### REPORT DOCUMENTATION PAGE

**Title and Subtitle:** Massively Parallel Iterative Methods: Multiscale Preconditioners and Implicit Methods

**Author:** Tony F. Chan

**Performing Organization:**
- Department of Mathematics
- UCLA
- 405 Hilgard Ave.
- Los Angeles, CA 90095-1555

**Sponsoring/Monitoring Agency:**
- U.S. Army Research Office
- P.O. Box 12211
- Research Triangle Park, NC 27709-2211

**Funding Numbers:** DAAL03-91-G-0150

**Distribution/Availability Statement:** Approved for public release; distribution unlimited.

<table>
<thead>
<tr>
<th>Number of Pages</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price Code</td>
<td>UL</td>
</tr>
</tbody>
</table>

**Security Classifications:**
- Report: Unclassified
- This Page: Unclassified
- Abstract: Unclassified

**DTIC Quality Inspected:** 5

**Report Date:** 03/22/95

**Report Type and Dates Covered:** Final Report 6/17/91 - 9/30/94

**Subject Terms:**

| Subject Terms | 15950630 | 149 |

**Notes:** The views, opinions and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation.
ARO Contract DAAL03-91-G-0150

Massively Parallel Iterative Methods:
Multiscale Preconditioners and Implicit Methods

Final Report

Grant Period: 6/17/91 - 6/16/94. No cost extension to: 9/30/94.

Principal Investigator: Tony F. Chan

Department of Mathematics
Univ. of California at Los Angeles

1 Personnel Supported by this Contract

Faculty: Tony F. Chan, Tarek Mathew, Beth Ong, Barry Smith, Panayot Vassilevski.

Postdoctoral fellow: Patrick Ciarlet, Jr., Francoise Lamour, Jun Zou
Graduate students: Jennifer Y.C. Chang (Ph.D. 92), Jianping Shao (Ph.D. 93), Tedd Szeto (Ph.D. 94).

2 Description of Research:

Nonlinear and linear systems of equations often arise in scientific computation, for example in implicit methods in Computational Fluid Dynamics (CFD). It is important to find cost-effective and accurate methods to solve such systems. Iterative methods are among those widely used, especially for 3D problems. In this project, we consider iterative methods which are especially suited to massively parallel architectures.

To accelerate convergence of these iterative methods, preconditioners are often used. Good preconditioners reduce the number of iterations and involves few arithmetic operations per iteration. Effective parallel preconditioners must account for the global coupling inherent in elliptic problems. On the other hand, efficient parallel implementation often favors local computations. Multiscale iterative methods represent a good compromise between these two conflicting goals. We focused our attention on two classes of multiscale preconditioners: multilevel basis preconditioners and domain decomposition preconditioners.
Domain decomposition methods are techniques which solve a boundary value problem posed on a general domain by patching together solutions on small subdomains, usually in an iterative fashion. They seem ideally suited for coarse grain parallel machines, with each subdomain assigned to a single processor. We derived efficient methods in this class, for overlapping and nonoverlapping subdomains, for convection-diffusion problems, for unstructured meshes and for partitioning meshes.

For the successful application of preconditioned iterative methods, in addition to efficient preconditioners, robust and efficient convergence acceleration and stabilization methods are also needed. We investigated ways to construct conjugate gradient like methods for nonsymmetric systems which possess some of the following desirable properties: short recurrences, transpose-free, no breakdowns, smooth convergence and efficient use of matrix-vector multiplies. We have developed a simple technique called composite step, which can overcome one kind of breakdowns in BCG methods. We also give one of the first convergence proofs of the BCG method by analyzing as a Petrov-Galerkin method defined on Krylov subspaces and use the theory of Babuska and Azis. Moreover, we have now extended this composite step technique to stabilize other product methods derived from BCG, such as CGS, BiCGSTAB and several other variants.

Finally, we also studied some basic linear algebra problems, including rank-revealing QR factorizations and stable Toeplitz solvers, which have applications to signal and image processing.

3 Summary of Most Important Results

1. We extended existing domain decomposition and multigrid algorithms for elliptic problems to unstructured grids. The key issues are how to decompose the grid into subgrids, and how to construct the coarse grid. We proved convergence without assuming the space of coarse grid functions are nested within the fine grid function spaces and we also allow the coarse grid's boundary to not match that of the fine grid.

2. We proved that the "Recursive Spectral Bisection" method for partitioning unstructured mesh is optimal in the sense that it computes the nearest discrete feasible solution to the optimal solution of a continuous form of the discrete graph optimization problem (which is computed by solving an eigenvalue problem). Based on this result, we developed a more efficient variant of the algorithm based on using the sign of the eigenfunction instead of its median.

3. We developed "boundary probing" interfacial preconditioners in domain decomposition, which adapts to the coefficients of the differential operator.
and the geometry of the subdomains. We extended this to derive a more efficient version of the vertex space method of Smith and Widlund.

4. We have developed a simple technique called composite step, which can overcome one kind of breakdowns in BCG methods. We also give one of the first convergence proofs of the BCG method by analyzing as a Petrov-Galerkin method defined on Krylov subspaces and use the theory of Babuska and Azis. Moreover, we have now extended this composite step technique to stabilize other product methods derived from BCG, such as CGS, BiCGSTAB and several other variants.

5. Using a result on the relationship between the overlapping and nonoverlapping methods in domain decomposition, we proved that many domain decomposition algorithms have convergence rates that are independent of the aspect ratios of the subdomains or the amount of overlap.

6. We developed a block-ILU factorization technique for block-tridiagonal discretization matrices that need not necessarily be M-matrices. It is based on combining the multigrid idea with the block ILU factorization. We proved the existence of the factorization and derived estimates for the spectral equivalence relation between the original matrix and the proposed preconditioner.

7. Developed look-ahead Levinson algorithms which are stable for a much more general class of Toeplitz systems, including indefinite ones, while retaining the efficiency of the classical algorithm. Fortran and MATLAB codes have been developed also.

8. Developed rank-revealing QR factorization algorithms, which can be viewed as inexpensive alternatives to the singular value decomposition. We developed applications to total least squares, subset selection, rank-deficient least squares.

4 List of papers/reports supported by this contract

4.1 Ph.D. Thesis


4.2 Papers in domain decomposition and multigrid methods


7. The application of a domain decomposition method to solving singular Neumann boundary problems, by J.P. Shao, UCLA CAM report 93-09.


5 Papers in Mesh Partitioning


5.1 Papers in Nonsymmetric Krylov subspace methods


5.2 Other papers
