Abstract

The concept of crack growth resistance curve (R-curve) has been applied to random fiber glass composites and analytical relations for R-curves have been obtained. The effect of thickness and the procedure of lamination is studied. It is found that thickness does not affect significantly the fracture characteristics. The analytical relations for R-curve are used to predict the residual strength characteristics and the fracture toughness.

Introduction

Geiger and Breitman (1) have applied the crack growth resistance method for random fiber polystyrene composites and showed that the $K_R$-curve is independent of initial crack length. Based upon their study they concluded that the $K_R$-curve concept could be an useful approach to study the fracture behaviour of such materials since substantial amount of slow crack growth occurs prior to unstable fracture.

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More recently Morris and Hahn\(^{(2)}\) have applied the \(K_R\)-curve approach to graphite/epoxy composites where they have shown that the effective increment of crack length at fracture and the corresponding \(K_R\) are independent of initial crack length.

This paper too attempts to characterize the fracture behaviour of fiber reinforced glass composites using \(R\)-curve approach and obtains the analytical relations for \(R\)-curves. An approximate relation for critical stress intensity factor (fracture toughness \(K_c\)) is presented.

**Material Preparation and Experimental Procedure:**

The specimens used in the experimental program were prepared by Paderite Ltd., U.K., using Chopped Strand Mat (CSM) in 2-7 Rigid resin. The laminations designated as A, B and C were prepared in the following manner:

- **A:** 4 layers of \(0.45 \text{ kg/m}^2\) CSM in 2-7 Rigid resin (thickness \(\approx 3.5\) mm).
- **B:** 16 layers of \(0.45 \text{ kg/m}^2\) CSM in 2-7 Rigid resin (thickness \(\approx 13\) mm).
- **C:** 16 layers of \(0.45 \text{ kg/m}^2\) CSM in 2-7 Rigid resin, prepared over a period of 4 days curing 4 layers of CSM in resin each day (thickness \(\approx 13\) mm).
The specimens were cut to size of about 125 x 600 mm and provided with an edge notch by means of .5 mm saw. This was further sharpened using a .15 mm saw. All specimens were tested under load-controlled conditions in an Avery Denison fatigue testing machine. During each test the applied load and pseudocrack opening displacement (COD) were monitored and recorded continuously on an X-y plotter. The COD was measured by a double cantilever clip gage (3) as shown in Fig.1.

The load-COD curves as recorded are given in Figures 2, 3, and 4 for laminates A, B and C respectively. These curves are found to be linear initially followed by almost continuous deviation from linearity indicating a slow crack growth prior to fracture. Since the composites tested do not show any visible self similar crack growth such as occurs in metal, an effective crack length matching the compliance based on COD was used to construct crack growth resistance R-curves. The compliance was obtained using the initial straight portion of load displacement records at various initial crack lengths. This compliance was plotted against a/w as shown in Fig.5 to obtain the effective crack lengths. The procedure to obtain effective crack length is as follows:

1) As shown in Fig.6, a straight line is drawn from the origin to the selected point on the load-displacement curve. The Inverse of the slope of this line is the compliance.
2) Using this compliance together with the calibration curve (Fig. 5) gives the effective crack length. The procedure can be repeated for other points on the load-COD curve to get additional values of effective crack lengths.

**EVALUATION OF R-CURVES:**

Crack growth resistance \( R \) is defined as

\[
R = \frac{K_R^2}{E} = \frac{1}{3} \cdot \frac{Y^2}{a^2} \cdot a, \quad \ldots \quad (1)
\]

where,

- \( a \) is the effective instantaneous crack length corresponding to stress \( \sigma \),
- \( Y \) is finite width correction factor defined by

\[
Y = 1.99 - 0.41 \left( \frac{a}{w} \right) + 13.7 \left( \frac{a}{w} \right)^2 - 38.48 \left( \frac{a}{w} \right)^3 + 53.85 \left( \frac{a}{w} \right)^4 \quad \ldots \quad (2)
\]

and \( E \) is the Young's modulus of the material.

The variations of \( K_R^2 \) with effective crack length \( a \) are shown in figure 7 to 9 for laminates A, B, and C respectively. The figures indicate that the \( K_R \) - effective crack length relationships appear to be nonlinear. From Fig. 9 it is seen that the \( K_R \) - curves are similar for all the initial crack lengths. The maximum value of \( K_R \) do not vary significantly with initial crack length but \( K_R \) at crack growth initiation varies to some extent. This
indicates that crack growth resistance may be independent of initial crack length which is confirmed by Fig. 12 where $K^2_R$ has been plotted as a function of crack extension ($\Delta a = a - a_0$) for laminates B. The scatter from mean line is small.

