### Title and Subtitle

Development of "Smart" Piezothermoelastic Laminae: Theory and Applications

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### Abstract

Active "smart" structures with self-sensation, action and reaction capabilities can lead to a major technology breakthrough for the next-generation high-performance structures and mechanical systems. This new emerging area encompassing sensors, actuators, electromechanical systems, active materials, controls, and structural continua can be defined as the structure-electronics — StrucTronics. Among the commonly used sensor/actuator materials, piezoelectric materials possess unique electromechanical properties (the direct and converse piezoelectric effects) which can be respectively applied to sensor and actuator applications. The main objective of this research is to study the multi-field piezothermoelastic phenomena of piezoelectric laminae and to investigate the thermal effects to the performance of distributed piezoelectric sensors and actuators. There are three research components: 1) to develop new generic piezothermoelastic shell lamination theories, 2) to formulate piezothermoelastic finite elements and associated sensing/control numerical capabilities, and 3) to validate analytical and/or finite element solutions via laboratory experiments. This report includes all three aspects of the piezothermoelastic research. As described in the standard ARO Report Instructions, this report begins with Statement of Problems Studied, and followed by Summary of Research Results. Lists of technical publications and participating personnel are also provided.

### Subject Terms

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Final Report

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Development of "Smart" PiezoThermoElastic Laminae: Theory and Applications

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FORWARD

Active "smart" structures with self-sensation, action and reaction capabilities can lead to a major technology breakthrough for the next-generation high-performance structures and mechanical systems. This new emerging area encompassing sensors, actuators, electromechanical systems, active materials, controls, and structural continua can be defined as the structure–electronics – Structronics.

Among the commonly used sensor/actuator materials, piezoelectric materials possess unique electromechanical properties (the direct and converse piezoelectric effects) which can be respectively applied to sensor and actuator applications. The main objective of this research is to study the multi-field piezothermoelastic phenomena of piezoelectric laminae and to investigate the thermal effects to the performance of distributed piezoelectric sensors and actuators. There are three research components: 1) to develop new generic piezothermoelastic shell lamination theories, 2) to formulate piezothermoelastic finite elements and associated sensing/control numerical capabilities, and 3) to validate analytical and/or finite element solutions via laboratory experiments. This report includes all three aspects of the piezothermoelastic research.

As described in the standard ARO Report Instructions, this report begins with Statement of Problems Studied, and followed by Summary of Research Results. Lists of technical publications and participating personnel are also provided.

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STATEMENT OF PROBLEMS STUDIED

In this section, specific problems studied in this research are outlined. Brief background introductions are provided. Summaries of important research findings are presented in the next section.

A.1 Anisotropic Piezothermoelastic Composite Shell Lamination Theory

Piezoelectric materials are very useful in dynamic sensing, actuation, and control of active structural and mechanical systems. The purpose of this study was to derive a comprehensive thermo–electromechanical theory of anisotropic piezoelectric thin/thick shell laminae and laminates subjected to mechanical, electric, and temperature excitations (Tzou and Bao, 1994a). Piezothermoelastic constitutive equations of anisotropic piezoelectric materials were defined, and governing thermo–electromechanical equations and boundary conditions were derived using Hamilton's principle. In general, the resultant forces/moments and electric displacements have three components contributed by the elastic, electric, and thermal fields, respectively; they interact closely in the system thermo–electromechanical equations and boundary conditions. Applications of the theory to dynamic sensing and control were also discussed and explored. Due to the generalities of the material and geometry in the mathematical model, the derived theory can be widely used in many piezoelectric materials, e.g., piezoceramics, piezopolymers, etc., and also geometries, e.g., shells, plates, rings, beams, etc (Tzou, 1993a). Application examples were given in case studies. Other piezothermoelastic shell theories evolved from the generic anisotropic lamination theory were also investigated (Tzou and Bao, 1994b; Tzou and Howard, 1994a; Tzou, 1993a,b and 1994). Applications to distributed sensing and control of continua were also explored (Tzou, 1993a,b).

A.2 Hybrid Composite Plates

1) Classical Lamination Theory

Analytical research of composite plates during the project period has focused on the piezothermoelastic response of hybrid composite plates (combination of fiber–reinforced and piezoelectric laminae) subject to mechanical, thermal and electrical fields. This work initially focused on the influence of piezoelectric layers on the response of hybrid composite plates governed by classical lamination theory (Tauchert, 1992). Tauchert (1992) established the initial equations for piezothermoelastic response of hybrid plates. Solutions were obtained for a "free" plate of arbitrary contour and for a simply supported, rectangular plate. Numerical results illustrated that thermally induced deformation of a structural laminate can be reduced significantly through the addition of a layer of piezoelectric material under an applied electric potential. Further investigation demonstrated that reduced material stiffness coefficients, rather than full three-dimensional stiffness, provide greater solution accuracy and consistency with classical theory.
2) Mindlin Plate Theory

The developments of Tauchert (1992) were extended to the analysis of moderately thick plates using Reissner–Mindlin plate theory (Jonnalagadda, Blandford and Tauchert, 1994). This paper considered the response of composite plates constructed of graphite/epoxy laminae with an attached piezoelectric polyvinylidene fluoride layer subjected to mechanical, thermal and electric field loading for various length-to-depth and aspect ratios. First-order shear deformation theory was extended to include the piezothermoelastic respond of composite plate structures. Analytic results were obtained for a plate simply supported along all edges.

3) High–order Plate Theory

Following the Reissner–Mindlin formulation, a general third–order displacement formulation that incorporates transverse normal and shear strains was developed and applied to cross–ply laminates by Jonnalagadda, Tauchert and Blandford (1993a,b). The high–order displacement theories for thermoelastic composite plates were compared with some of the published high–order theories. Analytical solutions were developed for specially orthotropic plates using the Navier approach. Numerical results illustrate the importance of in

The response of an antisymmetric, hybrid composite plate to stationary mechanical, thermal, and electric loads was examined by Xu, Tauchert and Blandford (1993) using the third–order displacement formulation. An exact Navier–type solution was developed for a simply–supported plate, and corresponding numerical results were compared with those obtained by the finite element method and other plate theories. Inclusion of transverse normal strains was found to be important in cases of thermally or electrically loaded thick plates. Furthermore, it was found that application of electric potentials to the piezoelectric layers provides an effective means of reducing mechanically induced deformation and, to a lesser degree, in reducing thermal deformation.

