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SUPG FINITE NUMERICAL ELEMENT SOLUTIONS FOR
NON-COMPRESSIBLE NAVIER-STOKES EQUATION SETS

by

Xu Guoqun, Zhang Guofu

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SUPG FINITE NUMERICAL ELEMENT SOLUTIONS FOR NON-COMPRESSIBLE NAVIER-STOKES EQUATION SETS

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Xu Guoqun Zhang Guofu

ABSTRACT

This article sets out from steady state non-compressible Navier-Stokes equation sets and constructs weighted residual SUPG formulations. In order to guarantee the precision of numerical value solutions, this article--with regard to speeds--selects eight points for the interpolation of values and maintains the second order derivative quantities in perturbation terms. Looking from the point of view of calculation instances made using the methods of this article, the calculated results have proved very satisfying.

INTRODUCTION

When one has the existence of convection terms, the coefficient matrices associated with Galerkin methods are asymmetrical. This will very often give rise to oscillations in numerical value solutions. As far as problems with high Pe numbers or Re numbers are concerned, this type of phenomenon is particular severe. In order to control the convection effects associated with each single variable, it is possible to take grids and make them finer. However, this will cause the loss of large amounts of content. The most recent research clearly shows that, going through the construction of appropriate Petrov-Galerkin weighted residual formulae, finite element windward methods are capable of resolving very well the problems described above [1-2]. In 1982, Hughes and others put forward SUPG (Streamline Upward/Petrov-Galerkin) methods [3]. The methods in question not only possess the strengths of the original windward

* Numbers in margins indicate foreign pagination. Commas in numbers indicate decimals.

methods, they also solve false diffusion problems associated with windward methods. Reference [4] uses SUPG finite element methods to solve non-steady state, non-compressible N-S equation sets. In conjunction with this, it uses penalty methods to deal with non-compressibility conditions (continuity equations). In order to eliminate second order derivative terms in variation equations, the references in question, as far as speeds are concerned, opt for linear interpolations.

This article sets out directly from steady state non-compressible Navier-Stokes equation sets (belonging to the elliptical type) in order to construct SUPG variation equations. In order to guarantee the precision of solutions, the article in question opts for four node point interpolations in the case of pressures and eight node point interpolations in the case of speeds. In this way, it is necessary to consider the influences of windward flow directions on viscosity diffusion terms. Looking in terms of the calculated cases which have already been completed, the methods of this article are precise and accurate, convergent and stable. Calculation results are fully satisfactory.

I. MODEL EQUATIONS

We now discuss the steady state constant coefficient model advection diffusion equation

$$u\frac{\partial\phi}{\partial x}-\frac{1}{Re}\frac{\partial^2\phi}{\partial x^2}=0 \qquad (1.1)$$

It possesses the boundary conditions

$$\phi(0)=0,\ \phi(1)=1 \qquad (1.2)$$

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First of all, consideration is given to the Galerkin steady state numerical value solutions associated with the equations discussed above at times when u = 1. The solution in Fig.1a is obtained for Re = 1. The solution is a smooth one. Following along with increases in Re, the cross section associated with ϕ is blown down the flow. Fig.1b is the solution when Re = 20. However, when Re is increased a step further, the bending places in the cross sections are blown past the inverse second node point i = 10. At this time, the lattice Reynolds number R_c is greater than 2. Numerical value solutions show the appearance of oscillations, simulating equation (1.1) in an excessively early manner. When Re -> infinity, one has the appearance of singularity. See Fig.1c. One can clearly see that Galerkin solutions and center difference solutions are identical in their natures.

