Special Publication ARFSD-SP-95003

FUZZY LOGIC

Sanjay Tailor

May 1996

U.S. ARMY ARMAMENT RESEARCH, DEVELOPMENT AND ENGINEERING CENTER

Fire Support Armaments Center

Picatinny Arsenal, New Jersey

Approved for public release; distribution is unlimited.

19960627 001
The views, opinions, and/or findings contained in this report are those of the authors(s) and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation.

The citation in this report of the names of commercial firms or commercially available products or services does not constitute official endorsement by or approval of the U.S. Government.

Destroy this report when no longer needed by any method that will prevent disclosure of its contents or reconstruction of the document. Do not return to the originator.
# FUZZY LOGIC

## AUTHOR(S)
Sanjay Tailor

## PERFORMING ORGANIZATION NAME(S) AND ADDRESS(S)
ARDEC, FSAC
Precision Munitions/Mine and Demolitions Division (AMSTA-AR-FSP-E)
Pica tinny Arsenal, NJ 07806-5000

## SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(S)
ARDEC, DOIM
Information Research Center (AMSTA-AR-IMC)
Pica tinny Arsenal, NJ 07806-5000

## DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution is unlimited.

## ABSTRACT (Maximum 200 words)
This report deals with the concepts of fuzzy logic and its use in modern commercial systems. The fundamental differences between conventional and fuzzy logic are explained. The discussion also includes procedures used to design systems with fuzzy logic and the capability of modern design tools. Some examples are provided to further clarify the concepts of fuzzy logic and to show how to develop fuzzy models using mathematical sets. The overall advantages and disadvantages of this technology are discussed. A brief history is provided to show the origin of this technology and its relative popularity in different parts of the world.

## SUBJECT TERMS
- Fuzzy
- Defuzzification
- Vague
- Membership function
- Patch
- Crisp
- Multivalued

## SECURITY CLASSIFICATION OF REPORT
UNCLASSIFIED

## SECURITY CLASSIFICATION OF THIS PAGE
UNCLASSIFIED

## SECURITY CLASSIFICATION OF ABSTRACT
UNCLASSIFIED

## LIMITATION OF ABSTRACT
SAR
<table>
<thead>
<tr>
<th>CONTENTS</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Introduction</td>
<td>1</td>
</tr>
<tr>
<td>Discussion</td>
<td>1</td>
</tr>
<tr>
<td>History</td>
<td>1</td>
</tr>
<tr>
<td>Fuzzy Models</td>
<td>1</td>
</tr>
<tr>
<td>Fuzzy Systems</td>
<td>3</td>
</tr>
<tr>
<td>Design of Fuzzy Systems</td>
<td>4</td>
</tr>
<tr>
<td>Conclusions</td>
<td>7</td>
</tr>
<tr>
<td>References</td>
<td>13</td>
</tr>
<tr>
<td>Bibliography</td>
<td>13</td>
</tr>
<tr>
<td>Distribution List</td>
<td>15</td>
</tr>
</tbody>
</table>
INTRODUCTION

The basis for the development of fuzzy logic was the shortfall of conventional binary logic when describing some aspects of the real world. Conventional logic is based on principles of precise data such as 0 or 1, true or false, or on or off. This represents extreme values which is not always sufficient when describing some aspects of the real world.

Fuzzy logic is a mathematical representation of vague real world information. Instead of just two data points representing extremes as in binary logic, there can be many "in-between" data points. This idea stems from the way the human brain reasons using imprecise information. For example, when pulling out of a parking spot, "common sense" logic is used to determine whether to go straight, turn slightly, turn sharply, etc. Here, we are not simply saying turn or do not turn; we are saying turn slightly or turn sharply. Fuzzy logic applies this type of logic to make decisions about vague situations. Conventional logic, in this situation, would fall short because we are no longer dealing with precise information and extreme decisions would not work well.