A.3 High–order Plate Theory — Thermal Buckling

A third–order theory was also used for the prediction of critical buckling temperatures of simply supported laminated plates under uniform temperature loading (Tauchert, Jonnalagadda and Blandford, 1993). Exact Navier–type solutions were derived for simply supported, symmetric specially orthotropic plates. Numerical results were compared with those obtained using classical, first–order, and other high–order formulations. Results indicate that, regardless of the mode of buckling, transverse normal strains have little effect on the critical temperatures. While this is true for the case of a uniform temperature field, further study is needed to determine if transverse normal strains influence buckling produced by nonuniform thermal or electrical excitations.
A.4 High–order Theory — Free Vibration

Progress was also made on the development and application of a third–order theory for the free vibration of laminated plates (Jonnalagadda, Tauchert and Blandford, 1994). Exact Navier–type solutions were derived for simply supported, symmetric, specially orthotropic plates. Calculated natural frequencies were compared with those obtained using classical bending theory and Reddy's high–order formulation. The present theory is shown to yield vibration modes not obtainable by theories which ignore transverse normal strains, i.e., "thickening–type" modes of vibration.

A.5 Hybrid Composite Plates (Finite Element Analysis)

Numerous finite element codes have been developed as part of the research project. The first code is based on classical lamination theory and uses nonconforming C continuous rectangular and triangular plate bending elements with five degrees of freedom per node (three translational displacements and two rotational displacements). This program is based on the Tauchert (1992) formulation and has been used to confirm a number of analytical developments.

1) Mindlin Plate Theory (Finite Element Analysis)

A nine–node Lagrangian finite element formulation has been established using C continuous Mindlin plate theory, which is valid for moderately thick composite plates, again using five displacement degrees of freedom per node. Numerical results were presented for simply supported or fixed boundary conditions by Jonnalagadda, Blandford and Tauchert (1994). These results showed that moderately thick piezothermoelastic composites were sensitive to shear deformation, but increasing plate aspect ratio reduces the shear deformation influence.

2) High–order Plate Theory (Finite Element Analysis)

Furthermore, the nine–node Mindlin plate program formed the basis for the development of a high–order plate theory finite element program. High–order theory refers to using displacement expansions of the form:

\[ u_i(x,y,z) = \sum_{k=0}^{N_i} z^k U_{ik}(x,y) , \]  

where \( N_i \) is the number of desired expansion terms in each coordinate displacement (\( i=1,2,3 \)) direction; \( U_{ik}(x,y) \) = translational displacement functions for \( k = 0 \); \( U_{ik}(x,y) \) = rotational displacement functions for \( k = 1 \); and \( U_{ik}(x,y) \) = second– and third–order rotational displacement functions for \( k = 2 \) and \( 3 \). The developed version is based on using \( N^1 = N^2 = N^3 = 3 \), but can be modified to consider \( N^1 \) and \( N^2 \) between 1 and 3, and \( N^3 \) between 0 and 3. One advantage of this scheme is that shear deformation is more
accurately represented and shear correction factors are unnecessary. Shear deformation in laminated plates and shells is more significant than for homogeneous plates and shells. Initial testing of the program has been completed. The finite element version of the theory has been used extensively to verify both cross-ply and angle-ply analytical developments.

3) Layer-by-layer 3-D Model (Finite Element Analysis)

The completed high-order program has demonstrated the need to use a quasi-three-dimensional model for modeling coupled piezothermoelastic response. One such program is based on a 27 node hexahedron element for the coupled piezothermoelastic analysis of composite plates. This program development is forming the basis of an h-version layer-by-layer 3D model, i.e.

\[ u_i(x,y,z) = \sum_{j=0}^{N_i^j} \varphi_j^i(z) U_{ik}(x,y) \]  \hspace{1cm} (2)

in which \( N_j^i \) is the number of desired expansion terms in each coordinate displacement \((i=1,2,3)\) direction for layer \( j \); \( U_{ik}(x,y) \) = translational displacement interpolation functions for layer \( j \); and \( \varphi_j^i(z) \) are piecewise continuous interpolation functions through the layer depth. The current version of the program has \( N_j^i = 2 \) for all three values of \( i \). Layer-wise theories provide an intermediate discretization level between a complete 3D representation and an equivalent single-layer 2D model. Depending on the choice of the interlayer interpolation functions, layer-by-layer formulations can mimic first-order shear deformation as well as high-order shear deformations.

The displacement expansions of Eq.(2) were also used to model the electromagnetic potential and temperature fields in the finite element code.

Additionally, a conjugate gradient iterative solution scheme has been implemented into the layer-by-layer program to iteratively solve the resulting simultaneous equations. This scheme results in a substantial reduction in storage requirements and a decrease in computational effort compared to a direct solution algorithm.

Testing of this program is currently being pursued. Furthermore, this version of the program will form the development base for a hierarchical or p-version finite element code.

A.6 Segmentation of Distributed Sensor and Actuator Patches on Plates

It was noted that a fully (symmetrically) distributed piezoelectric sensor/actuator could lead to a minimal, or zero, sensing/control effect for anti-symmetrical modes of structures, especially with symmetrical boundary conditions (Tzou, 1993a). One method to improve the performance was to segment the symmetrically distributed sensor/actuator layers into a number of colocated sub-segments. In this study, Tzou and Fu (1994a,b) investigated the distributed vibration sensing and control of plates using segmented
distributed piezoelectric sensors and actuators. Sensing and control effectiveness of the segmented sensors and actuators were studied. Mathematical models of a plate with a single–piece symmetrically distributed and quarterly segmented–distributed sensors/actuators were formulated and analytical solutions were derived. Modal sensitivities and modal feedback factors for the two sensor/actuator configurations were defined, and the modal displacement and velocity feedbacks were formulated. Analytical solutions proved that the single–piece symmetrically distributed sensor/actuator layers are deficient for anti–symmetrical modes, all even modes, of the plate. The quarterly segmented distributed sensors/actuators can sense/control most of the natural modes, except the quadruple modes. Detailed parametric studies were also conducted and design parameters were evaluated.

