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By M. A. Tsikulin

- USSR -

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ON THE ANALOGY TO EXPLOSION OF SUPERSONIC FLOW AROUND BODIES

[Following is a translation of an article by M. A. Tsikulin entitled "K vzryvnoi analogii pri sverkhzvukovom obtekanii tel" (English version above) in Izvestiya Akademii Nauk SSSR, Otdel. Tekhnicheskikh Nauk -- Energetika i Avtomatika (News of the USSR Academy of Sciences, Technical Sciences Section -- Energetics and Automatics), No 1, Jan-Feb 1961, pp 91-96.]

The shock wave formed during flow around rotating bodies of a different form by a stream of gas with great supersonic speed can in the limits of applicability of the law of plane sections be presented as a result of two factors [1]: 1) blunting of the forward end equivalent to the action of an explosion and 2) subsequent increase in the cross section of the body equivalent to the action of an expanding round cylindrical piston. For blunt-nosed rotating bodies with constant cross section and for bodies with small length, with the ratio of the length to the transverse dimensions in the order of one, when the effect of the equivalent expanding piston can be ignored, the departing shock wave is equivalent to the shock wave of an explosion. In general, for bodies of complex form and especially for thin sharp bodies with considerable length the explosive analogy is correct at distances which are great in comparison with the transverse dimensions of the body. In this case energy due to the action of an equivalent cylindrical piston is added to the energy of the explosion, equivalent to the effect of blunting.

The use of the solution to the problem of a forceful explosion to describe the departing shock wave when a stream of gas flows with great supersonic speed around bodies gives a parabolic form of the front [1-2]

$$\frac{r}{d} = \bar{\varkappa}_1 c_x^{1/4} \left(\frac{z}{d}\right)^{1/2}$$

which agrees well with the experimental data [3, 4]. Here d end v_x are the diameter and the coefficient of frontal resistance of the body, z and r the distance along and perpendicular to the axis of the body, and $\bar{\varkappa}_1$ the coefficient.

But the numerical value of the coefficient of proportionality $\bar{\varkappa}_1$ obtained experimentally is 30% greater than that according to the precise solution. At small velocities of the flowing stream, and likewise at a removal from the body where the excess pressure in the wave is comparable to the original pressure of the gas, the form of the front is still not parabolic.

It is interesting to use for the description of a leading shock wave the experimental results obtained in investigating the explosion in air of long VV cord charges [5]. The characteristic value in the explosion which defines all the parameters of the shock wave at a given distance from the axis of the explosion, is the linear scale of a cylindrical explosion

$$\lambda = \sqrt{\frac{q_l}{p_0}} \qquad (1)$$

where q_l is the energy of the explosion at a unit of length and p_o the initial pressure of the gas. The dependence of the excess pressure on the front of the shock wave $\Delta p_m / p_o$ upon the dimensionless distance to the axis of the explosion $\xi = r/\lambda$ in the range of $25 < \Delta p_m/p_o < 0.25$ is well described by the empirical formula

$$\frac{\Delta p_m}{p_0} = \frac{0.24}{\xi^2} + \frac{0.48}{\xi^{3/4}} \qquad (2)$$

In comparing the experimental results for the parameters of shock waves obtained in the explosion of cylindrical VV charges and those for flow at great supersonic velocity around bodies it is necessary to express the equivalent energy q_1 and the characteristic linear scale of a cylindrical explosion λ by the parameters of the flowing stream and by values characterizing the dimensions and shape of the body.

The equivalent energy of explosion per unit of length q_1 is proportional to the full force of the frontal resistance E

$$q_1 = \eta E \tag{3}$$

which is a projection of the forces of the pressure acting on the body in the direction of the flowing current. At a great supersonic velocity of the flowing stream the pressure on the surface of the body determined, with an accuracy acceptable for practical purposes, according to the Newtonian formula, is proportional to $\cos^2 \Theta$ (Θ is the angle between the normal to the surface of the body and the direction of the flowing stream). In this case the full force of the frontal resistance of a blunt-nosed rotating body is expressed by the formula:

$$E = 2\pi p_k \int_0^R \cos^2 \theta r\, dr = c_E \pi R^2 p_k \tag{4}$$

where R is the greatest radius of the body and p_K is the pressure at the leading point of the body, which is equal to the pressure behind the forceful shock wave $p = 2\rho_0 V^2/(\gamma + 1)$ on account of increase in pressure due to stoppage of gas on the section from the front of the wave to the surface of the body. In this the precise factor $[(\gamma+1)^2/4\gamma]^{\gamma/\gamma-1}$ for practical purposes does not differ from the factor $(\gamma + 3)/4$ which is obtained if the gas behind the front of the shock wave is considered an incompressible fluid.