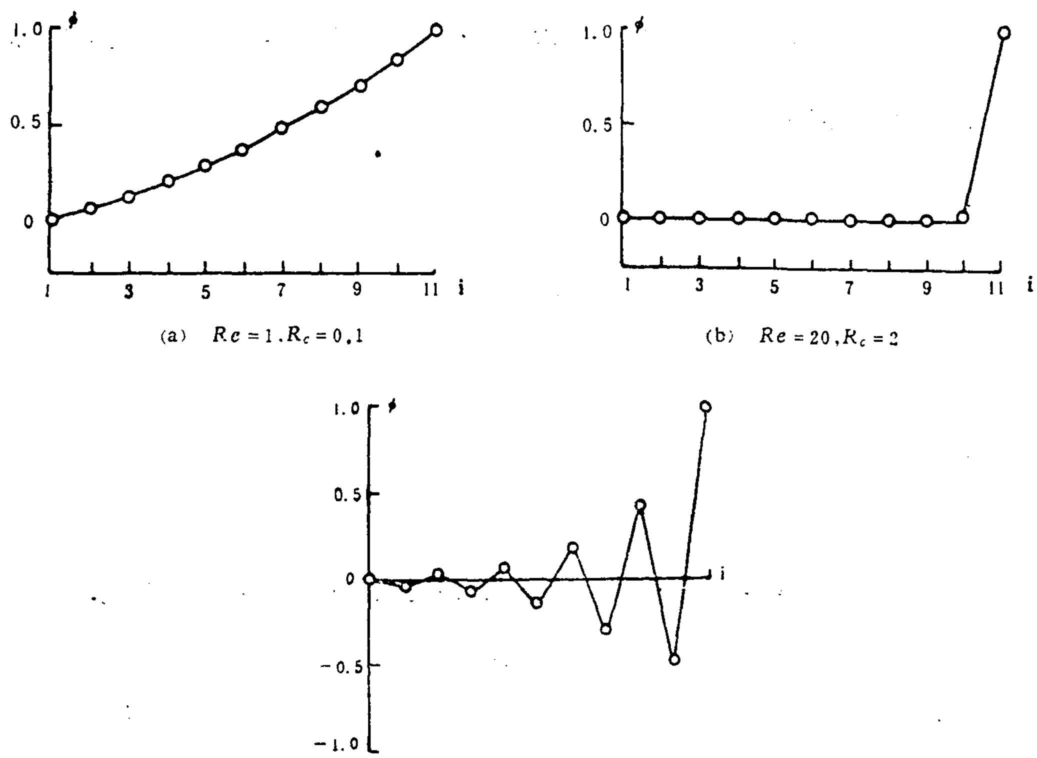

Fig. 1 Galerkin Finite Element Numerical Results

In order to eliminate oscillation, besides the possibility of methods for fining up grids, there is another type of method for rectification, that is, opting for the use of Petrov-Galerkin methods. At present, we will define the weighted function in one variable W' to be

$$W' = M + P' \qquad (1.3)$$

In this, M is an interpolation function. It is a function Co. This is continuous on single variable boundaries. P' is the weighted function perturbation amount. It is a function C^{-1}. It is not continuous on single element or variable boundaries. If P' = 0, the methods discussed above then turn into Galerkin methods.

As far as writing out the Petrov-Galerkin forms to express equation (1.1) is concerned, in conjunction with this, one takes the boundary conditions and makes the basic boundary conditions for treatment. Thereupon, one has the weak formulation

$$\int_{\Omega}\left(Mu\frac{\partial \phi}{\partial x} - \frac{1}{Re}\frac{\partial \phi}{\partial x}\frac{\partial M}{\partial x}\right)d\Omega + \sum_{e=1}^{n_{el}} \int_{\Omega^e} P' u\frac{\partial \phi}{\partial x} d\Omega = 0 \qquad (1.4)$$

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This article--as far as ϕ is concerned-- selects linear interpolation, that is, M is a mountain shaped function. Because this is the case, there is no second order derivative term in the second integral term of the equation above. n_{el} is a single element or variable number. The weighing function perturbation amount P' is taken to be

$$P' = \tau \cdot u \frac{\partial M}{\partial x} \qquad (1.5)$$

τ is taken to be

$$\tau = \alpha \bar{\xi} h/u \qquad (1.6)$$

In this equation, $h=\Delta x$, α is a calculation method parameter. As far as equation sets with different natures are concerned, it is possible to have different values taken. The derivation of the form for the selection of $\bar{\xi}$ is as follows.

The center difference FDE associated with equation (1.1) is capable of being expressed as

$$\frac{2}{\Delta x\ Re}\phi_i=\left(\frac{1}{\Delta x\ Re}-\frac{u}{2}\right)\phi_{i+1}+\left(\frac{1}{\Delta x\ Re}+\frac{u}{2}\right)\phi_{i-1} \tag{1.7}$$

The elegant solution is then

$$(\phi_i-\phi_{i-1})/(\phi_{i+1}-\phi_{i-1})=(1-e^{\Delta x * Re})/(1-e^{2\Delta x * Re})$$

Let

$$\beta=\frac{1}{2}\ \Delta x\ u\ Re=\frac{1}{2}hu\ Re$$

The equations above are capable of being written as

$$u\frac{e^{2\beta}+1}{e^{2\beta}-1}\phi_i=\frac{u}{e^{2\beta}-1}\phi_{i+1}+\frac{ue^{2\beta}}{e^{2\beta}-1}\phi_{i-1} \tag{1.8}$$

