Ferroelectric Ceramic/Polymer Bimorph Sensor for
Strain Measurement in Laminates

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Second Interim Report

Associate Investigator: M.P. Wenger

Principal Investigator: D.K. Das-Gupta
School of Electronic Engineering and
Computer Systems
University of Wales
Dean Street
BANGOR
Gwynedd
LL57 1UT

Phone No: (01248) 382 696
Fax No: (01248) 361 429
E-Mail: mattw@sees.bangor.ac.uk (M.P. Wenger)
dilip@sees.bangor.ac.uk (D.K. Das-Gupta)

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1. Introduction

Work has been done by many workers in the past to investigate the bending (or flexural) piezoelectricity of poled polymer or ceramic films in the form of monomorphs or bimorphs (1-9). A single uniformly poled film when subjected to a uniform bending stress or strain will exhibit little piezoelectric effect due to the fact the average stress/strain across a cross-section of the film will tend to zero, the compressional stress/strain towards the inside of the bend cancelling with the extensional stress/strain towards the outside of the bend. The bending piezoelectric activity can be greatly enlarged by bonding the piezoelectric film to an electromechanically neutral material, thus forming a monomorph, or by bonding two piezoelectric films together to form a bimorph. Other methods used by other workers have been to produce a polarisation gradient within a single piezoelectric film by an application of temperature gradients during poling or poling by electron-beam irradiation.

This report describes the results of investigations into the bending piezoelectricity of monomorphs and bimorphs constructed from ferroelectric composite materials of PTCa/P(VDF-TrFE) and PTCa/Epoxy. The theory of a piezoelectric bimorph acting as a cantilever is outlined and the results of the measurement of the piezoelectric strain coefficient $h_{33}$ are given.

2. Construction of a bimorph sensor/actuator

A monomorph is constructed by bonding a piezoelectric material to a non-piezoelectric material. The non-piezoelectric material can be conducting or non-conducting but electrical contact must be made to the electrode on the surface of the piezoelectric material. A bimorph on the other hand is constructed from two piezoelectric materials bonded together with either their polarisation directions opposing or the same. With the polarisations opposing each other the bimorph is said to be connected in series whilst with the directions the same it is said to be connected in parallel, figure 1.

Monomorphs and bimorphs have been constructed from 0-3 composite materials of PTCa/P(VDF-TrFE). The non-piezoelectric material used in the construction of the monomorphs was thin sheets of brass, which were cleaned and polished prior to bonding with the piezoelectric material to remove oxidisation deposits. Aluminium electrodes of 1cm x 3.5cm were vacuum deposited onto the piezoelectric film, of thickness ~65-100μm, prior to poling of the ferroelectric material. The films were poled in a DC electric field of approximately 20MV-1 for about 1hr and at a temperature of ~100°C. The temperature of the films was allowed to return to room temperature whilst in the presence of the field.
Bonding of the materials was achieved by the use of a fast setting epoxy resin (Araldite Rapid). The resin was applied to one surface to be bound whilst the hardener was applied to the other surface. Both the excess resin and hardener were almost completely removed before the two surfaces were joined together. The two halves of the bimorph were allowed to adhere whilst being held between two metal plates. This process ensured the bimorph was flat and the epoxy bonding layer was as thin as possible as it does not play an active role in the operation of the bimorph.

![Diagram of Series and Parallel connected bimorphs.](image)

**Figure 1** Series and Parallel connected bimorphs.

In the bending mode, the series connected bimorphs will produce a signal between the outer electrodes, thus there is no need for the inner electrode. Therefore, in the construction of a series connected bimorph the electrodes on the surfaces to be bonded were removed with a dilute solution of sodium hydroxide (NaOH). This procedure was also carried out in the construction of the monomorphs. For a parallel connected bimorph the inner electrodes are
not removed prior to assembly thus allowing a signal to be monitored between the outer electrodes and the inner electrode. Quick drying silver paint (Agar) was tried as a conducting adhesive for constructing the parallel connected bimorphs and the monomorphs but was found to provided inadequate adhesive properties. In total, for each of the different arrangements, five bimorphs or monomorphs were constructed for investigations.

