FREQUENCY IN FORMATION OF CAVITATION POCKETS IN TURBULENT BOUNDARY LAYERS AND SATELLITE FLOWS

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The experimental studies of K. K. Shal'nev (1), Knapp (2), and Daily (3), have forced the conclusion that the formation and growth of cavitation pockets in turbulent boundary layers and satellite flows is connected with the pressure pulsations in the turbulent flow. By making use of this conclusion, it is possible to use the following diagram as representing the dynamics of cavitation pockets in a turbulent flow: at a given moment the nucleus, for which the amount of critical pressure $P_k$ is greater than $P$, falls into the rarefaction field, which is formed to the pressure pulsation. The bubble grows, while the amount $P_k - P$ remains positive. Falling into the zone of increased pressure, the bubble closes up. In this way the frequency in formation of cavitation pockets is determined by the frequency with which the rarefactions zones appear, the pressure in which is lower than a certain critical pressure level, and also by the probability of the cavitation nucleus, with this critical pressure level, falling into the zone of local pressure reduction. (See Figure 1)

In order to simplify the task in the future, it will be assumed that density of the nucleus is great in the liquid being studied, that the liquid is uniform in terms of stability, and that any volume of it can be characterized by the critical pressure $P_k$, at which the cavitation pocket is formed. This cavitation pocket can be made up of several gas or vapor bubbles, the dynamics of which is connected with one and the same discharge of pressure. Let us also assume that the air bubbles which are formed do not have an actual effect on the characteristics of the turbulent flow.

Taking into account all of these factors, we can
formulate the problem of the frequency of formation of cavitation pockets in the following way: it is necessary to
determine the frequency of formation of cavitation pockets
in a certain volume of turbulent flow \( \Delta V, \) which equals the
volume of the flow correlation, if this frequency agrees
with the frequency with which the pressure is lowered to the
critical pressure level \( P_k. \) The symbols which are used for
pressure in the turbulent flow being studied are given in
Figure 1. The problem is analogous to that worked out by
Rice (4) for the frequency \( N_1 \) of excesses in discharges of
electric noise with amplitudes \( A, \) which are distributed
according to normal laws above a certain fixed level \( A_k. \)

The result which Rice obtained can be illustrated as
follows: see (1)
where \( \omega \) is the circular frequency; \( S(\omega) \) -- the spectral
density of a random value of \( A; \) \( \zeta \) -- the average square
deflection; \( \bar{A} \) -- average value of a chance value of \( A. \) The
brackets in Formula (1) represent the average square, cir-
cular frequency of the random process.

It can be assumed that the pressure pulsations in a
turbulent flow are distributed according to normal laws (5):
see (2)
where \( U_1 \) -- the critical speed (with \( U > U_1 \) the flow is tur-
bulent); \( K --/a \) constant; \( l \) -- characteristic length of the
body; \( \nu \) -- kinematic viscosity; \( Re -- \) Reynolds number.
Therefore, it is possible to use formula (1) to calculate
the frequency of formation of cavitation pockets \( \Delta V \)
equal to the volume of correlation. In order to discover
the general number of cavitation pockets which are formed
around the body in one second, it is necessary to sum up the
values of \( N_1 \) according to all the elementary volumes of \( \Delta V, \)
taking into account the fact that the values of the average
rarefaction \( P_1 = P_0 - \frac{\omega \Delta V}{2} \) and of the average square
deflection \( \zeta (Re) \) depend on the distribution of the elemen-
tary volume \( \Delta V; \) see (3)

Formula (1) can be verified by experimental data (1).
In this work it is assumed that gas bubbles are built up
under the cavitation pocket into a vortex, which breaks away
from a circular cylinder. Therefore, we can determine the
average square frequency of pressure pulsations in formula (1)
as the average square frequency of formation of vortices
around the cylinder, leaving out small-scale pulsations. It
is well known that when \( Re = 10^3 \rightarrow 10^5, \) the B"yeyar-Karman
trail is formed behind a circular cylinder, and the formation
process for the vortices at a constant flow speed is close to
being periodical, due to which the mean square frequency of
the process is close to the mean, and the relationship of the
mean square frequency to the flow speed at infinity \( U_0 \) and to
the diameter of the cylinder \( l \) can be described as followz:
see (4)
where $0.18 \pm 0.22$ is the Strukhal St number for the vortices which break away from the circular cylinder (6).

