Handbook

Reliability Engineering

PUBLISHED BY THE DIRECTION OF
CHIEF OF THE BUREAU OF NAVAL WEAPONS

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FOREWORD

The Reliability Engineering Handbook has been prepared to fill an increasing need for a manual of reliability methods suitable for application by project management and engineering personnel within the Bureau of Naval Weapons—to assist in the full exploitation of available reliability assurance measures in the planning, direction, and monitoring of their respective development programs.

This handbook differs from others in that it demonstrates step-by-step procedures for the *application* of methods to fulfill specific program requirements, and it references other documentation for more detailed treatment of the principles which underlie the methods. The handbook attempts to satisfy the need for a "digest" of these principles, however, through practical examples drawn from the several phases of the system life cycle. This first edition presents specific procedures for effective planning, achievement, management, and control of reliability, with emphasis on the conceptual and early design phases of system development.

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INTRODUCTION

1-1 RELIABILITY AS AN ENGINEERING PRACTICE

1-1-1. The Importance of Reliability to System Effectiveness

Reliability is generally defined as the "*probability of successful performance under specified conditions of time and use.*" As it relates to military products – weapon systems, equipments, components, parts, and even the processes by which these products are combined into a tactical entity – reliability is one of the important characteristics by which the tactical suitability of a product is judged.

In the case of the weapon system, tactical suitability is measured in terms of its operational effectiveness in the intended tactical role, and reliability is one of the more consequential of the effectiveness parameters. As tactical roles and "mission" requirements become more sophisticated – to keep pace with the changing threat – weapon systems become more complex in the functional configuration necessary to satisfy increased performance requirements. As system complexity increases, system reliability invariably becomes more problematical – more elusive as a design parameter. Not only does it become more difficult to define and achieve as a design parameter, reliability becomes more difficult to control and *demonstrate* in production and thus more difficult to *assure* as an operational characteristic under the projected conditions of use. These difficulties can, at most, only be minimized. They can never be completely eliminated, for it has become apparent that a predictable upper limit of reliability feasibility exists for a given system concept or design approach.

It is also now recognized, however, that with the exercise of very deliberate and positive reliability engineering methods throughout the evolutionary life cycle of the weapon system – from the early planning stages through design, development, production, and the inevitable product improvement phases – this upper limit of reliability feasibility can be attained, and perhaps exceeded. Like other system characteristics, reliability is a *quantitative* characteristic – predictable in design, measurable in test, assurable in production, and maintainable in the field. Reliability is thus *controllable* throughout the system life cycle and can, then, be monitored and guided at each step of system development to assure a high probability of program success long before delivery of the system to the Fleet.

1-1-2. Purpose and Scope of the Handbook

This handbook provides step-by-step procedures for the definition, pursuit, and acquisition of required reliability and maintainability in Naval weapon systems, equipments, and components. The methods presented are generally applicable to all categories of weapon system elements – electronic, electro-mechanical, mechanical, hydraulic, chemical, etc. – although the examples chosen to illustrate the application of specific procedures are drawn largely from experience with electronic and electromechanical systems because of the ready availability of documented experience with these systems.

Although the handbook is primarily a "reliability" handbook, considerable attention has been given to maintainability as a second important ingredient in the system effectiveness equation. Procedures are therefore included for the computation, assessment, measurement, and specification of maintainability as a design controlled characteristic essential to overall system operational effectiveness.

The handbook is written to fill three basic needs within the Bureau of Naval Weapons and its contractor facilities:

Project Management –
general guidance for the *implementation* of selected reliability program functions and engineering procedures at appropriate points in the system life cycle.

Project Engineering –
step-by-step demonstration of the engineering procedures used in the actual *performance* of these reliability program functions.

Design Engineering –
procedures and technical detail sufficient for *design guidance* in the actual achievement of required reliability and maintainability, as inherent features of design.

1-1-3. Reliability is a "Growth" Process

Reliability and maintainability are characteristics that can be both created and destroyed. The creation of a reliable product comes from planning, designing, testing, producing, and ultimately using the product according to a set of preconceived "effectiveness-oriented" procedures. The destruction or degradation of reliability in an otherwise satisfactory product comes from ignorance or disregard of these same procedures at any single point in the evolutionary "growth" cycle of the reliability-acquisition process.

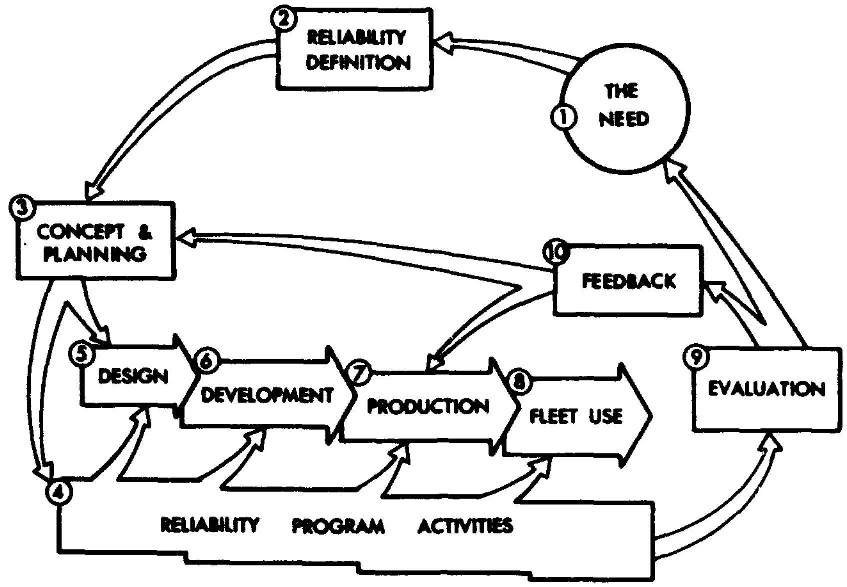
Reliability-oriented procedures, then, are the important tools by which reliability instinctiveness and craftsmanship are fostered, evaluated, and guided to assure a prescribed rate or reliability growth during the life cycle of the weapon system, from the conceptual stage through design, development, production, and Fleet use phases. Orderly reliability growth does not come about without the continuous application of effective reliability assurance measures. Nor can it survive without the decisive guidance of an enlightened management.

1-1-4. Organization and Use of the Handbook

Figure 1-1 identifies applicable chapters within the handbook corresponding to major reliability assurance functions to be performed in the design and development of a reliable weapon system. The functions are listed in the approximate chronological order of their application during the development phase, and for this reason the figure also serves as a basic checklist of things to be done in planning a new program.

