

IDA DOCUMENT D-1457

ON THE FEASIBILITY OF USING  
EXTREMELY LOW FREQUENCY ELECTROMAGNETIC FIELDS  
TO DETECT MINES IN SHALLOW WATER

Bohdan Balko  
Irvin Kay

January 1994

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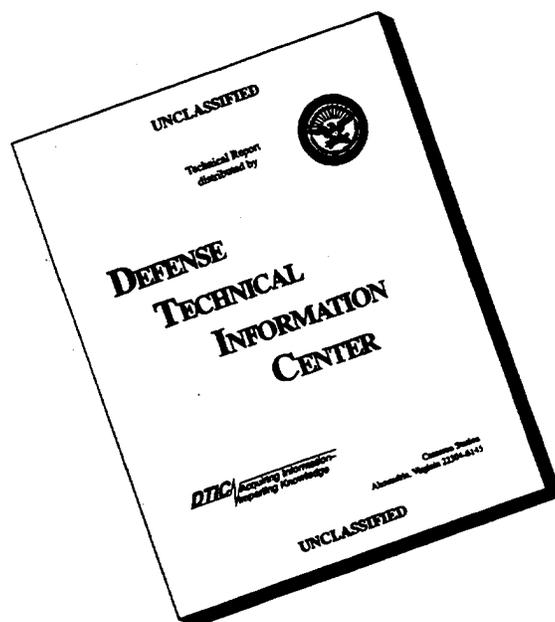
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INSTITUTE FOR DEFENSE ANALYSES

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Task T-Q2-893

## **PREFACE**

This document was prepared in partial fulfillment of contractual obligations for Contract MDA 903-89-C-0003, Task Order T-Q2-893, performed for the Office of the Assistant Secretary of Defense, Program Acquisition and Evaluation.

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## ABSTRACT

This document considers an approach to mine detection based on electromagnetic detection. It involves the use of a magnetic or electric dipole antenna to transmit a low frequency signal that would illuminate and interact with an object, such as a mine, all dimensions of which are small compared to the wavelength as well as to the distance between the object and the antenna. A dipole antenna, which may be the same as or different from the transmitter, would receive the signal due to the interaction of the illuminated body with the incident electromagnetic field.

Past analyses have demonstrated the feasibility, even at fairly long ranges, of using blue-green lasers to detect bodies immersed in pure water. Unfortunately, this detection method fails in turbid water, which is not transparent to the short wavelength laser radiation, and in cases where the mines have become buried. Sonar techniques, while useful in deeper water, also have problems in the shallow water regime because of clutter and reverberations.

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# ON THE FEASIBILITY OF USING EXTREMELY LOW FREQUENCY (ELF) ELECTROMAGNETIC FIELDS TO DETECT MINES IN SHALLOW WATER

## A. INTRODUCTION

Past analyses have demonstrated the feasibility, even at fairly long ranges, of using blue-green lasers to detect bodies immersed in pure water. Unfortunately, this detection method fails in turbid water, which is not transparent to the short wavelength laser radiation and in cases where the mines have become buried. Sonar techniques, while useful in deep water, also have problems in the shallow water regime because of clutter and reverberations.

The approach considered in the present document is based on electromagnetic detection. It involves the use of a magnetic or electric dipole antenna to transmit a low frequency signal that would illuminate and interact with an object, such as a mine, all dimensions of which are small compared to the wavelength as well as to the distance between the object and the antenna. A dipole antenna, which may be the same as or different from the transmitter, would receive the signal due to this interaction of the illuminated body with the incident electromagnetic field. Figure 1 shows one possible operation of the proposed system with a dipole transmitting antenna mounted on an airplane and a dipole receiving antenna attached by a cable to the airplane and pulled above the water surface (Ref. 1).

Figure 2 gives a schematic of the shallow water regime, which extends from the beach to 180 ft in depth and is divided for convenience into three regions: shallow water (SW), very shallow water (VSW), and surf zone (SZ). Each region presents its own special problems for the mine hunter, with the SZ especially troublesome.

The useful ELF frequency range is determined by the depth of water penetration required to reach the mine. The penetration depth of the EM fields as a function of frequency  $f$  can be obtained from the skin depth formula given by (cf. Ref. 2, p. 252)

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} = 0.25 \times 10^3 \frac{1}{\sqrt{f}}, \text{ where } \mu = 4\pi \times 10^{-7} \text{ henrys/m and the conductivity of sea}$$

water is taken to be  $\sigma = 4\Omega^{-1}/m$ . Table 1 gives the frequency range required for penetration to the depth of interest and lists the regions of applicability.

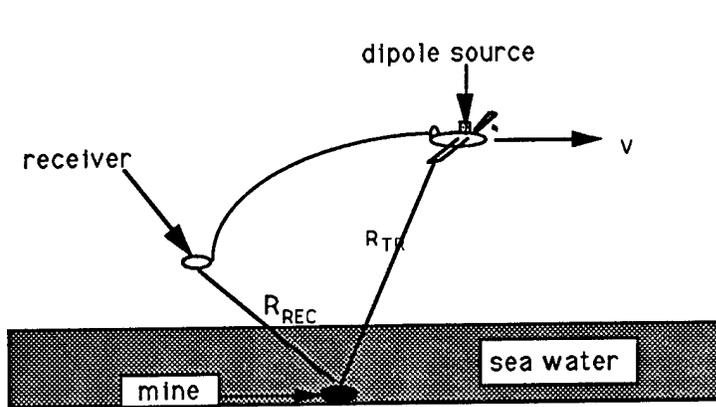


Figure 1. A possible realization of the ELF mine detection concept

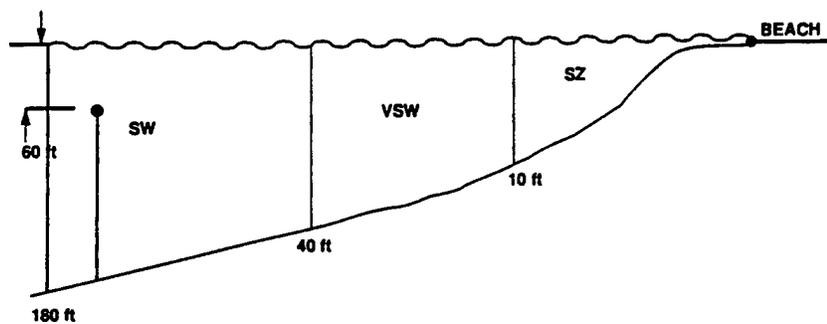


Figure 2. Shallow water regime showing the shallow water (SW), very shallow water (VSW) and surf zone (SZ) regions

Table 1. Frequency range of interest determined from skin depth in water

f	$\delta$	region of applicability	effectiveness against mines
0.1 kHz	25m	SW/VSW	moored mines
1.0 kHz	8 m	SZ	bottom mines
10 kHz	2.5m	SZ/beach	buried mines

In addition to the penetration requirement, detectability will also depend on the difference between the electromagnetic properties of the body and those of the surrounding

medium. The body is assumed to have finite conductivity  $\sigma_2$  and permittivity  $\epsilon_2$ , and the medium surrounding it (e.g., sea water or wet sand) is assumed to have the conductivity  $\sigma_1$  and permittivity  $\epsilon_1$ .

