AN APPROXIMATE THEORY OF FLUIDIZED BED COATING

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AN APPROXIMATE THEORY OF FLUIDIZED BED COATING

by

W. H. Chen and Chaim Gutfinger

Introduction

The fluidized-bed system for coating metals with plastics has de-
volved from a laboratory curiosity barely thirteen years ago to a routine
process in operation today in more than 360 major companies. (7) Applica-
tions are increasing in the appliance, chemical processing, electrical,
power distribution, and pipeline fields.

In the fluidized-bed coating process, a fusible polymeric resin in
powder form is applied to the surface of an object that is immersed in a
bed or chamber of powder through which a current of gas is passed. The
gas serves to levitate the resin powder in such a manner that it resembles
a boiling liquid in appearance. The object is heated to a temperature high
enough above the melting or softening point of the resin so that, after the
object is removed from the heat source, it retains enough heat on its sur-
face to melt the resin powder particles, which then stick fast, melt, and
flow together to form a coating.

In fluidized-bed coating, there are a number of variables that can
affect the thickness and uniformity of coating layers applied to the objects.
The major variables affecting the thickness of coating layer are object
temperature, immersion time, bed temperature, velocity of fluidizing gas,
particle size, shape and size distribution of the resin powder and the phy-
sical properties of object, powder and carrier gas.

In spite of the widespread use of the fluidized-bed coating process
and the many experiments done, no coating theory has been developed to
date. In the present paper, an attempt has been made to present a theory
that will correlate the coating thickness with the other coating parameters.

Fluidized-bed Coating - Statement of the Problem and Assumption

The discussion presented in this paper deals with the growth of
casting films on vertical plates in a fluidized bed. We consider one-
dimensional heat conduction in a coating film that extends from x=0 to x=X(t).
The face x=0 is the object surface. If the surface temperature is at or above
the melting or softening point, T_m, the coating commences. If the surface
temperature of the film drops below T_m, the growth of the coating film stops
and X(t) remains constant. The thickness of the coating film X(t) as a
function of time is the quantity we wish to find.

The equation describing the process is as follows:

\[ \frac{\partial X}{\partial t} = k \frac{\partial ^2 X}{\partial x^2} \] (1)

with boundary conditions

\[ T(x, 0) = T_0 > T_f \] (2)

\[ T(x, t) = T_{in} \] (3)

\[ T(X(t), t) = T(f) \] (4)

where \( T_f \) is the temperature of the film surface.
\[-k \frac{\partial T}{\partial x} = h\left( T - T_w \right) + \rho c \left( \bar{T} - T_w \right) \frac{\partial X}{\partial \bar{t}} \]  

(5)

Equation (5) expresses the fact that the coating film absorbs the heat flow into the fluidized bed by convection and conduction plus the heat absorbed by the bed particles that adhere to the object and form the film. In this energy balance, we neglect the heat loss by radiation. It is apparent that the convective transfer of heat far outweighs all other types of heat transfer in a fluidized bed system.

The essential difficulty in the problem is in the determination of the unknown moving boundary, $X(t)$. This is a non-linear problem because it involves a moving boundary whose location is unknown a priori. We were unable to treat it in an exact analytical manner; thus, we had to choose between alternative methods of either using a high-speed computer or of finding an approximate solution under some simplifying assumptions. The latter course was taken.

In the present paper the discussion is limited to cases where the following assumptions apply:

1. The thermal properties of material, $\rho$, $c$, $k$, are constant for a particular material during the coating.
2. The temperature within the fluidized bed is uniform throughout and constant.
3. The temperature of the particles and the fluid is the same.
4. The object temperature, $T_w$, is constant during the coating process.
5. The surface temperature of the coating film, $T_{s1}$, is constant and equals the melting or softening point of the material, $T_f$.
6. The thickness of films does not depend on the orientation of the coated object in the fluidized bed.
7. The heat transfer coefficient between the object and the fluidized bed is constant during coating.
8. Changes in the heat transfer coefficient over the height of the object are negligibly small.
9. The existence and uniqueness of $T(x, t)$ and $X(t)$ are assumed.