A.7 Spatial Filtering and Orthogonal Sensors/Actuators with Temperature Excitations

1) Spatial Filtering of Distributed Sensors/Actuators with Temperature Effects

Temperature effects to piezoelectric transducers were evaluated in this study (Tzou and Bao, 1994c). Mathematical modeling and analysis of a laminated piezothermoelastic cylindrical shell exposed to mechanical, electric, and thermal fields were investigated. Generic shell equations and solution procedures were derived. Contributions of the spatial and time components resulting from the mechanical, electric, and temperature excitations were discussed, and their analytical solutions derived. A laminated cylindrical shell composite laminate with fully distributed piezoelectric layers was used in a case study, and its multi–field step and impulse responses were investigated. Analyses suggested that the fully distributed actuators are insensitive to even modes due to a strain averaging and cancellation. Accordingly, these even modes are filtered out from the total response and only the modes that are combinations of m = 1,3,5,... and n = 1,3,5,... participate in the dynamic response of the shell laminate.

2) Orthogonal Sensors and Actuators for Independent Modal Sensing and Control

i) Spatial Characteristics of Distributed Sensors

Tzou, Zhong (1991a), and Natori (1993) investigated distributed spatial filtering characteristics of distributed piezoelectric sensors. In general, a sensor output signal is contributed by a membrane strain and a bending strain. Depending on the sensor placement, a distributed sensor can be only sensitive to either membrane or bending modes — membrane or bending sensor. In addition, a piezoelectric sensor can be sensitive to a mode or a group of modes due to a signal average on electrode surface, especially anti–symmetrical modes. Accordingly, the sensor layer can be spatially shaped such that it is sensitive to a mode or a group of modes. These spatial filtering characteristics were discussed and examples demonstrated.

ii) Orthogonal Actuators

Distributed structural control of elastic shell continua using spatially distributed orthogonal piezoelectric actuators was proposed and three generic distributed feedback algorithms with spatial feedback functions were formulated (Tzou, Zhong, and Hollkamp, 1994). In order to prevent spillovers from the modal couplings of feedback forces, the distributed actuators can be spatially shaped to be orthogonal to those undesirable modes.
Consequently, the distributed feedback functions evolve to the modal feedback functions which can be represented as a gain factor and a spatially distributed mode shape (actuator) function. In practice, the actuator can be shaped and convolved based on the prescribed mode shape function and the gain factor can be treated as a weighting factor. To demonstrate the utilities of the generic theory, distributed orthogonal convolving modal actuators designed for circular ring shells were proposed and their control effectiveness evaluated.

A.8 Laminated Shells Subjected to Mechanical, Electric, and Temperature Excitations

1) Oscillation and Control of Ring Shells

Tzou and Howard (1994b) studied the piezothermoelastic characteristics and open-loop control of a circular piezoelectric ring shell subjected to mechanical, temperature and electric excitations. Individual responses of these excitations were investigated. Active control of mechanical and temperature excitations were also studied.

2) Multi-field Responses of Cylindrical Shells

Tzou and Bao (1994c) investigated the multi-field responses of a five-layer piezoelectric laminated composite cylindrical shell subjected to mechanical, electric, and thermal step excitations. This study was concerned with a mathematical modeling and analysis of a laminated piezothermoelastic shell composite exposed to mechanical, electric, and thermal fields. Generic shell equations and solution procedures were derived. Contributions of spatial and time components in the mechanical, electric, and temperature excitations were discussed, and their analytical solutions derived. A laminated cylindrical shell composite was used in a case study; its multi-field step responses were investigated. (Note that this subject was also discussed in A.7–1 in which the spatial filtering characteristics were emphasized.)

A.9 Piezothermoelasticity and Static Control: Pyroelectric Effect, Thermal Strain Effect, and Static Deflection Control

Tzou and Ye (1994) studied the piezothermoelasticity of piezoelectric devices in a changing temperature environment. Temperature induced voltage generations of piezoelectric sensors were investigated, and the pyroelectric effect and the thermal strain effect were evaluated. These two temperature induced components of distributed piezoelectric sensors respectively made of polyvinylidene fluoride (PVDF) and lead-zirconate-titanate (PZT) were compared. A piezoelectric laminated square plate laminated with the distributed piezoelectric PVDF and PZT patches was investigated in a case study. Analyses suggested that the pyroelectric effect of PVDF sensors is much more prominent than the thermal strain effect. However, the PZT sensors exhibit the opposite phenomena. Distributed control of the plate with a temperature induced deflection was also studied.
A.10 Dynamics and Active Damping of Adaptive Shells

1) Active Piezoelectric Shells

Distributed actuators offer spatially distributed actuations and they are usually effective to multiple modes of a continuum. Tzou and Zhong (1991b,c; 1993) investigated the spatially filtered distributed vibration controls of a laminated cylindrical shell and a piezoelectric shell, and also evaluated their control effectiveness. There are two control actions, i.e., the in-plane membrane control forces and the counteracting control moments, induced by the distributed actuator in the laminated shell. There is only an in-plane circumferential control force in the piezoelectric shell. Analyses suggested that in either case, the control actions are effective to odd natural modes, and ineffective to even modes. Spatially filtered control effectiveness and active damping of both shells were studied.

2) Cylindrical Shells with Geometry Transformations

Adaptive structures with controllable geometries and shapes are rather useful in many engineering applications, such as adaptive wings, variable focus mirrors, adaptive machines, micro–electromechanical systems, etc. In this study, dynamics and feedback control effectiveness of adaptive shells whose curvatures are actively controlled and continuously changed were evaluated (Tzou and Bao, 1994d). An adaptive piezoelectric laminated cylindrical shell composite with continuous curvature changes was studied, and its natural frequencies and controlled damping ratios were evaluated. The curvature change of the adaptive shell started from an open shallow shell (30°) and ended with a deep cylindrical shell (360°). Dynamic characteristics and control effectiveness (via the proportional velocity feedback) of this series of shells were investigated and compared at every 30° curvature change.