3. Bimorph acting as a cantilever

When considering a bimorph suspended vertically from its upper end by a clamping system it can be thought of as a cantilever beam. With the bimorph in this position the effects due to gravity can be neglected. Assumptions are then made that the bimorph acts as a straight beam of constant cross-section and is composed of a homogenous isotropic material. A further assumption is made that the bimorph behaves in a linear (elastic) manner.

![Diagram of bimorph cantilever](image)

Figure 2  Cantilever arrangement of a bimorph.
Consider a bimorph clamped as shown in figure 2 with a force \( F \) acting at its unclamped end. For equilibrium to exist the horizontal reaction \( S \) at the clamp equals the applied force \( F \) and considering a cross-section \( mn \), the internal shear force \( V(x) \) at a distance \( x \) along the bimorph from the clamp will be given by

\[
V(x) = S = F \quad \text{(1)}
\]

Thus the internal shear force is a constant along the length of the bimorph. For rotational equilibrium to exist the bending moment \( M(x) \) at \( mn \) is given by

\[
M(x) = M_s - Sx = FL - Fx = F(L - x) \quad \text{(2)}
\]

Where \( M_s \) is the bending moment due to the clamp. The longitudinal axial strain \( \zeta(y) \) occurring, within the bimorph, at a distance \( y \) from the neutral surface of the bimorph is given by

\[
\zeta(y) = -\frac{y}{R(x)} \quad \text{(3)}
\]

where \( R(x) \) is the radius of curvature of the bimorph. Due to the symmetry of the bimorph the neutral surface lies through the centre. Since the response is linear then the strain, \( \zeta(y) \), is related to the stress, \( \sigma(y) \), by Hooke’s law such that

\[
\sigma(y) = E\zeta(y) \quad \text{(4)}
\]

where \( E \) is the Young’s modulus of the bimorph.

\[
\therefore \sigma(y) = -\frac{Ey}{R(x)} \quad \text{(5)}
\]

The stress distribution over the cross sectional area must have a sum of zero i.e.

\[
\int \int_{\text{area}} \sigma(y) dA = 0 \quad \text{(6)}
\]

but its moment effect must be equal to the internal bending moment at that point, then

\[
\int \int_{\text{area}} \sigma(y) y dA = -M \quad \text{(7)}
\]
or

\[ -M = \iint_{\text{area}} -\frac{Ey}{R(x)} \, y \, dA = -\frac{E}{R(x)} \iint_{\text{area}} y^2 \, dA \]  \quad \text{(8)}

\[ \therefore M = \frac{E}{R(x)} I \]  \quad \text{(9)}

where \( I = \iint_{\text{area}} y^2 \, dA \) is the second moment of the cross-sectional area with respect to the neutral surface of the bimorph. Also

\[ M(x) = \frac{E}{R(x)} I = -\frac{\sigma(y)}{y} I \]  \quad \text{(10)}

or

\[ \sigma(y) = -\frac{M(x) y}{I} \]  \quad \text{(11)}

which is the basic moment-stress relation for an elastic (linear) behaving bimorph.

As the bimorph is deflected by the force \( F \) acting at the unclamped end it will be deformed in a certain manner. Assumptions are made that the effects of the transverse shear on the deformation of the beam are negligible and the bending effects are dominant. From the above discussion

\[ \frac{1}{R(x)} = \frac{M(x)}{EI} \]  \quad \text{(12)}

and from elementary calculus

\[ \frac{1}{R(x)} = -\frac{d^2 y/dx^2}{\left[1 + (dy/dx)^2 \right]^{3/2}} \]  \quad \text{(13)}

which relates the curvature to the deflection \( y \) and location \( x \) along the length of the bimorph. For small deflections the quantity \( (dy/dx)^2 \) is small compared to unity then

\[ \frac{1}{R(x)} = -\frac{d^2 y}{dx^2} \]  \quad \text{(14)}
the negative sign is because positive $y$ is taken as to the right. So

$$\frac{1}{R(x)} = -\frac{d^2y}{dx^2} = \frac{M(x)}{EI} \quad \ldots (15)$$

$$\therefore EI \frac{d^2y}{dx^2} = -M(x) \quad \ldots (16)$$

which is the general differential equation of a deflection curve. For a bimorph such as that in figure 2 this equation becomes