TO PERFORM THESE RELIABILITY FUNCTIONS	USE THESE CHAPTERS OF THE HANDBOOK									
	1	2	3	4	5	6	7	8	9	10
Define requirements		●								
Estimate feasibility		●								●
Allocate reliability		●								●
Prepare a TDP				●						●
Prepare a specification				●						
Prepare an RFP				●						
Estimate time and cost									●	
Prepare contract task statement				●						
Formulate a design		●			●					●
Review a design				●	●					
Evaluate design problems					●	●				●
Evaluate a product or process			●			●		●		
Design an acceptance test							●			
Plan a reliability program			●							
Monitor a reliability program			●					●		
Use a reliability "feedback" loop								●		
Make a failure analysis								●		
Make a field evaluation								●		
Conduct a training course	●	●	●	●	●	●	●	●	●	●
Manage a reliability program		●								

Figure 1-1. Ready-Reference Index for the Performance of Specific Reliability Functions



(1)(2) The tactical NEED for reliability must first be anticipated, and Specific Operational Requirements (SOR's) must reflect this need.

(3) Plans must then be laid to fulfill the reliability need (i.e., the TDP):
Reliability requirements defined and specified;
Reliability program plans formalized;
Proposal requests and contracts documented;
Reliability is thus "planned-for" from the start.

(4) The reliability program is implemented:
Reliability is "monitored" continuously.

(5) The *conceptual* system is designed:
Reliability is assessed in design review;
Design is revised to correct deficiencies;
Reliability becomes "designed-in" by requirement.

(6) A *prototype* is developed according to the design:
Reliability is evaluated by test;
Design is refined to correct deficiencies;
Reliability is "proven-in" by demonstration.

(7) The system is *produced* from the prototype model:
Parts, materials, and processes are controlled;
Equipment acceptability is determined by test;
Reliability is "built-in" by control.

(8) The system is *deployed* to the Fleet:
Operators and maintenance technicians are trained;
Operating and maintenance instructions are distributed;
Reliability is "maintained-in" by procedure.

(9)(10) The system is *evaluated* to determine that the original need has been *met*, and the feedback loop completes the cycle:
To guide product improvements;
To guide future development planning.

Figure 1-2. Points of Reliability Practice in the System Life Cycle

1-2 RELIABILITY DOCUMENTS APPLICABLE TO THE SYSTEM LIFE CYCLE

1-2-1. The System Life Cycle

The major points of reliability practice in a typical "system life cycle" are shown in Figure 1-2. Several reliability specifications and documents have been adopted by the Bureau of Naval Weapons to support its overall reliability assurance program – to give assurance that the life cycle of each system under its cognizance does ultimately satisfy the "need" as initially anticipated. These reliability documents can be arranged according to their applicability at different points in the life cycle, as shown in Figure 1-3.

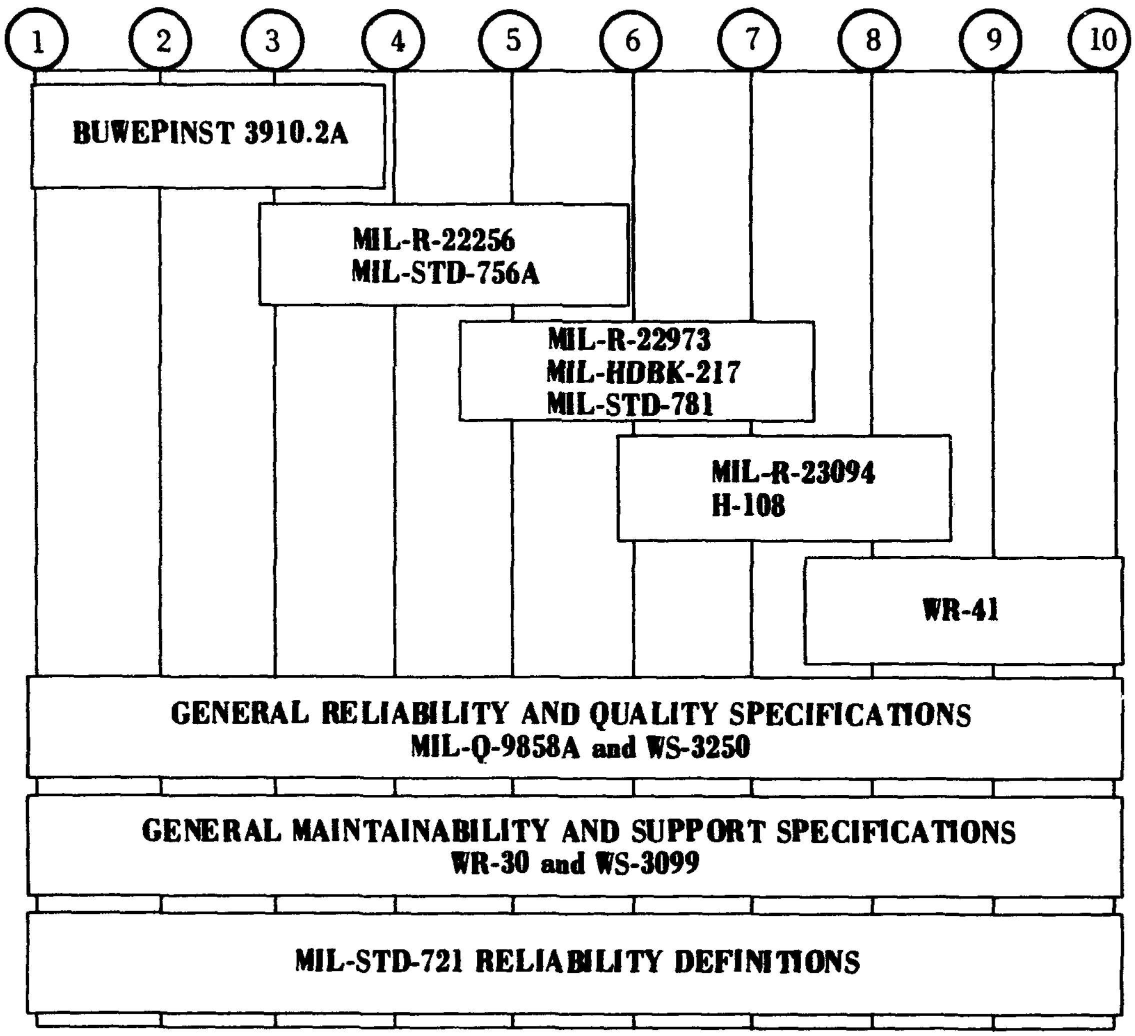


Figure 1-3. Documents Applicable to the System Life Cycle

The handbook has been prepared in support of Bureau of Naval Weapons policies and concepts as expressed or implied in these documents. Other documentation – handbooks, technical reports, and basic reference texts – has also been considered in the application of engineering and management procedures. A brief description of these documents follows.

