**Title and Subtitle**
Wavelets and Scattering (U)

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**Supplementary Notes**

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**Abstract**
This is the final report on the project entitled Wavelets and Scattering supported under AFSOR grant 90-307. During this project wavelets were used to analyze several problems in signal processing, quantum optics, elastic wave nondestructive evaluation, electromagnetic scattering and the dielectric response of water. The grant supported two students. One of them, Dr. C. R. Thompson is now an AF Captain in the ODEL Division at Brooks AFB in Texas.

A number of research papers were published including the first calculation of p-wavelets. Another publication shows the scale change of wavelet theory corresponds to the squeezing operation in quantum optics. A wavelet approach to visual recognition of faces was completed and has been submitted for publication. The Calderón-Grossmann-Morlet reproducing formula was shown to hold for the two-sided ideal of Hilbert-Schmidt operators. In elastic wave NDE, the frequency scales in phase space for the front face echo were shown to require a very different compression from the other scales. New results on Maxwell's equations in regions with Lipschitz boundaries were published.

**Subject Terms**
Wavelets, signal processing, optics, nondestructive evaluation, electromagnetism, inverse problems and applied mathematics.

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INTRODUCTION

This is the final project report for AFOSR grant 90-0307, Wavelets and Scattering. The original goals of this proposal included:

- Obtaining a better understanding of wavelets, including their strengths and weaknesses in areas which lie in the mission of AFOSR.

- Applying wavelets to electromagnetic scattering problems, ultrasonic nondestructive evaluation, quantum optics, signal analysis and the dispersive structure of the dielectric response of water.

Wavelets are very promising mathematical and computational objects. A single function, together with its dilations (translations and scale changes), generates a basis for function spaces used in many common physical situations. Scale changes provide a multiresolution structure which clearly and concisely describes elements of a solution or operator at different frequencies and thereby unifies diverse areas and applications. This facilitates the transfer of methods and techniques among problems, which explains the appeal of wavelet methods. Some of their useful properties include data compression, noise reduction, edge detection, and feature extraction.
In the next section, Publications, the 1994 publications which appeared, are in press, or submitted are listed and comments are given on what has been learned. In the section on Follow-On Projects, the next studies which are made possible are presented. The last section gives the Conclusion.

PUBLICATIONS 1994

Welland:


W3 M. Mitrea, R. Torres and G. V. Welland, Regularity and approximation results for the Maxwell problem on $C^1$ and Lipschitz domains, To appear in the proc. of conf. on Clifford Algebras held in April 1993, (12 typed pp).

W4 M. Mitrea. Torres and G. V. Welland, Layer potential techniques in electromagnetism, submitted to J. of Int. Eq. and Applic. (29 pp)

W5 S. K. Bhatia, V. Lakshminarayanan, A. Samal, G. V. Welland Parameters for Human Face Recognition submitted to J. of Visual Communicaton and Image Representation (26 typed pp)

DeFacio:


FOLLOW-ON PROJECTS

- Continuation of work with (McDonnell-Douglas Corp. CEM group); Wavelet methods, matrix sparsening methods for 3-D electromagnetic scattering. (GVW)

- Statistical mechanics of the dielectric response of water (BDF with G. Vignale, Cai and Welland).

- NDE using ultrasound, especially in composites, layered materials, bond adhesion (with Martin-Marietta, Baltimore).

- p-biorthogonal wavelet project with B. Jawerth with the boundary adjustments for edge sets. (GVW)

- feature extraction using wavelet packets for the development of a face recognition system with the target system to be competitive with the human capability for face recognition.

- Novel dielectrics including microwave Penrose tiling structures, wire structures, periodic arrays of spheres, honeycomb structures. The band-gap or pseudo-gap, switching properties, defect and the dispersion will be studied for each of these new materials (BDF with Satpathy).

- Inverse Neutron Optics of Layered Materials (BDF with H. Kaiser and D. Worchester).

