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Observation of Acoustic Scattering from Turbulence

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Abstract
An examination is made of the use of broad-bandwidth high-frequency acoustic scattering to infer remotely the spatial structure of the temperature field of a thermally driven buoyant plume.

INTRODUCTION

This grant provided student support for the Ocean Acoustics Turbulence Study (OATS), which was originated and directed by Louis Goodman at the Naval Un-
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dersea Warfare Center, Newport, RI. The initial goal of the OATS program was to examine whether high frequency acoustic scattering from ocean turbulence is strong enough to be observed in competition with typical scattering from biological and particulate matter. An acoustic scattering model based on the Born approximation that suggested a positive result had been developed by Goodman (1). A particular goal of this project has been to make laboratory measurements to study scattering from temperature variability in water using laminar, unstable, and turbulent thermal plumes.

THEORETICAL BACKGROUND

The weak scattering theory applicable to this project was developed by Goodman (1),(2),(3) and will be mentioned here only briefly. The wave equation for the scattered field is given by (4)

$$\left( \nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) P(r, t) = L(r, t) P(r, t),$$

where $P$ is the acoustic pressure and $L$ is the scattering operator. For scattering dominated by temperature-induced compressibility variations, the latter is given by

$$L(r, t) = \frac{-2\alpha T(r, t)}{c^2} \frac{\partial^2}{\partial t^2},$$

where $T$ is the temperature field relative to ambient and $\alpha = (dc/dT)/c_0$. We now make the following definitions and approximations: use the Born approximation where $L$ operates on the incident wave $P_0(r, t)$; express the equation in frequency space through the Fourier transform $P(r, t) = \int p(r, \omega) \exp(-i\omega t) d\omega$; let the $z$-axis be vertical, along the axis of the thermal plume, and let the $x$-axis be in the direction of $K = k_x - k_z$, where $K$ is the Bragg wavevector with magnitude $K = 2k \sin(\theta/2)$; and include first-order terms in the phase factor that express wavefront curvature.
The scattered signal is then given by the transfer function (3)

$$\Gamma(K) = \frac{p(r, \omega)}{p_0(r, \omega)} = \frac{ak^2}{2\pi r_0} \int T(r) B(r) \exp(iKx + i k z^2/r_0) dr,$$

(3)

where $B(r)$ describes the source/receiver beam pattern. Without wavefront curvature, the $kz^2/r_0$ phase term goes to zero. Equation (3) is a prescription for inferring important aspects of the temperature field from the scattering data. When wavefront curvature is negligible, the integral is just the beam-pattern weighted Fourier transform of the temperature field at the Bragg wavenumber $K$.

A simple test to examine the validity of the far-field Born approximation is to compare the magnitudes of the transfer functions for measurements made in opposite scattering directions. From Eq. (3) without the curvature term, $|\Gamma(K)| = |\Gamma(-K)|$. With wavefront curvature, this relationship no longer holds and Eq. (3) no longer expresses a Fourier transform relationship between $\Gamma(K)$ and $T(r)$.

**EXPERIMENTS**

**Acoustics**

Laboratory experiments were performed with the system shown in Fig. 1. The technique employs simultaneous measurements with multiple broad-bandwidth source and receiver transducer pairs to fully resolve the spatial and temporal variability of the scattering field. The scattering temperature variabilities are produced by a heater at the bottom of the tank. A laser shadowgraph system monitors the nature of the thermal plume; examples are shown in Fig. 2.

Two types of experiments were performed during the tenure of this grant. In basic
scattering experiments designed to infer the temperature field from Eq. (3), a broad range of $K$ values is needed for high resolution. This is accomplished by making simultaneous measurements with three sets of source/receiver transducers arranged to give different scattering angles, $\theta$, but with the same scattering-wavevector direction. This is shown in Fig. 3.

