Problems in Nonlinear Acoustics: Surface Acoustic Waves, Nondestructive Testing, and Acoustic Streaming

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Three projects in nonlinear acoustics are described: (1) Surface Acoustic Waves [derivation of theoretical models for Stoneley and Scholte waves in isotropic solids, surface waves in crystals, and surface waves in piezoelectric materials]; (2) Nondestructive Testing [theory and experiment for ultrasonic measurement of third order elastic moduli via immersion techniques]; (3) Acoustic Streaming [numerical modeling of acoustic streaming, at high Reynolds numbers, produced by focused sound beams containing shocks].
ANNUAL SUMMARY REPORT
ONR Grants N00014-89-J-1003 (primary grant)
and N00014-93-1-1135 (AASERT)
1 June 1995 to 31 May 1996

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AND ACOUSTIC STREAMING

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The research projects fall into three general categories:

I. Surface Acoustic Waves (N00014-89-J-1003)

II. Nondestructive Testing (N00014-93-1-1135)

III. Acoustic Streaming (N00014-89-J-1003)

Project descriptions, approaches taken, and accomplishments during the past year follow.

I. Surface Acoustic Waves

A. Project Description

The objective is to develop and interpret mathematical models for a wide variety of nonlinear surface acoustic waves, in particular, Stoneley and Scholte waves in isotropic solids, surface waves in crystals, and surface waves in piezoelectric materials. The theoretical model for each case would be the first that depends explicitly on all relevant material properties.

B. Approaches Taken

The Hamiltonian formalism developed by Zabolotskaya to describe nonlinear Rayleigh waves in isotropic solids is being extended to include each type of surface wave identified above. Analysis of each wave type requires the derivation of a new Hamiltonian function. With the Hamiltonian expressed in terms of appropriate generalized coordinates, spectral evolution equations are derived for the variations in harmonic amplitudes as a function of distance from the source.

For Stoneley and Scholte waves, derivation of the Hamiltonian functions for two infinite half-spaces in contact is a direct, although more complicated, extension of the calculations performed by Zabolotskaya for Rayleigh waves.
For surface waves in crystals, the eigenvalues for the linear solutions, which are required to evaluate the Hamiltonian, are considerably more difficult to calculate than for surface waves in isotropic solids. Therefore, analysis of this case begins with development of computer codes to perform these calculations (reported last year).

For surface waves in piezoelectric materials, calculation of the eigenvalues for the linear solutions must now include coupling with the electric field, and the Hamiltonian must account for piezoelectric, dielectric, and electrostrictive nonlinear effects, in addition to those for nonlinear elasticity.

C. Accomplishments

Stoneley and Scholte waves

All technical aspects of this project were completed during the past year. Accomplishments included additional numerical simulations of harmonic generation and waveform distortion to assist in physical interpretation of the model equations. The search for roots of the dispersion equations to determine eligible material pairs that support the existence of small-signal Stoneley and Scholte waves was also completed. Approximately 20 material pairs were identified for each case (for Scholte waves, the liquid medium was assumed to be water). The main effort during the past year involved documentation of all results associated with this project in a Ph.D. dissertation by G. D. Meegan. The dissertation is currently under review, and Meegan is planning to defend his dissertation this summer. Meegan did not draw any support from this ONR grant after December 1995.

Surface waves in crystals

Linear theory for surface waves in crystals, necessary for development of the nonlinear theory, was completed during the previous, 1994–95 contract year. This included development of a computer code for crystals having arbitrary symmetry, and which describes surface wave propagation in arbitrary directions in planes having arbitrary orientations with respect to the crystallographic axes. During the 1995–96 contract year, the eigenvalues for the linear theory and the third order elastic constants for the crystal were used to evaluate the nonlinearity matrix that appears in the spectral evolution equations.

Development of the theory for nonlinear surface waves in crystals is now completed. This model is the first to be based explicitly on the fundamental second and third order elastic moduli of crystals, i.e., there is no dependence on ad hoc or empirical parameters. Furthermore, the model is not subject to any restrictions on the direction of wave propagation. Numerical simulations were performed to model the distortion of surface waves that propagate in planes formed by two different cuts of KCl, the (111) and (001) planes. We are aware of no published numerical simulations of nonlinear surface wave distortion in crystals (despite the relevance to SAW devices!).

For the simulation in the (111) plane, the direction of propagation is the projection of any one of the three cubic axes onto that plane. In this case, the waveform
distortion was found to be opposite that of Rayleigh waves in common isotropic solids, and likewise opposite that of compression waves in both solids and fluids. Specifically, in the horizontal component of the particle velocity waveform, positive portions were found to recede in time, negative portions to advance. In short, expansion shocks are predicted, whereas only compression shocks occur in sound waves (apart from a few extraordinary cases).