CONCLUSION

The study of the frequency dependence of the dielectric response of water is a first step towards a quantitative understanding of the propagation of electromagnetic waves in tissue. Such understanding is required to answer
health and safety questions what are the responsibility of the Radiation Analysis Division of the Laboratory at Brooks AFB in Texas. The squeezed states developed for quantum optics will be studied for noise reduction in NDE. It is expected that this work will enhance the collaboration of BDF with scientists at Martin-Marietta. The study of bubble formation and their decay addresses questions concerning the health and safety of ultrasound in medicine, since these decays are either via shock waves, or initially high velocity jets of gas and matter.

Numerical work based on multileveling, sparsening, and conditioning methods are used in electromagnetic scattering problems which are very difficult to resolve because of technical problems. The technical problems derive from large high density matrices which require inversion. Large matrices occur in cases for which there is a large ratio of the characteristic length of the scattering body to wavelength of an illuminating source in problems involving body element methods. This is at the heart of the collaboration of GVW with Lou Mitschang, D.-S. Wang, et al. at McDonnell-Douglas (MDA). This has been extended to the use of local SVD methods and the use of wavelet packets to provide a controlled sparsening. The goal is to be able to solve very large problems using sparsening techniques in conjunction with other available methods or methods under development, while maintaining an understanding of implications with respect to invertibility and reliability of solutions obtained using sparsening techniques. These methods are to be developed with the objective for more general applicability. Other groups at MDA are involved in this project, e.g. the CFD group.

Methods using wavelet packets are being developed for application to a machine based recognition system for human faces. This is the first step in a project to obtain good representation of this class of images. One must clearly distinguish recognition from identification. The goal of this effort will be to develop a system that rivals the power of the human system reported in the paper, Parameters for Human Face Recognition. The methods will require registration of images through shifts and rotations, which requires a generalization of existing methods. This problem gathers together all of the ideas in the present project, and more.

These techniques will also play an important role in simulation of statistical mechanics models and, in particular, in the simulation of the dielectric response of water to electromagnetic pulses. Our studies are ongoing and we are making substantial progress toward the goals stated above.

The studies completed, so far, have developed methods and understanding which uniquely qualify the PI's for these follow-on studies. The follow-on
studies will incorporate refinement of analytic modeling of physical problems and numerical simulation.

The publications and preprints listed in section 2 add to the understanding of several problems of interest to the AFOSR. The p-wavelets in (WI) provides new possibilities for segmenting, compression and denoising signals and one of us (GVW) is using these wavelets to study edge sets with Prof. B. Jawerth. The papers on the dispersion of water, which appear in references, [D6], [D8], [D10] and [D11] show established structures such as the Debye decrease in dielectric response function, and such potentially new effects which have not yet been experimentally observed such as the collective dipolaron mode. A free rotor peak was found in the far infrared frequency region at exactly the frequency of the Simpson et al. peak which was experimentally reported, but had not been obtained in other theoretical models. However, the peak which was found was much too large and quantitative agreement has not been obtained. The dipolaron, if correct, would totally change the accepted nature of the relaxation mechanism. This collected mode cannot be ruled out by the data which exists today.

The publication listed under [D7] presented one operator-valued wavelet in quantum optics. There are two kinds of low-noise quantum states in optics, coherent states and squeezed states, and it is well known no additional such objects are possible. In a group theoretic approach, the coherent state is a translation in Fock space. In this paper, the scale change of wavelet theory is shown to be implemented by the squeeze operator. This opens several paths for study of noise in scattering, in addition to optics.

The publications listed in [D4], [D12] and [D13] apply wavelet analysis to inverse scattering of elastic waves for NDE and to glancing angle thermal neutrons from layered materials including both high technology metal composites and lipid bilayers.

We take the opportunity to point out that Brian DeFacio was elected to Fellow of the American Physical Society in November, 1993.

We have not applied nor obtained any patents. We have made no inventions. The support of this research by AFOSR is greatly appreciated by both of the PI's.

Brian DeFacio
Professor

Grant Welland
Professor