a_{12} & a_{13} \\
a_{12} & a_{22} & a_{23} \\
a_{13} & a_{23} & a_{33}
\end{bmatrix}
= F(k_f, k_h, k_s, k_g, \sigma_{12}, m_{21}') = 0
\]

The other relation between the coefficients \( a_{11}, a_{12}, a_{13} \) may be determined considering that

\[
\tan \eta_1 = \frac{v_f^{(1)}}{v_g^{(1)}}
\]

where \( \eta_1 \) is the assigned direction at \( M \) of the tangent to the path of contact on the pinion surface \( \Sigma_1 \).

Using the relations between the coefficients of linear equation (9) discussed above, we are able to determine the sought-for pinion principal curvatures \( k_f, k_h \) and orientation angle \( \sigma_{12} \).

Step 6: We consider now that pinion principal curvatures \( k_f, k_h \) and angle \( \sigma_{12} \), and the principal directions \( e_f \) and \( e_h \) on surface \( \Sigma_1 \) are known. The generating surface of the head-cutter is designed as surface of revolution (fig. 3). The pinion head-cutter surface and the pinion tooth surface are in line contact at every instant. Using the direct relations between the principal curvatures and directions for two surfaces being in line contact [9], we may determine the principal curvatures of the pinion head-cutter. Then, the desired mismatch of the surfaces of the gear and the pinion will be obtained by the generation of the gear and the pinion by the designed head-cutters.
Step 7: Knowing the principal curvatures and directions of the pinion and gear tooth surfaces, the elastic approach of the surfaces, we may determine the orientation and the axes of the instantaneous contact ellipse [9].

6 TOOTH CONTACT ANALYSIS

The purpose of TCA is to determine the influence of misalignment on the shift of the bearing contact and the transmission errors. This goal is obtained by the simulation of meshing and contact of pinion and gear tooth surfaces of a misaligned gear drive.

We consider that the pinion and gear tooth surfaces are analytically represented in coordinate systems $S_1$ and $S_2$ (see sections 3 and 4, respectively). The meshing of pinion and gear tooth surfaces is considered in fixed coordinate system $S_h$ (figs. 11 and 12). Auxiliary fixed coordinate system $S_a$ and $S_c$ are applied to describe the installation of the pinion with respect to $S_h$ (fig. 11). The pinion alignment error $\Delta A_p$ is the pinion axial displacement. The misaligned pinion in the process of meshing with the gear performs rotation about $Z_a$-axis. The current angle of rotation of the pinion is designated by $\phi_1$ (fig. 11).

Auxiliary coordinate systems $S_a$, $S_c$ and $S_d$ are applied to describe the installation of misaligned gear with respect to $S_h$. The errors of alignment are: the change $\Delta \gamma$ of the shaft angle (fig. 12), the offset $\Delta E$ and the gear axial displacement $\Delta A_g$ (fig. 13). The misaligned gear performs rotation about the $Z_a$-axis, and $\phi_2$ is the current angle of the gear rotation.

A TCA computer program was developed to simulate the meshing of pinion-gear tooth surfaces of the misaligned gear drive. The development of the TCA program is based on the following considerations:

Step 1. We consider that the pinion and gear tooth surfaces and the surface unit normals are represented in coordinate system $S_1$ and $S_2$ by vector functions

$$r_i(\theta_i, \psi_i) \quad (i = 1, 2) \quad (14)$$

$$n_i(\theta_i, \psi_i) \quad (i = 1, 2) \quad (15)$$

where $(\theta_i, \psi_i)$ are the surface parameters.

Step 2. We represent now the pinion-gear tooth surfaces and their surface unit normals in coordinate system $S_h$, and take into account that the surfaces are in continuous tangency. Then we obtain the following equations

$$r_h^{(1)}(\theta_1, \psi_1, \phi_1) - r_h^{(2)}(\theta_2, \psi_2, \phi_2) = 0 \quad (16)$$

$$n_h^{(1)}(\theta_1, \psi_1, \phi_1) - n_h^{(2)}(\theta_2, \psi_2, \phi_2) = 0 \quad (17)$$

Equations (16) and (17) represent the conditions that the contacting surfaces at the point of tangency have a common position vector and a common surface unit normal. Equations (16) and (17) yield a system of five independent scalar equations of the following structure

$$f_i(\theta_1, \psi_1, \phi_1, \theta_2, \psi_2, \phi_2) = 0 \quad f_i \in C^1 \quad (i = 1..5) \quad (18)$$

As a reminder, vector equation (17) yields only two independent scalar equations and not three, since $|n_h^{(1)}| = |n_h^{(2)}| = 1$.