For the simulation in the (001) plane, the direction of propagation is along one of the two cubic axes in that plane. In this case, because of the high degree of crystal symmetry with respect to these propagation axes, it was expected that the waveform distortion would resemble closely that which we have predicted for nonlinear Rayleigh waves in isotropic solids. However, the results are considerably different. The predicted distortion of the surface waveform in the crystal is highly irregular, containing unexpected ripples and with substantially less nonlinearity than in the (111) plane. Examination of the nonlinearity matrix revealed the reason for this discrepancy. For propagation along a cubic axis in KCl, the nonlinearity coefficient corresponding to second harmonic generation is anomalously small, preventing efficient transfer of energy to higher harmonics, and thus giving rise to the irregular wave shape.

Surface waves in piezoelectric materials

Development of a computer code that calculates all parameters required to describe small-signal surface waves in piezoelectric materials is completed, and the theoretical framework for construction of the nonlinearity matrix was formulated. Whereas two modes of propagation (one compression and one shear) combine to form a Rayleigh wave in an isotropic solid, three modes (one compression-like and two shear-like) are required to describe surfaces waves in crystals, and four modes are required for surface waves in piezoelectric materials. Despite the complexity of the piezoelectric case, the code developed for calculating the eigenvalues and eigenvectors possesses the same generality as the code developed by our group for the purely anisotropic case of nonpiezoelectric crystals, namely, no restrictions on crystal cut or propagation direction. The remaining task for completion of this project is explicit calculation of the terms in the nonlinearity matrix and interpretation of solutions predicted by the spectral evolution equation, which shall be accomplished during the coming year.

II. Nondestructive Testing

A. Project Description

The objective is to develop a quasilinear model for second harmonic generation in a sound beam that propagates through an isotropic solid immersed in a liquid, and to verify theoretical predictions with experiments. The theory would have no restrictions on the geometry or orientation of the sound source in the liquid, and it would take into account all possible nonlinear interactions among the compression
and shear wave modes of propagation in the solid. Both reflection from and transmission through the elastic solid would be modeled. A goal is to infer nonlinear elastic properties of the solid from measurements of the second harmonic field in the surrounding fluid.

B. Approaches Taken

The theoretical model is based on angular spectrum decomposition. Both the primary and second harmonic fields are represented as superpositions of plane waves. Analytic solutions were derived previously, during the 1994–95 contract year, for second harmonic generation due to interaction of any two plane-wave components in the primary beam. A computer code combines the plane-wave solutions and performs a spatial Fourier transform to construct the second harmonic field.

Experiments are being performed at megahertz frequencies in our ultrasonics water tank facility. Conventional ultrasonic sound sources, namely, circular piezoelectric transducers, and a pvdf membrane hydrophone are used for acoustic transmission and reception, respectively, in the water. Two different elastic solids were used, aluminum and lucite blocks, each having sufficient thickness that multiple reflections of acoustic pulses within the blocks is not a factor.

C. Accomplishments

Advances were made in both theory and experiment. The main advance in theory was development of the computer code for modeling second harmonic generation. Prior to June 1995 we had completed development of the computer code for describing the primary beam in both the liquid and the solid, and we had completed the derivation of analytic solutions for lossless second harmonic generation due to nonlinear interaction of the plane wave components in the primary beam. During the past year, losses were included in the plane wave solutions, and these solutions were incorporated in a computer code for describing the second harmonic beams both reflected from and transmitted through the solid.

Two sets of experiments were performed to test the theory, the first for reflection and the second for transmission. The reflection experiment involved second harmonic generation in a sound beam incident on a water-aluminum interface near the Rayleigh angle. Linear theory predicts strong, well-known interference phenomena in the reflected beam at these angles. The interference occurs between the specularly reflected component and the component reradiated by the Rayleigh wave. Theory and experiment were in excellent agreement for both the fundamental and second harmonic components in the reflected beam at these angles. Neither theory nor experiment has been reported previously for second harmonic generation in this case. The accurate predictions of the intricate second harmonic beam patterns lend a high degree of credibility to the theoretical model.

In the second set of experiments, which are still underway, measurements of second harmonic generation are made for a beam that propagates through a block of lucite. Three source configurations have been considered: (1) source located 5–
10 cm away from the block; (2) source in contact with the block; (3) source as close as possible yet not touching the block, i.e., much less that one millimeter away.

In case 1, second harmonic generation in the water between the source and the lucite block was of the same order as the second harmonic generation within the block. Consequently, both components were present in roughly equal strength in the signal transmitted from the far side of the block. Although theory and experiment are in good agreement for this case, the fact that the second harmonic component generated in the lucite does not the dominate the contribution from the water makes it difficult to accurately determine nonlinear properties of the lucite.

In case 2, second harmonic generation in the water can be virtually eliminated from consideration. Although some second harmonic generation takes place in the transmitted beam, the contribution can be made arbitrarily small by moving the receiving hydrophone sufficiently close to the solid. However, the physical contact between the transducer and the block created a strong source of second harmonic generation, significantly greater than that in the lucite.

Case 3 appears to be the most promising. The primary beam transmitted through the lucite was well modeled by theory for circular piston radiation, the second harmonic component generated in the lucite dominated that which was generated in the water, and because the source did not touch the block there was no second harmonic generation associated with contact nonlinearity. Preliminary results show good agreement between theory and experiment for the structure of the second harmonic beam pattern in the transmitted field. Neither theoretical predictions nor experimental observations of second harmonic beam patterns associated with nonlinearity of elastic solids have been reported previously in the literature.