$$EI \frac{d^2y}{dx^2} = -F(L-x) \quad \ldots (17)$$

Multiplying both sides by $dx$ and integrating with respect to $x$ gives

$$EI \frac{dy}{dx} = F\left(\frac{x^2}{2} - Lx\right) + C \quad \ldots (18)$$

where $C = 0$ because slope at $x = 0$ is zero. Integrating again with respect to $x$ yields

$$EIy(x) = F\left(\frac{x^3}{6} - \frac{Lx^2}{2}\right) + C \quad \ldots (19)$$

where again $C = 0$ because there is no deflection at $x = 0$, so

$$y(x) = \frac{F}{EI}\left(\frac{x^3}{6} - \frac{Lx^2}{2}\right) \quad \ldots (20)$$

is the deflection curve for a bimorph subjected to a force at its end. Maximum deflection occurs at the unsupported end i.e. $x = L$

$$\therefore y_{\max} = \frac{F L^3}{EI \cdot 3} \quad \ldots (21)$$

and relating the radius of curvature of the bimorph to the end deflection we obtain

$$\frac{1}{R(x)} = \frac{M(x)}{EI} = \frac{F(L-x)}{EI} \quad \ldots (22)$$

but
\[
\frac{F}{EI} = \frac{3y_{\text{max}}}{L^3}
\]  \quad \text{...}(23)

so

\[
\frac{1}{R(x)} = \frac{3(L-x)y_{\text{max}}}{L^3}
\]  \quad \text{...}(24)

\[
\therefore R(x) = \frac{L^3}{3(L-x)y_{\text{max}}}
\]

Thus knowing the relation of the maximum end deflection to the radius of curvature we can now relate the maximum deflection to the strain within a bimorph acting as a cantilever. The average strain in one half of the width \(H\) is given by

\[
\bar{\varepsilon}_x = \frac{H}{4R(x)} = \frac{H}{4} \frac{3(L-x)}{L^3} y_{\text{max}}
\]  \quad \text{...}(25)

If the voltage produced on the bimorph is sensed in open circuit then the electromechanical effects take place at constant \(D\). Therefore, from the definition of the piezoelectric strain constant \(h_{31}\), i.e.

\[
h_{31} = -\left(\frac{\partial E_3}{\partial \varepsilon_3}\right)_p
\]  \quad \text{...}(26)

the field \(E_3(x)\) developed at \(x\) is given by

\[
E_3(x) = h_{31} \bar{\varepsilon}_x
\]  \quad \text{...}(27)

or

\[
E_3(x) = h_{31} \frac{3}{4} \frac{H}{L} (L-x) y_{\text{max}}
\]  \quad \text{...}(28)

The total charge density \(\varphi(x)\) appearing on each surface of the bimorph half is

\[
\varphi(x) = \varepsilon_{33} \varepsilon_3(x)
\]  \quad \text{...}(29)

The charge \(Q_{\delta x}\) on the element \(\delta x\) is

\[
Q_{\delta x} = \varepsilon_{33} \varepsilon_3(x) w \delta x
\]  \quad \text{...}(30)
where \( w \) is the width of the bimorph. The total charge is

\[
Q_T = \int_0^L Q_{tx} \, dx
= \frac{3}{4} \frac{Hw}{L^3} y_{max} \varepsilon_{33} \hat{s} h_{31} \int_0^L (L - x) \, dx
= \frac{3}{8} \frac{Hw}{L} \varepsilon_{33} \hat{s} h_{31} y_{max}
\]

...(31)

If \( C^c \) is the clamped capacitance of one of the bimorph plates, then the voltage across it is \( U = Q_T / C^c \) and that across two oppositely poled plates connected in series is twice this, therefore

\[
U_T = \frac{3}{4} \left( \frac{H}{L} \right)^2 h_{31} y_{max}
\]

...(32)

Thus by measuring the voltage produced across the bimorph when an end deflection of \( y_{max} \) is applied the value of the piezoelectric strain constant \( h_{31} \) can be obtained from

\[
h_{31} = \frac{4}{3} \left( \frac{L}{H} \right)^2 \frac{U_T}{y_{max}}
\]

...(33)

4. Bimorph clamped at both ends

When the bimorph is clamped at both ends and a force \( F \) is applied to its centre to produce a deflection a similar argument can be used to relate the maximum deflection of the bimorph to the output voltage as follows. Consider the bimorph as shown in figure 3.