The objective of this document is to demonstrate the detectability, in principle, of such a body by estimating typical values of the electric and magnetic field components that would be observed at the receiver because of its presence. For the first order estimate considered here it is assumed that the surrounding medium is homogeneous and infinite in extent. In particular, the effect of an air-water interface is not taken into account.

## B. ANALYSIS

### 1. One Medium Field Calculation

In this Section it will be assumed that the transmitter and receiver, as well as the conducting body, are surrounded by a single, possibly conducting, propagation medium. The medium and the body have different conductivities, and perhaps different permittivities as well.

The electric field due to a vertical magnetic dipole antenna (coil in horizontal plane) is given in spherical coordinates (Fig. 3) by

$$E_{\phi} = \frac{k^2}{4\pi} \sqrt{\frac{\mu}{\epsilon}} \left( \frac{1}{R} + \frac{i}{kR^2} \right) \sin\theta |\mathbf{m}| e^{-i\omega t},$$

$$k = \omega\sqrt{\epsilon\mu} \quad (1)$$

(Ref. 3, p. 437), where  $\omega$  is the angular frequency and  $\mathbf{m}$  is the magnetic dipole moment. The electric field due to a vertical electric dipole is given by

$$E_R = \frac{1}{4\pi\epsilon} \left( \frac{1}{R^3} - \frac{ik}{R^2} \right) \cos\theta |\mathbf{p}| e^{-i\omega t},$$

$$E_{\theta} = \frac{1}{4\pi\epsilon} \left( \frac{1}{R^3} - \frac{ik}{R^2} - \frac{k^2}{R} \right) \sin\theta |\mathbf{p}| e^{-i\omega t}, \quad (2)$$

(Ref. 3, p. 436) where  $\mathbf{p}$  is the electric dipole moment. It is seen from Table 1 that the frequencies of interest are low, e.g., of the order of 100 Hz -  $10^4$  Hz, so that  $k \sim 10^{-6}\text{m}^{-1}$  -  $10^{-4}\text{m}^{-1}$ .

If the medium has a non-zero conductivity, it can be taken into account in (1) and (2) by using a complex permittivity  $\epsilon'$  in place of the real  $\epsilon$ .<sup>1</sup> That is,  $\epsilon$  would be replaced by

$$\epsilon' = \epsilon + i\frac{\sigma}{\omega},$$

where, for the frequency  $f$  in Hertz,

$$\omega = 2\pi f.$$

The magnetic dipole moment is proportional to the area  $\pi a^2$ , where  $a$  is the radius of the loop circuit element associated with the magnetic dipole, and to the current flowing around the circuit. The electric dipole moment is proportional to the length  $l$  of the linear circuit element associated with the electric dipole and to the charge accumulated at the endpoints of the element. Assuming a periodic alternating current in the electric dipole [multiplying and dividing (2) by  $-i\omega$ ], equations (2) can be written in the form

$$\begin{aligned} E_R &= \frac{I}{2\pi} \sqrt{\frac{\mu}{\epsilon}} \left( \frac{i}{kR^3} + \frac{1}{R^2} \right) \cos \theta e^{-i\omega t}, \\ E_\theta &= \frac{I}{2\pi} \sqrt{\frac{\mu}{\epsilon}} \left( \frac{i}{kR^3} + \frac{1}{R^2} - \frac{ik}{R} \right) \sin \theta e^{-i\omega t}, \end{aligned} \quad (3)$$

where  $I$  is the current equal to the charge in the electric dipole moment multiplied by  $i\omega$ . The electric field given by Eq. 1, due to the magnetic dipole, can now be compared to the electric field given by Eq. 3, due to the electric dipole.

Because the ratio of the field given by Eq. 1 to either component given by Eq. 3 is proportional to  $k^{-2}$ , if the conductivity in the medium is zero the electric field due to the magnetic dipole should be many orders of magnitude less than that due to the electric dipole. In fact, even in the case of a non-zero  $\sigma$ , for which

$$k^2 = \omega^2 \epsilon \mu + i\omega \sigma \mu \sim i\omega \sigma \mu$$

it may be observed that for  $f \sim 10^2 - 10^4$

$$|k|^2 \text{ in of the order } 10^{-4} \text{ m}^{-2} - 10^{-2} \text{ m}^{-2},$$

if, for example, as in the case of sea water,  $\sigma \sim 4$  mhos/m.

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<sup>1</sup> Cf. Ref. 1, p. 483.

The quantities  $\sigma_2$  and  $\epsilon_2$  are the conductivity and permittivity of the body. The corresponding quantities for the surrounding medium are represented by  $\sigma_1$  and  $\epsilon_1$ .

It is assumed that  $\sigma_1$  is large compared with  $\omega_1\epsilon_1$  and  $\sigma_2$  is large compared with  $\omega_2\epsilon_2$ . Otherwise, it is assumed that for the medium containing the antennas and the body the permittivity and permeability have the values associated with a vacuum (cf. Ref. 2); i.e.,