1-2-2. *Specifications*

MIL-R-22256 **Reliability Requirements for Design of Electronic Equipment or Systems**

Outlines procedures to insure that electronic equipment designs will have a high level of inherent reliability before release to production. Sets forth detailed requirements for feasibility study and planning of the proposed design, reliability assessment of the design, and report preparation in accordance with MIL-STD-441. Prescribes tests for parts, subassemblies, and assemblies, to determine compatibility with proposed application in the design. Prescribes requirements for construction of prototype models and updating of reliability predictions and "design approval" testing. Specifies requirements for design evaluation test reports, and final reliability report at completion of development.

MIL-R-22973 **Reliability Index Determination for Avionic Equipment**

Establishes requirements and procedures to determine mean life of avionics equipment, by testing models of prototype, preproduction, or production equipment. Specifies requirements for test facilities, test conditions and duration, and definition of test levels. Outlines procedures for test and failure data recording, failure analysis and corrective action, MTBF estimation for prescribed levels of confidence. Sets forth detailed requirements for engineering reports.

MIL-R-23094 **Reliability Assurance for Production Acceptance of Avionic Equipment**

Establishes requirements and procedures to assure compliance with a specified MTBF requirement for production acceptance of avionics equipment. Sets forth requirements for equipments to be tested, test equipment, test conditions and duration, debugging and thermal survey, maintenance rules, and test data records. Defines specific conditions for five test levels (Levels I through V) with respect to temperature, altitude, vibration, humidity, heating and cooling cycles, and input voltage. Provides samples of suggested test data logs, failure and repair records. Presents sequential plans for two levels of consumer risk. Establishes requirements for preproduction assurance, requirements apportionment among components and parts for acceptance criteria, vendor control, training, etc. Prescribes contractor responsibility for failure analysis and repair or corrective action. Outlines accept/reject decision criteria and procedures to be followed in the event of either decision.

TEST LEVELS

Level	Factor	Conditions
I	Temperature Input Voltage	25 ± 5°C (68°F to 86°F) Nominal (within range specified for equipment)
II	Temperature Vibration Input Voltage	40 ± 5°C (95°F to 113°F) ± 2g non-resonant frequency, 20 and 60 cps Max. specified voltage +0 -2% at max. temp.; Min. specified voltage +2 -0% at min. temp.
III	Chamber Temperature Vibration Heating Cycle Cooling Cycle Input Voltage	-54°C to +55°C (-65°F to 131°F) Same as Test Level II Time to stabilize, plus 3 hours Time to stabilize at the low temperature Same as Test Level II
IV	Temperature Vibration Heating/Cooling Cycles Input Voltage	-65°C to +71°C (-85°F to 160°F) Same as Test Level II Same as Test Level III Same as Test Level II
V	Temperature Altitude Humidity Vibration Input Voltage	50° ± 5°C (113°F to 131°F) Normal (0 – 5000 ft.) Room ambient (up to 90%) Same as Test Level II Nominal (within range specified for equipment)

MIL-S-23603 **System Readiness/Maintainability; Avionic Systems Design, General Specification for**

Specifies one of the major requirements for system effectiveness as it relates to avionics systems and subsystems. Equipment complying with these requirements shall be designed to meet the requirements for maintainability and system readiness without reduction in the functional system performance. All levels of maintenance, including certain airborne maintenance functions, are considered in this specification.

MIL-Q-9858A **Quality Program Requirements**

Specifies requirements for an effective and economical quality program, planned and developed in consonance with the contractor's other administrative and technical programs. Design of the program shall be based upon consideration of the technical and manufacturing aspects of production and related engineering design and materials. The program shall assure adequate quality throughout all areas of contract performance – for example, design, development, fabrication, processing, assembly, inspection, test, maintenance, packaging, shipping, storage, and site installation.

All supplies and services under the contract, whether manufactured or performed within the contractor's plant or at any other source, shall be controlled at such points as necessary to assure conformance to contractual requirements. The program shall provide for the prevention and ready detection of discrepancies and for timely and positive corrective action. The contractor shall make objective evidence of quality conformance readily available to the government representative. Instructions and records for quality must be controlled.

The authorities and responsibilities for those in charge of the design, production, testing, and inspection of quality must be clearly prescribed. The program shall facilitate determinations of the effects of quality deficiencies and quality costs on price. Facilities and standards necessary to the creation of the required quality such as drawings, engineering changes, measuring equipment, and the like, must be effectively managed. The program must include an effective control of purchase materials and subcontracted work. Manufacturing, fabrication, and assembly work conducted within the contractor's plant must be controlled completely. The quality program also encompasses effective execution of responsibilities shared jointly with the Government or relating to government functions such as control of government property and government source inspection.

MIL-D-8706 **Contract Requirements for Aircraft Weapon Systems *Engineering Data and Tests***

Specifies requirements for engineering data and tests, including reports of contractor reliability program plans, reliability analyses and allocations, reliability test plans, and flight test results, for aircraft weapon systems.

MIL-D-8684 **Contract Requirements for Guided Missile System Design Data**

Specifies requirements for design data to be furnished under contracts for guided missile systems, and outlines specific reliability monitoring, testing, evaluation, and reporting requirements.

WS-3250 **General Specification for Reliability**

Covers general requirements to assure that reliability is given adequate and uniform consideration in procurements sponsored by the Bureau of Naval Weapons. Requires the achievement of a prescribed level of reliability as set forth by the contract. Requires the establishment of a reliability assurance and monitoring program by the contractor, to assure that systems, equipments, and components meet the contract requirements. Prescribes, in general, also, the quality assurance provisions by which compliance with the requirements will be determined.

MIL-A-8866 **Airplane Strength and Rigidity Reliability Requirements, Repeated Loads, and Fatigue**

Contains the reliability criteria and repeated loads spectra applicable to procurement of airplanes, including a tabulation of service-life requirements for structural design of thirteen types of Navy aircraft expressed in terms of flight hours, flights, and landings, for particular flight maneuver spectra.

WS-3099 **General Specification for Maintainability**

Covers general requirements for contractors' maintainability programs and monitoring procedures, and prescribes requirements for maintainability prediction, evaluation, and reporting.

1-2-3. *Weapon Requirements Documents*

SAR-317 **Special Aeronautical Requirement: Reliability Analysis for Controls for Aeronautical Gas Turbine Power Plants**

Specifies a procedure for analyzing the power control systems of aeronautical gas turbine power plants for the effects of component malfunctions. Power control systems are here defined as all equipment used for measuring engine controlled variables and/or environment for control purposes and for manipulating engine variables for the purpose of maintaining engine operation within safe and satisfactory limits and for the purpose of establishing the required power or thrust condition. Included are such items as rpm, pressure, and temperature sensors; actuators for manipulating fuel flow and engine geometry for control purposes; computers with interconnect sensors and actuators; and power supplies such as electric generators or hydraulic pumps which are used *exclusively* for the control system. Control components used for auxiliary engine services other than producing thrust or power, such as anti-icing, afterburner cooling, bleed air for airplane services, fuel pumps, nozzles, manifold or fuel lines, are not included.