For equilibrium to exist

\[
S_A = S_B = \frac{F}{2}
\]

...(34)

also

\[
M_A = M_B
\]

...(35)

Considering the bimorph from \( x = 0 \) to \( x = L/2 \)
\[ V(x) = S_A = \frac{F}{2} \]

\[ M(x) = -S_A x + M_A \]
\[ = -\frac{Fx}{2} + M_A \]

\[ \text{(36)} \]

From the general differential equation of a deflection curve, equation 16, integrating both sides of equation 37 with respect to \( x \) we obtain
\[ EI \frac{dy}{dx} = -\frac{F x^2}{4} + M_A x + C \]  
\text{(38)}

By applying the boundary conditions that \( dy/dx = 0 \) when \( x = 0 \) and \( x = L/2 \) then \( C = 0 \) and \( M_A = FL/8 \). So

\[ EI \frac{dy}{dx} = -\frac{F x^2}{4} + \frac{FLx}{8} \]  
\text{(39)}

and integrating again with respect to \( x \)

\[ E I y(x) = -\frac{F x^3}{12} + \frac{FLx^2}{16} + C \]  
\text{(40)}

and since \( y = 0 \) when \( x = 0 \) then \( C = 0 \) and

\[ y(x) = \frac{F}{EI} \left[-\frac{x^3}{12} + \frac{Lx^2}{16}\right] \]  
\text{(41)}

The maximum deflection occurs when \( x = L/2 \) and is given by

\[ y_{\text{max}} = \frac{F}{EI} \left[-\frac{L^3}{96} + \frac{L^3}{64}\right] \]

\[ = \frac{F}{EI} \left[-\frac{L^3}{192}\right] \]  
\text{(42)}

The radius of curvature is found as before

\[ \frac{1}{R(x)} = \frac{M(x)}{EI} = \frac{F}{EI} \left[-\frac{x}{2} + \frac{L}{8}\right] \]  
\text{(43)}

but

\[ \frac{F}{EI} = \frac{192}{L^3} y_{\text{max}} \]  
\text{(44)}

so

\[ \frac{1}{R(x)} = \frac{192}{L^3} y_{\text{max}} \left[-\frac{x}{2} + \frac{L}{8}\right] = \frac{192}{L^3} y_{\text{max}} \left[-\frac{L - 4x}{8}\right] \]  
\text{(45)}

or
\[ R = \frac{L^3}{24(4x - L)y_{\text{max}}} \] ...\((46)\)

Due to the symmetrical nature of the geometry and the complexity of the deflection curve, when considering the origin of the co-ordinate axis at one end of the bimorph, the response from one half of the bimorph can be considered. The response of the whole bimorph can then be found from doubling the result. From above, the average strain in one half of the width of the bimorph is given by

\[ \bar{\zeta}_r = \frac{H}{4R(x)} = \frac{H}{4} \frac{24(4x - L)}{L^3} y_{\text{max}} \] ...\((47)\)

but with this type of deformation of the bimorph the average strain along half of the bimorph from \( x = 0 \) to \( x = L/2 \) is given by

\[ \zeta_r = \int_0^{L/2} \frac{H}{4} \frac{24(4x - L)}{L^3} y_{\text{max}} \, dx \]

\[ = \frac{6H}{L^3} y_{\text{max}} \int_0^{L/2} (4x - L) \, dx \]

\[ = \frac{6H}{L^3} y_{\text{max}} \left[ 2x^2 - Lx \right]_0^{L/2} = \frac{6H}{L^3} y_{\text{max}} \left[ \frac{L^2}{4} - \frac{L^2}{2} \right] \]

\[ = 0 \]

Hence, in theory for a uniform bimorph the output would be zero as the average strain is zero. Experiments were considered to measure the response of the bimorph when clamped at both ends and subjected to a central force but in light of the preceding analysis these experiments were not carried out. However, preliminary investigations showed that the only signals observable were very low, consisting mainly of noise, and are thought to be due to non-uniformity's of the bimorphs or of the accuracy of positioning the force with respect to the bimorph.