$$\epsilon_1 = 8.854 \times 10^{-12} \text{ farads/meter};$$

$$\mu_1 = 4\pi \times 10^{-7} \text{ henrys/meter}.$$

It follows from these values that

$$\frac{1}{\sqrt{\epsilon_1\mu_1}} = 3 \times 10^8 \text{ m/sec} = c;$$

$$\sqrt{\frac{\mu_1}{\epsilon_1}} = 377\Omega.$$

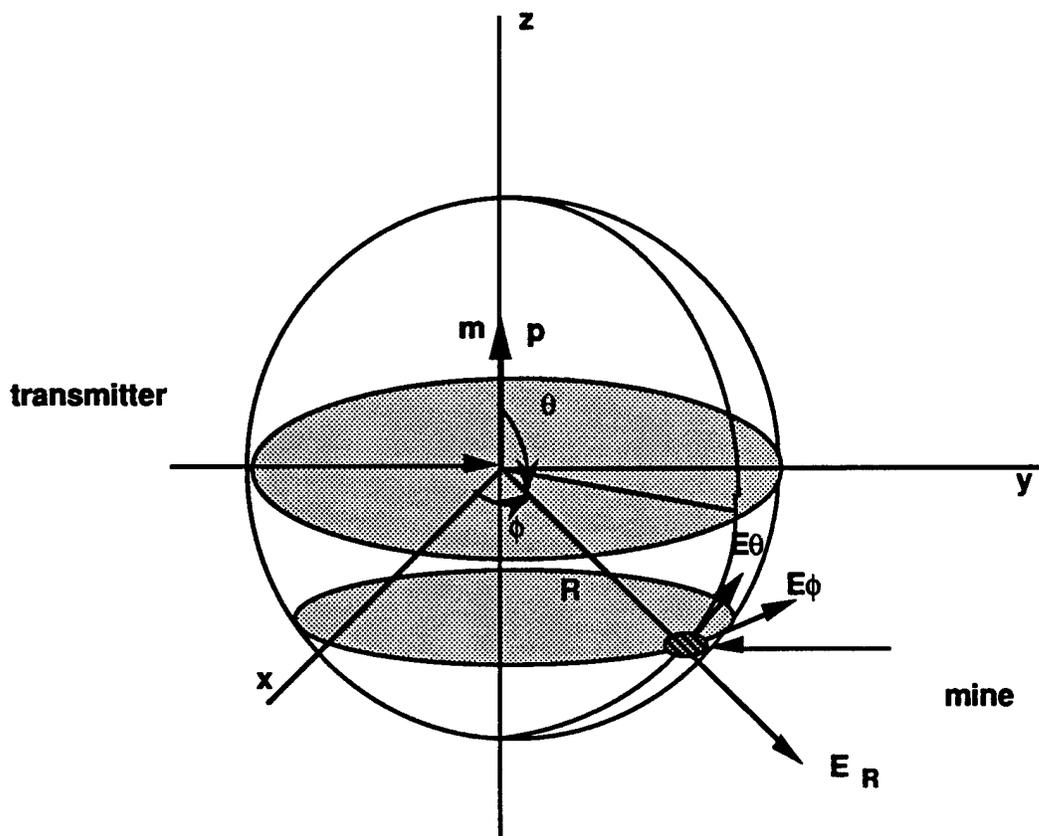


Figure 3. Electric field components in spherical coordinate ( $E_R, E_\theta, E_\phi$ ) originating from a magnetic dipole  $m$  (or electric dipole  $p$ ) located at the origin

The dipole antenna radiation interacts in the near field region with the body to be detected. Therefore, a reasonable way to estimate the electromagnetic signal due to the presence of the body, the effect of which depends largely on the fact that its surface is a discontinuity in the surrounding medium, is to treat it as a small source characterized by a dipole moment that the incident electric field induces in it. For this purpose, in a first order approximation the body can be regarded as a small sphere (of radius  $r$ ) immersed in a uniform field having parallel lines of force. If  $E_0$  is the incident complex electric field amplitude the corresponding dipole moment is given by (Ref. 3, p. 206)

$$|\mathbf{p}| = 4\pi r^3 \left| \frac{k_1 - 1}{k_1 + 2} \epsilon_1 E_0 \right|, \quad (4)$$

where

$$k_1 = \frac{\epsilon_2}{\epsilon_1}$$

and the permittivity is interpreted as a complex quantity taking into account the conductivity (Cf. Ref. 3, p. 483); i.e.,

$$\epsilon_k \Rightarrow \epsilon_k + i \frac{\sigma_k}{\omega}. \quad (5)$$

It is evident that terms involving the  $\epsilon_k$  can be neglected in comparison to the  $\sigma_k$ , so that (4) becomes

$$|\mathbf{p}| = 2 \frac{\sigma_1}{f} \left( \frac{\frac{\sigma_2}{\sigma_1} - 1}{\frac{\sigma_2}{\sigma_1} + 2} \right) r^3 |E_0|. \quad (6)$$

The electric field at the receiver in terms of the dipole moment given by (6) is given by (2) if the variables  $\theta$  and  $R$ , as well as the components  $E_\theta$  and  $E_R$ , are interpreted in terms of a coordinate system centered at the body.<sup>2</sup> (See Figure 4.) The magnetic field at the receiver is given by (Ref. 2, p. 436)

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<sup>2</sup> Although the  $1/f$  dependence of  $|\mathbf{p}|$  in eq. 6 seems to lead to unphysical results as  $f \rightarrow 0$ , in fact as the induced dipole moment given by Eq. 6 is introduced into Eq. 2 to obtain the actual measured quantity,

$$H_{\phi} = -\frac{if}{2} \left( \frac{1}{R_2} - \frac{ik}{R} \right) \sin\theta |p| e^{-i\omega t}. \quad (7)$$