DISCUSSION

History

Some of the principles of fuzzy logic have been in existence since the 1920s. At the time, it was not known as fuzzy logic and the values of these principles were not yet realized. During that time, these principles were known as multivalued logic. The philosopher, Max Black, was one of the first to apply these principles and he was the one who originated the term "vague." Unfortunately, since that time, not much work has been done in this area and not much interest was generated. Although the foundation of this theory dates back to 1920, it was relatively unknown in the decades to follow. It was not until the 1970s that fuzzy logic was put to practical use and even then it was still relatively obscure. The popularity grew at a slower than expected rate in the United States. Japan was one of the first and currently the most active to use fuzzy logic in commercial applications. The 1980s saw many commercial fuzzy products from Japan and this helped popularize fuzzy logic throughout the world. Today, Japan is still the overwhelming leader in the development and production of fuzzy products.

Fuzzy Models

Data, either conventional or fuzzy, can be modeled using mathematical sets. Conventional, or crisp, sets contain data that satisfy specific requirements for membership. For example, let set A be numbers from 1 to 10. A number, n, can be
either a member or non-member of the set. The number 5 is a member of set A, but 12 is a non-member. We'll then introduce a membership function, \( f(A) \), which is defined as follows

\[
f(A) = \begin{cases} 
1 & 1 \leq n \leq 10 \\
0 & \text{otherwise}
\end{cases}
\]  

where \( f(A) \) equals 1 when \( n \) is a member of set A, otherwise \( f(A) \) will be zero. These statements represent strict requirements for A and \( f(A) \). The graphs of A and \( f(A) \) are shown in figure 1.

Now, let's create a fuzzy model for this application. To do so, we must first apply "fuzzy" requirements for A and \( f(A) \). In this case, we would say, let A be a set of real numbers that are close to 5. The rules we would apply are the closer A is to 5, the closer \( f(A) \) is to 1 and \( f(A) \) approaches zero as A moves further away from 5. Based on these requirements, the graphs of A and \( f(A) \) are shown in figure 2. In this case, A is a fuzzy set because its boundaries are not precise. This example shows that with fuzzy sets, an object or number can belong to a set in varying degrees, whereas in conventional sets, a number either belongs to a set or it doesn't. The boundaries of conventional sets are exact, those of fuzzy sets are a function of the number.

Often, fuzzy logic is confused with probability. They are two distinctly different concepts. Probability measures the chance something will occur. Fuzzy logic measures the degree to which something occurred.

Fuzzy logic can be applied to many areas of technology. Quality control is one area that has benefited. Consider the following example where we have two groups of TV monitors (A and B). Each group contains five monitors. The customer requirement is that the power supply circuit for the monitors must output 115 ± 20 VDC. The monitors are made by two different manufacturers and our goal is to determine which manufacturer delivers the better product based on power circuit output accuracy. This quality control study will be performed using both conventional and fuzzy logic. Data for the two groups is provided in the table. An asterisk next to a number indicates that the reading was outside of the tolerance limits.

First, conventional logic will be used to compare the quality of the two groups of TV monitors. Based on this logic, a monitor would be either good or bad depending on whether or not it falls within the tolerance limits. We introduce a membership function, \( f(y) \), which is defined as follows

\[
f(y) = \begin{cases} 
1 & 95 < y < 135 \\
0 & \text{otherwise}
\end{cases}
\]  

(2)
y is the power supply output reading. An f(y) is associated with each TV set. An f(y) of 1 is good, whereas, an f(y) of 0 is bad. The f(y) will be totaled for both groups and the group with a larger f(y) will be considered better. Referring to the table, we see that the total f(y) for group B is equal to four and that for group A is equal to three. Thus, based on this information, group B is better than group A.

Next, we consider this example using fuzzy logic. Quality of each unit will be measured by how far the output voltage readings were from the target, not simply looking at whether or not the readings were within the tolerance limits. The delta, or deviation from the target, is calculated for each set and then totaled for the group. Here we see a completely different picture. The delta total for group A is 52, whereas, that for group B is 84. Looking at this parameter, it is clear that group A is better than B which contradicts what we concluded earlier.