The second type of experiment used two source transducers and two receivers, as shown in Fig. 4, to measure scattering with Bragg wavevectors that have equal magnitude and opposite directions. This was used to study wavefront curvature effects, as is suggested above. Contour plots of the scattered signal from an unstable plume for positive and negative Bragg wavevectors, respectively, are shown in Figs. 5 and 6. The vertical axis represents the time elapsed as successive measurements are made. If the plume rises steadily without distortion—the shadowgraph images indicate it does so, in this case, at 3 cm/s—then the vertical axis also represents the height of the plume. The two figures are qualitatively similar, but have a mirror image type of asymmetry. Both have approximately the same number of local maxima and very similar periodicity of about 1.5 seconds or 5 cm. Further analysis of the scattering from this unstable thermal plume (3) confirmed the failure of the far-field approximation. By calculating the transfer function of the scattering process for the measurements made in the plus and minus directions, $\Gamma_+$ and $\Gamma_-$, and comparing the time series at selected acoustic frequencies, the data show that $|\Gamma_+| \neq |\Gamma_-|$. The wavefront curvature term must be included to describe the scattering process. To the extent that the hypothesis of a uniformly advecting plume is valid, however, the Fourier transform of the scattering field can still be extracted from the scattering data. The length scale over which this appears to be true is equal to or greater than 2-3 cm. This distance corresponds to the order of the Fresnel radius of the transducers used at the center frequency of 500 kHz. For further discussion, see Ref. (3).
Optics

An optical annex to the OATS laboratory was established to investigate optical interactions with the thermal plume. Laser shadowgraphs of the plume are shown in Fig. 2. Another shadowgraph of a laminar plume is shown in Fig. 7. This pattern arises because the warm plume acts as a diverging lens. A typical analysis is shown in Fig. 8. If a Gaussian temperature profile is assumed, then the index of refraction also has a Gaussian profile (Fig. 8(a)). Here, r = 0 is the center of the plume, which extends out to about r = 0.4 cm. Light rays that pass through the plume are refracted by an amount that depends on the index gradients and the path lengths that they experience. The resultant deflection angles are shown in (b). Finally, (c) shows the intensity of light at a screen 2.74 m from the plume.

A Mach-Zehnder interferometer was also built to study the temperature profile of the laminar plume in more detail. This is shown in Fig. 9, where a laser beam is cleaned up and expanded by a spatial filter (sf), and is divided by a beamsplitter (bs1) into a sensing beam that passes by the plume and a reference beam. These are recombined by beamsplitter bs2 to produce an interference pattern at the camera. A typical result is shown in Fig. 10. Similar results can be obtained by assuming a Gaussian temperature profile for the plume. Figure 11 shows the results of the analysis. Again, (a) shows a Gaussian profile for the index of refraction (in this case, with an axial temperature ~ 4 K above ambient). The resultant phase shift for rays passing through different parts of the plume is shown in (b) and, finally, (c) shows the interference pattern.
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Xi Lin, M.S. 1994, non-thesis option.

REFERENCES


Fig. 1. Experimental system for acoustic scattering experiments. Up to three acoustic sources and receivers can be operated.
Fig. 2. Laser shadowgraphs of a laminar plume, an unstable plume and a turbulent plume.
Fig. 3. Configuration for three simultaneous scattering measurements with common direction, but differing magnitudes of $\mathbf{K}$, the Bragg scattering vector.
Fig. 4. Configuration for two simultaneous scattering measurements with opposite directions, but equal magnitudes of $\mathbf{k}$, the Bragg scattering vector.
Fig. 5: Contour plot of the magnitude of the scattered signal from an unstable plume with \( \mathbf{k} \) in the positive \( x \)-direction.
Fig. 6. Contour plot of the magnitude of the scattered signal from an unstable plume with $\mathbf{x}$ in the negative $x$-direction.
Fig. 7. Shadowgraph of a laminar thermal plume; (a) digitized version of (b) photograph of video monitor.
Fig. 8. Shadowgraph analysis of a laminar thermal plume. (a) Index of refraction from the plume axis outward for a Gaussian temperature profile. (b) Deflection angle due to refraction of rays passing through different parts of the plume. (c) Resultant light intensity at a screen.
Fig. 9. Mach-Zehnder interferometer for study of a thermal plume.
Fig. 10. Interferogram of a laminar thermal plume; (a) digitized version of (b) video monitor view.
Fig. 11. Interferogram analysis of a laminar thermal plume. (a) Index of refraction from the plume axis outward for a Gaussian temperature profile. (b) Phase shift for rays passing through different parts of the plume. (c) Resultant light intensity of the interference pattern.