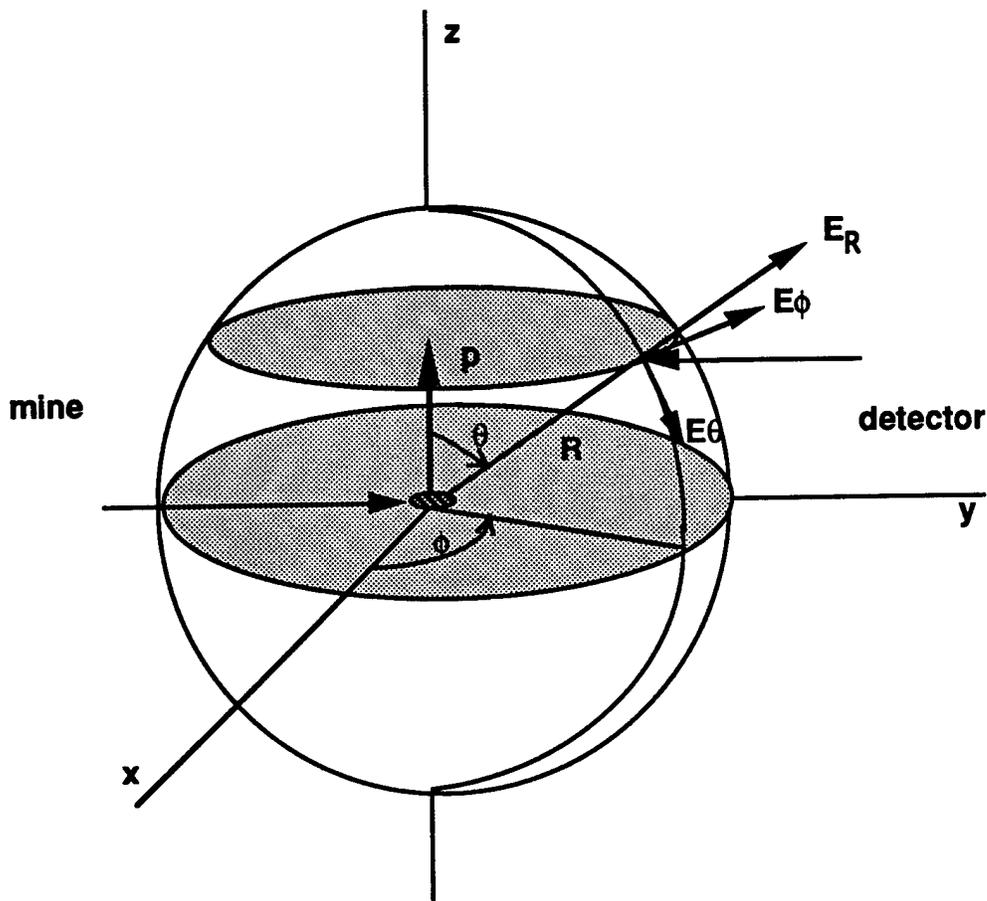


Figure 4. Electric field components in spherical coordinates ( $E_R$ ,  $E_{\theta}$ ,  $E_{\phi}$ ) originating from an induced electric dipole  $p$  located at the origin and representing the mine in an electromagnetic field

## 2. Two Media Field Calculation

### a. Basis for the Calculation and Assumptions

A cylindrical coordinate system will be used for the analysis in this case. The plane  $z = 0$  separates space into two regions, with  $z < 0$  defining Region 1 and  $z > 0$  defining Region 2.

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the field, the  $f$  dependence cancels out thus removing the singularity. The same happens when Eq. 7 for the magnetic field is used.

It is assumed that the transmitter and receiver are located in Region 2, while the conducting body is located in Region 1. It is also assumed that the propagation medium in Region 2 is air, for which the conductivity  $\sigma_2$  is 0, and that the medium in Region 1 is homogeneous and has a conductivity  $\sigma_1$  greater than 0, as well as a permittivity  $\epsilon_1$  that may differ from the air (or vacuum) permittivity  $\epsilon_2$ . However, for simplicity, in the numerical examples in Section IV it will be assumed that both permittivities have the same value, i.e.,  $8.854 \times 10^{-12}$  farads/meter.

Ref. 4 has already derived all of the mathematical results needed for calculating the electromagnetic fields in the theoretical model treated in this section. In particular, Ref. 4 (pp. 43-49) gives the electric and magnetic field components in cylindrical coordinates in Regions 1 and 2 for each of the following source and source position combinations:<sup>3</sup>

- (1) vertical electric dipole in Region 1,
- (2) vertical electric dipole in Region 2,
- (3) vertical electric dipole in Region 1,
- (4) vertical electric dipole in Region 2,
- (5) horizontal electric dipole in Region 1,
- (6) horizontal electric dipole in Region 2,
- (7) horizontal electric dipole in Region 1,
- (8) horizontal electric dipole in Region 2.

The effect of the body, located in Region 1, on the field at the receiver, located in Region 2, depends only on the dipole moment induced in the body by the local electric field, i.e., the field in Region 1. Therefore, in Region 1 only the electric field due to a source in Region 2 is of interest; i.e., only cases (1), (2), (4), (5), (6), and (8) are of interest here. Also, in a particular region the fields due to a source (including the dipole induced in the body) in the same region are of no interest here.<sup>4</sup>

Ref. 4 (pp. 28-33) gives all of the formulas for the field components in terms of certain universal functions, the definitions of which are given here in Appendix A. Appendix A also contains formulas for various required derivatives of these functions.

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<sup>3</sup> The definition of an electric dipole moment in Ref. 4 differs from the conventional definition used here by a factor  $-i\omega$ .

<sup>4</sup> This is true only because multiple scattering is neglected.

Each of the following subsections, using the Ref. 4 formulas, derives the electromagnetic fields in Region 2 due to the electric dipole moment induced in a small conducting sphere in Region 1 by a particular dipole source in Region 2. In all cases the (cylindrical) coordinate system will be such that the source is located on the z axis at a distance h above the z = 0 plane, and the center of the body is located at the point (r<sub>b</sub>, 0, z<sub>b</sub>).

### b. Vertical Electric Dipole in Region 2

According to Ref. 4, p. 43, the electric field components in Region 1 due to the vertical electric dipole in Region 2 are given by

$$E_{1r}(r,0,z) = \frac{\omega^2 p \mu}{4\pi} \left( \frac{\partial^2 V_{21}}{\partial r \partial z} \right),$$

$$E_{1z}(r,0,z) = \frac{\omega^2 p \mu}{4\pi} \left[ \left( \frac{\partial^2}{\partial z^2} + k_1^2 \right) V_{21} \right].$$
(8)

The relations in Appendix A applied to (8) lead to

$$E_{1r}(r,0,z) = -\frac{\omega^2 p \mu}{2\pi} \int_0^\infty \frac{\gamma_1 e^{\gamma_1 z - \gamma_2 h}}{k_2^2 \gamma_1 + k_1^2 \gamma_2} J_1(\lambda r) \lambda^2 d\lambda,$$

$$E_{1z}(r,0,z) = \frac{\omega^2 p \mu}{2\pi} \int_0^\infty \frac{e^{\gamma_1 z - \gamma_2 h}}{k_2^2 \gamma_1 + k_1^2 \gamma_2} J_0(\lambda r) \lambda^3 d\lambda.$$
(9)

The incident electric field in Region 1 has the components E<sub>1r</sub> and E<sub>1z</sub> given by (9). The first component induces a horizontal electric dipole moment and the second a vertical electric dipole moment in the body. Each component acts separately as the incident field in (6), which gives the corresponding magnitude of the dipole moment induced by the component in the body.

The total field in Region 2 resulting from these two induced sources is the sum of the fields due separately to each. The field in Region 2 due to the dipole moment induced

in the body by the component  $E_{1z}$  can be calculated using the formula (11). The field in Region 2 due to the dipole moment induced in the body by the component  $E_{1r}$  can be calculated using the formula (12). In these calculations it is important to keep in mind that those formulas are expressed in terms of a cylindrical coordinate system for which the  $z$  axis passes through the center of the body.