WR-30 **Integrated Maintenance Management for Aeronautical Weapons, Weapon Systems, Related Equipment**

Establishes the policy, terms, and conditions governing the implementation and execution of an integrated maintainability and support program for weapons, weapon systems, and related equipments to be procured under the contract in which this document is cited. It is the specific intent of this document to charter the Integrated Maintenance Management Team to manage the total Logistic Support Program. Accordingly, this document is designed to develop, early in a program, a maintenance plan which is tailored to specific commodities and contracts. The procedural details formerly spelled out in an effort to define all possible conditions have been deleted.

WR-41 **Reliability Evaluation**

Provides guidance in the collection and interpretation of failure data from tests and field evaluations to assess reliability achievement and problem areas.

1-2-4. Instructions

DOD INST 3200.6 **Reporting of RD and E Program Information**

Establishes requirements for the quantitative definition of reliability and mainatinability requirements in technical development plans (TDP's); requires a complete description of the program plan by which achievement of development goals are to be assured. Is applicable to all development programs. DOD RDT&E will base their approval of budget plans on the adequacy of the TDP as defined in this instruction.

OPNAVINST 3910.4A **Technical Development Plan**

Provides guidance for the preparation, submission, review, and implementation of technical development plans. Implements DOD Instruction 3200.6 within the Department of the Navy.

BUWEPINST 3910.2A **Instructions for Preparing Technical Development Plans (TDP's)**

Translates DOD and OPNAV Instructions for direct Bureau of Naval Weapons application.

1-2-5. Military Standards

MIL-STD-441 **Reliability of Military Electronic Equipment**

Describes factors to be considered in the study, planning, design, and construction of prototype models of new equipment. Provides an excellent outline of required contents for reports to be submitted during planning, design, and development phases. Equally applicable, in principle, to non-electronic systems.

MIL-STD-721 **Definition of Terms for Reliability Engineering**

Defines terms commonly used in reliability work. Important terms and symbols are presented in Appendix 1 of this handbook.

MIL-STD-756A **Reliability Prediction**

Establishes uniform procedures for predicting the quantitative reliability of aircraft, missiles, satellites, electronic equipment, and subdivisions of them throughout the development phases, to reveal design weaknesses and to form a basis for apportionment of reliability requirements to the various subdivisions of the product. Graphically portrays the effects of system complexity on reliability, to permit the early prediction of tolerance and interaction problems not accounted for in the simple multiplicative case and provides appropriate k factors by which to adjust MIL-HDBK-217 predictions for airborne, missile, and space environments.

MIL-STD-781 **Test Levels and Accept/Reject Criteria for Reliability of Non-Expendable Electronic Equipment**

Outlines a series of test levels for demonstration tests (also known as reliability index determination), longevity tests, the reliability qualification phase of production acceptance, and the sampling phase of production acceptance. Also outlines several test plans for use in the qualification phase and the sampling phase of production acceptance. The test plans are based on an assumption of an exponential distribution. This standard is intended to provide uniformity in reliability testing for the following purposes:

(a) Assist the preparation of military specifications and standards to the extent that standard test levels and test plans are used.

(b) Restrict the variety of reliability tests so that those conducting tests can better establish facilities therefor.

(c) Permit more realistic comparison of reliability data resulting from tests.

1-2-6. Handbooks

M-200A **Defense Standardization Manual**

Establishes format and general instructions for the preparation of specifications, standards, handbooks, and maintenance manuals. Appendix V-C suggests 60% confidence level for acceptance testing of parts for weapon systems, as a practical approach for reducing equipment development time and costs.

MIL-HDBK-217 **Reliability Stress and Failure Rate Data for Electronic Equipment**

Provides the procedures and failure rate data for the prediction of *part-dependent* equipment reliability from a stress analysis of the parts used in the design of the equipment. Must be used according to procedures outlined in MIL-STD-756A for estimates of MTBF and reliability, at the *system level,* on account for tolerance and interaction failures, and to adjust for the particular "use" environment.

NAVSHIPS 94324 **Maintainability Design Criteria Handbook for Designers of Shipboard Electronic Equipment**

The first part of the handbook discusses the maintainability concept. The second part presents a brief description of shipboard physical environment and a summary of maintenance personnel qualifications. Maintainability design criteria relating to equipment packaging, modularization and micro-miniaturization, testing, displays and controls, cables and connectors, and other design considerations are presented in the remaining six parts of the handbook. The design features recommended in this handbook are based almost entirely on maintainability considerations. Inasmuch as the final equipment design must also satisfy other requirements of the design specifications, such as those for reliability, operation, and size and weight, discussions of tradeoffs between maintainability and other specified requirements are included in various parts of the handbook.

H-108 **Sampling Procedures and Tables for Life and Reliability Testing**

This handbook describes the general principles and outlines specific procedures and applications of life test sampling plans for determining conformance to established reliability requirements.

1-2-7. Procedures Related to Specific Documents

Most of the reliability-maintainability-effectiveness documents described above explicitly define certain engineering or management procedures, test plans, and data requirements to be complied with in fulfillment of contractural requirements. Similar requirements are implicitly defined in others. All impose a responsibility upon the applicable project office, contractor, or contracting agency to do certain things to assure ultimate realization of ***known required*** system effectiveness in the Fleet. Figure 1-4 is an abbreviated directory for the guidance of those who become obliged to conform to the requirements of a particular document. Opposite each document identification number are indicated those sections of this handbook that will prove helpful in satisfying these requirements.

TO FULFILL REQUIREMENTS OF THESE DOCUMENTS — USE THESE CHAPTERS OF THE HANDBOOK

Document	1	2	3	4	5	6	7	8	9	10
MIL-R-22256		•	•	•	•					•
MIL-R-22973						•				
MIL-R-23094							•			
MIL-S-23063		•			•	•	•	•		
MIL-Q-9858A			•	•						
MIL-D-8706			•	•	•	•	•	•		
MIL-D-8648		•	•	•	•		•	•		
WS-3250			•	•		•	•	•		
MIL-A-8866		•			•		•	•		
WS-3099		•	•	•		•	•	•		
SAR-317				•		•	•			
WR-30		•	•	•	•	•	•	•		
WR-41								•		
BUWEPINST 3910.2A		•	•	•		•	•	•	•	•
MIL-STD-441		•	•		•					
MIL-STD-721				•						
MIL-STD-756A		•			•					
MIL-STD-781							•			
M-200A				•						
MIL-HDBK-217					•					
NAVSHIPS 94324		•	•							
OASD H-108							•			

Figure 1-4. Ready Reference Index for Compliance with Specified Documents

1-3 RELATIONSHIP OF RELIABILITY AND MAINTAINABILITY TO SYSTEM EFFECTIVENESS

1-3-1. System "Operational" Effectiveness

The "worth" of a particular system or piece of equipment is determined primarily by the effectiveness with which it does its job – its "operational" *effectiveness*. An acceptable level of effectiveness is required of every system destined for tactical use. Emphasis on reliability alone *does not* necessarily produce the required level of effectiveness. Other factors must be considered simultaneously. These are shown in Figure 1-5.