5. Monomorph acting as a cantilever

With a monomorph as with a bimorph the total stress distribution over a cross-sectional area must have a sum of zero such that

\[ \int \int_{\text{area}} \sigma(y) \, dA = 0 \] ...\((49)\)

or when considering the cross-sectional area in figure 4
\[ w \int \sigma(y) \, dy = 0 \quad \text{...(50)} \]

where \( w \) is the width of the monomorph. Since, from equation 5, we have \( \sigma(y) = -E y/R(x) \) equation 50 becomes

\[ \frac{w}{R(x)} \int E \cdot y \, dy = 0 \quad \text{...(51)} \]

![Figure 4 Cross sectional area of a monomorph.](image)

From this equation \( w \) and \( R(x) \) are considered non-zero constants and can be neglected from our argument. Because the monomorph is constructed of two different materials the integration of equation 51 becomes

\[ \int_{-y_c}^{y_t} E_b \cdot y \, dy + \int_{-y_r}^{y_r} E_c \cdot y \, dy = 0 \quad \text{...(52)} \]

where \( E_b \) and \( E_c \) are the Young's modulus of the brass and the piezoelectric composite material respectively. Solving this integration gives

\[
E_b \left[ \frac{y^2}{2} \right]_{-y_c}^{(y_t)} + E_c \left[ \frac{y^2}{2} \right]_{(y_r)} = 0
\]

\[
E_b \left[ \frac{(y_t - T)^2}{2} - \frac{y_c^2}{2} \right] + E_c \left[ \frac{y^2}{2} - \frac{(y_t - T)^2}{2} \right] = 0 \quad \text{...(53)}
\]
where we can multiply through by 2 and use the fact that \( y_i + y_e = H \), where \( H \) is the total thickness of the monomorph, to give

\[
E_b \left[ (y_i - T)^2 - (H - y_i)^2 \right] + E_c \left[ y_i^2 - (y_i - T)^2 \right] = 0 \quad \text{...(54)}
\]

Expanding and simplifying this equation to solve for \( y_i \), yields

\[
y_i = \frac{1}{2} \frac{E_b (H^2 - T^2) + E_c T^2}{E_b (H - T) + E_c T} \quad \text{...(55)}
\]

which gives us the position of the neutral surface from the face of the composite material which forms part of the monomorph. If the Young’s Modulus for the composite is very small compared to that of the brass then the terms containing \( E_c \) can be neglected; therefore, \( E_b \) will also cancel from the equation leaving

\[
y_i = \frac{\frac{1}{2} (H^2 - T^2)}{H - T} = \frac{1}{2} \frac{(H - T)(H + T)}{H - T} \quad \text{...(56)}
\]

\[
i.e. \quad y_i = \frac{1}{2} (H + T) \quad \text{...(57)}
\]

The shifting of the neutral surface will cause the piezoelectric material to experience a greater strain and will therefore increase the signal produced. The average strain experienced in the half of the monomorph containing the piezoelectric material is given by

\[
\bar{\xi}_z = \frac{y_i}{2R(x)} = \frac{(H + T) 3(L - x)}{4 \frac{L^3}{L^3}} y_{\text{max}} \quad \text{...(58)}
\]

Therefore, the average strain experienced by the piezoelectric material will be

\[
\bar{\xi}_z = \frac{2y_i - T}{2R(x)} = \frac{H 3(L - x)}{2 \frac{L^3}{L^3}} y_{\text{max}} \quad \text{...(59)}
\]

which is twice that which is experienced within one half of a bimorph of equivalent dimensions. If the voltage produced on the monomorph is sensed then the electromechanical effects take place at constant \( D \). Therefore, similar to the bimorph the electric field produced will be given by
\[ E_3(x) = h_{31} \frac{3}{2} \frac{H}{L^3} (L - x) y_{\text{max}} \] ...(60)

The total charge density \( \varphi(x) \) appearing on each surface of the monomorph is

\[ \varphi(x) = e_{33} \xi E_3(x) \] ... (61)