According to Ref. 4, p. 43, the electromagnetic field components in Region 2 due to a vertical electric dipole in Region 1 are given by

$$\begin{aligned} E_{2r} &= \frac{\omega^2 p \mu}{4\pi} \left( \frac{\partial V_{12}}{\partial r \partial z} \right), \\ E_{2z} &= \frac{\omega^2 p \mu}{4\pi} \left[ \left( \frac{\partial^2}{\partial z^2} + k_2^2 \right) V_{12} \right], \\ H_{2\theta} &= \frac{i\omega p k_2^2}{4\pi} \left( \frac{\partial V_{12}}{\partial r} \right). \end{aligned} \quad (10)$$

The relations in Appendix A applied to (10) lead to

$$\begin{aligned} E_{2r} &= \frac{\omega^2 p \mu}{2\pi} \int_0^\infty \frac{\gamma_2 e^{-\gamma_1 h - \gamma_2 z}}{k_2^2 \gamma_1 + k_1^2 \gamma_2} J_1(\lambda r) \lambda^2 d\lambda, \\ E_{2z} &= \frac{\omega^2 p \mu}{2\pi} \int_0^\infty \frac{e^{-\gamma_1 h - \gamma_2 z}}{k_2^2 \gamma_1 + k_1^2 \gamma_2} J_0(\lambda r) \lambda^3 d\lambda, \\ H_{2\theta} &= \frac{i\omega p k_2^2}{2\pi} \int_0^\infty \frac{e^{-\gamma_1 h - \gamma_2 z}}{k_2^2 \gamma_1 + k_1^2 \gamma_2} J_1(\lambda r) \lambda^2 d\lambda. \end{aligned} \quad (11)$$

According to Ref. 4, p. 46, the electromagnetic field components in Region 2 due to a horizontal electric dipole in Region 1 are given by

$$\begin{aligned} E_{2r} &= \frac{\omega^2 p \mu}{4\pi} \cos \theta \left( \frac{\partial^2 V_{12}}{\partial r^2} + U_{12} \right), \\ E_{2\theta} &= -\frac{\omega^2 p \mu}{4\pi} \sin \theta \left( \frac{1}{r} \frac{\partial V_{12}}{\partial r} + U_{12} \right), \end{aligned}$$

$$E_{2z} = \frac{\omega^2 p \mu}{4\pi} \cos \theta \left( \frac{\partial^2 V_{12}}{\partial r \partial h} \right), \quad (12)$$

$$H_{2r} = -\frac{i\omega p}{4\pi} \sin \theta \left( \frac{\partial U_{12}}{\partial z} - \frac{1}{r} \frac{\partial W_{12}}{\partial r} \right),$$

$$H_{2\theta} = -\frac{i\omega p}{4\pi} \cos \theta \left( \frac{\partial U_{12}}{\partial z} - \frac{\partial^2 W_{12}}{\partial r^2} \right),$$

$$H_{2z} = \frac{i\omega p}{4\pi} \sin \theta \left( \frac{\partial U_{12}}{\partial r} \right).$$

The relations in Appendix A applied to (12) lead to

$$\begin{aligned} E_{2r} &= \frac{\omega^2 p \mu}{4\pi} \cos \theta \left( \int_0^{\infty} \frac{e^{-\gamma_1 h - \gamma_2 z}}{k_2^2 \gamma_1 + k_1^2 \gamma_2} [J_2(\lambda r) - J_0(\lambda r)] \lambda^3 d\lambda + 2 \int_0^{\infty} \frac{e^{-\gamma_1 h - \gamma_2 z}}{\gamma_1 + \gamma_2} J_0(\lambda r) \lambda d\lambda \right), \\ E_{2\theta} &= \frac{\omega^2 p \mu}{2\pi} \sin \theta \left( \int_0^{\infty} \frac{e^{-\gamma_1 h - \gamma_2 z}}{\gamma_1 + \gamma_2} J_0(\lambda r) \lambda d\lambda - \frac{1}{r} \int_0^{\infty} \frac{e^{-\gamma_1 h - \gamma_2 z}}{k_2^2 \gamma_1 + k_1^2 \gamma_2} J_1(\lambda r) \lambda^2 d\lambda \right), \\ E_{2z} &= \frac{\omega^2 p \mu}{2\pi} \cos \theta \int_0^{\infty} \frac{\gamma_1 e^{-\gamma_1 h - \gamma_2 z}}{k_2^2 \gamma_1 + k_1^2 \gamma_2} J_1(\lambda r) \lambda^2 d\lambda, \\ H_{2r} &= \frac{i\omega p}{2\pi} \sin \theta \left( \int_0^{\infty} \frac{\gamma_2 e^{-\gamma_1 h - \gamma_2 z}}{\gamma_1 + \gamma_2} J_0(\lambda r) \lambda d\lambda - \frac{1}{r} \int_0^{\infty} \frac{(\gamma_2 - \gamma_1) e^{\gamma_1 h - \gamma_2 z}}{k_2^2 \gamma_1 + k_1^2 \gamma_2} J_1(\lambda r) \lambda^2 d\lambda \right), \\ H_{2\theta} &= \frac{i\omega p}{2\pi} \left( \int_0^{\infty} \frac{\gamma_2 e^{-\gamma_1 h - \gamma_2 z}}{\gamma_1 + \gamma_2} J_0(\lambda r) \lambda d\lambda + \int_0^{\infty} \frac{(\gamma_2 - \gamma_1) e^{-\gamma_1 h - \gamma_2 z}}{k_2^2 \gamma_1 + k_1^2 \gamma_2} [J_2(\lambda r) - J_0(\lambda r)] \lambda^3 d\lambda \right), \\ H_{2z} &= -\frac{i\omega p}{2\pi} \sin \theta \int_0^{\infty} \frac{e^{-\gamma_1 h - \gamma_2 z}}{\gamma_1 + \gamma_2} J_1(\lambda r) \lambda^2 d\lambda. \end{aligned} \quad (13)$$

### c. Vertical Magnetic Dipole in Region 2

According to Ref. 4, p. 44, the electric field has just one non-zero component in Region 1 due to a vertical magnetic dipole in Region 2. It is given by

$$E_{10} = -\frac{i\omega m\mu}{4\pi} \left( \frac{\partial U_{21}}{\partial r} \right). \quad (14)$$

The relations in Appendix A applied to (14) lead to

$$E_{10} = \frac{i\omega m\mu}{2\pi} \int_0^{\infty} \frac{e^{\gamma_1 z - \gamma_2 b}}{\gamma_1 + \gamma_2} J_1(\lambda r) \lambda^2 d\lambda. \quad (15)$$

In this case the incident electric field in Region 1 consists of the single component  $E_{10}$  given by (15). If the  $z$  axis passes through the center of the body in Region 1, then  $r_b = 0$ , so that, according to (15),  $E_{10} = 0$ , and thus, according to (6),  $p = 0$ . Therefore, the body will have no perceptible effect on the electromagnetic field.