A.11 Static and Dynamic Control of Piezothermoelastic Laminates

Piezothermoelastic effects of distributed piezoelectric sensor/actuator and structural systems were studied, and distributed controls (static and dynamic) of piezoelectric laminates subjected to a steady–state temperature field were investigated (Tzou and Ye, 1993). Temperature gradients, heat conductions, and heat convections were considered in the study. Piezothermoelastic constitutive equations were defined, followed by three energy functionals for the displacement, electric, and temperature fields, respectively. A new 3-D piezothermoelastic thin hexahedron finite element with three internal degrees of freedom was formulated using a variational formulation which includes thermal, electric, and mechanical energies. A system equation for the piezoelectric continuum exposed to combined elastic, electric, and thermal fields was formulated. Distributed sensing and control equations of piezoelectric laminates in a temperature field were derived. Thermal influences on the sensing and control of piezoelectric PZT/steel laminates were investigated in case studies.

A.12 Remarks

This section outlined the research problems investigated over the three year period. Important summaries and significant research findings of these studies are presented next. Bibliography information are provided at the end of this report.
SUMMARY OF RESEARCH RESULTS

Important results and research findings are summarized in this section.

B.1 Piezothermoelastic Composite Shell Lamination Theory

Tzou and Bao (1994a) proposed a generic piezothermoelastic shell lamination theory and also derived piezothermoelastic governing equations for generic anisotropic piezoelectric shell laminae and laminates. Applications of the generic theories were demonstrated.

As discussed previously, thin piezoelectric layers are widely used as distributed sensors and actuators in smart structural systems and thin-film electromechanical devices. Accordingly, there is a need to investigate detailed micro-thermo-electromechanical characteristics of piezoelectric laminae and laminates. There are a number of piezoelectric materials with different elastic and piezoelectric material matrices. The arbitrary anisotropic piezoelectric material used in the generic shell model is the most generic piezoelectric material which has full elastic and piezoelectric matrices. Depending on material symmetries, the elastic and piezoelectric matrices can be simplified to account for various piezoelectric material (Tzou, 1993). On the other hand, the deep thin shell defined in a tri-orthogonal curvilinear system can be easily simplified to apply to a large number of shell and non-shell continua (Tzou, 1993). Combining the material and geometrical generalities provides the versatility of the piezothermoelastic shell lamination theory presented in this study. (Note that elastic properties of electrodes or rigid adhesives can be accommodated; however, viscoelastic properties of bonding materials can not be accommodated in this theory.) System thermo-electromechanical equations and boundary conditions of the generic thin piezothermoelastic shell laminate were derived and their thermo-electromechanical characteristics discussed. Based on the derived system thermo-electromechanical equations and boundary conditions, it was observed that the elastic and electric components are closely coupled with the temperature induced component, and all three components are important to the overall electromechanical and dynamic behavior of the laminated shell. In general, the charge equation is used in sensor applications (Tzou, 1993; Tzou, Zhong, Natori, 1993; Tzou and Bao, 1994b, etc.), in which the signal is contributed by elastic strains (the direct piezoelectric effect) and temperature (the pyroelectric effect and thermal strain effect), while the electric displacement is usually neglected. The electric forces and moments in the dynamic equations can be used to control shell dynamics (Tzou, 1993; Tzou, Zhong, Hollkamp, 1994; Tzou and Bao, 1994d, etc.); these electric control forces and moments, in conjunction with appropriate control algorithms, are essential to counteract both the mechanical and thermal induced oscillations. The generic piezothermoelastic shell lamination theory can serve as a design and analysis tool when new piezothermoelastic laminated composites are designed and fabricated. With appropriate material and geometry properties specified, dynamic performance and control effectiveness of piezothermoelastic laminates can be evaluated. These detailed thermo-electromechanical interactions of specific piezothermoelastic continua made of various piezoelectric materials are to be reported in the near future.

Other piezothermoelastic shell and laminated shell theories evolved from the anisotropic piezothermoelastic composite shell theory include 1) a piezoelectric composite shell theory (Tzou and Bao, 1994b), 2) a piezothermoelastic shell theory (Tzou and Howard, 1994a), 3) piezoelastic thick and thin shell theories (Tzou, 1993), etc. Nonlinear piezothermoelastic shells and plates are being investigated.
B.2 Hybrid Composite Plates

1) Classical Lamination Theory and Mindlin Plate Theory

Jonnalagadda, Blandford and Tauchert (1994) analyzed a number of hybrid (nine layer composite with one PVDF layer across the top of the plate) composite plates using both analytic and finite element solutions. Analytic solutions were presented for both classical lamination theory (CLT) and first-order shear deformation theory (FSDT) for a simply supported plate; whereas the finite element formulation was based on FSDT and used to investigate the response of moderately thick plates for both simple and fixed edge conditions. Considering a square plate subjected to mechanical, thermal, and electrical loadings, it was demonstrated that the difference between classical lamination theory and first-order shear deformation theory increases with increasing plate thickness. Results show that the inclusion of shear deformation was important for the composite plate considered for $4 \leq \text{length/thickness (b/h)} \leq 15$. For $b/h > 15$, shear deformation becomes increasingly less important.

Finite element results for the simply supported plate problems matched the analytic solutions using a 4x4 mesh of nine node Lagrangian elements on symmetric quarter of the plate. Both selective (2x2 Gaussian quadrature rule for shear and a 3x3 Gaussian quadrature rule for bending) and full integration (3x3 used for all stiffness calculations) schemes were considered in the finite element simulations. The finite element results showed that the mechanically loaded composite plate was most sensitive to thickness changes from $b/h = 4$ to $b/h = 10$. The thermally loaded composite was more sensitive to plate thickness ratio as compared to the electrically loaded composite plate. This pattern of behavior persists with changing aspect ratio ($b/a = 1, 2, 3$) with the plate displacements increasing with increasing aspect ratio. Changing the plate aspect ratio effectively shifts the location of maximum displacement from the center of the plate to off-center locations. Again, as the plate thickness ratio decreases from $b/h = 10$ to $b/h = 40$, the influence of plate thickness becomes minimal. However, increasing the b/h ratio has a pronounced effect on the numerical calculations of the finite element stiffness coefficients, i.e., for $b/h > 40$, a selective reduced integration scheme should be used to avoid the 'shear locking' phenomenon.