From Figures 7 and 9 it is difficult to make such conclusive statement as to whether one could consider $K_R$ to be independent of initial crack length as all the panels do not lead to consistent $K_R$ behaviour, though maximum $K_R$ does not vary significantly except in a few cases.

Superposition of Figures 7 and 8 reveals that the $K_R$ curve for at least $a_0/W = 0.2$ is almost identical, indicating that crack growth resistance may be independent of laminate thickness. However, more data will be needed to substantiate this statement. Despite such variations, the interesting feature of these results is that the average of maximum $K_R$ (denoted as $K^{\text{max}}_R$ in Table III) is practically same for laminates A, B and C respectively.

The R-curves can be used to predict the crack instability point by plotting the crack driving force curves with $\sigma$ as a parameter (Fig. 10) using the equation

$$K^2 = Y^2 \sigma^2 a$$

... (3)
In Figures 7, 8 and 9 where such curves have not been actually shown. The point of tangency between R-curves and crack driving force curve determines the point of instability. For the present cases such points of tangency were not observed in all cases. Thus in such cases the maximum value of $K_{\sigma}$ (denoted as $K_{\sigma f}$) is considered as critical which are represented by $K_{\sigma f}$ in Table III. The average $K_{\sigma f}$ are found to be practically same for laminates A, B and C indicating that $K_{\sigma}$ value at instability is invariant with thickness and the procedure of lamination.

These R-Curves can also be used to calculate candidate stress intensity factor $K_{\sigma}$ using a crack extension of 2\% of initial crack length similar to one used by Jones and Brown (14) for some metallic materials. The values of $K_{\sigma}$ so determined are indicated in Table III and are found to be varying with initial crack length. Due to such variation probably $K_{\sigma}$ may not be treated as a characteristic parameter as critical stress intensity factor. Also indicated in this table are the values of $K_{\max}$ obtained on the basis of maximum load and initial crack length. The average $K_{\max}$ do not differ very much between laminates A, B and C.

Kraft et al. (5) have proposed R-curves to be invariant i.e. independent of initial crack length. Then R-curve would be a function only of the amount of
slow crack growth \( \Delta a \). To verify this statement, the variation of \( K_R^2 \) with effective crack extension \( \Delta a^2 \) are plotted on double logarithm scale in Figs. 11, 12 and 13. It is depicted from these figures that there is linear relationship between \( \log K_R^2 \) and \( \log (\Delta a^2) \). Using the method of least squares the mean lines were determined and are shown on these figures. It is obvious that the deviation from the mean lines is small and is almost negligible in the case of laminates B. In view of small deviation, \( R \)-curve can be considered to be function of \( \Delta a \) only and may be represented by simple power law

\[
R = \frac{K_R^2}{E} = \frac{1}{E} \beta (\Delta a/w)^\alpha \quad \cdots (4)
\]

where \( \alpha \) and \( \beta \) for laminates A, B and C are given by Table I.

**TABLE -1**

<table>
<thead>
<tr>
<th>Laminates</th>
<th>( \alpha )</th>
<th>( \beta (\text{ksi/m}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>.40</td>
<td>539.5</td>
</tr>
<tr>
<td>B</td>
<td>.29</td>
<td>398.1</td>
</tr>
<tr>
<td>C</td>
<td>.46</td>
<td>501.2</td>
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</table>
R-curves so determined are shown in Fig. 14 as $R^2_2$ vs. $a/w$. It is observed that R-curves for laminates A and B do not differ significantly from each other but differ considerably from laminates C. As a result, R-curve may be considered invariant with thickness but not with procedure of laminations. In view of small variation between A and B an equation

$$R = 4.67.7 \left( \frac{a}{w} \right)^{3.5} \quad \ldots (5)$$

can be used to represent R-curve for laminates of any thickness without causing appreciable error. This curve is shown in Fig. 14 as an average of A and B.

The analytical expression (4) for $R$ can be used to derive fracture criterion by using the fracture conditions

$$G = R$$

$$\frac{2G}{\gamma a} = \frac{2R}{\gamma a}$$

where $G$ is the energy release rate and is given by,

$$G = \frac{K^2}{E} = \frac{\nu^2 v^2 a}{E}$$

These equations lead to the following fracture criterion

$$a_c = a_c (1 - \chi) F_2/F_1$$

$$a_c \sqrt{\sigma} = \sqrt{\frac{\pi}{\beta}} (\chi a_c/F_1 \omega)^{\nu/2} y^{-1}$$
where,

\[ F_1 = 1 + 2 \alpha_c Y (w_i)^{-1} \quad \ldots \quad (10) \]
\[ F_2 = 1 + 2 Y' \alpha_c Y (1 - \omega) v^{-1} \quad \ldots \quad (11) \]
\[ F' = \gamma Y' \gamma (a/v) \quad \alpha = \alpha_c \quad \ldots \quad (12) \]

\( \alpha_c \) and \( a_c \) are initial and critical crack length respectively and \( Y \) is given by equation (2).

