THE OSCILLATORY BEHAVIOR OF A

TWO-PHASE NATURAL CIRCULATION LOOP

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ABSTRACT

A natural circulation loop with water as the
circulating fluid was studied for a range of operation
covering two-phase flow. Periodic oscillations of the
flow rate and fluid temperature occur even with constant
heatinp and constant cooling water properties for the
heat exchangers. Several conclusions concerning the
stability of operation are given. Use is made of the
theoretical analysis of an open-ended system, and an
analogue computer. For use in a more detailed numerical
analysis, the equation of motion, the continuity equation,
and the energy equation are presented for a transient two-
phase flow model.
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Introduction

A program of study of the transient operation of
natural circulation loops has been underway at the University
of Minnesota (1), and this paper is concerned with the
oscillatory behavior of a two-phase natural circulation
loop. These studies are of interest for the emergency
cooling of nuclear reactors and in the design of boiling
water reactors. The literature survey pertaining to the
transient operation of a natural circulation loop is
given by Alstad, Iabin, Amundson and Silvers (1), and a
survey on two-phase flow is given by Iabin, Moen and Kosher (2).

Experimental Loop

Figure 1 is a schematic diagram of the natural
convection loop which was studied. The loop was constructed
primarily of 16 gage, 1 inch O.D. (0.872 inch I.D.) hard
drawn brass tubing. The major features of the loop are
described in reference (1). During a natural circulation
run, the flowmeters were by-passed and only the electro-
magnetic flowmeter was used. Normally the surge tank,
MINNESOTA NATURAL CIRCULATION LOOP NO. 3

SCALE: 1" = 40"

KEY:
A HEATERS
B PRESSURE DROP TEST SECTION
C HEAT EXCHANGER
D FLOW METERS
E SURGE TANKS
F PUMP
G ELECTROMAGNETIC METER
H COOLANT FLOW METER
J COLD WATER
K COOLANT OUTLET
O OVERFLOW
P PRESSURE TAPS
S STEAM
T THERMOCOUPLES

Figure 1: The Natural Circulation Loop.
E₂ was not used for two-phase natural circulation runs. For those runs in which the pressure at one point in the loop was held constant, the gate valve between the surge tank, E₂, and the loop was opened; for the constant volume run, the gate valve was closed. A heater and pump were installed to maintain the cooling water supply at 5 gpm and at 130°F.

Theoretical Analysis

General Equations

The continuity equation, the equation of motion and the energy equation for a viscous fluid flowing in a region of general geometry were formulated for a phase having continuous properties. A similar set of equations were derived for flow across a surface of discontinuity. The combination of these two sets of equations permits one to write the equations for the two-phase flow. An annular flow model was selected to illustrate some of the essential properties of two-phase flow. The model is sufficiently simple to permit attempting a numerical solution. Each phase is assumed to flow through a well defined cross-sectional area with a uniform velocity with the reservation that the liquid velocity at the wall must equal zero. Further, the steam and water phases are assumed to be in equilibrium; that is, p and T correspond to the saturation pressure and temperature.
For these conditions, the continuity equation may be written as

\[ \frac{\partial p}{\partial t} = -\frac{\partial w}{\partial x} \]  

(1)

The equation of motion for upward flow through a pipe of constant diameter becomes

\[ \frac{1}{\varepsilon_c} \frac{\partial w}{\partial t} + \frac{1}{\varepsilon_c} \frac{\partial (w\phi_v)}{\partial x} = -A \frac{\partial p}{\partial x} - A \frac{\varepsilon}{\varepsilon_c} \rho z^* - A F' \]  

(2)

Finally, the energy equation is

\[ A \frac{\partial (\rho H)}{\partial t} + \frac{\partial (w \phi_H)}{\partial x} - \frac{A}{J} \frac{\partial p}{\partial t} = Q' + \frac{\partial}{\partial x} \left[ (k_w A_w + k_s A_s) \frac{\partial T}{\partial x} \right] \]  

(3)

where

\[ 2 \pi r_k k_w \left. \frac{\partial T}{\partial x} \right|_{r_p} = Q' \]  

de ducted through the pipe.

The terms \( \phi_1 \) and \( \phi_2 \) represent the ratio of the true rate of transport to the rate of transport of the mean flow for momentum and energy respectively. If both phases have the same linear velocity, \( \phi_1 \) and \( \phi_2 \) are unity.

