Final Report on Research Achievements

(May 1989 to May 1992)

ONR Grant Information

Grant Title:	Nonlinear Waves, Vortex Dynamics and Stability
Grant Number:	N00014-89-J-1971 R&T4322-737
Program Director:	Dr. Edwin P. Rood
Principal Investigator:	T. Y. Wu

1. The scientific research objectives and goals

Under the present three–year ONR grant, research has been actively pursued by Dr. Wu's group in the general field of nonlinear waves, vortex dynamics and hydrodynamic stability theory with the following scientific objectives and goals:

1. Nonlinear waves under resonant forcing — to continue the group research studies on forced emission of nonlinear waves by resonant excitation; to develop analytical and computational methods for solving the general initial–boundary–value problems involving forced solitons and nonlinear waves.

2. Hydrodynamic stability theory for forced nonlinear waves — to investigate the eigen–value problems pertaining to the linear stability analysis of forced nonlinear waves; to determine the bifurcation diagram for unstable forced steady waves; and to develop nonlinear stability theory for both slow and rapid growth of unstable waves under resonant forcing.

3. Forced production of nonlinear vortex waves — to explore the generation of vortex solitons and nonlinear vortex waves in free–surface flows, in sheared streams and in stratified and rotating fluids; to ascertain the basic mechanism underlying the phenomena of vortex breakdown, vorton–vorton interaction and their relationship with coherent structure or chaos that may emerge as a result.

2. Research results and achievements

Under the present ONR Grant support, studies have been carried out on a number of important problems in the general areas outlined above. This section serves to describe these studies, both accomplished and continuing, and to report significant results achieved. Publications of new results so obtained are listed below under each major project. The participating graduate and undergraduate students and collaborating scientists are listed in §3.

2.1 Forced emission of solitary waves in shallow water

The remarkable phenomenon of genesis of nonlinear dispersive waves by a resonant forcing was first identified by Wu & Wu (1982). It reveals that a surface pressure or a submerged topography moving steadily with a critical velocity in a shallow channel of uniform water depth generates an unsteady response with a sequential upstream emission of solitary waves. This discovery was further pursued by Lee (1985) and Lee et al. (1988, 1989) who carried out perhaps the first known experimental validation of the principal theoretical models that predict this remarkable phenomenon. A broad agreement between experiment and both the generalized Boussinesq (gB) model and the forced Korteweg–de Vries (fKdV) equation developed by this group was found for the depth Froude number ($F = U/\sqrt{(gh)}$, U being the constant velocity of the forcing disturbance, g the acceleration of gravity, and h the undisturbed water depth) lying within the narrow transcritical range of $0.9 < F < 1.1$. Outside this range, wave breaking was found generally to occur in our laboratory experiment, but the basic phenomenon may persist over the extended range of $0.2 < F < 1.2$, as indicated by both our numerical and experimental observations.

Continued effort has been devoted to explore the basic mechanism that underlies the intriguing phenomenon in question. To provide a solid foundation for investigation of the hydrodynamic stability of such forcibly produced waves, two families of exact solutions to the forced KdV equation have been found, one by Wu (1987) for stationary forcing effects with fore–aft symmetry and the other by Yates (1990) for skewed forcing distributions. Once established initially with a proper stationary forcing, each of these waves will remain permanent in form provided it is stable. The existence of these solutions is significant in providing basic flows necessary for stability and bifurcation studies and in applications to examine certain types of forced disturbances that yield only local (non–propagating) effects, including oceanographic studies on tidal currents over topographies and meteorological observations of mountain waves (e.g. Patoine & Warn 1982).

Publications

Lee, S.J. 1985 Generation of long water waves by moving disturbances. Ph.D. Thesis, California Institute of Technology, Pasadena, CA.

Lee, S.J., Yates, G.T. & Wu, T.Y. 1989 Experiments and analysis of upstreaming–advancing solitary waves generated by moving disturbances. *J. Fluid Mech.* **199**, 569-593.

Wu, D.M. & Wu, T.Y. 1982 Three–dimensional nonlinear long waves due to moving surface pressure. In *Proc. 14th Symp. on Naval Hydrodynamics*, 103-125. Washington, DC. National Academy Press.

Wu, T.Y. 1987 Generation of upstream–advancing solitons by moving disturbances. *J. Fluid Mech.* **184**, 75-99.

