13. ABSTRACT (Maximum 200 words)

Cellular Nonlinear Networks (CNNs) are large arrays of nonlinear circuits coupled to their immediate neighbors. In the past year we have made many advances in understanding the pattern-forming dynamics of such circuits and their relationship to problems in physics and biology.

Large arrays of complex cells have been shown to demonstrate many interesting pattern forming behaviors. Celebrated examples include the reaction-diffusion systems of Turing used to explain animal markings, the propagation of autowaves and the ‘synergy’ effect, and the Ising spin system and discrete bistable systems used to describe magnetic media and metal alloys. In the past year, we have shown that the simple first-order CNN is capable of exhibiting features found in these systems. However, due to the continuous-time nonlinear dynamics and general neighborhood weights the patterns formed by the CNN are a study in their own right. In fact, the piecewise-linear sigmoid allows many theorems to be derived, which are not otherwise possible, about stable patterns supported by the CNN medium.
Pattern Formation Properties of Cellular Neural Networks

Pattern formation in the CNN is a result of the mechanism of symmetry breaking around an unstable equilibrium. The symmetry can be broken by either small random noise, a 'seed' or site defect, or a systematic disturbance. The parameters which affect the type of pattern formed are the bias and interconnection weights.

Random Perturbation

When the symmetry breaking is introduced as a small random perturbation, our research has led us to identify the following three epochs of behavior in the time formation of patterns.

- Linear System leading to noise shaping.
- Separation of Spatial Frequencies and Phases leading to meta-stability.
- Boundary Negotiation leading to stability.

Various observations and theoretical results have been made for each of these epochs, which are summarized below:

Linear System

If the system is halted at the moment when the first cell of the array enters the saturation region, the linear system theory will be exact. The states at this time will be the result of a linear filtering operation which can be found in terms of the template weights. Such a filtering operation can be understood to enhance certain spatial frequencies while suppressing others.

Motif Separation

In regions where a significant number of cells have reached saturation, the linear analysis does not even hold approximately. At this point, arbitrary combinations of the unstable modes can no longer be maintained. The states begin to separate themselves into regions with motifs that are at least locally stable. These regions grow until their boundaries meet with other regions of different motif. At this point the system is considered to be meta-stable in the sense that most cells are unchanging in time (the interior cells of the regions) and only the boundaries between regions are moving.

It is difficult to determine what the possible meta-stable motifs are. We have shown that if any combination of equally unstable modes has a purely binary representation, then in the saturation region, that motif is stable, at least in the sense that a big enough patch of it will not change unless outside influences force it to. It is not certain that a finite array containing this motif would be stable, however.

For a $3 \times 3$ template of weights (i.e. immediate neighbors) we have fully characterized the stable motifs. In addition, simple inequalities allow the determination of the dominant stable motif, which will generally arise from random initial conditions.

Boundary Negotiation

Even though the array has separated into regions that are locally stable, the boundary between them may prove to be unstable. Thus begins a long process where most of the cells in the array are not changing in time, but slowly the boundaries are negotiated. In some cases there may not be
any acceptable boundary, and eventually one motif must cover the whole array. The whole process may take many order of magnitudes longer than the first two steps.

Bias

We have studied some of the effects of using the bias term in the dynamics. The bias changes the allowed meta-stable motifs. For instance, we have used this fact to demonstrate the formation of ‘stripes’ of a tiger when no bias is used and the ‘spots’ of a leopard when bias is added. Varying the bias in time can simulate some of the effects of striping patterns in the angelfish. Finally, we have begun investigations into the use of space-varying bias for image processing applications such as fingerprint enhancement.

Site Defects

Finally, we have begun understanding the mechanism involved in the production of spiral and target patterns in the CNN by using ‘seeds’ to break the unstable equilibrium. When one site of the array is disturbed from equilibrium, a pattern begins propagating outward. This will usually take the form of a circular blob containing the dominant motif. However, when more than one seed is used in the same array, these motifs must join up in a consistent way, which can cause the formation of spirals. The patterns formed are reminiscent of the aggregate growths of bacteria colonies, for instance.

List of Publications