Step 3. System (18) of five nonlinear equations contains six unknowns, but only one of the unknowns, say $\phi_1$, may be considered as the input parameter. Our goal is the numerical solution of nonlinear equations (18) by functions

$$\{\theta_1(\phi_1), \psi_1(\phi_1), \theta_2(\phi_1), \psi_2(\phi_1), \phi_2(\phi_1)\} \in C^1 \quad (19)$$

The sought-for numerical solution is an iterative process that requires on each iteration the observation of the following conditions [9][3]:

(i) There is a set of parameters (the first guess)

$$P(\theta_1^{(0)}, \psi_1^{(0)}, \theta_2^{(0)}, \psi_2^{(0)}, \phi_1^{(0)}, \phi_2^{(0)}) \quad (20)$$

that satisfies the equation system (18).

(ii) The Jacobian taken at $P$ differs from zero. Thus, we have

$$\Delta s = \frac{D(f_1, f_2, f_3, f_4, f_5)}{D(\theta_1, \psi_1, \theta_2, \psi_2, \phi_2)} \neq 0 \quad (21)$$

Then, as it follows from the Theorem of Implicit Function System Existence, equation system (18) can be solved in the neighborhood of $P$ by functions (19).

Using the obtained solution, we can determine the path of contact on the pinion-gear tooth surface, and the transmission errors caused by misalignment. The path of contact on surface $S_i$ $(i = 1, 2)$ is determined by the expressions

$$r_i(\theta_i, \psi_i) = \phi_i(\psi_i) \quad (i = 1, 2) \quad (22)$$

The transmission errors are determined by equation

$$\Delta \phi_2 = \phi_2(\phi_1) - \frac{N_1}{N_2} \phi_1 \quad (23)$$

The dimensions and orientations of the instantaneous contact ellipse at the contact point may be determined considering that the principal curvatures and directions of the contacting surfaces, and the elastic approach of the surface [9] are known.

7 NUMERICAL EXAMPLE

The blank data is given in Table 1.

The gear head-cutter is a cone (figs. 2, 3 and 4), the cutter radius is designated by $R_g$ (fig. 1), the radial setting of the head-cutter is $|O_C O_2|$ (fig. 5(b)), and the installment angle is $\tau_2$ (fig. 5). The data for the gear head-cutter that generates the gear concave side are represented in Table 2.

The parameters of the pinion head-cutter were determined by application of the method of local synthesis (section 5). The data for the pinion head-cutter that generates the pinion convex side are represented in Table 3. We
considered in the numerical examples the meshing of the gear tooth concave side with the pinion tooth convex side.

Case 1 corresponds to the orientation of the bearing contact across the surface, case 2 corresponds to the orientation of the bearing contact in the longitudinal direction.

The application of TCA for the simulation of meshing and contact permits the determination of misalignment effects on the transmission errors and the shift of the bearing contact. It has been shown that in the case of application of ideal generating surfaces (without mismatch, figs. 1 and 2) the errors of misalignment cause indeed discontinuous almost linear transmission errors as shown in fig. 14 for shaft angle error $\Delta \gamma$. Similar functions of transmission errors are caused by errors $\Delta A_p$, $\Delta A_s$ and $\Delta E$. Table 4 shows the maximal transmission errors caused by misalignment.

The results of TCA for the properly mismatched generating surfaces (see section 5) confirmed that a predesigned parabolic function indeed absorbs the transmission errors caused by misalignment, and the resulting function of transmission is a parabolic one (fig. 15). The absorption of linear function of transmission errors is carried out as well in other cases of misalignment: $\Delta A_p$, $\Delta A_s$ and $\Delta E$. The bearing contact of the drive is stabilized, and its shift is permissible (fig. 16). Model 2 of the gear drive (with longitudinal direction of the bearing contact) is preferable due to the lower level of transmission errors caused by misalignment.

8 CONCLUSION

From the study conducted the following general conclusions can be drawn:

1. An approach has been developed for the synthesis of spiral bevel gears that provides: (i) localized bearing contact, and (ii) low level of transmission errors of a parabolic type. The approach developed permits two possible directions of the bearing contact: across the tooth surface or in the longitudinal direction.

2. A Tooth Contact Analysis (TCA) computer program for the investigation of the influence of misalignment on the shift of the bearing contact was developed.

3. The low level of transmission errors, the parabolic type of the function of transmission errors, and the localization of the bearing contact are achieved by the proper mismatch of contacting surfaces.

4. The influence of the following errors of alignment was investigated: (i) for axial displacement of the pinion, (ii) axial displacement of the gear, (iii) offset, and (iv) change of the shaft angle. These types of misalignment were proven to cause discontinuous almost linear functions of transmission errors, but they are absorbed by the predesigned parabolic function of transmission errors.