Assumption 9 is adopted from the classical moving boundary problem known as the Stefan problem. (3)

Using the assumptions made above, the equations describing the process are:

\[ \frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} \]  

(6)

with the boundary conditions

\[ T(x, 0) = T_w > \bar{T} \]  

(7)

\[ T(x, t) = \bar{T} \]  

(8)

\[ T(\infty, t) = \bar{T} \]  

(9)

\[-k \frac{\partial T}{\partial x} = h\left( \bar{T} - T_w \right) + \rho c \left( \bar{T} - T_w \right) \frac{\partial X}{\partial \bar{t}} \]  

(10)
These equations can be further limited to a narrower class of heat transfer problems by neglecting the convective term in Equation (10). This will be done later on in a solution of a special case in order to demonstrate a limiting behavior of the general solution.

Approximate Solution of the Fluidized Bed Coating Problem

As mentioned above, the heat transfer problems involving a moving boundary are non-linear, and, except in very special cases\(^{(1)}\), can be solved either by using high-speed computers or by some approximate technique. In this paper, we solve the heat transfer problem in fluidized-bed coatings by using the heat balance integral method\(^{(2)}\). For the one-dimensional case, the equation determining the thickness of the coating film reduces to an ordinary differential equation when this method is applied. Thus, it can be solved analytically or numerically. These solutions, although not exact, are accurate enough to be of practical use. We may also note that our main interest is the determination of film thickness as a function of time, rather than the temperature distribution in the film. Minor variations in the temperature profile inside the film are of secondary importance to the build up of the film thickness taking place on its surface.

Returning to the unbounded problem, we now multiply both sides of Equation (6) by \(dx\) and integrate from \(x=0\) to \(x=X(t)\).

\[
\int_0^X \frac{\partial T}{\partial x} \, dx = \int_0^X \frac{\partial T}{\partial x} \, dx
\]

Equation (11) is called the heat balance integral.

Applying Leibnitz’s rule on the left-hand side of Equation (11) and integrating the right-hand side, one obtains

\[
\frac{dT}{dt} + \nu \frac{d^n T}{dx^n} = \rho \left( \frac{\partial T}{\partial x} \right)_{x=0} - \rho \left( \frac{\partial T}{\partial x} \right)_{x=\infty}
\]

where

\[
\frac{dT}{dt} = \frac{\partial T}{\partial x} \int_0^X \frac{\partial T}{\partial x} \, dx
\]

Applying Equation (11) and letting

\[
A = \frac{\mu c^2 (T - T_0)}{\bar{K}}
\]

the following equation results:

\[
\frac{dT}{dt} - \nu \frac{d^n T}{dx^n} = \sigma \left( A \left( \frac{\partial T}{\partial x} \right)_{x=0} + \frac{\partial T}{\partial x} \right)_{x=\infty}
\]

Now we assume that the temperature profile within the film can be represented by a second-degree polynomial in the form:

\[
T = c + a(x - X(t)) + b(x - X(t))^2
\]

where the coefficients \(a\), \(b\), and \(c\) depend on time, \(t\). Since there are three coefficients in Equation (16), three conditions are necessary. Equations (8) and (9) constitute two conditions; the third one is the combination of Equations (6), (9), and (10). Substitution and simplification results in the following equations for the coefficients:

\[
a = \frac{\bar{K} \rho (T - T_0) - \rho c}{24A(t)}
\]

\[
b = \frac{T - T_0 + \rho c}{4A(t)}
\]

\[
c = \frac{T}{4}
\]
Where

\[ F(x) = \frac{hA}{Pc} x(1 - 2A\alpha) + 8A\alpha (T_c - T_f) \]

Substituting the values of \( a, b, \) and \( c \) into Equation (16), one obtains

an expression for the temperature profile in the film

\[ T = T_f + \left[ \frac{hA}{Pc} x(1 - 2A\alpha) + F(x) \right] (x-x(t)) \]

\[ + \frac{T_t}{2} \left[ \frac{hA}{Pc} x(1 - 2A\alpha) + F(x) \right] (x-x(t)) \]