The three functions \( \phi_1, \phi_2 \), and \( F' \) were determined experimentally from steady state data. It was found that \( \phi_1 \) and \( \phi_2 \) could be correlated as functions of \( \rho \) alone, and that

\[ F' = a(\rho) \nu^{1.79} \]  

(4)

where \( a(\rho) \) is a function of \( \rho \). Figures 2, 3, and 4 illustrate the variation of \( \phi_1, \phi_2 \), and \( a(\rho) \) with \( \rho \).
Figure 4. The Friction Coefficient, $a$. 

\[ \rho: \frac{\text{lb}}{\text{cu.ft.}} \]
Equations (1) through (4) have been applied to a natural circulation loop in the form of finite difference equations. The discussion of these equations is not included in this paper for the numerical calculations using the SEAC (National Bureau of Standards' digital computer, Standard Eastern Automatic Computer) have not been successfully completed.

**Stability Analyses**

An insight on the factors which determine the stability of a natural convection system is gained through the analysis of an open-ended loop, such as shown in Figure 5. The fluid entering the heater always has the constant temperature $T_0$, and the velocity of the stream is fixed by the density difference between the hot and cold leg. For any constant heat input, one may define a state of equilibrium in which the difference in weight of the two legs is just equal to the frictional resistance to flow. Under certain conditions this system may be unstable; that is a small deviation from the equilibrium temperature distribution or the equilibrium velocity may be propagated in space or time with increasing amplitude. For example, if a velocity perturbation of the form, $\xi_y \sin \omega t$, is present, under what conditions could the temperature perturbation which is generated cause the driving force that sustains the velocity
disturbance? The problem has been treated analytically for a one-phase fluid (3), and the following conclusions were obtained:

a) If an oscillatory flow rate is to be possible, the force cannot be generated in the heater; it must be generated in the vertical riser.

b) The product of the coefficient of expansion of the fluid and the vertical height of the riser must exceed a certain value (defined by an analytical expression) if the velocity perturbation is to be sustained.

c) The period of oscillation will be approximately equal to the residence time of the fluid in the heater and the vertical riser.

In order to predict the period for a closed loop, a problem was solved on a Reeves Electronic Analogue Computer. The number of non-linear terms was limited to 16, and the model used was necessarily a simple one. For such a problem, the period of the oscillations should be meaningful even though the wave shape is not correct, if the period is a function of only the geometry and the mean velocity. The computer could handle only ten subdivisions, with the driving force expressed as a linear function of all ten enthalpies, and the frictional force as a quadratic function of the velocity. The number
of available summing amplifiers limited the problem to
the case in which boiling occurs only at the top sub-
division of the vertical riser. The equations solved
were of the following form:

\[
\frac{dH_n}{dt} = -\frac{V}{(\Delta x)_n} (H_n - H_{n-1}) + \frac{Q_n}{(\Delta x)_n} \rho p
\]  

\[
\frac{dV}{dt} = \frac{-\rho_n \Delta t \Delta x_n - F}{\frac{F_n}{\frac{c}{C}} \frac{\Delta x_n}{\Delta n}}
\]

(5)  

(6)

The loop subdivisions are given in Figure 6.
For section 1, \( Q_n = q_1 \), for section 5, \( Q_5 = -6V \left( \frac{H_5 + H_6}{2} - h_c \right) \)
where the cooler heat transfer coefficient is taken to be
a linear function of velocity, and all remaining Q's are
taken equal to zero. The density at point \( n \) is written as

\[
\rho_n = \rho_{n_0} + \alpha_n (H_n - H_{n_0})
\]

(7)

and the mean density of the \( n \)th subinterval is set equal
to the average of the densities at \( n \) and \( n-1 \).

A stable solution was found if the coefficient
of expansion of water is used for all values of \( \alpha_n \). An
oscillatory solution was obtained, Figure 7, if vaporization
to a few per cent quality were assumed in the top section
of the vertical riser ( \( \alpha_4 = -9.500 \), all other \( \alpha's =
-0.024 \text{ lb/ft}^2 \text{F}. \) \).
Figure 7. Computer of the Analog Computer Problem.
Results

Two steady equilibrium modes of operation were possible when the pressure at one point in the loop is held constant (surge tank $E_2$ open to the atmosphere). For a very low heat input the water temperature in the riser never exceeds the boiling point and a state of stable equilibrium may be defined. For a very high heat input, the entire riser contains both steam and water, and a maximum flow rate is obtained. Oscillatory modes of operation result for the intermediate heat input. An illustration of the manner in which the period and amplitude depend on the heat input is given in Figures 8 to 12. The period and amplitude of the oscillations are determined by the mean temperature level of the fluid in the vertical riser.

