THE INITIAL VELOCITIES OF FRAGMENTS FROM
BOMBS, SHELL, GRENADES

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The Ballistic Research Laboratory has determined that the subject report, titled, "The Initial Velocities of Fragments from Bombs, Shell, Grenades" is now distribution Statement A.

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Abstract

To assess the efficiency of a projectile, it is often required to predict the initial velocities of the fragments from a knowledge of the dimensions of the metal casing and the character and quantity of explosive. Between a grenade containing 1/2 ounces of R.E. and a bomb containing 3000 pounds of R.E. the difference in scale is so great, that it is a question whether any simple scheme will apply over the whole range. A theory is put forward, making the following assumption, that the contribution to the total kinetic energy made by the detonation of unit mass of explosive is independent of the size of the projectile. In a large bomb the explosion gases have actually more kinetic energy than the fragments. A simple expression is found for the average initial velocity in terms of the charge-weight ratio; C/l; this expression is found to agree with the experimental data fairly well over the whole range from C/l = 0.06 to C/l = 5.6.

1. To compare the efficiency of fragmentation of different types of projectiles, one needs to know the velocities of the fragments at suitable distances from the explosion. For this purpose one needs to know both the retardation in air and the initial velocity v of the fragments. We are concerned here with the question, whether the value of v for any projectile can be predicted from the dimensions of the metal casing and from the character and quantity of the explosive contained in it.

We have data on projectiles containing from one pound to 3000 pounds of R.E. Preliminary tests have also been made on grenades containing only 1 1/2 or 2 ounces of R.E. The range from 1 1/2 ounces to 3000 pounds being so great, it is
a question whether any simple scheme will be found to apply to the whole set of results.

It is well known that, though a cylindrical shell is detonated from one end, both the nose-spray and the tail-spray are feeble compared with the side-spray; it is in the radial velocities of fragments from the cylindrical walls that we are interested.

2. Let $C$ be the mass of explosive and $M$ the mass of the metal casing containing it. (When treating a cylindrical projectile we consider mass per unit length). We discuss a metal casing with walls of uniform thickness, and assume first the following simple picture of fragmentation:

Before the metal casing breaks into fragments, it expands to some extent. Let the radius at the moment of fragmentation be $a$, and let the density of the explosion gases at this moment be $\rho$. The metal casing is everywhere moving outward with radial velocity $v_0$, which at this moment becomes the velocity of the metallic fragments, (the same for all). Now $v_0$ is also the radial velocity of that part of the explosion gases which are in contact with the metal casing. On the axis of the cylinder the radial velocity of the gases is zero. Elsewhere the gases are moving outwards with velocity intermediate between zero and $v_0$; we shall take the velocity at any point to be proportional to the distance from the axis of the cylinder (or from the center of a spherical grenade)—that is,

$$v = \frac{\rho v_0}{a}$$

Consider now different types of projectiles, shell, bomb and grenades all containing the same explosive, say TNT. We try the assumption that the contribution to the kinetic energy made by the detonation of each unit mass of this explosive is the same in all types of projectile. Let this contribution per unit mass of explosive be $E$. Equating the total kinetic energy to $EC$ we have for a cylinder per unit length

$$EC = \frac{M}{2} \sum m_i v_0^2 + \frac{1}{2} v_0^2 \int_0^a 2\pi r \rho^2 \frac{r^2}{a^2} dr$$
and for a spherical casing

\[ EC = \frac{1}{2} \frac{\pi m}{1} v_o^2 + \frac{1}{2} v_o^2 \int_0^a 4\pi r^2 \rho_r^2 dr \]  

Both these expressions reduce to the simple form

\[ v_o = \sqrt{2EC} \]

where \( R \) is the function:

For a cylinder

\[ R = \frac{C}{M + \frac{C}{2}} = \frac{C/M}{1 + \frac{C}{M}} \]

and for a sphere

\[ R = \frac{C}{M + \frac{3C}{5}} = \frac{C/M}{1 + \frac{3C}{5M}} \]

For small values of \( C/M \) we see that \( R \) is approximately equal to \( C/M \); hence for small values of the charge-weight ratio, the value of \( v_o \) varies as \( \sqrt{C/M} \). On the other hand, for very large values of \( C/M \), such as are found in large bombs, we see that \( R \) tends asymptotically to the value 2 for a cylinder, and to the value 5/3 for a sphere.