To find a relationship for the coating thickness, \( X(t) \), as a function of time, we integrate Equation (15) after substituting \( T \) and \( T_c \) into it and performing the necessary algebra. The resulting expression is:

\[ X(t) = \frac{hA}{Pc} \left[ 2\Theta + \frac{h}{k} \Theta + \frac{1}{2} F(T) + \frac{hA}{Pc} T \left( \frac{hA}{Pc} x - 2A\alpha \right) \right] \sqrt{t} \]

\[ -3 \frac{hA}{Pc} - F(T) + 4\Theta + 2 \]

where \( \Theta \) is a dimensionless temperature defined as

\[ \Theta = T_t - T_f \]

The integral in Equation (21) can be most easily evaluated by graphical integration.

When the denominator of Equation (21) is set to equal zero, the value of the integral or the time will be infinity. This means that the growth of the coating film stops and that the final coating thickness, \( X_f \), is reached. Thus, for final thickness

\[ -3 \frac{hA}{Pc} X_f = \frac{hA}{Pc} T_f - 2A\alpha + 8A\alpha (T_c - T_f) + 4\Theta + 2A\alpha = 0 \]

substituting for \( A \) and \( \Theta \) from Equations (14) and (22), respectively, and simplifying one obtains

\[ 4\Theta + 2 - 3 \frac{hA}{Pc} X_f - \left( \frac{hA}{Pc} x - 2A\alpha \right) + 8A\alpha = 0 \]

Introducing the Biot Number, \( Bi = \frac{h}{k} X_f \), into Equation (24) results in

\[ 4\Theta + 2 - 3Bi \sqrt{(B_i - 2)^2 + 8\Theta} = 0 \]

Solving Equation (25) we get the very simple relationship between the Biot Number and dimensionless temperature

\[ Bi = \Theta \]

From Equation (26), we see that, for a given \( \Theta \), the Biot Number is fixed and is equal to it. This means that the final thickness is proportional to \( k \) and inversely proportional to \( h \). In other words, a plot of \( X_f \) versus \( k \) will give a straight line, while \( X_f \) versus \( h \) will yield a hyperbola. Equation (27) may be used for finding the heat transfer coefficient in the fluidized bed from experimental data. The detailed procedure is described in Appendix A.

The general solution of the coating problem as given by Equation (21), is plotted in Figures 1 and 2.

Figure 1 presents the coating thickness versus \( t \) for various values of the \( h/k \). A plot of coating thickness versus immersion time for various dimensionless temperatures, \( \Theta \), is shown in Figure 2 for a typical coating process. As seen in Figure 2, we can obtain a desired coating thickness at a fixed immersion time by adjusting the surface temperature of the object, the fluidized-bed temperature or the softening point of the coating material; i.e., change the coating material.
To demonstrate the sensitivity of the solution to various parameters, we plotted the coating thickness versus immersion time for typical working conditions.

Figure 3 presents coating thickness versus immersion time curves for various object temperatures. From Figure 3, it is obvious that the coating thickness is a strong function of object temperature. The relationship between coating rate and the object temperature is linear. The object temperature has a more pronounced influence on coating rate in the case of short immersion times.

A plot of final coating thickness, \( X_f \), versus softening point, \( T_f \), with the heat transfer coefficient as a parameter is shown in Figure 4. As seen in this figure, the change in final coating thickness with the softening point of the coating material is more pronounced at lower heat transfer coefficients than at high heat transfer coefficients in the range of softening points of 200-360°F.

Figure 5 shows the variation of coating thickness with immersion time for different values of the fluidized-bed heat transfer coefficient. A more direct evaluation of the influence of heat transfer coefficient, \( h \), on coating thickness is given in Figure 6 which presents the final coating thickness, \( X_f \), versus \( h \) for typical coating conditions. These figures show that the fluidized-bed heat transfer coefficient plays a very important role in fluidized-bed coating. The higher the heat transfer coefficient, the thinner the coating thickness because of higher heat loss to the surroundings.