The charge \( Q_{\delta x} \) on the element \( \delta x \) is

\[ Q_{\delta x} = e_{33} \xi E_3(x) w \delta x \] ... (62)

where \( w \) is the width of the bimorph. The total charge is

\[ Q_T = \int_{0}^{L} Q_{\delta x} \, dx \]
\[ = \frac{3}{2} \frac{Hw}{L^3} y_{\text{max}} e_{33} \xi h_{31} \int_{0}^{L} (L - x) dx \]
\[ = \frac{3}{4} \frac{Hw}{L} e_{33} \xi h_{31} y_{\text{max}} \] ... (63)

If \( C^5 \) is the clamped capacitance of the monomorph, then the voltage across it is \( U = Q_T / C^5 \) and is given by

\[ U = \frac{3}{4} \left( \frac{H}{L} \right)^2 h_{31} y_{\text{max}} \] ... (64)

Thus by measuring the voltage produced across the monomorph when an end deflection of \( y_{\text{max}} \) is applied the value of the piezoelectric strain constant \( h_{31} \) can be obtained from

\[ h_{31} = \frac{4}{3} \frac{L^2}{H} \frac{U}{y_{\text{max}}} \] ... (65)

Comparing equation 64 to equation 32 it is noted that for the same overall dimensions of sensor the magnitudes of the signals are the same for both the monomorph structure and the bimorph structure.
6. Measurement of bending piezoelectric coefficient

![Diagram of the device](image)

**Figure 5** Device for measuring the bending piezoelectric coefficient of a bimorph.

The bimorph was suspended vertically by clamping it’s upper end, see figure 5. A known horizontal deflection was produced at the lower end which was generated by the rotation of an eccentric cam. The cam was constructed from a circular piece of PTFE with the central hole offset by a certain amount which produced an eccentricity of twice this amount. The cam was driven by a DC motor connected to an appropriate gearbox. The speed of the motor was
selected by adjusting the driving voltage and current. Speeds of up to 7 revolutions per minute could be produced in this way. The signal produced by the bimorph monomorph was observed directly on a digital storage oscilloscope (DSO). The trigger signal for the DSO was produced by using a micro-switch connected to a 5V supply which was activated by the eccentric cam.

The noise present in the signals of the bimorphs monomorphs was due mainly to the various vibrations of the measuring device. The signals were averaged over 32 captures of the DSO and then downloaded to a computer. A cosinusoidal curve of the form

\[ y = A + B \cos \left( \frac{2\pi x}{C} + D \right) \]  

was then fitted to determine the amplitude of the signal, given by the parameter \( B \). The peak to peak amplitude of \( U_T = 2B \) was used to determine the value of \( h_{31} \) using equations 33 and 65.

7. Results and Discussion

An application of a sinusoidal oscillating displacement at the end of a cantilever arrangement produces a sinusoidal output from the sensor. Any vibrations generated by the moving parts of the measuring device will also be transmitted to the sensor and will manifest themselves as noise on top of the signal of interest. These added vibrations can be minimised by ensuring that the moving parts are suitably lubricated but the inevitable vibrations from the gearbox and the other mechanical parts could not be avoided and are present in the final signal as shown in figures 6 to 9.

Typical responses of series connected bimorphs can be seen in figures 6 and 7. Tables 1 and 2 give average values of the piezoelectric \( h_{31} \) coefficient for these sensors while showing typical output voltages seen from the sensors investigated. The frequency response in the range 1 - 7 Hz was found to be linear as can be observed from figure 10.

The monomorphs investigated, which were constructed from a piezoelectric composite film adhered to a brass backing, can be seen to produce higher outputs than the bimorphs due to their extra thickness. The bimorphs constructed had an average thickness of 142\( \mu \)m while the average thickness of the brass backing was 119\( \mu \)m giving an average thickness of the monomorph construction of 190\( \mu \)m. Responses to measurements performed on monomorphs can be seen in figures 8 and 9. The signal to noise ratio of the monomorphs appear to be greater than those of the bimorphs, this may arise
from the extra rigidity of the construction reducing the amount of flexing of
the sensor. Average values of the output signal and the calculated values of $h_{31}$
for monomorphs constructed from composites of PTCa/P(VDF-TrFE) are
shown in tables 3 and 4. The typical values of output, shown in tables 1 to 4,
are a representation of the magnitudes seen from the bimorphs and
monomorphs tested. The output of a certain sensor will obviously depend on
its dimensions.