For other positions of the body the induced dipole moment will be that of a horizontal electric dipole. Therefore, the resulting electromagnetic field added in Region 2 can be calculated using the formula (13).

## C. NUMERICAL EXAMPLES

### 1. One Medium

To illustrate the effect that a small body might have on the electromagnetic field created and observed by dipole antennas under conditions such as those mentioned in the last section, the following parameters will be assumed for the numerical examples of this section:

frequency  $f = 100$  Hz;

electric dipole moment  $|\mathbf{p}| = 1$  coulomb meter;<sup>5</sup>

magnetic dipole moment  $|\mathbf{m}| = 1$  ampere meter<sup>2,3</sup>

---

<sup>5</sup> The electric dipole and the magnetic dipole have different units and thus cannot be compared directly. We picked the particular values of  $\mathbf{p}$  and  $\mathbf{m}$  given in the text for illustration only. A magnetic dipole antenna with  $|\mathbf{m}| = 1$  ampere meter<sup>2</sup> can be visualized as a single loop of wire carrying a current of 1 ampere and describing an area of 1 meter<sup>2</sup>. The electric dipole can be visualized as a center fed antenna of length 1 meter with a current  $I = I_0 e^{i\omega t}$  where  $I_0 = 2\pi(100)$  amps minimally. The current of 600 amps was obtained by assuming all the separated charges appears at the extremities of the center fed antenna. Even this current presents a design problem and a more realistic estimate would require a higher current. In both the magnetic and electric dipole cases we have ignored the radiative efficiency of the simple antennas which in the electric dipole case is particularly restrictive.

distance between antennas and object  $R = 10$  meters;  
 permeability of body and external medium  $\mu = 4\pi \times 10^{-7}$  henrys/meter;  
 permittivity of body and external medium  $\epsilon = 8.854 \times 10^{-12}$  farads/meter;  
 conductivity of external medium  $\sigma_1 = 4 \Omega^{-1}\text{m}$  meter;  
 conductivity of body  $\sigma_2 = 3.72 \times 10^7 \Omega^{-1}$  meter;<sup>6</sup>  
 the body radius  $r = 0.5$  meters.

The dipole moment  $\mathbf{p}$  induced in the body is given by (4), in which the incident electric field  $\mathbf{E}_0$  will be given by (1) if the transmitting antenna is a magnetic dipole or by (2) if it is an electric dipole.

As observed in Section II, the effect of the body is assumed to be that of an electric dipole of moment  $\mathbf{p}_1$ . Therefore, (2) and (7) will determine the resulting field at the receiving antenna, which is assumed to be located at the same position as the transmitting antenna.<sup>7</sup>

The body location relative to the antennas is determined by the polar angle and radial spherical coordinates  $\Theta$  and  $R$ . Since  $R$  is assumed to be 10 meters in the examples, only  $\Theta$  changes.

If the body is directly below the antennas,  $\Theta = \pi$  radians. Then it is evident from (1) that in the case of a horizontal magnetic dipole transmitting the value of the incident field, and therefore the induced dipole moment of the body, will be zero. Thus, no effect due to the body will be observed at the receiver.

If the body is off to one side, so that  $\Theta = 3\pi/4$  radians, then the induced dipole moment due to the magnetic dipole transmitting is of the order of  $10^{-10}$  coulomb meters. In this case the resulting non-zero electromagnetic field components at the receiver consist of a complex azimuth component of magnetic field, and, since the polar angle of the receiver relative to the body dipole moment is  $\pi/2$  radians, a complex polar component of electric field. They are given by

$$H_\phi = -3.998 \times 10^{-10} + 1.371 \times 10^{-9}i \text{ amp m}^{-1}, \quad (16)$$

<sup>6</sup> This is actually the conductivity of aluminum (cf. Ref. 2, p. 289).

<sup>7</sup> It should be kept in mind that when using (2) and (7) to calculate the field at the receiver due to the induced dipole moment of the body, the center of the coordinate system is located at the body; e.g., if  $\Theta = 3\pi/4$  when calculating the induced dipole moment at the body, with the center at the transmitter, then  $\Theta = \pi/4$  when calculating the field at the receiver (compare figures 3 and 4).

$$E_{\theta} = -2.174 \times 10^{-12} + 3.568 \times 10^{-11}i \text{ v m}^{-1}.$$

In the case of a vertical electric dipole antenna transmitting, for  $\Theta = \pi$  radians the induced dipole moment is the order of  $10^{-5}$  coulomb meters. The only non-zero electromagnetic field component at the receiver is a complex radial electric field component given by

$$E_R = -1.49 \times 10^{-6} + 5.128 \times 10^{-7}i \text{ v m}^{-1}. \quad (17)$$

In the case of a vertical electric dipole antenna transmitting, for  $\Theta = 3\pi/4$  radians the induced dipole moment is of the order of  $10^{-5}$  coulomb meters. The corresponding electromagnetic field observed at the receiver is given by

$$\begin{aligned} H_{\phi} &= -8.495 \times 10^{-6} + 6.676 \times 10^{-6}i \text{ amp m}^{-1}, \\ E_R &= -7.449 \times 10^{-7} + 2.564 \times 10^{-7}i \text{ v m}^{-1}, \\ E_{\theta} &= -1.704 \times 10^{-7} + 2.1 \times 10^{-7}i \text{ v m}^{-1}. \end{aligned} \quad (18)$$

From a comparison of (8) and (10) it is evident that the effect of the body on the electromagnetic field observed at the receiver is 4 orders of magnitude less when the transmitter is a horizontal magnetic dipole than when it is a vertical electric dipole. However, because of the difference in definition between a magnetic dipole moment and an electric dipole moment the two cases are not really comparable. What is important to consider is the engineering problem of designing and building a magnetic or electric dipole with the required power at the selected frequency. This is the basis on which the design decision should be made.

## 2. Two Media

In the case of two regions with different media, separated by a plane interface, the medium in Region 2 where the dipole is located is air, while the medium in Region 1 where the spherical body is located has the same conductivity and permittivity as in the single medium case. It is assumed that all other parameters are the same as in the single medium case.