Each of these characteristics can be expressed as a "probability" of successful fulfillment of requirements, defined as follows:

Performance capability is the probability that the system[1/] will satisfy mission performance requirements when operating within specified design limits – a measure of "how well" it does its job when working properly.

Operational reliability is the probability that the system will maintain a specified level of performance throughout a given mission – a measure of "how long" it is capable of working without failure.

Tactical availability, or operational readiness, is the probability that at any point in time the system will be ready to operate at a

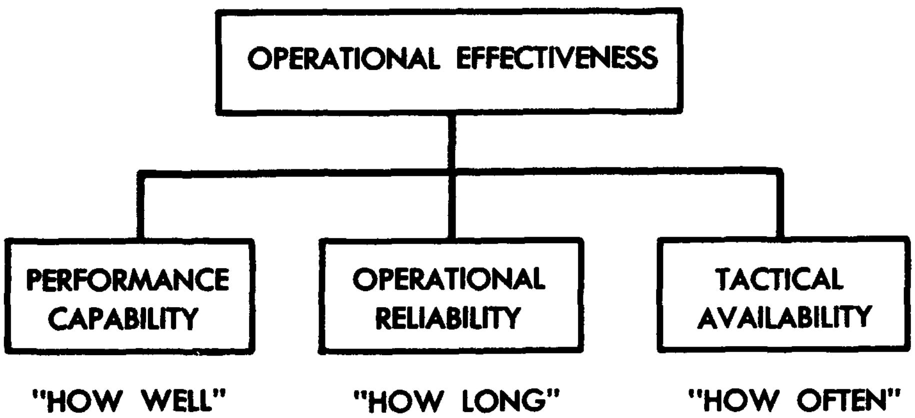


Figure 1-5. Definition of Operational Effectiveness

[1/] The words "equipment" and "system" can be used interchangeably.

specified level of performance, on demand – a measure of "how often" the system is ready when needed.

Operational effectiveness is the product of these three characteristics, i.e.:

Effectiveness = Performance × Reliability × Availability

Operational effectiveness of an equipment or system is then the *probability* that it can successfully meet an operational requirement, for the duration of the need, when the need arises.

Other factors – development time and cost, logistic supportability – also enter into an evaluation of "worth" during system planning. Within the bounds set by these other factors, however, operational effectiveness must be optimized by judicious *balance among attainable performance, reliability, and availability* characteristics, taking care not to stress the importance of one at the exclusion of the other two.

EXAMPLE: A VHF transceiver designed for 100-mile air-to-ground line-of-sight range is found to work over this range 90% of the time when properly tuned. The performance capability of the equipment with respect to its design specification is thus P = .9.

The transceiver has also demonstrated that in 9 flights out of 10, on the average, it will remain in operation for 5 hours without failure. Its reliability for a 5-hour mission is thus R = .9.

It has also been observed on the flight-line that 1 set in 10 is usually being worked on, and consequently would not be considered operationally ready for use if needed. Availability of the transceiver for flight operations is thus determined to be A = .9.

Overall effectiveness of the transceiver for 5-hour missions of 100-mile range is then estimated from

$$E = P \times R \times A$$

$$= (.9) \times (.9) \times (.9) \approx .73$$

In other words, the tranceivers in 7 aircraft in a flight of 10 could be expected to be ready, willing, and able to satisfy the specified tactical communication requirement when called upon.

1-3-2. The Concept of "Operational" Reliability

The reliability characteristic of an equipment or system – its "operational reliability" – is often described as the product of two constituent factors:

- An inherent reliability achieved in design and manufacture; and
- A use reliability degradation factor attributable to the shipping, handling, storage, installation, operation, maintenance, and field support of the system.

These are shown in Figure 1-6.

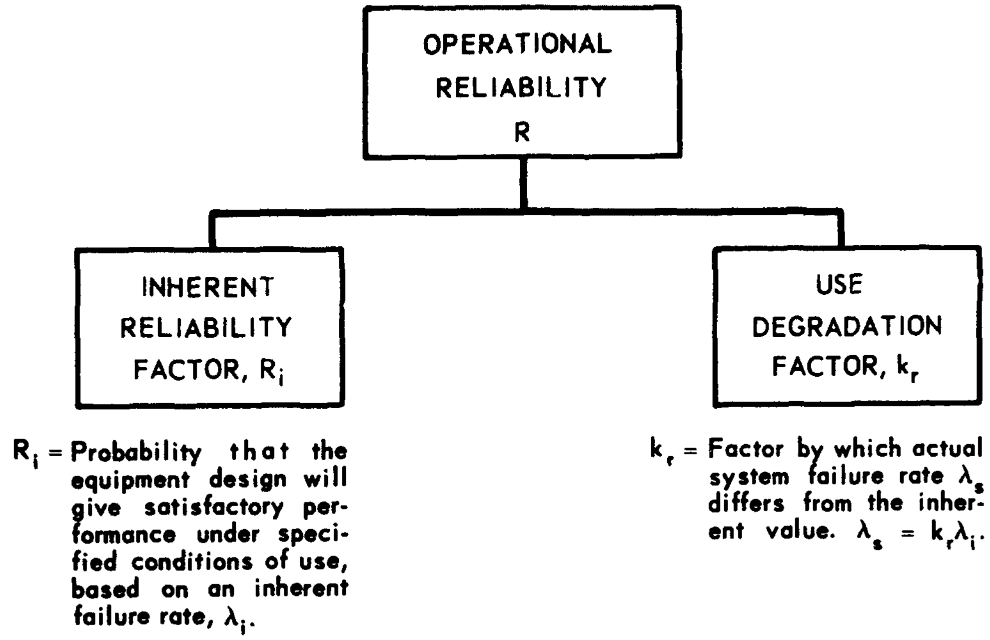


Figure 1-6. The Concept of Operational Reliability as a Combination of Two Factors

Operational reliability approaches the value of inherent reliability as development *test* conditions approach and more nearly simulate *use* conditions and, conversely, as *use* conditions approach the idealized *test* conditions under which the value of inherent reliability is measured, i.e., $k_r = 1$. It is quite obvious that the design concept, the development program, the manufacturing process, and the reliability test program must realistically anticipate, and "build to", the use requirement.

It is equally important that the tactical user understand the design intent of the equipment if its inherent reliability potential is to be fully exploited in the field.

EXAMPLE: The "bench-test" reliability of an airborne equipment is measured repeatedly and found to demonstrate a mean life (MTBF) of 50 hours. In actual Fleet use, however, the same equipment repeatedly demonstrates an MTBF of only 25 hours. This indicates a 2-to-1 reduction in equipment life, due to differences between "test" conditions and "use" conditions. These differences are reflected in the factor k_r[2/] due in this case to a value of $k_r = 2$.