This is an example that illustrates the basic difference between conventional and fuzzy logic. Conventional logic looks at something as simply good or bad, 0 or 1, off or on. It ignores the “in-between” data which accounts for the inaccurate conclusion earlier. Thus, fuzzy logic is more appropriate for this application because it takes into consideration this “in-between” data which provides the needed information to make accurate conclusions.

**Fuzzy Systems**

The example just discussed demonstrates fuzzy logic applied to the measurement of quality. One of the most popular applications is in control systems. Fuzzy systems use microprocessor chips that store and process fuzzy rules. The design of such systems involves specifying input and output variables and by defining membership functions as we have done with the previous examples. This process can be difficult when dealing with complex systems. There are, however, fuzzy design tools that can aid the design process considerably. This will be explained in further detail in the design section of the report.

Fuzzy control systems are now used in many different commercial products. Some cameras, camcorders, washing machines, and refrigerators use this technology. Air conditioners use fuzzy logic with vague data such as “warm” or “cool” to determine the speed at which they should operate. Panasonic was the first to introduce the “digital image stabilization” system for their camcorders which is accomplished using fuzzy logic. This system cancels image jitter normally caused by a shaking hand and produces a more stable image. Fuzzy rules are established to help the system determine where the image will shift before it shifts. Slight changes in the image are used to predict the direction of further shifting and then compensate for it. Other video camera manufacturers, such as Sony, have caught on and implemented this technology. Also some camcorders use fuzzy logic for the autofocus. Image data is linked to various lens settings. Sensors measure the clarity of the images and the fuzzy rules are used to make the adjustments.
Fuzzy products use microprocessors that run fuzzy algorithms and sensors that measure changing input conditions (ref 1). Fuzzy logic also has applications in other products such as automobiles. There are fuzzy fuel injectors. A set of fuzzy rules are used by a microprocessor to adjust the fuel flow. Some auto manufacturers are applying this technology to transmissions, braking systems, and engines. Nevertheless, there are still other areas of the automobile industry to which fuzzy logic can make great impacts. Specifically, this technology can be used in the development of autonomous vehicles (vehicles that require no human involvement). The input to the system would be fuzzy, common sense data about road conditions, the location and relative speeds of nearby cars and other objects, the weather, etc. Fuzzy rules would be used to make decisions based on this vague input data.

There are many benefits of applying fuzzy logic to systems. Fuzzy systems are generally more energy efficient than conventional systems “because they calculate more precisely how much power is needed to get the job done” (ref 1). They provide improved safety, accuracy, and overall quality. It can even be used to measure quality more accurately as was demonstrated in an earlier example.

However, there is one drawback to this technology, and that is complexity. Fuzzy products rely on its rules. It is more complicated and time consuming to write fuzzy rules and create fuzzy sets than to write conventional rules and create conventional sets. “The number of fuzzy rules tend to grow exponentially as the number of system variables increase” (ref 1). There is a tradeoff between complexity and performance. The consensus is that, in most cases, the improved performance outweighs the increase in complexity.

Design of Fuzzy Systems

To design a fuzzy system, one must first establish the fuzzy rules. This will provide the relationship between input and output variables. This relationship is then plotted on a graph with the inputs on one axis and the outputs along the second axis. The product of the input and output sets forms a fuzzy patch (ref 1). This patch is an area on the graph that represents the relationships between the inputs and outputs.

The rules that were established at the beginning of the design are used to map the inputs to the outputs to create a set of patches. A mathematical function or equation can then be generated based on the patches. The microprocessor uses this function to generate the appropriate output instructions based on the inputs received.