By substituting the expressions for the mean pressure in the chosen elementary volume for the mean square pressure deflection and for the mean square circular frequency, we obtain: see (5)

For a circular cylinder, the critical Reynolds' number is $Re_c = 50$. The value $(P_0 - P_k)/\gamma(u_\infty - u_1)^2 = Q$, i.e., the cavitation number. When $u_\infty > u_1$, this is identical to that which is used in hydrodynamics literature.

Formula (5) can be written in another way: see (6)

From formulas (4) and (5), it is possible to determine the relationship between the St number for the cavitation pockets and the parameters of the flow and the form of the body around which it is flowing: see (7)

Constants $\alpha$ and $k$ occur in formulas (6) and (7). We can determine the value of the rarefaction coefficient for the formation point of the vortices (the point at which the turbulent boundary layer breaks away from the wall of the circular cylinder) from the distribution of the pressure, which has been determined experimentally (7), around the circular cylinder when $Re = 10^5 \pm 10^6$. The value of the proportionality coefficient between the mean square deflection of pressure and the speed of the flow $k$ was taken to be equal to 0.58. It was obtained by Betchelor (5) for the case of isotropic turbulence. It is apparent that the value of the mean square deflection of pressure in the boundary layer will be 0.58.

The speed of the flow at infinity in an infinite medium occurs in formulas (6) and (7). Experiments by K. K. Shal'vev were carried out making use of a hydrodynamic tube; therefore, in comparing theoretical and experimental data it is necessary to take into account a correction for the effect of boundaries. Since the speed of flow $u$ has been determined (1) as the mean speed for every cross section of a hydrodynamic tube in the absence of a model, in calculating the frequency of formation of cavitation pockets according to formula (6) it is necessary to assume $u = 1.17u_\infty$, and in calculating the St number according to formula (7) for cavitation pockets -- $u_\infty = 1.26 u_\infty$. These were determined experimentally by K. K. Shal'vev.

In figure 2, the results of calculating the frequency of formation of cavitation pockets and the St number for them are compared with the experimental data given in the reference (1). The relationship of the St of cavitation pockets to the cavitation number Q is set forth in figure 3. The number of cavitation pockets was calculated according to formula (7). see Figures 2 and 3.
Usually in hydrodynamics the number of cavitations is used to represent cavitation phenomena. Formulas (1)-(7) show that uniformity in the number of cavitations does not guarantee a similarity in frequencies of formation of cavitation pockets, as their number also depends on the size of the body around which the flow is passing and, through the coefficients for the mean square deflection of pressure, the mean rarefaction and the mean square frequency of pulsation on the Reynolds' number. Thus, in order to have similarity in frequencies of formation of cavitation pockets, there must be a uniformity in the cavitation number, the Reynolds' number, and the Strukhal number.

Let us consider the scale factor which can be observed by patterning the critical speed of formation of cavitation after the cavitation number. If the speed at which the number of cavitation pockets sharply increases is used for the critical speed of cavitation, it can then be determined from the formula: see (8)

where \( \alpha \) -- is a certain constant which has been selected at random. The formula determining \( u_k \) can be written thus: see (9)

where \( u_1 = \text{Re}_1 v/l \). It can be concluded from formula (9) that an increase in the size of the cylinder leads to a decrease in the critical speed for formation of cavitation (scale factor).

Above we have considered the use of formula (1) in the case of cavitation about a circular cylinder; however, it can be used in the case of cavitation along boundaries. To do this it is only necessary to determine the mean square frequency of pressure pulsation. Using data from the theory of isotropic turbulence of Kolmogorov-Obukhov, it can be shown that the value of enormous pulsations of pressure is proportional to \( u_1/\ell \). Then, for the frequency of pocket formation with cavitation along boundaries, the following formula must be correct: see (10)

This formula shows that at a constant cavitation number the frequency of formation of cavitation pockets is proportional to the flow speed -- a conclusion which is confirmed by the experimental work of Knapp (2). The linear relationship of \( N \) to the flow speed can be observed when the value of the exponent is close to one, which, for example, is realized when values \( Q \) and \( \ell \) are small.

In conclusion, we can state that the use of formulas cited above makes it possible to calculate the relationship of the intensity of cavitation noise and cavitation erosion to characteristics of the flow and of the body around which the flow is passing.