2) High-order Displacement Theories

Jonnalagadda, Tauchert and Blandford (1993a) considered various high-order displacement theories for the thermoelastic analysis of non-hybrid composite plates. Analytic result comparisons were provided for the high-order displacement theories recommended by Reddy (1984) and Chang and Leu (1991), quadratic and cubic variations through the thickness, as well as Reissner-Mindlin and classical lamination theories. The reported comparisons, based upon the response of a symmetric eight-layer graphite/epoxy square laminate to a depth-linear temperature field, demonstrate the importance of accounting for the transverse normal strain behavior for moderately thick plates. Reddy neglected the transverse normal strain effect and Chang's theory incorrectly models the effect, except when the composite plate is subjected to an antisymmetric thermal load. Furthermore, Jonnalagadda, Tauchert and Blandford (1993a) showed that a quadratic deflection variation in the thickness direction yields results equal to those obtained using a cubic variation when the in-plane displacements are assumed to vary cubically with the thickness coordinate in both formulations. However, studies on a hybrid composite plate
consisting of laminae made of different materials, revealed that the cubic theory predicts results which show a substantial deviation from the quadratic theory. Moreover, composite plates subjected to a through–thickness temperature field of order higher than linear may require the use of a cubic through–thickness displacement representation.

The variation in plate deflections through the thickness investigated by Jonnalagadda, Tauchert and Blandford (1993b) points out an important consideration regarding suppression of thermally induces deflection via an imposed electric field. Considering a laminate with a length–to–thickness ratio of 10, numerical results showed that with the given electric field load, the deflections due to thermal load are essentially eliminated at the middle and lower surfaces, but the deflection at the upper surface are reduced by only about 50%. An increase in electric field (if permissible) to reduce the top surface deflection would result in a relatively large positive deflection of the middle and lower surfaces. This behavior is overlooked in classical lamination theory and some high–order theories (e.g., Reddy, 1984) in which the deflections are depth–invariant. Therefore, based on the problem to which the present theory is applied, a criterion needs to be developed to maximize the effectiveness of the electric field loading.

Furthermore, the Jonnalagadda, Tauchert and Blandford (1993b) high–order plate theory results conflict with the results obtained in their (1993a) paper. The (1993a) paper found that the high–order solution converges to the classical solution for thin, single–material composite plates, which is not the case for a hybrid laminate. Numerical experiments with b/h = 400 indicated that for thermally induced deflections, the solution inconsistency between the classical and present theories for hybrid laminates vanishes only when the coefficients of thermal expansion for both the composite and piezoelectric layers have the same values, with the other elastic properties retaining their reported values. When the z–axis coefficient of thermal expansion for the piezoelectric and structural layers is taken to be the same, the present theory predicts a mid–plane deflection which is 6% less than that predicted by classical lamination theory. This compares with a 13% difference when the published properties were used. Based on these discrepancies, further results based on analytical or numerical solutions of the three–dimensional elasticity equations are required.

Xu, Tauchert and Blandford (1993) considered the response of a square, antisymmetric (15°/−15°), graphite/epoxy (T300/5208) laminate subject to mechanical, thermal, and electric field loading. A double thick layer of polyvinylidene fluoride (PVDF), a piezoelectric polymer, was added to both the lower and upper surfaces to make a ten–layer hybrid composite structure. The plate dimensions are a = b with various plate thicknesses considered.

It was determined that the influence of transverse normal strains was important for a moderately thick laminate subjected to thermal or electric loads. For example, at a thickness ratio (a/h) of 10, the surface deflection (maximum) under thermal loading was about 11% greater than the mid–plane deflection. Similarly, when an electric field load was imposed on the same plate, the surface deflection exceeds the mid–plane value by about 12%. However, the high–order theory was seen to have no effect on the deflections induced by mechanical loading. This is consistent with the expected behavior since the coefficient of thermal expansion in the thickness direction α3 and the piezoelectric constant d33, the main terms causing normal self–strains, do not affect mechanically loaded plates. It was also shown that the influence of transverse normal strain decreases with increasing a/h ratio, as should be expected.
Furthermore, this study revealed that due to the temperature sensitivity of the piezoelectric polymer polyvinylidene fluoride (PVDF), alternate piezoelectric materials should be considered for suppression of thermal induced deflections.

### B.3 Thermal Buckling — High–order Theories

Tauchert, Jonnalagadda and Blandford (1993) investigated the thermal buckling characteristics of a square fiber–reinforced laminate using various plate theories. The critical buckling temperatures of simply supported plates under uniform temperature loading were determined using a general third–order displacement theory and compared to some of the published plate theory solutions. Numerical results indicate that, regardless of the mode of buckling, transverse normal strains have little effect on the critical temperatures. Thus, Reddy’s (1984) theory is adequate for the present class of problems. Chang’s (1991) theory, which uses the trivial solution to satisfy the zero top and bottom plate shear stress condition, has been found to underpredict the buckling temperatures. Although the present high–order theory was not required for the buckling analysis of uniform temperature loaded plates, this conclusion may not extend to plate response under non–uniform thermal load.

### B.4 High–order Theory — Free Vibration

Jonnalagadda, Tauchert and Blandford (1994) considered the natural frequencies of simply supported, isotropic and specially orthotropic laminates using a general third–order displacement theory. They compared their solutions to solutions obtained using classical lamination theory and Reddy’s (Reddy and Phan, 1985) high–order theory. Numerical results demonstrated that the classical theory overpredicts the frequencies. Frequencies predicted by Reddy’s theory, which ignores transverse normal strain, were in close agreement with corresponding values obtained from the present formulation. This indicates that transverse normal strain has little effect on those modes, and that Reddy’s theory is adequate for their prediction. However, the present formulation has the advantage that it describes plate–thickening modes not accounted for in other theories.