The quantity \( E \) has the dimensions of energy per unit mass; therefore \( \sqrt{E} \) has the dimensions of a velocity; in fact, \( v_o \) is equal to \( \sqrt{E} \) when \( R = 1/2 \). We conclude then, that for large values of \( C/M \) the value of \( v_o \) tends asymptotically to the value \( 2\sqrt{E} \), or to the value \( \sqrt{10E/3} \) for a sphere.

*The conventional charge-weight ratio \( C/M \) takes into account the whole mass of metal in the projectile, while our \( C/M \) takes into account only the metal in the walls of the casing. If \( p \) is the ratio of the external to the internal diameter, and \( \rho_c \) and \( \rho_m \) are the densities of the explosive and metal, respectively, we have for cylindrical walls

\[ C = \frac{\rho_c}{\rho_m(\frac{1}{p} - 1)} \]
For simplicity $v_0$ was assumed to have the same value for all the fragments. Even for cylinders with walls of uniform thickness there is always some spread in the initial velocities, at any rate when the cylinder is short. We may therefore take $v_0$ to be a mean of the initial velocities of all the fragments which contribute to the total kinetic energy; i.e. the smallest fragments may be excluded since they make a negligible contribution to the total kinetic energy.

3. Measuring the initial velocities of the leading fragments from large bombs, Schwarzschild and Sachs** found that $v_0$ appeared to increase very slowly with $C/M$, and proposed the relation

$$v_0 = a C^{0.22}$$

(6)

which is inconsistent with the observed fact that for small projectiles $v_0$ varies as rapidly as $\sqrt{C/M}$. We are able to remove the discrepancy, since for large value of $C/M$ the velocity given by (3) varies as slowly as that given by (6).

4. The expression (3) may be written in the form

$$v_0 = v_1 R^{1/2}$$

(7)

where the parameter $v_1$ depends on the particular explosive used. It is difficult to know to what extent we ought to expect the velocities of fragments in the side-spray of a shell or bomb to agree with (4). But figs. 1 and 2 show that for projectiles containing TNT, using the value $v_1 = 8000$ feet/sec, the formula fits the experimental data fairly well over the whole range from $C/M = 0.06$ to $C/M = 5.62$.

It seems then that the basic assumption may be correct that for a series of projectiles containing different quantities of the same explosive, the contribution made to the total kinetic energy by the detonation of each unit mass of explosive is the same.

The reason why for large values of $C/M$ the initial velocity fails to increase as rapidly as $\sqrt{C/M}$ is clear. In a shell with a relatively thick and heavy casing, nearly the whole of the kinetic energy is possessed by the metal casing, as it breaks up into fragments. But for projectiles with $C/M$ greater than 2 there is actually more energy in the

**BRL Report No. 347.
The kinetic energy of radial motion of the explosion gases inside the bomb is less than in that of the metal casing which contains them; this severely limits the value of \( v_0 \) for the fragments.

In deriving (4) and (5) it was assumed that \( p \) was constant, and that inside the projectile the radial velocity \( v \) of the explosion gases was proportional to \( r \). It may be that this assumption overestimates the amount of kinetic energy of radial motion retained by the explosion gases. If this is so, the numerical factor \( 1/2 \), which occurs in the denominator of (4) should be replaced by a somewhat smaller value, such as 0.45. At the same time the value of \( v_1 \) in (7) would have to be reduced. Experimental data on initial velocities are at present too scanty to decide this point; but with the present data no significant improvement is obtained by replacing \( 1/2 \) by a different factor.