The period was inversely proportional to the mean velocity providing some steam is in the riser at all times. When the system does not contain steam during most of the cycle, the period is considerably longer than that predicted by the extrapolation of the higher flow rate periods.

In the analogue computer problem, $Q_1 = 3.16$ Btu/sec, and the period was $149$ seconds, which is about $21$ seconds.
less than the experimentally observed value for the lowest flow rate. The computed period would be expected to be less than the observed value, since the density function used in the top subinterval in the riser was a two-phase density function. Further, the density function used for the boiling subinterval was linear and it was not possible to exclude densities greater than the density of saturated water. A special non-linear element is required to generate a density function with the correct properties. As a result, the computed mean flow rate is less than that which could actually exist; however, the flow rate curve and enthalpy curves have essentially the same shape as the experimentally determined ones. A larger computer is required for solving the stability problem at the higher heat fluxes.

Conclusions

A natural circulation loop can be made unstable in the sense that a small displacement from the equilibrium state leads to undamped oscillations. Stable operations result when the fluid temperature in the riser is restricted to values less than the boiling point, and when the heat input reaches a value such that the frictional force
changes more rapidly than the driving force. The theoretical treatment of an open-ended natural circulation system and the solution of a simple problem using an analogue computer have lent support to the general conclusions of the stability analyses. A detailed numerical analysis is in progress on a large digital computer.
Acknowledgments

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NOTATION

Equations 1 through 4

A = cross-section area for flow
W = total mass flow rate
x = distance along streamline
t = time
\( g = \) local acceleration of gravity; \( g_0 = \) conversion factor in Newton's law of motion
\( z' = \) slope of pipe at \( x \)
\( F_f = \) wall of frictional force
\( \rho = \) fluid density. \( \rho = \frac{1}{A} (A_w \rho_w + A_b \rho_b) \)
\( V = \) volumetric flow rate. \( V = \frac{A_w \rho_w V_w + A_b \rho_b V_b}{A} = \frac{W}{A} \)
H = specific fluid enthalpy. \( H = \frac{1}{A} [A_w \rho w H_w + A_b \rho_b H_b] \)
\( \phi_1 = \frac{A_w \rho w V_w^2 + A_b \rho_b V_b^2}{W} \); \( \phi_2 = \frac{A_w \rho w V_w H_w + A_b \rho_b V_b H_b}{W} \)
k = thermal conductivity
T = fluid temperature
p = fluid pressure
r = radius, \( r_p \) = pipe radius

Q' = rate at which heat is conducted through pipe.

\( s(\rho) \) = an empirically defined function for Eq.4 for two-phase pressure drop.

Subscripts

s = steam phase; \( w \) = liquid water phase

Equations 5 through 7

The energy balance is reduced to a heat balance

\( H \) = specific enthalpy

Q = rate at which heat is added to fluid

V = volumetric flow rate

F = frictional force,

\[
F = \frac{N}{V} \left\{ \frac{\Delta P}{V} + \frac{V}{V_o} (V-V_o) + \frac{0.72F_o}{V_o} (V-V_o)^2 \right\}
\]

\( F_o \) is the frictional force corresponding to a volumetric flow rate \( V_o \).

\( h_c \) = cooler heat transfer coefficient

\( \Delta x \) = length of subinterval; \( z_n \) = vertical height of subinterval

\( \rho \) = density, \( \bar{\rho} \) = mean density in a subinterval

\( C_p \) = heat capacity

\( c_n \) = a constant related to the coefficient of expansion of water and defined by Eq. 7

For the computer problem, for \( n = 1, 2, \) and 3, \( H_{n_o} = 18 \text{Btu/lb}, \)

\( \rho_{n_o} = 59.781 \text{ lb/cu ft.}, \ c_n = - 0.024 \text{ lb/cu ft.}^2 \text{F}; \ n = 4, \)
\[ h_{n_0} = 182, \quad \rho_{n_0} = 37.500, \quad \alpha_n = -9.560; \quad n = 5 \text{ through } 10, \]
\[ h_{n_0} = 153, \quad \rho_{n_0} = 60.477, \quad \alpha_n = -0.024. \quad V_0 = 0.0025 \text{ cu.ft./sec,} \]
\[ Q_1 = 3.16 \text{ Btu/sec, and } Q_2 = -25.0V(\frac{h_5 + h_4}{2} - 89.6) \text{ Btu/sec.} \]

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