In order to guarantee absolutely the production of strict solutions for any Pe number, it is possible on the two sides of the equation above to simultaneously add and take away equation (1.7). One then has

$$\left(\frac{2}{\Delta x\ Re}+u\bar{\xi}\right)\phi_i=\left(\frac{1}{\Delta x\ Re}-\frac{u}{2}+\frac{1}{2}u\bar{\xi}\right)\phi_{i+1}$$
$$+\left(\frac{1}{\Delta x\ Re}+\frac{u}{2}+\frac{1}{2}u\bar{\xi}\right)\phi_{i-1}$$

In this

$$\bar{\xi}=\frac{e^{\beta}+e^{-\beta}}{e^{\beta}-e^{-\beta}}-\frac{1}{\beta} \tag{1.9}$$

The changes versus β are as shown in Fig.2.

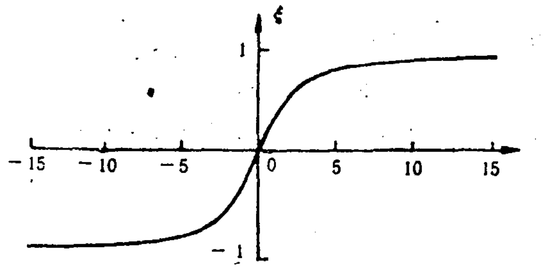

Fig.2

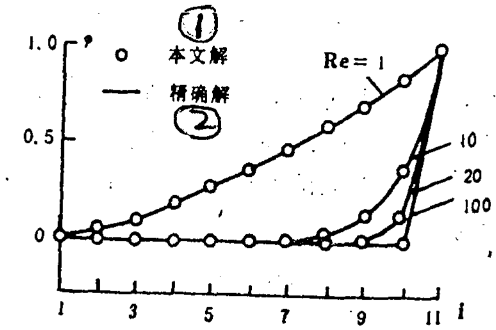

Fig.3 SUPG Finite Element Numerical Results

Key: (1) This Article's Solution (2) Precise Solution

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SUPG numerical value solutions are set out in Fig.3. From the Fig., it is possible to see that SUPG finite element methods not only effectively restrain the oscillations associated with numerical value solutions but also guarantee the precision of numerical value solutions.

II. VARIATION EQUATIONS

The steady state non-compressible N-S equation set is

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2.1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\frac{1}{Re}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \tag{2.2}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{\partial p}{\partial y}+\frac{1}{Re}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \tag{2.3}$$

In these equations, u and v are velocity components. p is pressure.

As far as the equation set discussed above is concerned, this article constructed a variation equation set of the form below.

Continuity Equation

$$\int_{\Omega} (M+P_1')\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)d\Omega=0 \tag{2.4}$$

Momentum Equations

x direction

$$\begin{aligned}&\int_{\Omega}\left[N\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+\frac{\partial p}{\partial x}\right)+\frac{1}{Re}\left(\frac{\partial N}{\partial x}\ \frac{\partial u}{\partial x}+\frac{\partial N}{\partial y}\ \frac{\partial u}{\partial y}\right)\right]d\Omega\\&+\sum_{e=1}^{n_{el}}\int_{\Omega^e}P_2'\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+\frac{\partial p}{\partial x}-\frac{1}{Re}\nabla^2u\right)d\Omega\\&=\int_{\Gamma_n}N\frac{\partial u}{\partial n}d\Gamma\end{aligned} \tag{2.5}$$

y direction

$$\begin{aligned}&\int_{\Omega}\left[N\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+\frac{\partial p}{\partial y}\right)+\frac{1}{Re}\left(\frac{\partial N}{\partial x}\ \frac{\partial v}{\partial x}+\frac{\partial N}{\partial y}\ \frac{\partial v}{\partial y}\right)\right]d\Omega\\&+\sum_{e=1}^{n_{el}}\int_{\Omega^e}P_2'\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+\frac{\partial p}{\partial y}-\frac{1}{Re}\nabla^2v\right)d\Omega=\int_{\Gamma_n}N\frac{\partial v}{\partial n}d\Gamma\end{aligned} \tag{2.6}$$

In these equations, M and N are respectively the basic interpolation functions associated with pressure and speed. They are continuous on each element boundary. In order to raise the precision of solutions, this article has opted for four node point interpolation with regard to pressure and chosen eight node point interpolation for speeds.

$$p=\sum_{i=1}^{4}M_i\,p_i \tag{2.7}$$

$$u,\ v=\sum_{i=1}^{8}N_i\,u_i,v_i \tag{2.8}$$

In this way, in equations (2.5) and (2.6), one thereupon has the appearance of second order derivative terms.