A specific input will then trigger the corresponding patches. The graphs usually contain multiple patches that overlap each other. Due to this overlapping, one input can sometimes trigger more than one patch simultaneously. The patches would not trigger fully, but only to some degree or percentage. With fuzzy logic, items can belong
partially to a set and can also belong to more than one set. For instance, the air can be 60% warm and 40% not warm. The rule to keep in mind is that an object's total membership in complementary sets must be equal to one. An object's membership in sets that are not complementary do not have to equal 1.

Take, for example, a fuzzy air conditioner. The inputs to the air conditioner are temperature readings and the outputs are corresponding motor speeds. We have four fuzzy sets for the input and four sets for the output. The four input sets are cool, just right, warm, and hot. The corresponding output sets are slow, medium, fast, and very fast. Suppose, we receive a temperature reading of 74°F and the fuzzy rules indicate that this is 80% just right and 10% warm. These represent vague inputs to the system and would correspond to the motor speed outputs of medium and fast. These outputs are plotted on a graph as shown in figure 3. The curve for the medium speed would be 80% high and the curve for the fast speed would be 10% higher. The area under both curves is totaled and the center of the mass is computed. This center of mass represents a specific motor speed that corresponds to the input temperature reading of 74°F. Note, that we started out with a specific input temperature and converted it to fuzzy form and then ended up with a specific motor speed output.

Both fuzzy and conventional air conditioners aim to reach and maintain a specific temperature, but they accomplish this through different means. The conventional unit would simply turn the motor on and off depending on whether the temperature is above or below the target. The fuzzy unit, on the other hand, will determine the exact motor speeds needed to reach the specified temperature. As a result, the fuzzy unit will be more efficient.

Although there are many benefits of using fuzzy logic, it is not meant to completely replace conventional logic. It is more practical to use fuzzy logic in combination with conventional logic. The idea is to use fuzzy logic to represent input variables and conventional logic to represent the outputs. First, precise, non-fuzzy data from the real world is inputted to the system. This data is converted to fuzzy form. Fuzzy rules are applied which produces an output in fuzzy form. The fuzzy output is later converted to conventional form as was shown in the previous example when a specific motor speed was selected as the output. This process is known as defuzzification. Defuzzification is necessary because the output usually represents some form of instructions. Instructions should not be fuzzy or vague; they must be specific. This example shows that fuzzification determines the degree of membership of input data in fuzzy sets. The degree of input membership determines the degree of output membership.

Thus, to be able to design a fuzzy system, engineers not only need to make a transition to a new domain, but must be able to work with both domains interchangeably. Fuzzy computer tools can help greatly in this design process. They are used specifically for the following tasks:
• Establish fuzzy rules

• Generate code for implementing the fuzzy design

• Merge fuzzy code with the conventional code

• Debug the system

Once the fuzzy rules are established, a code must be written for processing the input data. This code can be written in any higher level language such as C, FORTRAN, Basic, or ADA. The purpose of the code is to execute fuzzy operations based on the fuzzy rules that were established early in the design process.

A fuzzy tool, which is a form of computer simulation, is used to help tune a fuzzy design (ref 2). The initial design usually never produces the optimum results, so parameters need to be changed repeatedly, processing the code each time to produce an output. Manually, this is a very time consuming process. Computer simulation allows you to make the changes to any parameter and see the results immediately. Also, fuzzy tools provide a convenient method of defining the rules. They contain editors that let you express rules linguistically with “if-then” statements (ref 2).

Since, the inputs to the system are fuzzy, and the established rules are fuzzy, the outputs will be fuzzy. So, the latter stages of the system are used to defuzzify the data. The air conditioner example provided earlier in the report is good in demonstrating some of the design steps. Once defuzzification has been accomplished, the final step would be to debug the system.

Thus, it has been shown that virtually each stage of the design process can be eased by the help of the tools that are available today. Most tools still use 8-bit microcontrollers. Nevertheless, systems are becoming more complex, so 32-bit devices will soon become the predominant choice.