From Equation (21), it seems that the factors that influence the coating thickness are only the object temperature, fluidized-bed temperature, and the properties of the coating material. Actually, there are some factors that affect the coating thickness indirectly because the heat transfer coefficient is governed by the following factors:

1. Properties of the materials
   a. Fluidising gas - thermal conductivity, density, viscosity
   b. Fluidised powder - thermal conductivity, shape, size, size distribution, density, specific heat

2. Design of fluidized bed
   Location and geometry of heat transfer surface, size of fluidized bed

3. Operating conditions
   Flow rate of fluids, feed or recycle rate of the powder, concentration of the powder in the bed, temperature level and magnitude of the temperature driving forces, etc.

Thus, these variables are indirect factors that can affect the coating thickness.

**Solution for the Case of No Convection**

In this section, we derive the solution for the coating thickness for the case where heat transfer by convection into the fluidized bed can be neglected. This solution will provide an upper boundary of coating thickness that can be achieved in fluidized-bed coating; that is, one can find what is the maximum coating thickness that can be obtained by changing the
conditions of fluidization in the direction of reducing h.

Letting $h=0$ in Equation (21), we get

$$
\int \frac{1}{3a} \left( \frac{20+5+\sqrt{30+1}}{40+2-2\sqrt{30+1}} \right) \, dt = t
$$

(28)

Integrating and rearranging, we get

$$
\frac{X}{\sqrt{4t}} = \left\{ \frac{12(20+1-\sqrt{30+1})}{20+5+\sqrt{30+1}} \right\}^{\frac{1}{2}}
$$

(29)

A plot of $X/\sqrt{4t}$ versus $\theta$ is shown in Figure 7. Looking at Equation (29), we see that, for a given $\theta$, $X/\sqrt{4t}$ is a constant. This means that the coating thickness is proportional to the square root of time. For this case there is obviously no final thickness, as for constant object temperature and no heat convection to surroundings, thus the thickness of coating film will grow infinitely.

**Comparison of the Theoretical Solution with Experimental Data**

The theoretical solution for the coating thickness was developed under the simplified assumptions discussed above. In this section we compare the simplified theoretical solution with some experimental data from the literature. This comparison will show us whether or not the approximate theoretical solution can provide answers for practical coating problems.

Experimental studies of fluidized-bed coating processes were carried out by Pettigrew(6), Richard(7), and Lee(8). In reporting the experimental data, Pettigrew gave more details on the operating conditions than the others. Thus, the comparison of the theoretical solution and Pettigrew’s experimental data is straightforward, while, for the other data, one has to estimate some of the coating parameters.

The experimental data given by Pettigrew are shown as coating thickness versus immersion time in Figure 8 and Figure 9 with the coating thickness calculated from theoretical Equation (21). The operating conditions are:

1. **Coating material**
   - The experiments were performed using Corvel vinyl resin
   - VCA-1289.

2. **Object material and size**
   - The objects were made of 4-3-1/16 in. cold-rolled steel.

3. **Preheat cycle**
   - The preheat temperature is 650°F for Figure 8 and 550°F for Figure 9.

4. **Final temperature**
   - The final temperature of object is 590°F for data in Figure 8 and 510°F for those in Figure 9.

5. **Fluidizing air velocity**
   - The air velocity is 4.9 ft/min.

As mentioned above, the heat transfer coefficient is a major factor in the fluidized-bed coating process. The coating data reported in the literature do not generally give the heat transfer coefficient in the fluidized bed; thus, we have to estimate the heat transfer coefficients of fluidized beds.
from their operating conditions. The heat transfer coefficient used in
the calculation of the theoretical solution from Equation (21) as presented
in Figures 8 and 9 is found by a graphical method from experimental data
of final coating thickness. The detailed procedures are described in
Appendix A. The value of the heat transfer coefficient used in Figures
8 and 9 is 52.5 Btu/hr-sq. ft-°F. Richart's \(^7\) experimental data are
shown as plots of coating thickness versus immersion time in Figures 10
and 11 together with the curves for coating thickness calculated from
Equation (21). The operating conditions are shown in the figures. Since
Richart's \(^7\) experimental data do not give final coating thicknesses, we
cannot find the heat transfer coefficient by a graphical method as we did
in the case of Pettigrew's \(^6\) data. Thus, we used the operating conditions
given by Richart \(^7\) to estimate the heat transfer coefficient from the
literature. We found the heat transfer coefficient for similar conditions
from Mickley and Trilling \(^5\) as 34 Btu/hr-sq. ft-°F and used this value
to calculate coating thicknesses shown in Figures 10 and 11.