[2/] The factor k_r operates on equipment failure rate. In the reliability expression for the exponential case,

$$R_i = e^{-\lambda_i t}$$

and $$R_s = e^{-k_r \lambda_i t} = e^{-\lambda_s t}$$

where $$\lambda_s = k_r \lambda_i$$

1-3-3. Reliability Definitions

The reliability of a product is generally defined as the *probability* that the product will give *satisfactory performance* for a *specified period of time* when used under *specified conditions*. When applied to a specific equipment or system, reliability is frequently defined as

- "the probability of satisfactory performance for specified time and use conditions"; or
- "the probability of a successful mission of specified duration under specified use conditions"; or
- "the probability of a successful [event]under specified conditions", where the event may be a missile launch, a flight, an intercept, or an "actuation" independent of time.

Whenever the definition is worded to fit a particular system or device, it is always necessary to relate *probability* to a precise definition of "success" or *"satisfactory performance"*; to specify the *time base* or operating cycles over which such performance is to be sustained; and to specify the environmental or *use conditions* which will prevail during this time period.

As a general rule, applicable to most electronic equipment of conventional design,[3/] a simple relationship exists between the reliability of an equipment and its mean life, or mean-time-between-failures (MTF or MTBF).[4/] This relationship is the "exponential" case, which holds when the "failure rate" of the equipment is constant during its service life, shown by the following equation:

$$R \text{ (for "t" hours)} = e^{-t/MTBF}$$

Because of this relationship, reliability may be expressed in terms of an allowable mean-time-between-failures (MTBF) or mean life (θ). An exponential function is illustrated in Figure 1-7.

Failure rate in the above exponential case is the reciprocal of mean life, represented by FR or λ (lambda):

$$FR = \frac{1}{MTBF} = \frac{1}{MTF} = \frac{1}{\theta} = \lambda.$$

3/ Designs in which redundancy has not been used extensively.

4/ "MTF" and "MTBF" are frequently used interchangeably, although MTF usually applies to the mean life of "one-shot" or non-repairable items.

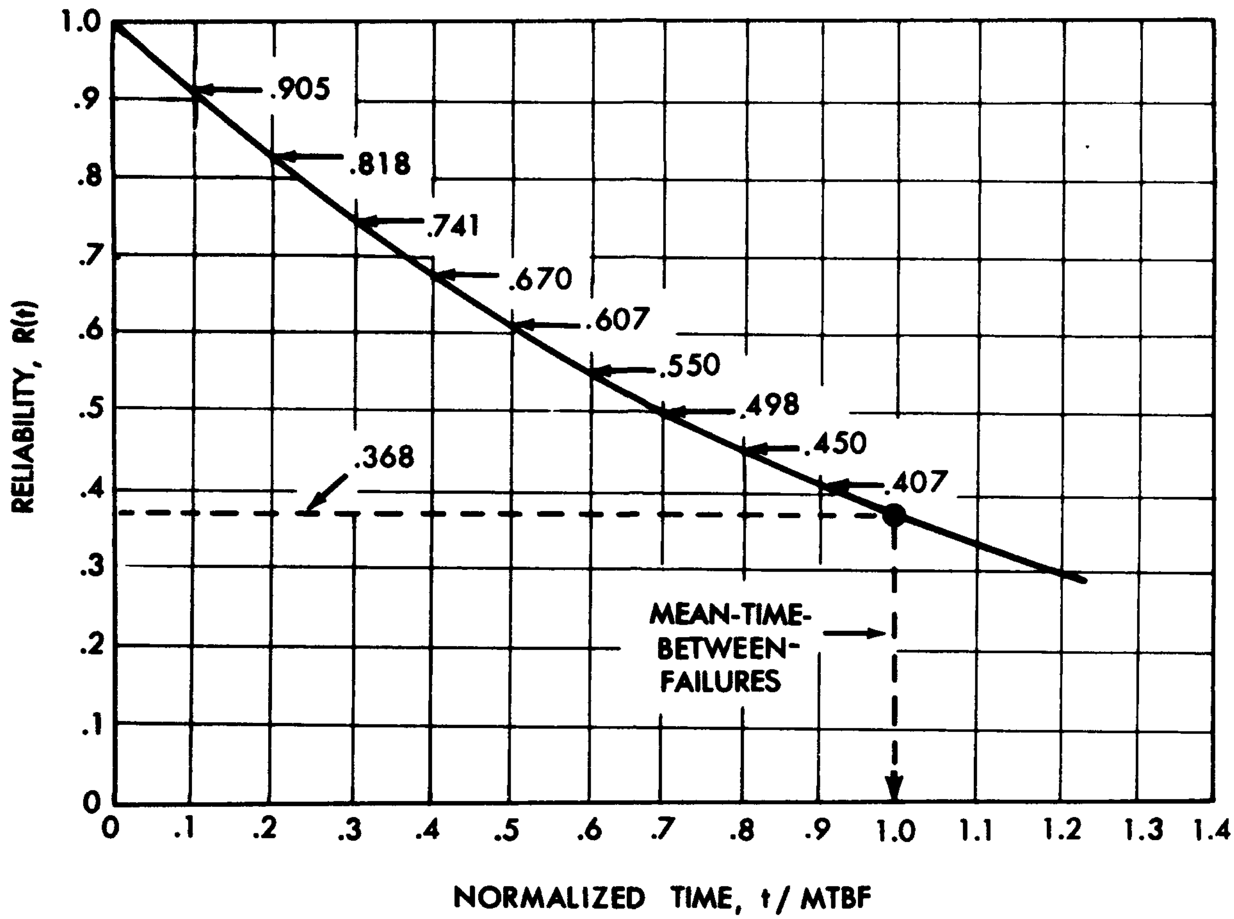


Figure 1-7. Exponential Reliability Function

1-3-4. Describing the Reliability Requirement

Figure 1-8 shows the relationship among the three basic definitions, applied to the same equipment at points 1, 3, and 4. Point 2 shows the choice of a probability definition to describe a high reliability requirement for a short mission, where time-to-failure thereafter is of secondary importance.