Yates, G.T. 1990 Some antisymmetric solutions with permanent form of the forced KdV equation. In *Engineering Science, Fluid Dynamics* (ed. George T. Yates), 119-131. World Scientific.

2.2 Stability of the solitary waves forcibly generated

Linear and nonlinear stability analyses have been performed by Camassa (1990a,b) and Camassa & Wu (1991a,b) for two particular classes of stationary solitary waves generated by steadily moving disturbances at transcritical speeds. Singular perturbation methods are used first to obtain local spectral analysis of the fixed–point solutions to the perturbation equation which governs small perturbations of the stationary wave, and are complemented by numerical analysis of the global spectral properties of the characteristic perturbation solutions. Fundamentally new results have been achieved showing that three distinct regimes exist in the pertinent parametric domain, called the periodical bifurcating transcritical regime, the aperiodical bifurcating regime, and the stable supercritical regime, each characterized by distinct stability properties.

In summary, they have the following salient features. In the periodical bifurcating regime, the eigen–value σ of the linearized perturbation equation has two pairs of complex conjugate branches, $\sigma = \pm\sigma_r \pm i\omega$, with $\sigma_r \ll \omega$. The smallness of the ratio σ_r/ω (of order 10^{-5} in the low range) infers that waves evolve under forcing with a slow growth rate of σ_r and that new waves are produced periodically. (This also explains the failure reported by Wu (1988) in ascertaining the real component σ_r by using the Galerkin modal expansion method due to its lack of the high accuracy required for the calculation.) In the aperiodical bifurcating regime, σ has only a pair of purely real branches, $\sigma = \pm\sigma_r$, signifying wave growth at an exponential rate in this regime and bifurcating to a new solution which was discovered numerically to be stable. In the stable supercritical regime, no roots of σ can be found, but a nonlinear stability analysis was developed to show the global stability of the steady state solutions. Thus, these properties and their corresponding bifurcation diagrams are new and drastically different from the classical cases previously known.

For the anti–symmetric family of steady solutions to the forced KdV equation, their linear and nonlinear stability properties have been investigated by Yates (1990) and Yates & Wu (1991) over a wide range of transcritical speeds and wave amplitudes for some typical cases. The study has been made largely with computational efforts since no analytical method is available for evaluating local stability of the fixed–point solutions due to a higher degree of singularity possessed by the perturbation equation. In spite of this deficiency, the same three regimes of stability as those for the case of symmetric forcing are identified, with their similarities and dissimilarities distinguished in refined details.

These results have thus greatly elucidated the different types of instability and bifurcation of nonlinear waves produced by forcing effects. However, further research will still be needed to explain the basic mechanism underlying the perplexing phenomenon in question.

Publications

Camassa, R. 1990a Part I: Forced generation and stability of nonlinear waves. Ph.D. thesis, California Institute of Technology.

Camassa, R. 1990b Multiple stationary solutions for the forced KdV equation. In

Engineering Science, Fluid Dynamics (ed. George T. Yates), 45-58. World Scientific.

Camassa, R. & Wu, T.Y. 1991a Stability of some stationary solutions for the forced KdV equation. *Physica D***51** 295-307.

Camassa, R. & Wu, T.Y. 1991b Stability of forced steady solitary waves. *Phil. Trans. Roy. Soc. Lond. A* **337** 429-466.

Wu, T.Y. 1988 Forced generation of solitary waves. In *Applied Mathematics, Fluid Mechanics, Astrophysics — A Symposium to honor C.C. Lin* ed. D.J. Benney, F.H. Shu, C. Yuan. 198-212. World Scientific.

Yates, G.T. & Wu, T.Y. 1991 Stability of solitary waves under skewed forcing. In *Mathematical Approaches in Hydrodynamics, ed. T. Miloh*, 193-206. SIAM.

2.3 Forced nonlinear waves in a channel with variable cross section

Two types of theoretical wave models, namely, the generalized channel Boussinesq (gcB) two–equation model and the forced channel Korteweg–de Vries (cKdV) one–equation model, have been developed to simulate the generation and propagation of nonlinear water waves in channels of arbitrary cross section whose shape and dimensions may vary in space and time. Analytical solutions for four specific cross–sectional shapes, namely, the variable rectangular, triangular, parabolic and semi–circular cross sections, have been obtained in closed form. In particular, uniform channels of different cross–sectional shapes are characterized by a single shape factor κ. For this case, a uniform–channel analogy theorem has been enunciated stating that long waves of equal (mean) height in different uniform channels of equal mean depth but distinct κ–shape factors propagate with equal velocity and with their effective wavelengths appearing κ times of that in the equivalent rectangular channel, for which $\kappa = 1$. It also shows that the more the channel shape departs from the rectangular, the greater the value of κ. On this basis, the solitary and cnoidal waves in a κ–shaped channel have been found in good agreement with experiment on wave profiles and wave velocities. Further, the three–dimensional features of these solitary waves have been specifically determined.