Thus we get

$$p_k = \frac{\gamma+3}{2(\gamma+1)}\rho_0 V^2 = \frac{\gamma(\gamma+3)}{2(\gamma+1)} p_0 M_a^2 \qquad (5)$$

and further,

$$E = c_E \frac{\pi\gamma(\gamma+3)}{2(\gamma+1)} p_0 M_a^2 R^2 \qquad (6)$$

$$\lambda = \sqrt{\frac{\eta E}{p_0}} = \sqrt{\eta \frac{c_E \pi\gamma(\gamma+3)}{2(\gamma+1)}} M_a R \qquad (7)$$

In these formulas V and M_a are the velocity and Mach number of the flowing stream; p_0, ρ_0 and c_0 are the pressure, density and speed of sound in an undisturbed gas; γ is the ratio of the individual heat capacities of the gas.

The coefficient of the form c_E equals

$$c_E = \frac{2}{R^2}\int_0^R \cos^2\theta(r)\, r\, dr \qquad (8)$$

The coefficient of proportionality η in (3) and (7) between the energy of explosion per unit of length and the force of the frontal resistance is determined in a comparison between the experimental data according to the explosion and according to the supersonic flow around blunt-nosed bodies.

The value of $1/\eta$ acquires significance relative to the propor' nal pressure of trinitrotoluene explosion in air.

For the angle of inclination of the front of the wave to the axis within the limits of applicability of the law of plane sections we have approximately

$$\frac{d\xi_s}{d\zeta_s} = \frac{M}{M_a} \qquad (9)$$

at conditions $M^2/M_a^2 \ll 1$, where $\zeta = z/\lambda$ is a dimensionless distance along the axis, $\xi = r/\lambda$ a dimensionless distance along the perpendicular to the axis of the explosion, $M = D/c_0$ and $M_a = V/c_0$ are the velocity of the front of the shock wave and the source of the wave related to the velocity of sound in an undisturbed gas. For the front of the shock wave we have

$$M = \sqrt{1 + \frac{\gamma+1}{2\gamma}\frac{\Delta p}{p_0}} = \sqrt{1 + \frac{\gamma+1}{2\gamma} f(\xi)} \qquad (10)$$

Substituting (10) in (9) and considering $\xi = 0$, if $\zeta = 0$, for M_a = const we obtain the equation for the line of the front

$$\frac{\zeta}{M_a} = \int_0^{\xi} \frac{d\xi}{\sqrt{1 + \frac{\gamma+1}{2\gamma} f(\xi)}} = \Phi(\xi) \qquad (11)$$

Formula (11), when (7) is substituted in it, expresses the law of similarity of the flow around axially symmetric bodies with the blunting obtained by G. G. Cherniy [1].

The table presents the results of approximate numerical integration, made at $f(\xi)$, given by the empirical formula (2).

To compare the leading wave in the experiments on supersonic flow around bodies with that calculated according to (11) (see Table) the data in the literature have been extracted [4, 6-10]. In these works they investigated the flow of a stream of gas (air, helium, carbon tetrachloride vapor) with Mach numbers from 5.8 to 22.6 around spheres ($c_E = \frac{1}{2}$) and cylinders of different length with a hemispheric ($c_E = \frac{1}{2}$) and plane ($c_E = 1$) leading part (the stream parallel to the axis of the cylinder).

Comparison showed that when the value of the coefficient $\eta = 2.0 \pm 0.2$ so that

$$q_1 = (2.0 \pm 0.2) E \qquad (3')$$

ξ	$\frac{\zeta}{M_a}$	ξ	$\frac{\zeta}{M_a}$	ξ	$\frac{\zeta}{M_a}$
0	0	1.0	0.555	4.5	3.705
0.05	0.002	1.1	0.634	5.0	4.173
0.10	0.010	1.2	0.714	5.5	4.643
0.15	0.023	1.3	0.796	6.0	5.116
0.20	0.040	1.4	0.879	6.5	5.592
0.25	0.060	1.5	0.963	7.0	6.069
0.30	0.082	1.6	1.047	7.5	6.546
0.35	0.108	1.7	1.133	8.0	7.024
0.40	0.135	1.8	1.219	8.5	7.502
0.45	0.165	1.9	1.306	9.0	7.981
0.50	0.195	2.0	1.393	9.5	8.461
0.60	0.261	2.5	1.841	10.0	8.943
0.70	0.330	3.0	2.288		
0.80	0.403	3.5	2.756		
0.90	0.478	4.0	3.249		

excellent agreement of the shape of the front of the leading wave is observed in experiments on the flow around bodies with the line of the front calculated according to the data for the explosion of a cylindrical VV charge. In Figure 1 the continuous line shows the form of the front according to (11) (see Table) and the symbols show the form of the front in experiments on supersonic flow around bodies. For bodies with a hemispheric tip in air $\gamma = 1.4$ $\eta = 2.0$ according to (7) there occurs a simple expression for the linear scale of the cylindrical explosion

$$\lambda = M_a d \qquad (12)$$

where d is the diameter of the body. The coincidence of the experimental data indicates that other parameters of the leading wave (pressure, duration, etc.) can also, within the limits of the law of plane sections, be calculated according to the formulas for the shock wave of explosion of a cylindrical VV charge.