III. Acoustic Streaming

A. Project Description

A theoretical model is under development for acoustic streaming in focused sound beams. The main contribution of this work is that the model takes into account the combined effects of diffraction and shock formation in sound beams radiated by conventional piezoelectric (piston-like) sources, as well as hydrodynamic nonlinearity that becomes important at the high Reynolds numbers associated with recent experiments reported in the literature.

B. Approaches Taken

Both the sound beam and the acoustic streaming are described by equations derived in the paraxial approximation. The KZK equation is used to describe the sound beam, and it is solved numerically with the time domain computer code developed previously in our group. Time domain codes are more appropriate than frequency domain codes for modeling shock structure, and acoustic streaming is very sensitive to shock rise time. The hydrodynamic streaming equations include nonlinear terms that are essential for proper modeling of flow described by high Reynolds numbers,
and recent experiments on acoustic streaming report Reynolds numbers on the order of 100.

C. Accomplishments

The main difficulty faced during this past year was numerical calculation of the forcing function in the streaming equations. This function is dependent on the time derivative of the acoustic waveform, and its greatest effect on streaming is associated with shocks, where the steepest slopes occur in the waveform. The problem boils down to needing a sufficiently fine sampling of the waveform to accurately resolve shock structure at high amplitudes. For the thin shocks encountered in recent experiments, and given the zero padding required in our time domain code for modeling pulsed nonlinear sound beams, together with the number of cycles needed to simulate the cw conditions of most experiments, the computation time appeared prohibitive.

The problem was solved by modifying the time domain code to accept periodic waveforms, which reduced the computation time by an order of magnitude. How to make this modification was not obvious, because of difficulties in introducing periodic boundary conditions in the algorithm associated with the diffraction operator. The problem was solved, and the resulting code is at least an order of magnitude more efficient for calculations of cw radiation. Therefore, the code can now be used not only to solve the desired acoustic streaming problems, but its usefulness in general for modeling intense sound beams radiated by cw sources has been enhanced considerably. Specifically, the time domain code can now calculate finite amplitude sound fields radiated by monofrequency and bifrequency sources with an efficiency that compares with competing frequency domain codes, yet the time domain code remains far more efficient for modeling pulsed sound beams.

Additional accomplishments include modification of the algorithm to model radiation from Gaussian sources (for comparison with calculations performed by others), extensive analysis of numerical errors associated with the parameter ranges of typical streaming experiments, and coupling of the output from the KZK code with the computer code developed previously in our group to solve the acoustic streaming equations.
OFFICE OF NAVAL RESEARCH
PUBLICATIONS/PATENTS/PRESENTATIONS/HONORS REPORT
for
01 June 95 through 31 May 96

Contract/Grant Number: N00014-89-J-1003

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a. Number of papers submitted to refereed journals but not yet published: 1

b. Number of papers published in refereed journals (ATTACH LIST): 1

c. Number of books or chapters submitted but not yet published: 0

d. Number of books or chapters published (ATTACH LIST): 0

e. Number of printed technical reports & non-refereed papers (ATTACH LIST): 2

f. Number of patents filed: 0

g. Number of patents granted (ATTACH LIST): 0

h. Number of invited presentations at workshops or professional society meetings: 3

i. Number of contributed presentations at workshops or professional society meetings: 3

j. Honors/awards/prizes for contract/grant employees, such as scientific society and faculty awards/offices (ATTACH LIST): 0

k. Number of graduate students supported at least 25% this year this contract/grant: 4

l. Number of post docs supported at least 25% this year this contract/grant: 0

How many of each are females or minorities? These six numbers are for ONR's EEO/Minority Reports. Minorities include Blacks, Aleuts, Aminidians, etc., and those of Hispanic or Asian extraction/nationality. The Asians are singled out to facilitate meeting reporting semantics re “underrepresented”.

Graduate student FEMALE: 1 Post doc FEMALE: 0
Graduate student MINORITY: 0 Post doc MINORITY: 0
Graduate student ASIAN E/N: 0 Post doc ASIAN E/N: 0
Attachment to
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for
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a. Papers submitted to refereed journals but not yet published: 1


b. Papers published in refereed journals: 1


c. Books or chapters submitted but not yet published: 0

d. Books or chapters published: 0

e. Printed technical reports & non-refereed papers: 2


E. Yu. Knight, “Generalization of the theory for nonlinear Rayleigh waves to nonplanar and transient waveforms, and investigation of pulse propagation,” M.A. Thesis, Department of Physics, The University of Texas at Austin (August 1995).

f. Patents filed: 0

g. Patents granted: 0

h. Invited presentations at workshops or professional society meetings: 3


i. **Contributed presentations at workshops or professional society meetings:** 3


j. **Honors/awards/prizes:** 0

k. **Number of graduate students supported at least 25% this year this grant:** 4

   E. Yu. Knight
   B. J. Landsberger
   G. D. Meegan
   S. J. Younghouse

l. **Number of post docs supported at least 25% this year this grant:** 0