The equations (3) and (9) lead to the determination of critical stress \( \sigma_c \) and critical crack length \( a_c \).

When \( \sigma_{\text{act}} \) effects are negligible i.e. \( Y' = 0 \) and \( Y = \text{const} \), the equations (3) and (9) yield

\[ a_c = \alpha_c / (1 - \alpha) \quad \ldots \quad (13) \]

and

\[ \sigma_c a_c^{(1 - \omega) / 2} = \text{const} \quad \ldots \quad (14) \]

which are identical to one derived by Brock(6,p.183).

To show the usefulness of equations (8) and (9) \( a_c/v \) and \( \sigma_c \) have been plotted as functions \( a_c/v \) in Figs. 15 and 16 where they are compared with test results. The agreement between calculated and test results is good. Figure 16 also indicates that \( \sigma_c \) varies little with thickness. These figures may be useful in predicting the residual strength of such laminates.
The equations (7) and (8) can be used to determine critical stress intensity factor $K_c$, corresponding to critical strain energy release rate $G_c$ as a function of $a_c$. Such behaviour of $K_c$ is shown in Figure 17 which indicates that the variation of $K_c$ with $a_c/w$ is small.

Figure 15 depict the linear behaviour of $a_c/w$ with $a_c/w$ for $a_c/w > 0.15$ so that

$$a_c/w = m (a_c/w) + C_1, \quad a_c/w > 0.15$$

... (15)

where $m$ and $C_1$ are given by Table II.

**TABLE II**

Constants for eqn. for critical crack length $a_c$

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<tr>
<th>Laminates</th>
<th>$m$</th>
<th>$C_1$</th>
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<tr>
<td>A</td>
<td>0.955</td>
<td>0.0625</td>
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<tr>
<td>B</td>
<td>0.970</td>
<td>0.045</td>
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<tr>
<td>C</td>
<td>0.965</td>
<td>0.0675</td>
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Since variation of $a_c/w$ with thickness in Figure 15, appears to be small, a single line

$$a_c/w = 0.9625 (a_c/w) + 0.05375, \quad a_c/w > 0.15$$

... (16)

may be used to represent laminates A and B within small
error. Thus we may get an approximate relationship for
critical stress intensity factor $K_c$ using equations
(15) and (5) as

$$K_c = 13.06 \left[ 1 - 0.7 \frac{a_0}{w} \right]^{0.1725}, \frac{a_0}{w} > 15$$

which may be considered as a fracture toughness parameter.

CONCLUSIONS:

1. Due to slow crack growth prior to fracture, R-curves
are found to be useful to provide full information
on the fracture resistance of the material up to the
final fracture.

2. Average value of $K_R$ at point of instability is
practically same for all laminates.

3. The effective crack growth at instability point
varies with initial crack length and the extension
in general is large in case of laminates C. This
indicates that the critical stress for laminates C
should be smaller than laminates A or B. Thus it
appears disadvantageous to prepare laminations with
large interval of times.

4. A simplified expression for critical stress intensity
factor or fracture toughness $K_c$ is given by eqn.(17)
which may be found useful for quick estimation of
fracture toughness for such materials.
ACKNOWLEDGEMENTS:

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REFERENCES:


Fig. 1 Specimen and the clip gage.
Fig. 2. Load displacement record for laminate A.
Fig. 3. Load displacement record for laminate B.
Fig. 4. Load displacement record for laminate C.
Fig. 10. Schematic representation for determination of crack instability from R-curve.
Fig. 11. $K_R^2$ as a function of crack extension for laminates A.
Fig. 12. $K_R^2$ as a function of crack extension for laminates B.
Fig. 13. $K_R^2$ as a function of crack extension for laminates C.
Fig. 14. R-curves.

Fig. 15. Critical crack length $a_c/w$ as a function of initial crack length $a_0/w$. 

$K_R^2 (MN/m^3)$ vs $(\Delta a/w)$

$\frac{a_c}{w}$ vs $\frac{a}{w}$
Fig. 16 Critical stress as function of $a_0/w$.

Fig. 17 Critical stress intensity factor as function of $a_0/w$. 
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<th>A</th>
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**TABLE 3**

Stress intensity factors