### B.5 Segmentation of Distributed Sensors and Actuators

It was noted that a fully (symmetrically) distributed piezoelectric sensor/actuator could lead to a minimal, or zero, sensing/control effect for anti–symmetrical modes of structures, especially with symmetrical boundary conditions (Tzou, 1993). One method to improve the sensing/actuation performance is to segment the symmetrically distributed sensor/actuator layers into a number of colocated sub–segments. In this study, Tzou and Fu (1993a,b) studied the mathematical models and analytical solutions of a simply supported plate with a single–piece distributed sensor/actuator and multiple segmented sensors/actuators. *Modal sensitivities* and *modal feedback factors* for the two sensor/actuator configurations were defined; and modal displacement and velocity feedbacks were formulated. Derivations and solutions suggested the following conclusions.
1) A single-piece symmetrical distributed sensor layer has sensing deficiencies (i.e., an observation deficiencies) for all even modes because the locally generated positive and negative charges could be canceled out on the effective sensor surface.

2) A single-piece symmetrical distributed actuator layer is also ineffective for controlling even modes (i.e., a controllability deficiency) due to similar reasons stated above.

3) Quarterly segmented sensors and actuators can sense and control most of the natural modes, except the quadruple modes, of the plate. The sensing and control effects for all odd modes are identical to the single-piece sensor/actuator configuration.

4) Segmenting distributed sensor and actuator layers into a number of sub-segments does improve the observability and controllability of the plate system.

The segmented sensor/actuator design improves the observability/controllability for even modes without degrading the sensing/control merits for all odd modes of the simply supported plate. In general, the lower modes are more important than the higher modes in structural monitoring and control. Thus, only several lower modes were considered in this study. Note that finer segmentation of sensors/actuators are possible and might provide better structural observability/controllability.

Design parameters to the sensing and control effectiveness were also evaluated. Experimental study of distributed segmented sensor successfully revealed the mode shapes of a cantilever beam.

B.6 Spatial Filtering and Orthogonal Sensors/Actuators

1) Spatial Filtering of Distributed Sensors/Actuators with Temperature Excitations

External temperature fluctuation can significantly change the piezoelectric and control characteristics of piezoelectric structures. Tzou and Bao (1994c) investigated the multi-field step and impulse responses of a laminated piezothermoelastic cylindrical shell composite and studied the spatially filtered phenomena.

Generic solution procedures for piezothermoelastic shells subjected to mechanical, electric, and temperature excitations were presented. Detailed definitions of their spatial and time components were discussed. The spatial function was assumed uniformly distributed and the time functions were a step function and an impulse function, respectively. Analytical derivations suggested that the uniformly distributed excitation can only excite odd modes, and are ineffective to all even modes. Accordingly, these even modes are filtered out from the total response. Since the system is linear, the total response is a summation of all individual modal component responses.

Multi-field step and impulse responses of the laminated cylindrical shell laminate showed that the displacements induced by mechanical and temperature excitations are in phase, and that induced by the electric excitation is out-of-phase. Accordingly, the electric excitation can be used to compensate the mechanical and temperature induced excitations and active control can be successfully achieved.
2) Orthogonal Sensors and Actuators for Independent Modal Sensing and Control

i) Spatial Characteristics of Distributed Sensors

Tzou and Zhong (1993) studied the distributed spatial filtering characteristics of distributed piezoelectric sensors. In general, these filtering characteristics can be divided into three categories depending on: 1) sensor placement, 2) signal average, and 3) sensor shaping. Depending on the placement of the distributed sensor and induced strains, there are bending sensors and membrane sensors. Due to the signal averaging on the surface electrode, sensor signals from different strain regions can be averaged and canceled out and result in a zero or minimal output, e.g., anti-symmetrical modes of a symmetrical structure. To overcome this problem, a distributed sensor can be spatially shaped or segmented to be sensitive to a mode or a group of modes.

As to demonstrate the classifications, distributed sensors laminated on a cylindrical shell were proposed and their spatial distributed filtering characteristics investigated. It was noted that the fully distributed sensor is only sensitive to odd modes and insensitive to even modes, due to the signal cancellation of modal anti-symmetry. A distributed line sensor was studied and it was observed that the line sensor is only sensitive to all m = n modes. The transverse sensitivities of these two sensors were also studied. Analysis results suggested that the sensitivity increases when the sensor and the shell become thicker, due to an increase of bending strains in the sensor layers.

Tzou, Zhong, and Natori (1993) also proposed distributed orthogonal sensors for ring shells and evaluated their modal sensitivities. Membrane sensitivity is a function of membrane strains, which is independent of ring thickness. Bending sensitivity, however, is a function of bending strains, which is a function of ring thickness.

ii) Orthogonal Actuators

Tzou, Zhong, and Hollkamp (1994) investigated the generic distributed orthogonal piezoelectric shell actuators, and formulated generic distributed control algorithms (displacement, velocity, and acceleration) using the spatially distributed feedback functions. To prevent spillover from other residual modes, the feedback functions evolved to the modal feedback functions which were further expressed in a spatial mode shape function (the distributed actuator shape) and a weighting factor (gain constant). Their corresponding modal forces and equations were derived accordingly.

To demonstrate the utilities of the proposed distributed orthogonal shell actuators, distributed cosine shaped modal orthogonal ring actuators were designed and their control performances were analyzed. In practice, the actuator is shaped and convolved based on the prescribed mode shape (strain) function and the gain factor can be manipulated in the feedback system. Control effectiveness of distributed modal actuators was evaluated. Analyses suggested that the primary control action comes from the in-plane membrane control forces, and the contribution from the electric bending moment is relatively insignificant for lower natural modes n < 10. In addition, structural flexibility was noted to be of importance in feedback controls. It was observed that the control effect increases as the structural stiffness decreases. Increasing control moment arm (as the ring becomes thicker) of the actuator layer seems insignificant in overall control effects for lower natural modes.
Theoretical and parametric studies of distributed orthogonal actuators, carried out in this study, provided a better understanding of functionality and designs of the distributed piezoelectric shell actuators. Designing the actuator shapes based on the orthogonal functions leads to the spatial modal filtering and consequently control spillovers can be prevented in a distributed structural control system.