The results of this investigation show that a predesigned parabolic function can indeed absorb the linear functions of transmission errors caused by misalignment. The design of gears with a longitudinal bearing contact (in comparison with the bearing contact across the surface) is preferable since a lower level of transmission errors can be obtained.

REFERENCES


APPENDIX 1

Equations of Gear Generating Surfaces

The generating cone represented in $S_t$ is

$$r_t(s, \theta) = \begin{bmatrix} (R_s - s \sin \alpha_s) \cos \theta_s \\ (R_s - s \sin \alpha_s) \sin \theta_s \\ s \cos \alpha_s \end{bmatrix}$$

(24)
where \( s_g \) and \( \theta_g \) are the surface coordinates; \( \alpha_g \) is the blade angle; \( R_g \) is the radius of the head-cutter at mean point. Equations (24) may also represent the convex side of the generating cone considering that \( \alpha_g \) is negative.

Coordinate system \( S_{s_2} \) is rigidly connected to coordinate system \( S_{s_1} \), and the unit normal to the gear generating surface is represented by the equations

\[
\mathbf{n}_{s_2}(\theta_g) = \frac{\mathbf{N}_{s_2}}{|\mathbf{N}_{s_2}|}, \quad \mathbf{N}_{s_2} = \frac{\partial \mathbf{r}_{s_2}}{\partial \theta_g} \times \frac{\partial \mathbf{r}_{s_2}}{\partial s_g}
\]  

Equations (24) and (25) yield

\[
\mathbf{n}_{s_2}(\theta_g) = \begin{bmatrix} \cos \alpha_g \cos \theta_g \\ \cos \alpha_g \sin \theta_g \\ \sin \alpha_g \end{bmatrix}
\]  

(26)

APPENDIX 2

Equations of Pinion Generating Surfaces

The generating surface of revolution is represented in \( S_{t_1} \), as

\[
\mathbf{r}_{t_1}(\lambda_p, \theta_p) = \begin{bmatrix} [R_p - R_1(\cos \alpha_p - \cos(\alpha_p + \lambda_p))] \cos \theta_p \\ [R_p - R_1(\cos \alpha_p - \cos(\alpha_p + \lambda_p))] \sin \theta_p \\ -R_1(\sin \alpha_p - \sin(\alpha_p + \lambda_p)) \end{bmatrix}
\]  

(27)

where \( \lambda_p \) and \( \theta_p \) are the generating surface coordinates; \( \alpha_p \) is the profile angle at M point; \( R_p \) is the radius of the head-cutter at mean point; \( R_1 \) is the radius of surface of revolution. Equations (27) can also represent the concave side of the generating surface of revolution if we substitute \( \alpha_p \) as \( 180^\circ - \alpha_p \).

Coordinate system \( S_{t_1} \) is rigidly connected to coordinate system \( S_{c_1} \), and the unit normal to the pinion generating surface is represented by the equations

\[
\mathbf{n}_{t_1}(\lambda_p, \theta_p) = \frac{\mathbf{N}_{c_1}}{|\mathbf{N}_{c_1}|}, \quad \mathbf{N}_{c_1} = \frac{\partial \mathbf{r}_{t_1}}{\partial \theta_p} \times \frac{\partial \mathbf{r}_{t_1}}{\partial \lambda_p}
\]  

Equations (27) and (28) yield

\[
\mathbf{n}_{c_1}(\lambda_p, \theta_p) = \begin{bmatrix} \cos \theta_p \cos(\alpha_p + \lambda_p) \\ \sin \theta_p \cos(\alpha_p + \lambda_p) \\ \sin(\alpha_p + \lambda_p) \end{bmatrix}
\]  

(29)

<table>
<thead>
<tr>
<th>Table 1: Blank Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pinion</td>
</tr>
<tr>
<td>( N_1, N_2 ), Number of teeth</td>
</tr>
<tr>
<td>( \gamma_1 ), Shaft angle</td>
</tr>
<tr>
<td>Mean spiral angle</td>
</tr>
<tr>
<td>Hand of spiral</td>
</tr>
<tr>
<td>Whole depth (mm)</td>
</tr>
<tr>
<td>( \gamma_1, \gamma_2 ), Pitch angles</td>
</tr>
</tbody>
</table>

![Figure 1: Generating cones](image1.png)

![Figure 2: Generating cone and generating surface of revolution](image2.png)

![Figure 3: Mismatched generating surfaces](image3.png)

![Figure 4: Cones for gear generation](image4.png)
FIGURE 5: Coordinate systems $S_c$ and $S_m$