Membership functions are really the heart of any fuzzy design. They can take on many different shapes. The most common are triangles and trapezoids. The example of the mathematical sets discussed earlier involved a triangular membership function. For more complex systems, membership functions can take on various different shapes that can be unsymmetrical and nonlinear. The more complex the applications, the more complex the shapes. Nonlinear membership functions usually require the use of a workstation.
CONCLUSIONS

Fuzzy logic is a concept that deviates from the conventional binary logic that is well known and accepted in the world today. Fuzzy logic is a mathematical representation of vague real world information. Conventional logic uses two basic data points such as 1 or 0. Fuzzy logic can have many "in-between" data points.

There are many benefits of this technology such as increased efficiency, accuracy, and safety. The values of fuzzy logic become evident when trying to model the vague aspects of the real world. In these situations, traditional logic falls short. It is not, however, meant to replace conventional logic altogether, but rather to be used in conjunction with it. Fuzzy logic is used to represent vague input data and conventional logic represents precise outputs.

There is a drawback to this technology and that is complexity of the design process. We must deal with a tradeoff between performance and complexity. For most systems, the improvement in performance would far outweigh the increase in complexity, especially with the rapid development of design tools.

Fuzzy logic is most popular in Japan and most of the fuzzy products in existence today have originated from Japanese companies. The popularity of this field is growing in the United States and other countries. The last 5 or 10 years have seen the greatest incorporation of fuzzy logic in commercial products worldwide.
Figure 1
A and f(A) for conventional model

Figure 2
A and f(A) for fuzzy model

Figure 3
Fuzzy set relationship
Table 1
Conventional and fuzzy logic for quality of TV monitors

<table>
<thead>
<tr>
<th>Group A</th>
<th>Group B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>(y)</td>
<td>(f(y))</td>
</tr>
<tr>
<td>121</td>
<td>1</td>
</tr>
<tr>
<td>136*</td>
<td>0</td>
</tr>
<tr>
<td>116</td>
<td>1</td>
</tr>
<tr>
<td>94*</td>
<td>0</td>
</tr>
<tr>
<td>112</td>
<td>1</td>
</tr>
<tr>
<td>Total</td>
<td>3</td>
</tr>
</tbody>
</table>

\(y\) = power supply output voltage (VDC)
Target \(y\) is 115VDC
Lower limit for \(y\) is 95VDC
Upper limit for \(y\) is 135VDC
REFERENCES


BIBLIOGRAPHY

DISTRIBUTION LIST

Commander
Armament Research, Development and Engineering Center
U.S. Army Tank-automotive and Armaments Command
ATTN: AMSTA-AR-IMC
       AMSTA-AR-GCL
       AMSTA-AR-FSP-E (3)
Picatinny Arsenal, NJ 07806-5000

Defense Technical Information Center (DTIC)
ATTN: DTIC-OCC (12)
8725 John J. Kingman Road, Ste 0944
Fort Belvoir, VA 22060-6218

Director
U.S. Army Materiel Systems Analysis Activity
ATTN: AMXSY-MP
Aberdeen Proving Ground, MD 21005-5066

Commander
Chemical/Biological Defense Agency
U.S. Army Armament, Munitions and Chemical Command
ATTN: AMSCB-CII, Library
Aberdeen Proving Ground, MD 21010-5423

Director
U.S. Army Edgewood Research, Development and Engineering Center
ATTN: SCBRD-RTB (Aerodynamics Technology Team)
Aberdeen Proving Ground, MD 21010-5423

Director
U.S. Army Research Laboratory
ATTN: AMSRL-OP-Cl-B, Technical Library
Aberdeen Proving Ground, MD 21005-5066

Chief
Benet Weapons Laboratory, CCAC
Armament Research, Development and Engineering Center
U.S. Army Armament, Munitions and Chemical Command
ATTN: SMCAR-CCB-TL
Watervliet, NY 12189-5000

Director
U.S. Army TRADOC Analysis Command-WSMR
ATTN: ATRC-WSS-R
White Sands Missile Range, NM 88002

GIDEP Operations Center
P.O. Box 8000
Corona, CA 91718-8000