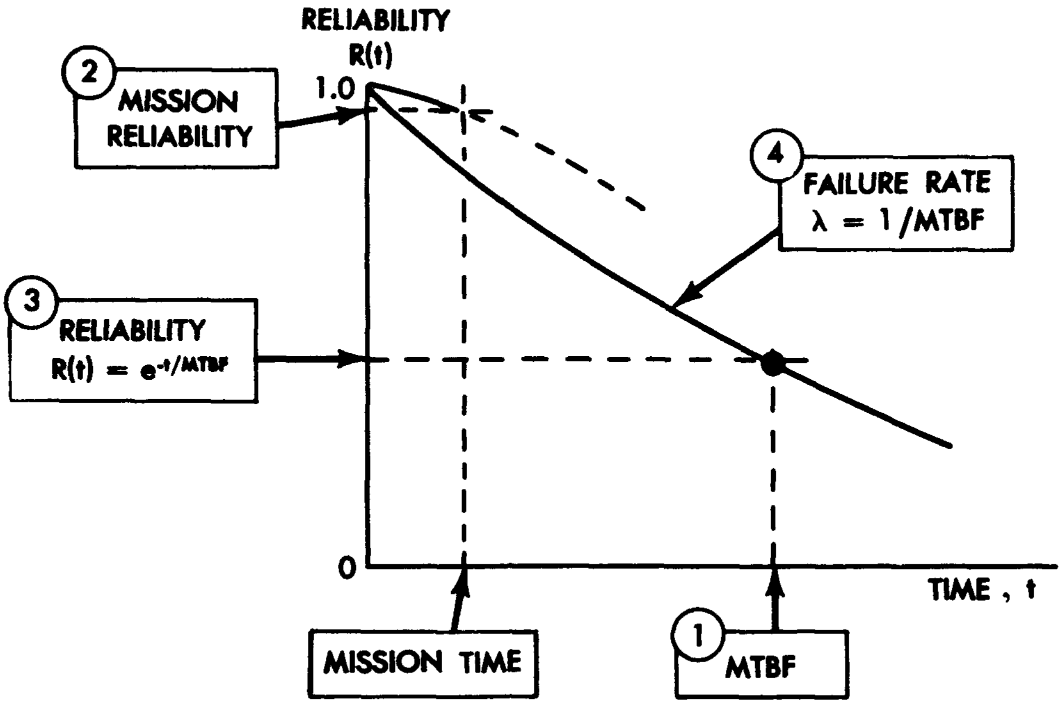


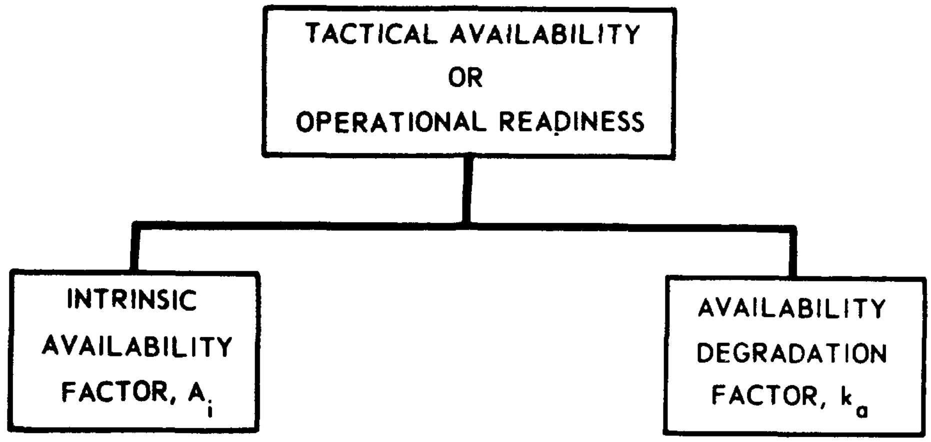
Figure 1-8. Probability and Time Relationships in the Definition of Reliability

1-3-5. Concept of "Tactical" Availability

Like reliability, tactical availability can be considered as a combination of two factors:

- *An "intrinsic" availability achieved in design* – the probability that the equipment will be operationally ready when needed at any point in time, under specified conditions of maintenance and logistic support; and
- *An availability degradation factor experienced in use* – the degrading effect of actual use conditions on the maintainability and supportability of the equipment, attributable to the degree of qualification of maintenance personnel, adequacy of test and repair facilities, sufficiency of spares provisioning, etc.

Figure 1-9 illustrates this concept.



A_i = Probability that the equipment design will satisfy availability requirements under specified conditions, based on an intrinsic repair rate, μ_i *achieved in design.*

k_a = Factor by which actual system repair rate differs from the intrinsic value. $\mu_s = k_a \mu_i$.

Figure 1-9. Concept of Tactical Availability as a Design Value and Use Factor

1-3-6. Availability as a Function of Equipment Maintainability and Mean Life

Availability is defined as:

$$A = \frac{MTBF}{MTBF + \overline{T}_r} = \frac{1}{1 + \overline{T}_r/MTBF} = \frac{1}{1 + \lambda/\mu}$$

where MTBF = Mean-time-between-failures or mean life;

$\overline{T}_r$ = Mean-time-to-restore equipment to operating status following failure

= Mean downtime for repair.

λ = Equipment failure rate

$$= \frac{1}{MTBF}$$

μ = Equipment repair (restoration) rate

$$= \frac{1}{\overline{T}_r}$$

($\overline{T}_r$ and μ include administrative and logistics downtime.)

If the ratio $\overline{T}_r$/MTBF is known, equipment availability can be derived from Figure 1-10.

EXAMPLE: A weapon control system has a mean-time-between-failures, MTBF = 20 hours. Maintenance logs show a mean-time-to-restore, $\overline{T}_r$ = 5 hours, including time required to localize the failure, obtain the replacement part, make the repair, and perform post-maintenance checkout. The ratio $\overline{T}_r$/MTBF = 5/20 = .25 is used to find availability in Figure 1-10. In this case, A = .8, i.e., the weapon control system can be expected to be in a state of operational readiness when needed 8 times in 10, on the average.

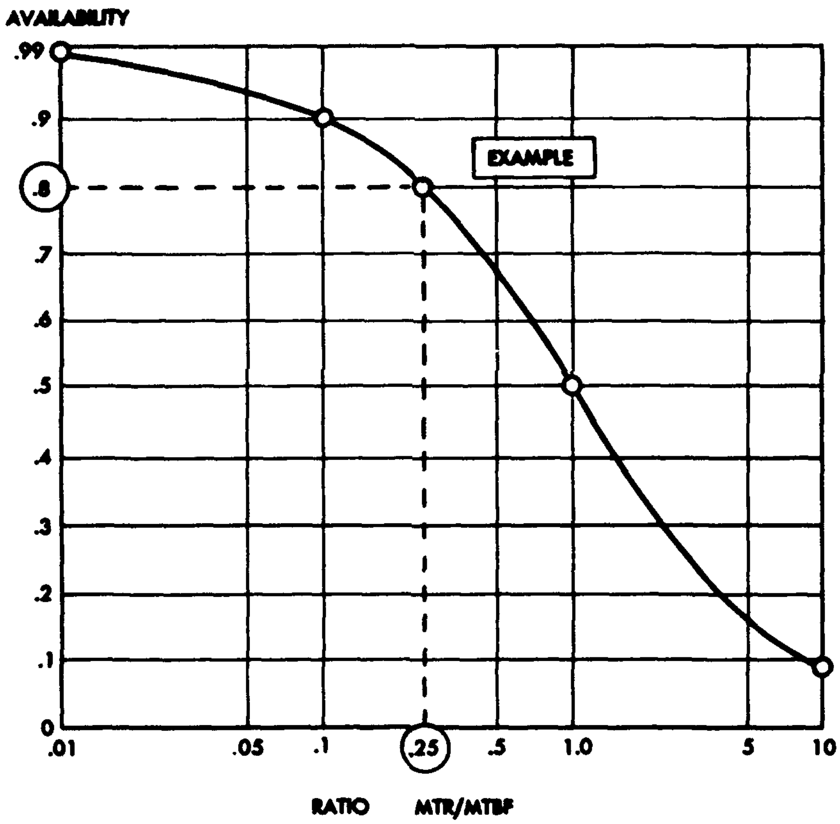


Figure 1-10. Availability as a Function of Mean-Time-To-Restore/Mean-Time-Between-Failures