These theoretical models are being applied to investigate the evolution of ocean long waves running up continental shelves and into coastal bays and harbors with variable depth and width. The new developments along this direction are proposed for further studies that will have useful applications to problems of maintaining and improving the quality of our coastal environment.

Publications

Teng, M.H. 1990 Forced emission of nonlinear water waves in channels of arbitrary shape. Ph.D. Thesis, California Institute of Technology, Pasadena, CA.

Teng, M.H. & Wu, T.Y. 1990 Generation and propagation of nonlinear water waves in a channel with variable cross section. In Engineering Science, Fluid Dynamics (ed. George T. Yates) 87-108. World Scientific.

Teng, m.H. & Wu, T.Y. 1991 Run–up of tsunami waves in coastal water of variable depth and width. In Proc. Intern. Workshop on Technology for Hong Kong Infrastructure Development (ed. J–C Chen & J.L. Beck) 529-544. Commercial Press.

Teng, M.H. & T.Y. Wu 1992 Nonlinear water waves in channels of arbitrary shape. (accepted to appear in *J. Fluid Mech.*)

2.4 Standing solitary waves and resonant Faraday waves

Theoretical, numerical and experimental efforts have been devoted to investigate the phenomenon of forced, nonlinear, standing waves sloshing on a shallow layer of water contained in a rectangular channel which undergoes vertical oscillations. Such a system is known to be capable of sustaining standing solitary waves within a certain parameter domain. A new mode of forced sloshing waves has been discovered to exist in a different parametric domain for rectangular tanks with the wave sloshing across the short side of the tank and with its profile modulated by one or more hyperbolic–tangent, or kink–wave–like envelopes. A theoretical explanation for the kink wave properties is provided. Experiments have been performed to confirm their existence. The stability criteria for the appearance of these nonlinear waves are being explored.

Publication

Guthart, G. & T.Y. Wu 1991 Observation of a standing kink cross wave parametrically excited. *Proc. Roy. Soc. Lond. A* **434**, 435-440.

2.5 Nonlinear vortex waves in stratified and rotating fluids

New research efforts have been focused on the production of nonlinear vortex waves, e.g. vortex solitons or the vortons in free–surface flows, in sheared streams and in stratified and rotating fluids. In particular, for axisymmetric flow of a non–uniformly swirling fluid bounded within a long cylindrical tube, periodic generation of vortex solitons have been discovered to undergo upstream emission by critical forcing. The numerical solutions of the forced KdV equation for the amplitude function of the stream function of the incompressible Euler model for the class of weakly nonlinear and weakly dispersive motions exhibit the well–defined axisymmetrical recirculating eddies being produced under the resonant forcing, advancing upstream and maintaining their entities with a permanent form. This interesting result has provided a base for investigating vorton–vorton interactions and vortex production due to localized forcings. This mode of vorticity growth is thought to be of vital importance in fluid dynamics and applied mathematics.

Publication

Choi, W. & Wu, T.Y. 1992 Upstream advancing vortex solitons in rotating fluids. (Submitted for publication.)

2.6 Generation and propagation of internal solitary waves through stratified fluids

Motivated by the observation of internal solitary waves moving within the pycnocline of the oceans, we have investigated the generation of internal waves by moving disturbances. Within a range of transcritical speeds (pertaining to the internal wave modes), periodic generation of internal solitary waves have been observed both experimentally and numerically, with salient features closely resembling those for the case of a single layer of homogeneous fluid. Both a forced KdV model, and a generalized Boussinesq model have been developed, each having respective advantages and restrictions on their application. Use of the actual density profile (rather than stepwise density distributions) is found to be important in determining the resonant normal mode with more highly accurate predictions of internal wave generation period.

From this research study an important correspondence rule has been found which correlates any resonant mode of internal waves in a layer of arbitrary density stratification with an equivalent critical mode of a single layer of homogeneous fluid. This rule is useful in greatly simplifying numerical computations for various resonance modes of internal waves in stratified fluids.