The expressions (1) and (2) are intended to express the fact that under optimum conditions of detonation a certain fraction of the chemical energy of explosion is converted into kinetic energy, other details being important. The integral is to be taken to a radius \( a \); and it was stated that this was the radius of the casing at the moment of rupture, (suggesting that this might depend on the strength of the metal forming the casing). This remark, however, was introduced only for the sake of simplicity; the kinetic energy of the metal should depend only on its mass. In the fragmentation of simulated shell at Bruceton, described below, steel casings of varying degrees of hardness were tried, ranging from Brinell 105 to 500. No significant effect of hardness on the initial velocities was found.

5. For each explosive the initial velocities will be determined by the characteristic value of \( E \). We have seen that for TNT the value of the constant \( v_1 \) is in the neighborhood of 8000 feet/sec. We have then

\[
\sqrt{2E} = v_1 = 8000 \text{ feet/sec} = 2.44 \times 10^5 \text{ cm/sec.}
\]  

Whence

\[
E = 3 \times 10^{10} \text{ ergs per gram} = 715 \text{ cal per gram}
\]  

The Report OSRD 1510 gives calculated values for a quantity \( W \), which is not directly comparable with \( E \); this \( W \) is the "reversible work per gram of products of adiabatic expansion from the adiabatic constant volume explosion state..."
to a pressure of one bar. For T.N.T. of density 1.59 the value given is
\[
\text{energy} = 3.72 \times 10^{10} \text{ ergs per gram} = 890 \text{ cal per gram.} \quad (10)
\]

6. The available data for T.N.T.-filled projectiles are as follows:

a. In the experiments by Division 8 of the N.D.R.C.* steel cylinders filled with T.N.T. or other explosive were used. The cylinders had an internal diameter of 2" and a uniform thickness of wall. The velocities of fragments were measured at a distance of about nine feet by means of the velocity camera. The values for different thicknesses of steel casing filled with T.N.T. are given in Table I, attention being paid only to the large and medium fragments for which the retardation in air will be negligible. As the number of fragments recorded was small, the probable error is large.

<table>
<thead>
<tr>
<th>Wall Thickness</th>
<th>1/2&quot;</th>
<th>3/8&quot;</th>
<th>5/16&quot;</th>
<th>3/16&quot;</th>
<th>1/8&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>C/H</td>
<td>0.165</td>
<td>0.231</td>
<td>0.286</td>
<td>0.500</td>
<td>0.775</td>
</tr>
<tr>
<td>(v_0)</td>
<td>2600</td>
<td>3240</td>
<td>3800</td>
<td>5110</td>
<td>6103 ft/sec</td>
</tr>
<tr>
<td></td>
<td>2870†</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

† This higher value was obtained when the experiment was repeated a month later (Interim Report May - June)

b. The 4000-lb bomb AN-M56, filled with T.N.T. The diameter of the central cylindrical part of this bomb was 34.25 inch, with a thickness of steel casing 0.51 inch. These values give C/H equal to 5.62. The velocities of some of the leading fragments only were measured; these had a mean value 10300 feet/sec. The mean velocity of all the large fragments must be somewhat less than this. We may take 9800 feet/sec as a value more suitable for comparison with the velocities obtained from other projectiles.

Further data for bombs, including T.N.T.-filled, will soon be available at the Ballistic Research Laboratory.

*Interim Reports of Division 8. April - June 1943.
c. For the 105-mm Howitzer shell MI and for the 75-mm shell M48 the velocities in the side-spray have been estimated from the change in angle of projection with change in residual velocity of the shell. The charge of TNT in these two shell had the values 4.93 pounds and 1.56 pounds, respectively; the total weights of the unfused shell are 30.625 pounds and 12.50 pounds, respectively. The thickness of the cylindrical walls, as in most modern shell, is variable. Before we could predict the resultant distribution of fragment velocities, we should have to answer the question, to what extent the wall acts as a whole during rupture. Instead of a complex theory, however, what is needed here is a formula by which the average fragment velocity can be rapidly estimated, when the total weight of the unfused shell, and the charge are given. If the ratio of these two quantities is taken as C/M (instead of the correct quantities) and $v_0$ is calculated from (4) and (7), setting $v_1 = 8000$, as before, one obtains the points plotted for the 105-mm and 75-mm shell in Fig. 2. It will be seen that these points lie on the straight line as well as, or better than, the neighboring points for the N.D.R.C. shell of constant wall thickness.