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REFERENCES

1. K. K. Shal'nev, Izv. AN SSSR (USSR Academy of Sciences News), Technical Sciences Division, No 1 (1956); No 1 (1958)


4. S. O. Rice, Bell System Techn. J., 23, 282 (1944); 25, 46 (1945)

5. D. K. Betchelor, Teoriya Odnorodnoy Turbulentnosti (Theory of Uniform Turbulence), (editor -- Obukhov), IL. 1955


7. B. A. Konstantinov, Izv. AN SSSR, TSD, No 10 (1946)
Figure 1. Distribution of Pressure in the Liquid Being Studied.

- $p_c$ - Hydrostatic Pressure;
- $p_k$ - Critical Cavitation Pressure;
- $p_a = p_0 - \rho_0 \frac{u_0^2}{2}$ - Average Pressure in Elementary Volume of the Liquid.
- $\alpha$ - Rarefaction Coefficient
- $\rho_0$ - Density of the Liquid
- $u_0$ - Speed of the Liquid to Infinity

Figure 2. Relationship of the number of cavitation pockets $N$ and the Strukhal number for the cavitation pockets $St$ to the flow speed $V$ at a constant cavitation number $\phi$. Points are the experimental data of reference (1). Lines are the values, which were determined theoretically, of $N$ and $St$ of cavitation pockets for two values of the Strukhal number for the vortices $0.18$ (1) and $0.22$ (2).
Figure 3. Relationship of the St number of cavitation pockets on the number of cavitations $Q$. --- is experimental data of the reference (1); --- the absence of cavitation according to data in a reference (7); --- the first tone indications of cavitation (7); --- broken pockets which can be seen visually (7); --- jump in resistance to the flowing (7); --- flow which separates.

Formulas:

1. \( N_i = \frac{1}{2\pi} \exp \left[ \left( \frac{\bar{R} - \bar{A}}{2\sigma} \right)^2 \right] \left\{ \int_0^\infty \Phi(\omega) \omega^2 d\omega \right\} \left\{ \int_0^\infty \Phi(\omega) d\omega - \bar{A}^2 \right\} \frac{1}{\sigma(Rc)\sqrt{2\pi}} \exp \left[ \frac{-(P - \bar{P})^2}{2\sigma^2} \right] \); 

2. \( \sigma = \begin{cases} 0, & Re < Re_c, u \frac{1}{\nu} \\ k \rho (u_\infty - u_1) \frac{1}{\nu}, & Re > Re_c, \end{cases} \)

3. \( N_v = \sum_{\alpha_i} \frac{1}{2\pi} \exp \left[ - \frac{(P - \bar{P})^2}{2\sigma^2} \right] \left\{ \int_0^\infty \Phi_\alpha(\omega) \omega^2 d\omega \right\} \left\{ \int_0^\infty \Phi_\alpha(\omega) d\omega - \bar{P}_\alpha \right\} \frac{1}{\sigma^2} \); 

4. \( \sqrt{\omega^2} \approx 2\pi (0, 18 \div 0, 22) u_\infty / \ell \).
(5) \( N_t(0, 18 + 0.22) \frac{u_\infty}{t} \exp \left[ -\frac{1}{2} \left( \frac{P_0 - P_k}{\rho(u_\infty - u_0)^2} - \frac{\sigma u_\infty^2}{2(u_\infty - u_0)^2} \right)^2 \right] \) при \( Re \).

(6) \( N_t = (0, 18 + 0.22) \frac{u_\infty}{t} \exp \left[ -\frac{(Q - q)^2}{8k^2} \right] \) при \( Re > 50, u_\infty \gg u_0 \).

(7) \( St = \frac{N_t}{u_\infty} = (0, 18 + 0.22) \exp \left[ -\frac{(Q - q)^2}{8k^2} \right] \) при \( Re > 50, u_\infty \gg u_0 \).

(8) \( \frac{(P_0 - P_k)}{\rho (u_\infty - u_0)^2} - \frac{\alpha u_\infty^2}{2(u_\infty - u_0)^2} = 2k^2 \sigma \).

(9) \( u_k = \frac{1.4k \alpha u_\infty (Re)}{a(Re)/2 + 1, 4k^2} + \left\{ \frac{P_0 - P_k}{\rho(Re)/2 + 1, 4k^2} - \frac{0.7k \alpha u_\infty^2 (Re)}{(a(Re)/2 + 1, 4k^2)} \right\}^{1/2} \).

(10) \( N \sim \frac{u_\infty}{t} \exp \left[ -\frac{(Q - q)^2}{8k^2} \right] \).

10,322