The values of $h_{31}$ calculated from the measurements on the monomorphs agree
well with the values calculated from results of the measurements on the
bimorphs. This is expected as $h_{31}$ is a property of the electroactive material
but it also reinforces the assumption that the Young’s Moduli for the two
materials of the monomorph are quite different. At this stage the Young’s
Modulus of the electroactive composite material is unknown but is assumed to
be similar to that of the host polymer at about 3GPa (10).
Figure 6  Typical response of a PTCa/P(VDF-TrFE) 50/50vol% series connected bimorph to cantilever measurements. Length 3.5cm, thickness 157μm, frequency 7Hz.

<table>
<thead>
<tr>
<th>Frequency [Hz]</th>
<th>Typical Output [mV]</th>
<th>$h_3$ [MVm$^{-1}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.53</td>
<td>0.098</td>
</tr>
<tr>
<td>2</td>
<td>5.12</td>
<td>0.199</td>
</tr>
<tr>
<td>3</td>
<td>7.72</td>
<td>0.299</td>
</tr>
<tr>
<td>4</td>
<td>10.27</td>
<td>0.398</td>
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<tr>
<td>5</td>
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<td>0.485</td>
</tr>
<tr>
<td>6</td>
<td>15.36</td>
<td>0.597</td>
</tr>
<tr>
<td>7</td>
<td>17.12</td>
<td>0.665</td>
</tr>
</tbody>
</table>

Table 1  Average derived values of $h_3$, and typical output voltages for bimorphs constructed from PTCa/P(VDF-TrFE) 50/50vol%
Figure 7  Typical response of a PTCa/P(VDF-TrFE) 60/40vol% series connected bimorph to cantilever measurements. Length 3.5cm, thickness 147µm, frequency 7Hz.

<table>
<thead>
<tr>
<th>Frequency [Hz]</th>
<th>Typical Output [mV]</th>
<th>$h_{33}$ [MVm$^{-1}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.40</td>
<td>0.181</td>
</tr>
<tr>
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<td>7.29</td>
<td>0.386</td>
</tr>
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<td>0.575</td>
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</tr>
<tr>
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<td>0.932</td>
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<td>21.27</td>
<td>1.128</td>
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<tr>
<td>7</td>
<td>24.80</td>
<td>1.314</td>
</tr>
</tbody>
</table>

Table 2  Average derived values of $h_{33}$ and typical output voltages for bimorphs constructed from PTCa/P(VDF-TrFE) 60/40vol%
Figure 8  Typical response of a PTCa/P(VDF-TrFE) 50/50vol% monomorph with brass backing to cantilever measurements. Length 3.5cm, thickness of brass 113μm, thickness of composite 114μm, frequency 7Hz.

<table>
<thead>
<tr>
<th>Frequency [Hz]</th>
<th>Typical Output [mV]</th>
<th>$h_{33}$ [MVm$^{-1}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8.99</td>
<td>0.109</td>
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<td>2</td>
<td>17.94</td>
<td>0.218</td>
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<tr>
<td>3</td>
<td>27.11</td>
<td>0.330</td>
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<tr>
<td>4</td>
<td>35.51</td>
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<td>0.639</td>
</tr>
<tr>
<td>7</td>
<td>60.65</td>
<td>0.740</td>
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</tbody>
</table>

Table 3  Average derived values of $h_{33}$ and typical output voltages for monomorphs constructed from PTCa/P(VDF-TrFE) 50/50vol%
Figure 9  Typical response of a PTCa/P(VDF-TrFE) 60/40vol% monomorph with brass backing to cantilever measurements. Length 3.5cm, thickness of brass 123µm, thickness of composite 72µm, frequency 7Hz.