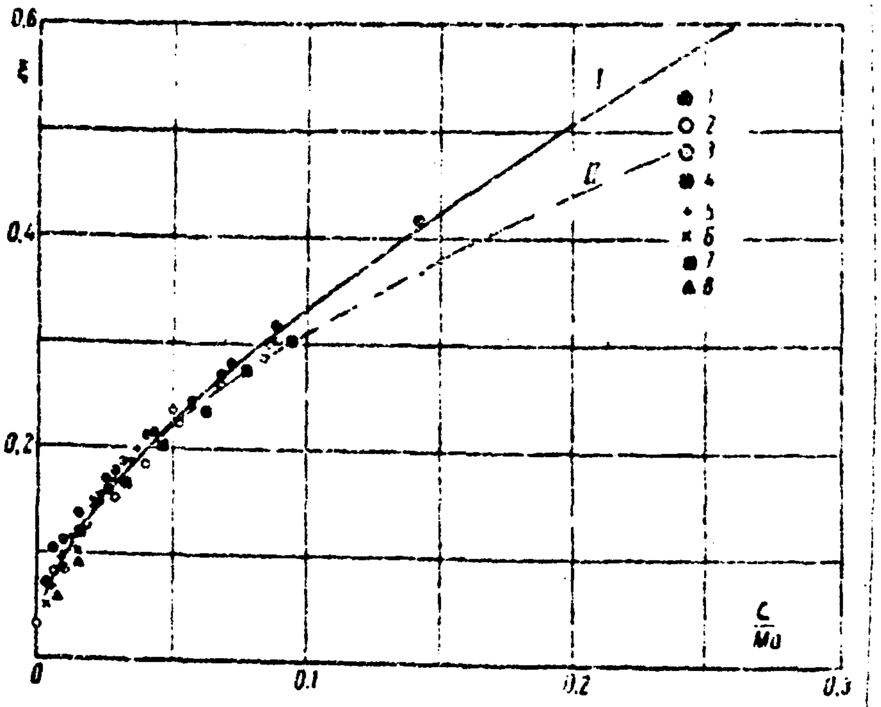

Fig. 1. Form of the front of a leading shock wave. I - according to the explosion date (Table 1), II - parabolic form of the front. Conventional symbols: 1) according to the data for the explosion of cylindrical charges; 2) according to [4], hemisphere-cylinder in air, M = 7.7; 3) according to [6], hemisphere-cylinder in air, M = 5.8; 4) according to [7], with end forward, M + 6.85; 5) according to [8], hemisphere-cylinder in helium, M = 22.0; 6) according to [9], hemisphere in air, M = 14.2; 7) according to [10], sphere in air, M = 8.1; 8) according to [10], sphere in CCl_4 vapors, M = 22.6.

Comparison of the form of the shock wave of explosion of a cylindrical VV charge with the parabolic form of the front obtained according to the precise solution for a forceful explosion shows that in the zone where the form of the front is close to the parabolic the results agree well in the ratio of the energy of the VV explosion q_1 and the forceful explosion E_0 per unit of length

$$q_l = 0.7E_0 \qquad (13)$$

A similar ratio was obtained earlier for a spherical explosion [5]. Hence for the force of the frontal resistance of the body E we have

$$E = 0.35E_0 \qquad (14)$$

The coefficient $\bar{\varkappa}_1$ in (A) should therefore be increased $(E_0/E)^{\frac{1}{4}} = 1.3$ times, which agrees with the experimental data.

At a considerable distance from the source of the explosion the shock wave obeys the asymptotic laws of propagation first obtained by L. D. Landau [11]. At the present time there exist methods of calculating the intensity of a shock wave remote from a body moving at supersonic speed [12]. On the score of application for the shock wave of an explosion, asymptotic formulas have been obtained in [13, 14] in a very convenient form. For a cylindrical explosion they can be substituted in the form

$$\frac{\Delta p_m}{p_0} = \frac{k}{\sqrt{\xi}\sqrt{\sqrt{\xi}-\sqrt{\xi_*}}} \qquad (15)$$

$$\frac{c_0\tau}{\lambda}\cos\Theta = \frac{\gamma+1}{\gamma}k\sqrt{\sqrt{\xi}-\sqrt{\xi_*}} \qquad (16)$$

where Δp and τ are the maximum excess pressure behind the wavefront and the duration of the phase of compression in the wave, Θ is the Mach angle such that $\sin\Theta = 1/M_a$, and K and ξ_* are constants determined from the experiment. As a result of establishing a connection between formulas