Publication

Zhu, J.L., T.Y. Wu & G.T. Yates 1990 "Internal solitary waves generated by moving disturbances", In *Stratified Flows* eds. E.J. List & G.H. Jirka, 74-83. ASCE.

2.7 The KdV model with boundary forcing

In sharp contrast with the theory of initial–value problems for the KdV model, which is well developed, no analytical methods are available for solving the initial–boundary–value problems of the KdV equation.

The problem of solitary waves generated by boundary forcing imposed on a KdV system has been analyzed by Camassa & Wu (1989a,b) with application of the inverse scattering transformation (IST) scheme (originally devised for unbounded space) to the $\frac{1}{4}$ – plane ($x \geq 0$ in space, $t \geq 0$ in time) problem with some simplifying assumptions in the spirit of Kaup & Hansen (1986). This approximate analytical method is found capable of providing quantitative predictions of such main variables as the total number of solitons generated by a specific boundary forcing, their amplitudes and phases, etc. These predictions are found to be in broad agreement with the numerical results.

From this study we have gained valuable insight into the basic structure of solution to this important problem of generation of nonlinear waves by boundary forcing. Some new light is directed towards improving the approximate IST scheme for the very important general class of boundary value problems based on the KdV model.

Publications

Camassa, R. & Wu, T.Y. 1989a The Korteweg–de Vries model with boundary forcing. *Wave Motion* **11**, 495-506.

Camassa, R. & Wu, T.Y. 1989b The Korteweg–de Vries model with boundary forcing. In *Proc. Euro Mech Colloquium 240* Dispersive waves in dissipative fluids. ed. D.G. Crighton & F. Mainardi. 20-21. University of Bologna, Bolongna.

2.8 Scattering and diffraction of solitary waves by a vertical cylinder

In this numerical and experimental study the generalized Boussinesq model has been applied to evaluate the scattering and diffraction of a solitary wave by a vertical circular cylinder. The three–dimensional wave patterns obtained numerically exhibit the scattered waves radiating outward with a nearly cylindrical symmetry centered at the cylinder. The forward scattered primary wave is found to soon recover its uniform phase after having left the cylinder, carrying with it a secondary mode along the nearly straight wave crest. The wave impact force has a positive first half and a weaker negative trailing half, and appears in variance with the classical Morison's formula, especially for waves of finite amplitudes. During the swift wave–body encounter, both the nonlinear and dispersive effects are found to vary rapidly in phase and magnitude relative to the net linear effects. Experimental results are found to be in good agreement with the numerical model.

Publication

Wang, K.H., Wu, T.Y. & Yates, G.T. 1989 Scattering and diffraction of solitary waves by a vertical cylinder. In *Proc. 17th Symp. on Naval Hydrodynamics*. 513-522.

3. Education and development of human resources

This research program has been actively participated in during the current three–year period by the following team members. The research work, devoted towards studies of physical phenomena which are of vital importance but little understood and towards development of new mathematical methods, has provided excellent opportunities for intellectual development of the graduate and undergraduate students and scientists who participate in this team work.

Faculty and research scientists

Wu, T.Y.	Professor, P.I.
Yates, G.T.	Senior Scientist
Murashige, S.	Research Fellow, Japan Soc. Promot. Sci. Fellow
Akylas, T.	Visiting Associate – Assoc. Prof., MIT
Teng, M.H.	Research Scientist
Shirose, Y.	Visiting Associate – Ishikawajima–Harima Inst., Japan
Wang, K.H.	Research Fellow

Graduate and undergraduate students

Camassa, Roberto	Ph.D. (1991)
Teng, Michelle H.	Ph.D. (1991, woman)
Choi, Wooyoung	Ph.D.
Guthart, Gary	Ph.D.
Kao, John	Ph.D.
Lin, Duo–Min	Ph.D.
Zhang, Jin	Ph.D.
Yang, Si–Long	M.S. (1991)
Choi, Jai–Sig	M.S.
Chao, Jing–Tying	Junior (woman)

This team is dedicated, with keen interest, to research studies in this new and fast growing field. Several problems of central importance have been investigated by these members with joint theoretical, numerical and experimental efforts following a well designed plan so that progress attained has been mutually beneficial. Some of the new research achievements are summarized in §2.