<table>
<thead>
<tr>
<th>Frequency [Hz]</th>
<th>Typical Output [mV]</th>
<th>$h_{33}$ [MVm$^{-1}$]</th>
</tr>
</thead>
<tbody>
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<td>7</td>
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<td>1.268</td>
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</table>

Table 4  Average derived values of $h_{33}$ and typical output voltages for monomorphs constructed from PTCa/P(VDF-TrFE) 60/40vol%
Figure 10  Variation of $h_{31}$ with frequency for bimorphs constructed from PTCa/P(VDF-TrFE) composites.

8. Converse effect

The converse effect of a piezoelectric bimorph will produce a bending of the bimorph when a voltage is applied to its electrodes. The deformation of a bimorph, clamped at one end as with the cantilever arrangement, will not be governed by the inverse of equation 32, but will be governed by the relationship $d_{31} = (\partial \zeta / \partial E_3)_a$, which is not the inverse of $h_{31} = - (\partial E_3 / \partial \zeta)_D$. Also the applied field will produce uniform strains along the length of the bimorph thus deforming it in a circular arc.

With a field applied to the bimorph the central plane will still be under a strain free condition as the stresses on opposite sides of the joining layer will be equal and opposed to each other. The two stress free planes will be on either side of the central plane and at equal distances, $a$, from it. The strain at the stress free planes will be $a/R$. Where $R$ is the radius of curvature. Under an applied field the relationship between the applied field and the strain at the stress free planes is given by

$$\frac{a}{R} = Ed_{31} \quad \ldots (68)$$
For a bimorph bent in a circular arc, the end deflection of the free end of a vertical cantilever such as in figure 2 is

\[ \delta y = \frac{L^2}{2R} = \frac{L^2 E}{2a} d_{31} \]  

...(69)

The value of \( a \) can be determined by taking moments in a cross-section of the bimorph and provided the thickness of the adhesive layer is very thin compared to the thickness of the bimorph then \( a \approx H/3 \), so that

\[ \delta y = \frac{3}{2} \left( \frac{L^2}{H} \right) E d_{31} \]  

...(70)

which in terms of an applied voltage is

\[ \delta y = \frac{3}{2} \left( \frac{L}{H} \right)^2 d_{31} U \]  

...(71)

The bimorphs fabricated during this work had an average thickness of 142\( \mu \)m and length of 3.5cm. Taking the values of the \( d_{31} \) coefficient of PTCa/P(VDF-TrFE) 60/40vol% and 50/50vol% as 4.6pC/N and 0.4pC/N respectively (11), end deflections of 42\( \mu \)m and 4\( \mu \)m will be produced by the application of 100V to the bimorph electrodes. These are the expected deflections from the bimorphs investigated to date. Due to our inability to measure such small deflections in the laboratory, there are no experimental data presented here.

9. Conclusion

Bimorphs, both series and parallel connected, and monomorphs have been fabricated from composite material of PTCa/P(VDF-TrFE). Five sensors of each arrangement were constructed and their behaviour was investigated over the frequency range 1 - 7Hz. The monomorphs, in the bending mode produced higher amplitude signals than the bimorphs, the highest being given by those constructed from the higher ceramic content composites. This was mainly due to the extra thickness of the monomorph construction. The results of the investigations have been given in this report. Further work will be undertaken to devise a method of measuring the value of \( h_{31} \) over a larger frequency range.

Theoretical values of the actuating properties of these bimorphs have been stated although no experimental data has been given. Further research in this area will be made by optimising the dimensions of the bimorphs to produced measurable deflections.
For embedded sensor applications, both the monomorph and the bimorph structure, of identical dimensions, will appear to be very suitable configurations giving equally high signal output with low noise content. However, with the bimorph structure it will be possible to separate the bending mode signal from that of the thickness mode operation, whereas for the monomorph such a separation of signal for the two different modes is not possible. The separation of the signals of these two different modes would be possible by employing the circuit shown in figure 11. Such a technique will make these sensors versatile for diverse applications. It may be worthwhile to embed both configurations into a laminate structure in order to monitor strain in different modes.

Figure 11  Duel operation of a parallel connected bimorph.

Characterisation of bimorphs constructed from PTCa/epoxy composites has yet to be completed. The problem of electrical breakdown during the poling process due to entrapped voids causes difficulty in consistently producing the larger area films needed for bimorph characterisation. This problem was addressed in our previous work on these composites (11). This work is currently in progress.

10. References