(15) and (2) the following values of the asymptotic coefficients were obtained:

$$k = 0.4, \qquad \xi_* = 0.5$$

so that the formulas for the wave in air ($\gamma = 1.4$) take the form

$$\frac{\Delta p}{p_0} = \frac{0.4}{\sqrt{\xi}\sqrt{\sqrt{\xi} - 0.7}} \qquad (15')$$

$$\frac{c_0 \tau}{\lambda} \cos\theta = 0.68\sqrt{\sqrt{\xi} - 0.7} \qquad (16')$$

Comparison of the results obtained for the shock wave of an explosion with a ballistic wave has been made according to the data of papers [13, 15] in which the results are given for measurements of the amplitude and duration of a wave from a charge moving with supersonic speed. In this the ratio between the energy of the equivalent cylindrical explosion and the force of resistance is assumed according to (3') to be in agreement with the results of the comparison of the experimental data according to the form of the shock wave during explosion and during supersonic flow around bodies. The force of the resistance was measured in [13] by its authors and for a given work [15] has been recalculated with the data of [13] according to a known velocity of movement and according to the diameter of the charge. The dependences of (15') and (16') are represented in Figure 2 by continuous lines. The duration of the compression phase (Fig. 2) is assumed to be half the time interval between the first and second front of the N-wave.

Comparison shows that at ratio (3') the parameters of the ballistic shock wave are well described by the asymptotic formulas for the shock wave of an explosion.

Thus the experimental results obtained for the shock wave of an explosion of a cylindrical VV charge are fully utilizable for description of a leading shock wave

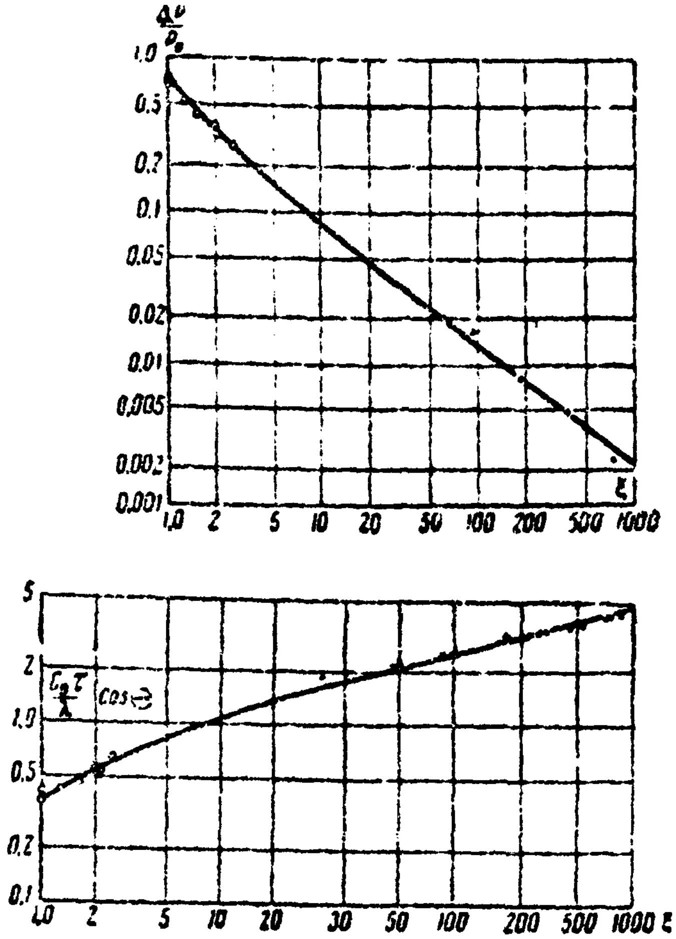

Fig. 2. a) dependence of pressure on the front of a shock wave on the distance given according to (15'), o - data for the explosion, + - data according to [13]; b) dependence of the duration of the compression phase in the shock wave on the distance according to (16'): o - data for the explosion, + - according to [13], x - according to [15].

both near (in the case of a blunt-nosed body) and remote from a body moving with supersonic speed. In this it is necessary to assume ratio (3') exists between the energy (heat) of the explosion of VV per unit of length and the force of resistance during the flight of the body. The mechanism of formation of the shock wave is approximately the same both in the case of explosion of a VV charge and in that of flight of a body with supersonic speed; in this and the other case the shock wave in the gas is formed under the action of a rapidly expanding piston which displaces the surrounding gas and forms the shock wave in it. In the case of an explosion the products of detonation of the VV are the piston, and in the case of flight of a body, the body itself. Ratio (3'), which turns out to be the same both near and far from the body, indicates that the process of formation of a shock wave during the flight of bodies is more "ideal" and linked with less "useless" losses of energy than in the propagation of products of an explosion.

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