As seen in Figures 8 through 11, the agreement between the theo-
retical predictions of Equation (21) and the experimental data given in the
literature is good. For object temperature, \(T_o = 600\)°F, the coating
thicknesses predicted by theory are, on the average, 10 per cent higher
than experimental data; for \(T_o = 510\)°F, they are 12 per cent higher. The
maximum deviation in coating thickness was less than 30 per cent. The
higher thickness predicted by theory is attributed to the assumption of
constant object and coating film surface temperatures. Other factors
that may account for deviations between the theory and the experiments are
the uncertainty of the heat transfer coefficient, and the temperature profile
within the coating film represented by a second-degree polynomial. As
seen in Figure 4, the coating thickness is a strong function of the heat
transfer coefficient. A small change in the heat transfer coefficient will
have a pronounced influence on the coating thickness. Since Richart's \(^7\)
operating conditions were not identical to those of Mickley and Trilling \(^5\),
the heat transfer coefficient found from the literature was not very accurate.
This could certainly introduce some error into Figures 10 and 11. One
could expect better agreement between theory and experiment if the heat
transfer coefficient data were available. Relaxing the assumption of constant
object and the coating film surface temperatures could also improve the
validity and range of the theoretical solution, but it would do so at the ex-
 pense of simplicity.

Discussion and Conclusions

In the present study an attempt was made to develop a theoretical
relationship between film thickness in fluidized-bed coating and the physical
properties of coating material, the object temperature, the fluidized-bed
temperature, and the coating time. The theoretical solution was compared
with experimental data. As seen in Figures 8 through 11, it can be stated
that this attempt was successful.

The weak points in the developed theory were the following assumptions:

1. The object temperature is constant.
2. The surface temperature of the coating film is constant and
equals its softening point.
3. The temperature profile within the coating film is represented
by a second-degree polynomial.

Assumption 1, which probably holds in the case of short immersion
times, is questionable for long immersion times.

Assumption 2 is the most questionable assumption in the developed
time theory. The temperature on the surface of the coating must remain higher
than the softening point of the polymer if the coating is to continue to build
up.

Following Goodman, it is expected that a cubic temperature pro-
file within the coating film would give a considerably more accurate solu-
tion than that using Assumption 3.

The following conclusions may be drawn from the present study:
1. Continuous coatings of less than 5 mils usually are difficult
   to apply because the rate of coating is very high in the first
   few seconds.

2. The heat transfer coefficient is a major factor in the fluidized-
   bed coating process. A small change in the heat transfer co-
   efficient (See Figure 5) will produce a large change in thickness.

   The smaller the heat transfer coefficient, the thicker the coating
   film. The maximum coating thickness is predicted by Equation
   (29) which was derived for the limiting case of no heat transfer

2. (contd)
to the fluidized bed.

3. We can obtain a desired thickness of coating film for a particular
   material at a fixed time by adjusting the object temperature, the
   fluidized-bed temperature, or the heat transfer coefficient. The
   heat transfer coefficient is governed by the properties of fluidizing
   fluid and fluidizing particles, the design of the fluidized bed, and
   the operating conditions.

4. The predicted coating thickness for higher object temperatures
   is better than that for lower object temperatures. This is seen
   in Figures 8 and 9.

5. The predicted coating thickness is slightly higher than experi-
   mental data because of the assumption of constant object and
   coating surface temperature.

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The authors are indebted to Professor Levenspiel for introducing
them to the field of fluidized-bed coating.
APPENDIX A

Method of Finding Heat Transfer Coefficients from Experimental Data

As mentioned above, if data of the final thickness of the film versus object temperature are available, we can find the heat transfer coefficient, \( h \), directly from the experimental data. The discussion presented in this Appendix provides a method of finding the heat transfer coefficient from experimental data.