For the case of a vertical electric dipole source located in Region 2 equations (9) give the vertical and horizontal components of the electric field incident on the body in Region 1, and (6) provides the dipole moment induced in the body by each component. Then (11) gives the electromagnetic field in Region 2 due to the vertical component of the

dipole moment induced in the body, and (13) gives the field in Region 2 due to the horizontal component of the dipole moment.

It will be assumed that the source is at the point  $(0,0,z_0)$ , in cylindrical coordinates, where  $z_0 = 1$  m, and the center of the body is at the point  $(0,0,z_1)$ , where  $z_1 = -10$  m. All other parameters remain the same as in the single medium example.

As in the case of a single medium, for a vertical electric dipole source, the only non-zero field component at the receiver is the vertical one, which in cylindrical coordinates is  $E_z$ . For the two medium case the component is given by

$$E_z = 2.263 \times 10^{-6} - 1.371 \times 10^{-5}i \text{ v m}^{-1}. \quad (19)$$

A comparison of (19) with (17) indicates that at the receiver the field due to a body directly below a vertical electric dipole source is somewhat larger when the source is in air and the body is in water than when they are both in water. Evidently, the conducting medium below the non-conducting medium focuses the electric lines of force like a lens.

An inspection of (15) indicates that for a vertical magnetic dipole source the electric field is zero at the body and therefore induces no dipole moment. Hence, as in the single medium case, the body has no effect on the electromagnetic field at the receiver.

#### D. CONCLUSIONS

A rough calculation indicates that the effect of a low frequency electromagnetic field interacting with a moderately conducting body immersed in water should be large enough for observation using presently available detectors, such as a SQUID gradiometer (cf. Ref. 5), at a distance of 10 meters.<sup>8</sup> Since this depends on differences in conductivity, insulating bodies such as fiberglass should also be detectable. However, the calculation ignores, among other things, the existence of an air-water interface, which is expected to reduce the strength of the observed signal.

The calculation also appears to indicate that if the transmitting antenna is a vertical electric dipole the electromagnetic disturbance attributed to the body's presence may be much larger than the disturbance that would occur if the transmitting antenna were a horizontal magnetic dipole. However, this conclusion is misleading since the electric and

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<sup>8</sup> The calculation indicates that a magnetic field intensity of 5 micro-amp/meter is easily obtainable. This corresponds to a magnetic flux density of about .06 micro-gauss, or 6 pico-tesla.

magnetic dipole moments assumed for the transmitters are not strictly comparable.<sup>9</sup> In fact, the true difference, if any, between the effects due to the different antenna types will depend on other factors not considered here, such as instrumental design details.

We are proposing a practical feasibility study of the proposed method of detecting submerged bodies which would extend the calculation described in the present document to include: (1) the effect of the air-water interface between the propagation media, (2) taking into account design details of available transmitters and receivers, and (3) estimating detectability based on signal-to-noise considerations. It would also (4) consider the possibility of separating the signals due to individual bodies using Doppler effects created by moving the transmitter-receiver platform, and (5) estimate the resolution limit that might be achievable by appropriate signal processing.

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<sup>9</sup> Because vertical electric dipole antennas may be difficult to implement at these frequencies one would select a horizontal antenna. The signal strength for this antenna can be obtained from the vertical antenna results by a simple transformation ( $\theta \rightarrow \theta + \pi/2$ ).

## APPENDIX A

Ref. 4 defines 12 two-dimensional Fourier transforms, called essential integrals.

Defining

$$\gamma_v = \sqrt{\lambda^2 - k_v^2},$$

where

$$\lim_{\lambda \rightarrow 0} \gamma_v = -ik_v,$$

the essential integrals can be reduced to:

$$U_{11} = 2 \int_0^{\infty} \frac{e^{-\gamma_1(h-z)}}{\gamma_1 + \gamma_2} J_0(\lambda r) \lambda d\lambda,$$

$$V_{11} = 2 \int_0^{\infty} \frac{e^{-\gamma_1(h-z)}}{k_2^2 \gamma_1 + k_1^2 \gamma_2} J_0(\lambda r) \lambda d\lambda,$$

$$W_{11} = 2 \int_0^{\infty} \frac{\gamma_2 - \gamma_1}{k_2^2 \gamma_1 + k_1^2 \gamma_2} e^{-\gamma_1(h-z)} J_0(\lambda r) \lambda d\lambda,$$

$$U_{12} = 2 \int_0^{\infty} \frac{e^{-\gamma_1 h - \gamma_2 z}}{\gamma_1 + \gamma_2} J_0(\lambda r) \lambda d\lambda,$$

$$V_{12} = 2 \int_0^{\infty} \frac{e^{-\gamma_1 h - \gamma_2 z}}{k_2^2 \gamma_1 + k_1^2 \gamma_2} J_0(\lambda r) \lambda d\lambda,$$

$$W_{12} = 2 \int_0^{\infty} \frac{(\gamma_2 - \gamma_1) e^{-\gamma_1 h - \gamma_2 z}}{k_2^2 \gamma_1 + k_1^2 \gamma_2} J_0(\lambda r) \lambda d\lambda,$$

$$U_{21} = 2 \int_0^{\infty} \frac{e^{\gamma_1 z - \gamma_2 h}}{\gamma_1 + \gamma_2} J_0(\lambda r) \lambda d\lambda,$$

$$V_{21} = 2 \int_0^{\infty} \frac{e^{\gamma_1 z - \gamma_2 h}}{k_2^2 \gamma_1 + k_1^2 \gamma_2} J_0(\lambda r) \lambda d\lambda,$$

$$W_{21} = 2 \int_0^{\infty} \frac{(\gamma_2 - \gamma_1) e^{\gamma_1 z - \gamma_2 h}}{k_2^2 \gamma_1 + k_1^2 \gamma_2} J_0(\lambda r) \lambda d\lambda,$$

$$U_{22} = 2 \int_0^{\infty} \frac{e^{-\gamma_2(h+z)}}{\gamma_1 + \gamma_2} J_0(\lambda r) \lambda d\lambda,$$

$$V_{22} = 2 \int_0^{\infty} \frac{e^{-\gamma_2(h+z)}}{k_2^2 \gamma_1 + k_1^2 \gamma_2} J_0(\lambda r) \lambda d\lambda,$$

$$W_{22} = 2 \int_0^{\infty} \frac{(\gamma_2 - \gamma_1) e^{-\gamma_2(h+z)}}{k_2^2 \gamma_1 + k_1^2 \gamma_2} J_0(\lambda r) \lambda d\lambda.$$