In equations (2.4) - (2.6), all the Dirchilet form boundary conditions act as the basic boundary conditions for treatment. /376

III. SELECTION OF WEIGHING FUNCTION PERTURBATION AMOUNTS

On the basis of the basic concepts associated with SUPG finite element methods, this article takes P1' and P2' and writes them as

$$P_1' = \tau\left(u\frac{\partial M}{\partial x} + v\frac{\partial M}{\partial y}\right) \quad (3.1)$$

$$P_2' = \tau\left(u\frac{\partial N}{\partial x} + v\frac{\partial N}{\partial y}\right) \quad (3.2)$$

In the equations, $\tau \cdot u$ and $\tau \cdot v$ are, respectively, artificial viscosity coefficients associated with the x direction and y direction. Referring to reference [4], one takes τ to be

$$\tau = \alpha\bar{\xi}\,h/a \quad (3.3)$$

The meaning of α is the same as that in equation (1.6). The value of $\bar{\xi}$ is still taken from equation (1.9). From this, it is possible to see that, when Re numbers are very large, the original equation set viscosity terms can be ignored, simplifying to an Euler equation set. At this time, $\bar{\xi} = 1$. When Re numbers are extremely small, the original equation set viscosity terms are far greater than inertial terms. At this time, there is no need, in calculations, to again add on artificial viscosities. Because this is the case, $\bar{\xi}$ tends close to zero.

As far as composite or synthetic speed a in single variable center calculations is concerned, the equation expressing it is

$$a = \sqrt{u^2 + v^2} \qquad (3.4)$$

h is the weighted average of the single variable characteristic lengths h_1 and h_2 (see Fig.4). The amount of weighing is a unit speed component, that is,

$$h = (h_1 u + h_2 v)/a \qquad (3.5)$$

In this

$$h_i = 2[(\partial x_i/\partial \xi)^2 + (\partial x_i/\partial \eta)^2]^{1/2}, \quad 1 \leq i \leq 2 \qquad (3.6)$$

In equation (3.6), ξ and η are single element or unit coordinates (Fig.4). Obviously, if $u > v$, then, h is primarily determined by h_1. In particular, when $v = 0$, the two dimensional problem becomes a one dimensional problem. At this time, h is then a unit width.

IV. NUMERICAL SUBSTITUTION METHODS OF CALCULATION AND INSTANCES OF CALCULATIONS

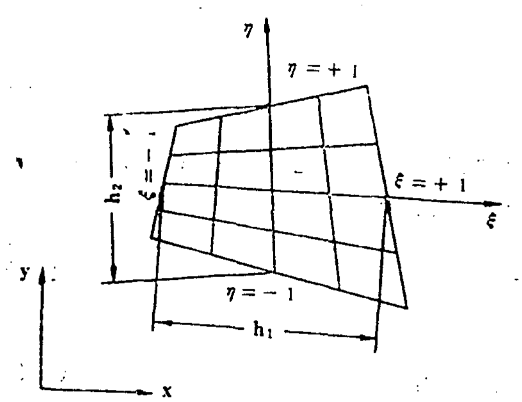
This article uses asymmetrical linear equation sets formed in association with the calculation methods put forward above. In conjunction with this wave matrix methods are used for solutions.

In order to empirically demonstrate the degree of usability and precision associated with the methods of this article, it makes the calculations below:

Calculation Case 1 Couette Flows

Fig.5 shows, with various types of pressure gradients, the calculation results associated with the flow movements between two parallel flat plates.

Fig. 4 Isoparametric Element



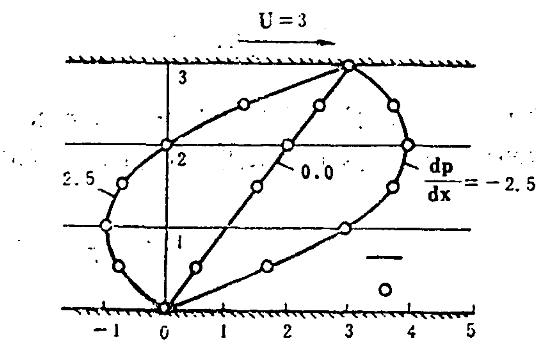


Fig.5 Couette Flow Between Infinite Parallel Plates

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Calculation Case 2 Recirculation

This article calculates a two dimensional rear step or recirculation problem for Re = 73 (see Fig.6). In the calculations, consideration is given to the influence of the effects of gravity. Flow calculation results within the base zone are shown in Fig.7.