Under the assumption we made above, the energy balance equation at the surface of the coating film is

\[
-k \frac{dT}{dx} = h (T_f - T_w) + \rho c \left( \frac{T_f}{T_w} - 1 \right) \frac{d\chi_f}{dx}
\]

(A-1)

When the coating film reaches its final thickness, the growth of coating film stops and Equation (A-1) becomes

\[
-k \frac{dT}{dx} = h (\bar{T} - T_w)
\]

(A-2)

Equation (A-2) expresses the fact that heat conduction at the surface of the coating film equals the heat flow into the fluidized bed by convection.

Integrating Equation (A-2) from \( x=0 \) to \( x=X_f \), and rearranging, we obtain

\[
\chi_f = \frac{k}{h(\bar{T} - T_w)} \int_{T_0}^{T_w} - \frac{k}{h(\bar{T} - T_w)} \int_{T_0}^{X_f}
\]

(A-3)

Equation (A-3) can also be obtained by rearranging Equation (27).

If we plot the variation of final thickness, \( X_f' \) versus the object temperature, a straight line is obtained. The intersection of the straight
Figure 1. Plot of coating thickness (x) versus at for various h/k ratios.
FIGURE 2. PLOT OF COATING THICKNESS (x) VERSUS IMMERSION TIME (t) WITH DIMENSIONLESS TEMPERATURE (θ) AS A PARAMETER FOR TYPICAL COATING CONDITIONS

\[ h = \frac{34}{hr \cdot \text{ft}^2 \cdot \text{F}} \]
\[ d = 0.00286 \frac{\text{ft}^2}{\text{hr} \cdot \text{F}} \]
\[ k = 0.0097 \frac{\text{Btu}}{\text{hr} \cdot \text{ft} \cdot \text{F}} \]

---

FIGURE 3. EFFECT OF IMMERSION TIME (t) AND OBJECT TEMPERATURE (θ_y) ON COATING THICKNESS (x)
FIGURE 4. FINAL COATING THICKNESS ($x_f$) AS A FUNCTION OF SOFTENING POINT OF COATING MATERIAL ($T_p$) FOR VARIOUS $h/k$ RATIOS

FIGURE 5. PLOT OF COATING THICKNESS ($x$) VERSUS IMMERSION TIME ($t$) FOR VARIOUS HEAT TRANSFER COEFFICIENTS ($h$)
FIGURE 7. DIMENSIONLESS COATING THICKNESS AS A FUNCTION OF DIMENSIONLESS TEMPERATURE FOR NO HEAT CONVECTION

FIGURE 6. EFFECT OF HEAT TRANSFER COEFFICIENT (h) ON FINAL COATING THICKNESS ($x_f$)
FIGURE 8. PLOT OF COATING THICKNESS (X) VERSUS IMMERSION TIME (t). COMPARISON BETWEEN THEORY AND EXPERIMENTAL DATA OF FETTIGREW (6).

Figure 8 shows a graph plotting coating thickness (X) against immersion time (t). The equation h = 52.5 Btu/hr ft² °F is given, along with other constants:

- k = 0.097 Btu/hr ft² °F
- q = 0.00286 Btu/hr ft²
- T = 290°F
- T = 75°F
- Immersion time (t) ranges from 0 to 35 seconds.

FIGURE 9. COATING THICKNESS (X) AS FUNCTION OF IMMERSION TIME (t). COMPARISON BETWEEN EXPERIMENTAL DATA OF FETTIGREW (6) AND THEORY.

Figure 9 illustrates the coating thickness (X) as a function of immersion time (t), comparing experimental data with theory.

- h = 52.5 Btu/hr ft² °F
- q = 0.00286 Btu/hr ft²
- k = 0.097 Btu/hr ft² °F
- T = 810°F
- T = 790°F
- T = 75°F
- Immersion time (t) ranges from 0 to 35 seconds.