B.7 Laminated Shells Subjected to Mechanical, Electric, and Temperature Excitations

1) Oscillation and Control of Ring Shells

Tzou and Howard (1994b) studied the piezothermoelastic characteristics and an open–loop control of a circular piezoelectric ring shell. Analysis results suggested that two control voltages are required to fully compensate the mechanical and thermal induced oscillations. Pressure and temperature induced oscillations and their respective controls were studied. However, the maximal control effect is limited by the breakdown voltage of the piezoelectric materials. When the breakdown voltage of the PVD actuator is considered, the temperature induced oscillation can be completely compensated and the pressure induced oscillation can be reduced by 64% of its original amplitude.

2) Multi-field Responses of Cylindrical Shells

Tzou and Bao (1994c) studied the multi-field responses of a laminated piezothermoelastic cylindrical shell composite. Analytical solutions of a piezothermoelastic laminated cylindrical shell subjected to the mechanical, electric, and temperature excitations were derived. Spatial and time components were analyzed using the modal expansion method which assumes the total response is composed of a temporal component (a modal participation factor) and a spatial component (the mode shape function).

In the forced vibration analysis, the spatial function was assumed uniformly distributed and the time function was a step function in the modal force integral expression. Analytical solutions suggested that the uniformly distributed excitation can only excite odd natural modes, and are ineffective to all even modes of the simply supported cylindrical shell. Multi-field step responses of the laminated cylindrical shell laminate showed that the displacements induced by mechanical and temperature excitations are in phase, and that induced by the electric excitation is out-of-phase. Accordingly, the electric excitation can be used to compensate the mechanical and temperature induced excitations and active control can be successfully achieved. (Note that detailed discussions on the spatial filtering characteristics were presented in B.6–1.)

B.8 Piezothermoelasticity and Static Control: Pyroelectric Effect, Thermal Strain Effect, and Static Deflection Control

Tzou and Ye (1994) studied the temperature effect of piezoelectric sensors and actuators using the finite element technique. Particularly, temperature related pyroelectric effect and thermal strain effect in distributed piezoelectric layers were investigated. Sensing and actuation effectiveness of distributed polymeric PVDF and piezoceramic PZT devices were quantitatively compared.
Finite element formulations were presented and applications to distributed sensing and control were introduced. A simply supported piezoelectric laminated plate was modeled and analyzed in a case study. Analyses suggested that the temperature can induce output voltages (the piezothermoelastic effect) via two temperature effects: 1) the pyroelectric effect and 2) the thermal strain effect. The pyroelectric effect is a direct temperature effect. The thermal strain effect is an indirect temperature effect, because the voltage generation is directly caused by the thermal strains introduced by the temperature variation. It was observed that the pyroelectric effect is significant in PVDF layers and insignificant in PZT layers. However, the thermal strain effect is insignificant in PVDF layers and significant in PZT layers. (Note that PVDF layers are used in thermal imaging devices and systems because of their significant pyroelectric effect.) It was also quantitatively proved that the PZT actuators are much more effective than the PVDF actuators in static deflection and position controls.

B.9 Dynamics and Active Damping of Adaptive Shells

1) Active Piezoelectric Shells

Tzou and Zhong (1993, 1994) investigated the spatially distributed active vibration controls of a laminated cylindrical shell composite and a piezoelectric cylindrical shell, and also evaluated their spatial filtering characteristics. It was observed that there are in–plane membrane control forces and out-of–plane counteracting control moments in the laminated shell case, and there is only an in–plane circumferential control force in the piezoelectric shell case. Accordingly, the laminated shell is controlled by both membrane control forces and control moments, and the piezoelectric shell is only influenced by the circumferential control force. In both cases, the control action was only effective to odd natural modes and ineffective to even natural modes, due to symmetries of natural modes and also control signal cancellations. Note that only the velocity feedback was considered and both cylindrical shells were simply supported.

Distributed filtering characteristics of the fully distributed actuator on the laminated elastic cylindrical shell were studied. Control actions induced by the fully distributed actuator were primarily for odd natural modes, and ineffective for even modes. Analyses suggested that the in–plane control forces are essential for controlling low natural modes, and this control effect decreases as the mode number increases. The control moment effects basically remain at the same level for all modes calculated. Controlled damping ratio linearly increases with the increase of feedback gains; it decreases as the mode number increases. Note that the overall convergence of a modal time history is determined by the combined effect of the modal natural frequency and the damping ratio. It was also found that the membrane control action remains identical, and the bending control action increases linearly when the elastic shell becomes thicker. However, the overall damping control decreases since the flexural rigidity is a cubic function of the shell thickness and the control moment is only a linear function.

Damping control of the piezoelectric cylindrical shell was also investigated. A distributed electric in–plane circumferential force, in the θ–direction, was kept in the piezoelastic equation and it was used as a control force in distributed shell controls. Simulation and experimental results suggested, in general, damping ratio was enhanced
when the feedback voltage increased (Tzou and Zhong, 1993, 1994). Analytical solutions also showed that this control action is only effective to odd natural modes and ineffective to even modes. Since this control force is primarily in the $\theta$-direction, control effects to circumferential oscillation modes were significant. It should be noted that in practical applications, the maximal control voltage is limited by a breakdown voltage of piezoelectric materials. In addition, a high feedback voltage could also heat up the piezoelectric actuator or shell.

2) Cylindrical Shells with Geometry Transformations

Adaptive structures with controllable geometries and shapes offer many advantages over conventional fixed-geometry structures. Tzou and Bao (1994d) investigated dynamics and control effectiveness of an adaptive cylindrical shell laminated composite which transforms from an open shallow shell (30°) to a deep cylindrical shell (360°). A mathematical model for the piezoelectric laminated cylindrical shell composite was formulated and natural frequencies and mode shapes were analyzed. These generic equations and solutions include a curvature angle which can be easily changed to accommodate the curvature transformations. Three generic feedback controls were proposed and only the velocity feedback was used in a case study. Control force level is determined by actuator material properties and also control voltages. Numerical analyses of the transforming cylindrical shell suggested that

1. Controlled damping ratio of the cylindrical shell decreases as the shell curvature increases for lower natural modes. However, controlled damping ratio keeps increasing for higher natural modes.