FIGURE 6: Coordinate systems $S_m$, $S_b$ and $S_2$

FIGURE 7: (a) Convex (inside blade) and (b) concave (outside blade) sides of the generating blades and generating surfaces of revolution

FIGURE 8: Coordinate systems $S_m$, $S_a$ and $S_1$

FIGURE 9: Transmission function and predesigned parabolic function of transmission errors, $\phi_1$-pinion rotation angle; $\phi_2$-gear rotation angle; $\Delta \phi_2$-transmission error

FIGURE 10: Unit vectors of principal directions of surfaces $\Sigma_2$ and $\Sigma_1$

TABLE 2: Parameters and Installment of Gear Head-Cutter on gear concave side

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_2$, Blade angle</td>
<td>20°</td>
</tr>
<tr>
<td>$R_{p}$, Cutter radius at mean point (mm)</td>
<td>78.52</td>
</tr>
<tr>
<td>$S_{r2}$, Radial setting (mm)</td>
<td>70.53</td>
</tr>
<tr>
<td>$\phi_2$, Installment angle</td>
<td>$-62^\circ14'$</td>
</tr>
</tbody>
</table>
TABLE 3: Parameters and Installment of the Pinion Head-Cutter on pinion convex side

<table>
<thead>
<tr>
<th></th>
<th>Case 1</th>
<th>Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_p )</td>
<td>20(^{\circ})</td>
<td>20(^{\circ})</td>
</tr>
<tr>
<td>( q_1 )</td>
<td>-61(^{\circ})51'</td>
<td>-51(^{\circ})24'</td>
</tr>
<tr>
<td><strong>INPUT</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \eta_1 )</td>
<td>171(^{\circ})</td>
<td>92(^{\circ})</td>
</tr>
<tr>
<td>( m_{21} )</td>
<td>-1.3e-3</td>
<td>-1.2e-3</td>
</tr>
<tr>
<td>( \Delta \phi_2 )</td>
<td>-10.94</td>
<td>-10.09</td>
</tr>
<tr>
<td><strong>OUTPUT</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M</td>
<td>(79.88, 0.39, 0.17)</td>
<td>(77.83, 1.64, 0.72)</td>
</tr>
<tr>
<td>( R_e (mm) )</td>
<td>78.0</td>
<td>64.7</td>
</tr>
<tr>
<td>( R_i (mm) )</td>
<td>235.0</td>
<td>765.0</td>
</tr>
<tr>
<td>( 2a (mm) )</td>
<td>12.54</td>
<td>4.5</td>
</tr>
</tbody>
</table>

TABLE 4: Maximum Transmission Errors for Generating Surfaces with Mismatch

<table>
<thead>
<tr>
<th>( \Delta \phi_2 ) in arc sec.</th>
<th>Case 1</th>
<th>Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta A_p = 0.1 \text{mm} )</td>
<td>8.8</td>
<td>16.2</td>
</tr>
<tr>
<td>( \Delta A_y = 0.1 \text{mm} )</td>
<td>11.5</td>
<td>12.5</td>
</tr>
<tr>
<td>( \Delta E = 0.1 \text{mm} )</td>
<td>11</td>
<td>15</td>
</tr>
<tr>
<td>( \Delta \gamma = 3' )</td>
<td>10.7</td>
<td>13.5</td>
</tr>
</tbody>
</table>

FIGURE 11: Simulation of pinion misalignment \( \Delta A_p \)

FIGURE 12: Simulation of gear misalignment \( \Delta \gamma \)

FIGURE 13: Simulation of gear misalignment \( \Delta E \) and \( \Delta A_y \)

FIGURE 14: Transmission errors for a misaligned gear drive with ideal surfaces: \( \Delta \gamma = 3 \text{ arc min.} \)

FIGURE 15: Transmission errors for a misaligned gear drive with mismatched gear tooth surfaces: \( \Delta \gamma = 3 \text{ arc min.} \)

FIGURE 16: Longitudinal bearing contact for a misaligned gear drive (\( \Delta \gamma = 3 \text{ arc min.} \))
**Computerized Design and Analysis of Face-Milled, Uniform Tooth Height Spiral Bevel Gear Drives**

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**ABSTRACT**
Face-milled spiral bevel gears with uniform tooth height are considered. An approach is proposed for the design of low-noise and localized bearing contact of such gears. The approach is based on the mismatch of contacting surfaces and permits two types of bearing contact either directed longitudinally or across the surface to be obtained. A Tooth Contact Analysis (TCA) computer program was developed. This analysis was used to determine the influence of misalignment on meshing and contact of the spiral bevel gears. A numerical example that illustrates the developed theory is provided.