The reason for this may be as follows. There is a theoretical objection to drawing the line through the origin, since this implies that an exceedingly small charge will be sufficient to rupture a heavy casing, and give the fragments an initial velocity. An expression of the form

$$v_0 = v_1 (R^{1/2} - \text{constant})$$

is more acceptable and seems to fit the facts better. For the practical purpose of estimating the $v$ for shell similar to the 75-mm and 105-mm, it seems, however, unnecessary to use an expression containing an additional new constant.

d. A British report* records measurements of fragment velocities for a 40-mm Bofors shell, which is interesting as its C/M is exceptionally low. The velocity was found to 650 metres/sec, or 2130 feet/sec. The charge of TNT was 56.4 grams, and the weight of casing excluding the brass cap and copper band was 820 grams; the ratio of these quantities is only 0.069. Taking this ratio as C/M, as in the case of the other shell, the point plotted in fig. 2 is obtained.

e. Although the initial velocities of fragments from grenades containing K.E. have not been measured, there is some indirect evidence that the expression (7) gives a correct estimate for grenades containing 1-1/2 to 2 ounces of TNT; calculations made on this assumption were in good agreement with direct panel tests.

* A.C. 3432

R. A. Tolch and R. Curney, BRL Memo Report No. 207.
7. A knowledge of the initial velocities of fragments is a first step towards the desired knowledge of the velocity at any distance \( r \) from the explosion. For a fragment of mass \( m \) this may now be obtained from the expression

\[
v = v_1 (1 - 1.315 e^{-0.315 e})
\]

where \( R \) is obtained from (14), \( g \) is expressed in grams, \( s \) is expressed in feet, and \( v_1 \) is given the value appropriate to the explosive used. For TNT \( v_1 \) has the value 8600, while for some other explosives recent experiments give values up to 20 per cent higher; the measurements, however, are not yet very consistent.

For cylindrical TNT-filled casings of constant wall thickness, the expected values of \( v_0 \) may be read from Table II. In the range of \( C/M \) less than 0.3 the values of \( v_0 \) have been adjusted to agree with the N.D.R.C. results plotted in Fig. 2.

8. In conclusion it may be mentioned that the fragmentation of an infinitely long cylinder, detonated from one end, was discussed by G.I. Taylor**, and an expression was obtained for the fragment velocities. It was assumed that the radial motion of the explosion gases was small compared with the longitudinal motion; and the results were not intended to apply to a projectile from which the end sprays are feeble compared with the side spray.

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Schwarzschild and Sachs, ERL Report No. 347.

** G. I. Taylor R.C. 193.
Table 2.

Fragment Velocities from Cylindrical Walls of Uniform Thickness.

Column 1 gives the ratio of the external to the internal diameter. In calculating $C/M$ the density of metal was taken to be 4.9 times the density of the explosive.

<table>
<thead>
<tr>
<th>$d_0/d_1$</th>
<th>$C/M$</th>
<th>$v_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.02</td>
<td>5.05</td>
<td>9560</td>
</tr>
<tr>
<td>1.04</td>
<td>2.50</td>
<td>8430</td>
</tr>
<tr>
<td>1.06</td>
<td>1.65</td>
<td>7610</td>
</tr>
<tr>
<td>1.1</td>
<td>0.97</td>
<td>6460</td>
</tr>
<tr>
<td>1.2</td>
<td>0.464</td>
<td>4910</td>
</tr>
<tr>
<td>1.3</td>
<td>0.296</td>
<td>4000</td>
</tr>
<tr>
<td>1.4</td>
<td>0.213</td>
<td>3400</td>
</tr>
<tr>
<td>1.5</td>
<td>0.163</td>
<td>2900</td>
</tr>
<tr>
<td>1.6</td>
<td>0.131</td>
<td>2580</td>
</tr>
<tr>
<td>1.7</td>
<td>0.108</td>
<td>2340</td>
</tr>
</tbody>
</table>
FIG. I

INITIAL VELOCITIES FROM TNT-FILLED PROJECTILES

N.D.R.C. EXPERIMENTS