2. Natural frequencies of lower natural modes increase and those of higher modes decrease in the process of curvature transformation from 30° to 360°. When the shell curvature increases, dynamic coupling between the circumferential and transverse modes becomes significant. Accordingly, the lowest mode is usually not the first mode for high-curvature shells.

Note that the shell dynamics and control were evaluated at (deformed) static equilibrium positions after the curvature transformation such that external actuation forces imposing the curvature change are not considered. The external actuation force retained in the transformed shell can significantly affect the stability of the transformed shell if considered. In addition, dynamics and control were evaluated in the linear range (small oscillation); large deformation and geometrical nonlinearity are not considered. These stability and nonlinear effects will be considered in future studies.

B.10 Static and Dynamic Control of Piezothermoelastic Laminates

Tzou and Ye (1993) studied the piezothermoelasticity of distributed piezoelectric sensor/actuator and control of piezoelectric laminates subjected to external thermal excitations. Temperature gradients, heat conductions, and heat dissipations were considered in this work. Linear piezothermoelastic constitutive relations were defined; three energy functionals, i.e., displacement, electric, and temperature, were formulated. A set of piezothermoelastic equations and boundary conditions of a piezothermoelastic continuum were derived, which can be simplified to the standard expressions published previously. A new 3-D piezothermoelastic thin hexahedron finite element was formulated using the energy functionals and the variational principle. Piezothermoelastic response and
control of laminated piezoelectric structures were then derived. The governing equation and the sensing/control equations showed the couplings and interactions of elastic, electric, and thermal fields.

To evaluate the piezothermoelastic behavior and control of piezoelectric laminates, a PZT laminated composite steel beam was used in case studies. Static deflections were first calibrated with experimental results. Thermally induced voltage generations (piezothermoelastic characteristics) of the distributed PZT layers were investigated. It was observed that the voltage is contributed by two effects: 1) the pyroelectric effect and 2) the thermal strain effect. The thermal strain effect is much more significant than the pyroelectric effect on the PZT layers. Thermally induced static deflections due to thermal gradients between the top and bottom surfaces were also analyzed. Static and dynamic controls of the cantilever beam were investigated. Analyses suggested that two voltages are required to fully control the oscillation in the thermal gradient condition, i.e., forcing the beam return to its original static equilibrium position with zero deflection. One voltage is used to compensate the thermally induced static deflection and the other voltage is used to control the dynamic oscillation.

Numerical results suggested that the temperature variation considerably influences the electric potential distribution on both piezoelectric sensor and actuator layers. In the earlier studies (Tzou and Tseng, 1991; Tzou, 1993a), it was assumed that the initial voltage on the actuator layer is much smaller than the feedback voltage and the initial voltage can be neglected in feedback controls. However, this analysis suggested that the thermal induced voltages are significant in both sensor and actuator layers. These voltages should be accounted for when considering sensing and control of piezoelectric laminated continua subjected to significant thermal excitations. This detailed piezothermoelastic sensing/control coupling needs to be further investigated.

The developed 3-D thin hexahedron piezothermoelastic element was generic, which can be used for various piezoelectric materials. The element was rather efficient, as compared with the conventional isoparametric hexahedron elements, in modeling thin planar piezoelectric laminates. For modeling curved piezoelectric laminates, e.g., shells, this element is usually inferior to higher-order elements or shell elements which need to be developed in the future. Note that all material properties were assumed constant in this study. These material constants may change when the temperature variation becomes significant. This temperature-dependent nonlinear control problem needs to be further studied. In addition, studies on the vibration control in a time varying thermal field are underway; their results will be presented subsequently.

B.11 Remarks

A detailed listing of technical publications related to the above mentioned studies is provided next. Other bibliographical information are listed at the end of this report.
RESEARCH PUBLICATIONS

Books


Monographs


Chapters in Books


**Refereed Journals**


Conference Papers


GRADUATE STUDENTS INVOLVED IN RESEARCH

1. Zhong, Jianping, Ph.D.: Dissertation title: "A Study on Piezoelectric Shell Dynamics Applied to Distributed Structural Identification and Controls," (6/1987 – 5/1991). A recipient of the President's Dissertation Year Fellowship at the University of Kentucky. Dr. Zhong was invited to give a talk at the Space Engineering Research Center in the Department of Aeronautics and Astronautics at Massachusetts Institute of Technology (MIT), Cambridge, MA on April 4, 1991. He is currently working for Allis–Chalmers Compressor, Appleton, WI.
5. Chian, C., M.S./Ph.D.: "Fabrication and Testing of Adaptive Composites," Department of Material Science (Co–advisor with Prof. L.S. Penn, Department of Chemical Engineering and Material Science)
6. A Ph.D. graduate student, Weimen Xiao, has been recruited to work on the research. His dissertation is tentatively titled "Geometric Non–linear Finite Element Analysis of Multi–Layer Piezothermoelastic Composite Plates". He is currently being supported by the University of Kentucky through the Center for Computational Sciences.

7. A graduate student, Krishna D. Jonnalagadda, received his M.S. degree in Engineering Mechanics in August 1993. The title of his thesis is "Analytic Solutions for Smart Composite Plate Structures". He is continuing his studies on smart materials as a Ph.D. student at the University of Illinois.

8. Three M.S. graduate students, Saikat Bhattacharjee, Xing Fu and Xiaoyao Shao have chosen to work on the research. Their thesis investigations entail further developments of analytical and numerical analyses of piezothermoelastic laminates, with concentration on discrete (patch) as well as continuous (full layer) piezoelectric elements. All three students are being supported by the Department of Engineering Mechanics.

9. Yi–Ping Xu, a Ph.D. student, left the University to pursue a career in practice.

REPORT OF INVENTIONS (None)

BIBLIOGRAPHY


(Aro–FnlRpt–10/94.Army8)