Calculation Case 3 Recirculation or Winding Flows Associated with Cylinders

Fig. 8 is the calculation results for noncompressible laminar flow recirculation or winding flows associated with cylinders for Re = 20. From the Fig., it is possible to see that stationary point wall surface pressures are the highest. Behind them, due to flow speeds being speeded up, wall surface pressures drop. This is a smooth pressure flow process. This section of the process advances directly to the place where cylinder surface flows are highest. From the place where the cylinder surface flows are highest to the wake or tail section, speeds gradually drop. Wall surface pressures rise. This is a reverse pressure flow process. Fig.9 is a pressure contour diagram associated with the vicinity of cylinder surfaces for cylindrical winding flows.

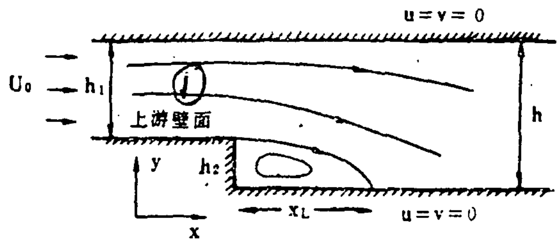

Fig. 6 Schematic Representation of Recirculation

Key: (1) Upper Flow Wall Surface

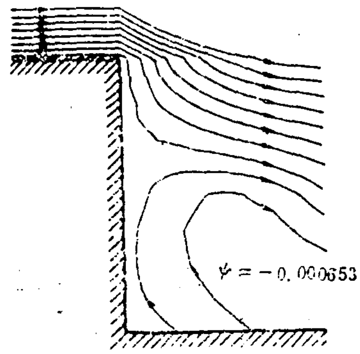

Fig.7 Stream Function Plot and Flow Direction Over Bottom of Backward Facing Step. Re = 73. $\Delta\psi = 0.000386$

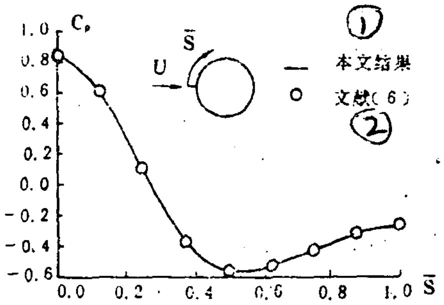

Fig.8 Pressure Coefficient on Cylinder. Re = 20 (Based on Cylinder Diameter)
Key: (1) Results from This Article
(2) Reference (6)

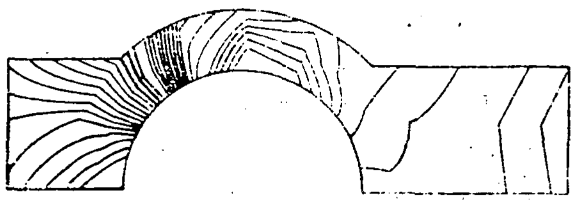

Fig.9 Pressure Contour (Re = 20)

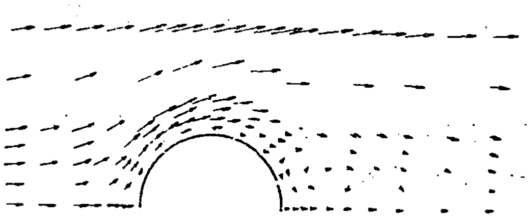

Fig. 10 Velocity Vector for Viscous Flow Past Cylinder (Re = 20)

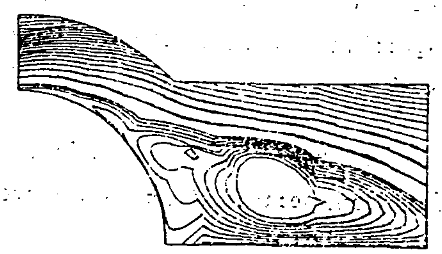

Fig.11 Stream Function Plot Behind Cylinder (Re = 20)

Flow functions obtained from calculations for cylinder winding flow speed fields and cylindrical wake regions are shown in Fig.'s 10 and 11. It is possible to see that, due to separation of object surface boundary layers, fluids, down the flow from cylinders, form vortices.

In the calculation cases which were made and discussed above, it is possible to see that the calculation results are satisfactory. This clearly shows that the methods in this article are precise and accurate, convergent, and stable.

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