### Abstract

The project had four major goals. The first was the development of distributed parameter models of the transient behavior of some or all of the state and internal variables describing the motion of multiple-link flexible structures. The structures under consideration consist of finitely many interconnected flexible elements such as strings, beams, plates and shells or combinations thereof and are representative of trusses, frames, robot arms, solar panels, antennae, deformable mirrors, etc. The second goal of the project was to provide rigorous mathematical analyses of the resulting models. Its third goal was to develop control-theoretic properties of, and control strategies for, multiple-link flexible structures based on the control-theoretic properties of the models. The fourth emphasis was on model validation and illustration with the aid of extensive numerical simulations of the predictive capabilities of the mathematical models.

### Subject Terms

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This report describes the accomplishments achieved in the research project titled “Modeling, Analysis and Control of Multiple-Link Flexible Structures”, supported under the above referenced research grant.

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Initial emphasis of the research was placed on the development of distributed parameter models and of a comprehensive theory of control of networks of one-dimensional elements, such as trusses and frames. Models describing the dynamic behavior of a 3-dimensional networks of thin, nonlinear thermoelastic beams were developed and tested by numerical simulations. The beams may be initially curved and twisted. In particular, the geometric and kinematic conditions which must hold at a junction where two or more beams are joined have been explicated in several situations of interest, including rigid joints and pinned joints. The very complicated interaction and transmission of elastic effects such as bending, torsion and axial deformation through the junction regions are clearly seen in the numerical simulations. A careful study of the nature of the spectrum of the such structures was carried out, and led to an explicit description of the spectrum and eigenfunctions as well as to techniques for their computation. A comprehensive theory of control and stabilization for such structures was developed. Various anomalies which may occur, such as the possibility of uncontrollable and unstabilizable modes, were pointed out and the underlying causes for such anomalies were clarified, as well as possible remedies.

Attention was next focused on the development of models of structures consisting of interconnected two-dimensional elements, such as thin plates or shells,
or combinations of one- and two-dimensional elements, such as linked beams and plates, and their associated control-theoretic properties. These problems are much more subtle than their one-dimensional analogs, beginning with the notion a joint, or junction, between two-dimensional elements, and new complicating phenomena arise in the analysis, such as the development of singularities (regions of high stress or where cracks occur). It turned out to be possible to give a complete controllability analysis in certain cases of interest, such as a thin elastic plate to which a thin elastic rod is rigidly attached either to its edge or to its face. However, a complete analysis of other configurations has proved to be more elusive. Nonetheless, while it cannot be claimed that all issues have been settled in either the modeling or control-theoretic aspects, substantial progress was made and the major remaining technical issues were elucidated. In addition, numerical simulations carried out on proposed models of certain configurations, such as those consisting of multiple interconnected elastic plates, lend credence to the models and to the control strategies suggested, even though the theoretical foundations are not yet completely in place.

**BOOK**


**RESEARCH PAPERS**


**INVITED PRESENTATIONS**

(A) Department of Mathematics, Rutgers University, March, 1995.

(B) Department of Mathematics, University of Delaware, March, 1994.


(D) Minisymposium on Control of Partial Differential Equations, College Station, TX, October 21, 1993.

(E) Special Session on Control Systems Governed by Partial Differential Equations, AMS regional meeting, College Station, TX, October 23, 1993.
(F) SIAM Conference on Control and Its Applications, Minneapolis, MN, September 1992. (Plenary Lecture)

(G) Joint AMS, IMS, SIAM Summer Research Conference, Mount Holyoke, MA, July, 1992. (Plenary Lecture)

(H) Special Session on Function Theoretic Methods in Differential Equations, Annual meeting of the AMS, Annapolis, MD, January, 1992.


