Final Report on
ONR GRANT–N00014–91–J–1700

Error Analysis in Numerical Solution
of Fluid–Structure Interaction Problems

G. C. Hsiao
R.E. Kleinman
Department of Mathematical Sciences
University of Delaware
The scope of the work under the subject grant encompassed two interrelated subjects: deriving integral equation formulations for the fluid structure interaction problem and obtaining measures of the error incurred in the numerical solution of these equations. The ultimate goal is a reliable method of converting the mathematical model of how an elastic body reacts to excitation in a surrounding fluid into a form amenable to numerical solution and furthermore ascertaining useful bounds on the error inherent in the numerical approximation. Significant progress was achieved in pursuit of these goals.

With regard to the fluid–structure problem a comprehensive treatment of weak solutions of this problem was completed. A number of formulations were discussed and the delicate proofs of existence and uniqueness were obtained. This work is fundamental to proving convergence of numerical approximations such as Galerkin methods. Moreover it establishes the function spaces in which solutions should be sought, based on the physical principle of conservation of energy. The manuscript describing these results, item 4 in the attached list of papers prepared under the grant, has now been completed and will be submitted to the Quarterly Journal of Applied Mathematics and Mechanics.

The error analysis of numerical solution of integral equations, which was originally thought of only in the context of fluid–structure interaction, was generalized to a broad class of integral equations. The importance of meaningful a–posteriori error estimates was recognized as having a profound impact on all numerical solutions. Rather than extrapolating from accurate error measures which are available
only for very simple boundaries, the sphere for example where exact analytic solutions are known, to much more complicated boundaries where exact solutions are not known, work accomplished under this grant (items 1, 6, 13) provides the means for obtaining error measures for any shape. A systematic study of the different integral operators which occur in mathematical physics was begun. Results have been obtained for both first and second kind equations involving weakly singular and hypersingular integral operators. The appropriate function spaces in which to measure residual error were determined. It was shown how first kind equations may be well conditioned when considered in the appropriate Sobolev spaces. Moreover the computability of the norms in some of these spaces was demonstrated. Work remains to be done in pre- and post-processing to make the computations tractable in some cases and more efficient in all cases. Nevertheless the utility of a-posteriori error estimation has been established under this grant, and most importantly is being adopted by the boundary and finite element community. This, perhaps more than all of the papers and presentations prepared under the grant, a list of which follows, attests to the degree to which the goals of the grant have been achieved.

In addition to the work on fluid-structure interactions and error analysis in approximate solution of integral equations, a number of related problems in integral equations were treated. Significant results were obtained in coupling finite and boundary element methods, multigrid and other iterative methods, domain decomposition as well as some integral equation based results in inverse scattering. Papers describing these results are included in the following list.


Presentations Under ONR Grant—N00014–91–J–1700


13. **G. C. Hsiao**, Boundary Integral Formulations for Plate Problems, SIAM 40th Annual Meeting, Los Angeles, CA,


22. **R. E. Kleinman and P. M. van den Berg**, Profile inversion by simultaneous error reduction, XXIVth General Assembly of URSI, Kyoto, Japan, August 1993.


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