These definitions and the well known Bessel function identities

$$\frac{\partial J_0(\lambda r)}{\partial r} = -\lambda J_1(\lambda r),$$

$$\frac{\partial^2 J_0(\lambda r)}{\partial r^2} = -\lambda^2 J_1'(\lambda r) - \frac{\lambda^2}{2} [J_2(\lambda r) - J_0(\lambda r)],$$

imply the following relations:

$$\frac{\partial U_{12}}{\partial r} = -2 \int_0^{\infty} \frac{e^{-\gamma_1 h - \gamma_2 z}}{\gamma_1 + \gamma_2} J_1(\lambda r) \lambda^2 d\lambda,$$

$$\frac{\partial U_{12}}{\partial z} = -2 \int_0^{\infty} \frac{\gamma_2 e^{-\gamma_1 h - \gamma_2 z}}{\gamma_1 + \gamma_2} J_0(\lambda r) \lambda d\lambda,$$

$$\frac{\partial V_{12}}{\partial r} = -2 \int_0^{\infty} \frac{e^{-\gamma_1 h - \gamma_2 z}}{k_2^2 \gamma_1 + k_1^2 \gamma_2} J_1(\lambda r) \lambda^2 d\lambda,$$

$$\frac{\partial^2 V_{12}}{\partial r^2} = \int_0^{\infty} \frac{e^{-\gamma_1 h - \gamma_2 z}}{k_2^2 \gamma_1 + k_1^2 \gamma_2} [J_2(\lambda r) - J_0(\lambda r)] \lambda^3 d\lambda,$$

$$\frac{\partial^2 V_{12}}{\partial r \partial h} = -2 \int_0^{\infty} \frac{\gamma_1 e^{-\gamma_1 h - \gamma_2 z}}{k_2^2 \gamma_1 + k_1^2 \gamma_2} J_1(\lambda r) \lambda^2 d\lambda,$$

$$\frac{\partial^2 V_{12}}{\partial r \partial z} = -2 \int_0^{\infty} \frac{\gamma_2 e^{-\gamma_1 h - \gamma_2 z}}{k_2^2 \gamma_1 + k_1^2 \gamma_2} J_1(\lambda r) \lambda^2 d\lambda,$$

$$\frac{\partial^2 V_{12}}{\partial z^2} = -2 \int_0^{\infty} \frac{\gamma_2^2 e^{-\gamma_1 h - \gamma_2 z}}{k_2^2 \gamma_1 + k_1^2 \gamma_2} J_0(\lambda r) \lambda d\lambda,$$

$$\frac{\partial W_{12}}{\partial r} = -2 \int_0^{\infty} \frac{(\gamma_2 - \gamma_1) e^{-\gamma_1 h - \gamma_2 z}}{k_2^2 \gamma_1 + k_1^2 \gamma_2} J_1(\lambda r) \lambda^2 d\lambda,$$

$$\frac{\partial W_{12}}{\partial z} = -2 \int_0^{\infty} \frac{\gamma_2 (\gamma_2 - \gamma_1) e^{-\gamma_1 h - \gamma_2 z}}{k_2^2 \gamma_1 + k_1^2 \gamma_2} J_0(\lambda r) \lambda d\lambda,$$

$$\frac{\partial^2 W_{12}}{\partial r^2} = \int_0^{\infty} \frac{(\gamma_2 - \gamma_1) e^{-\gamma_1 h - \gamma_2 z}}{k_2^2 \gamma_1 + k_1^2 \gamma_2} [J_2(\lambda r) - J_0(\lambda r)] \lambda^3 d\lambda,$$

$$\frac{\partial U_{21}}{\partial r} = -2 \int_0^{\infty} \frac{e^{\gamma_1 z - \gamma_2 h}}{\gamma_1 + \gamma_2} J_1(\lambda r) \lambda^2 d\lambda,$$

$$\frac{\partial V_{21}}{\partial r} = -2 \int_0^{\infty} \frac{e^{\gamma_1 z - \gamma_2 h}}{k_2^2 \gamma_1 + k_1^2 \gamma_2} J_1(\lambda r) \lambda^2 d\lambda,$$

$$\frac{\partial V_{21}}{\partial z} = 2 \int_0^{\infty} \frac{\gamma_1 e^{\gamma_1 z - \gamma_2 h}}{k_2^2 \gamma_1 + k_1^2 \gamma_2} J_0(\lambda r) \lambda d\lambda,$$

$$\frac{\partial^2 V_{21}}{\partial r^2} = \int_0^{\infty} \frac{e^{\gamma_1 z - \gamma_2 h}}{k_2^2 \gamma_1 + k_1^2 \gamma_2} [J_2(\lambda r) - J_0(\lambda r)] \lambda^3 d\lambda,$$

$$\frac{\partial^2 V_{21}}{\partial r \partial h} = 2 \int_0^{\infty} \frac{\gamma_2 e^{\gamma_1 z - \gamma_2 h}}{k_2^2 \gamma_1 + k_1^2 \gamma_2} J_1(\lambda r) \lambda^2 d\lambda,$$

$$\frac{\partial^2 V_{21}}{\partial r \partial z} = -2 \int_0^{\infty} \frac{\gamma_1 e^{\gamma_1 z - \gamma_2 h}}{k_2^2 \gamma_1 + k_1^2 \gamma_2} J_1(\lambda r) \lambda^2 d\lambda$$

$$\frac{\partial^2 V_{21}}{\partial z^2} = 2 \int_0^{\infty} \frac{\gamma_1^2 e^{\gamma_1 z - \gamma_2 h}}{k_2^2 \gamma_1 + k_1^2 \gamma_2} J_0(\lambda r) \lambda d\lambda,$$

$$\frac{\partial W_{21}}{\partial r} = -2 \int_0^{\infty} \frac{(\gamma_2 - \gamma_1) e^{\gamma_1 z - \gamma_2 h}}{k_2^2 \gamma_1 + k_1^2 \gamma_2} J_1(\lambda r) \lambda^2 d\lambda,$$

$$\frac{\partial^2 W_{21}}{\partial r^2} = \int_0^{\infty} \frac{(\gamma_2 - \gamma_1) e^{\gamma_1 z - \gamma_2 h}}{k_2^2 \gamma_1 + k_1^2 \gamma_2} [J_2(\lambda r) - J_0(\lambda r)] \lambda^3 d\lambda.$$

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