1-3-7. Describing the Availability Requirement

The availability requirement can be described either as a required probability of operational readiness, or as a maintainability requirement. The latter is usually more amenable to design interpretation and measurement, so long as care is exercised in keeping the maintainability requirement compatible with both the reliability and availability requirements.

Figure 1-11 illustrates a typical maintainability function, with two basic methods for defining the maintainability requirement:

(1) As a mean-time-to-restore requirement. This definition does not control the distribution of repair times. The definition is useful for specifying maintainability of long-life systems.

(2) As a probability of restoration within a specified period of repair time, t_r. This definition is useful for equipment to be designed for high maintainability, employing reliability-with-repair or module maintenance concepts.

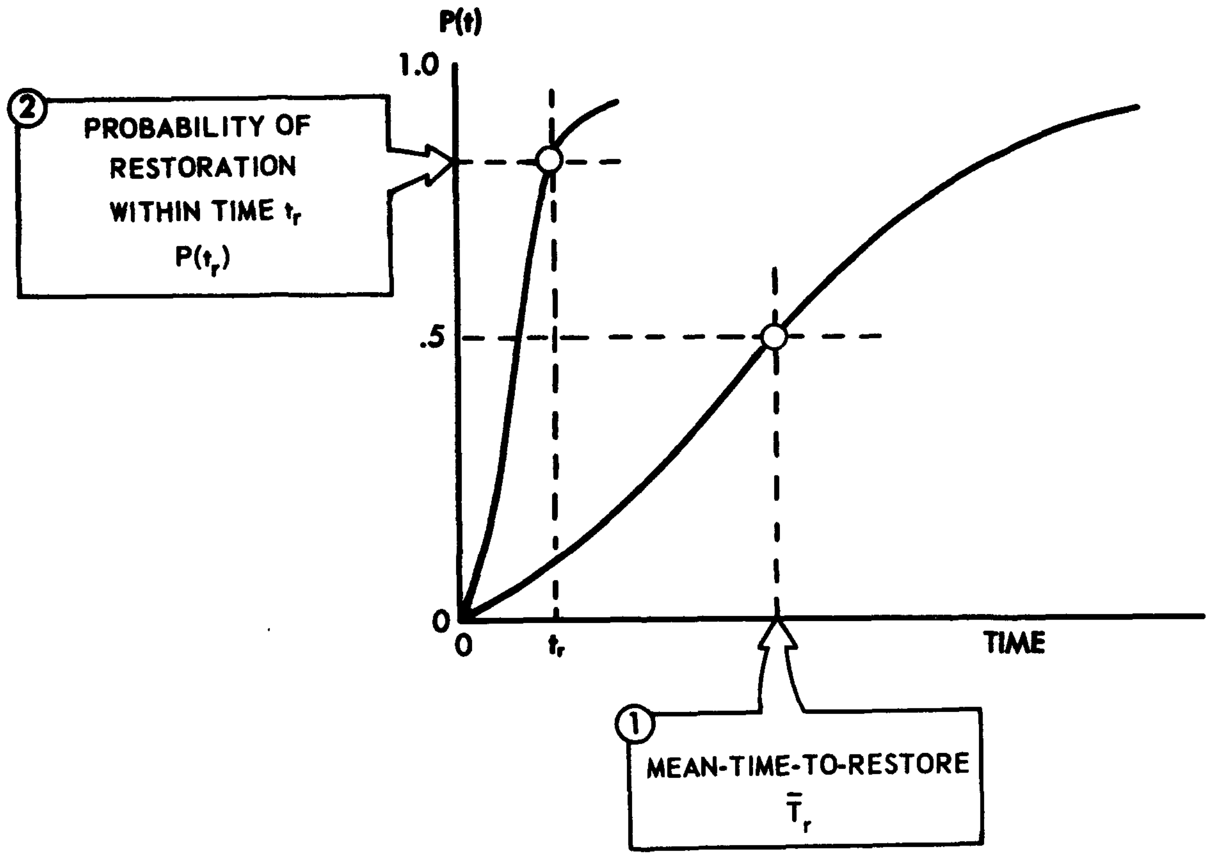


Figure 1-11. Two Ways to Define Maintainability

EXAMPLE: An airborne communications central is to be developed to meet an OPNAV specified effectiveness of 90% – i.e., it must be capable of operating on demand 9 times in 10 throughout a 5-hour mission. Initial tradeoff studies indicate a reliability feasibility of .92. Thus, an availability requirement of approximately .98 must be met to satisfy effectiveness requirements.

From Figure 1-10, a $\overline{T}_r$/MTBF ratio of .02 will be required.

From the nomograph of Appendix 3, an MTBF = 60 hours corresponds to R = .92 @ 5 hours.

Then $\overline{T}_r = .02 \times 60 = 1.2$ hours should be achieved if the availability requirement is to be simultaneously satisfied.

1-4 A REVIEW OF THE RELIABILITY SITUATION

1-4-1. Present Avionics Equipment

Figure 1-12 is plotted after the fashior of the familiar chart of MIL-STD-756A, showing several of today's avionics equipments superimposed for a graphical portrayal of "where we are *today,* on the basis of *yesterday's designs*". Each spot on the figure represents several equipments of a given type observed over an extended period in the Fleet. MTBF is measured as mean-time-between operator or pilot complaint. To convert to MTBF measured as mean-time-between technician-verified malfunction, add 25% to the MTBF shown in the figure.

The figure also shows a scale for computing reliability for 5-hour missions, R(5), and for determining mission operating time corresponding tc three different reliability requirements – .9, .95, and .99.

EXAMPLE: An avionics equipment (System "X") consists of 300 AEG's.[5] On the average, the equipment will demonstrate 10 hours MTBF. The 5-hour mission reliability of this equipment will then be .6. The equipment also can be predicted to demonstrate a reliability of .9 for a 1-hour mission, a reliability of .99 for a 6-minute mission. This assumes performance capability of 1.

[5] The AEG measure of complexity, discussed in Chapter 2, is based on the number of transistors, electron tubes, and power diodes used in the equipment.

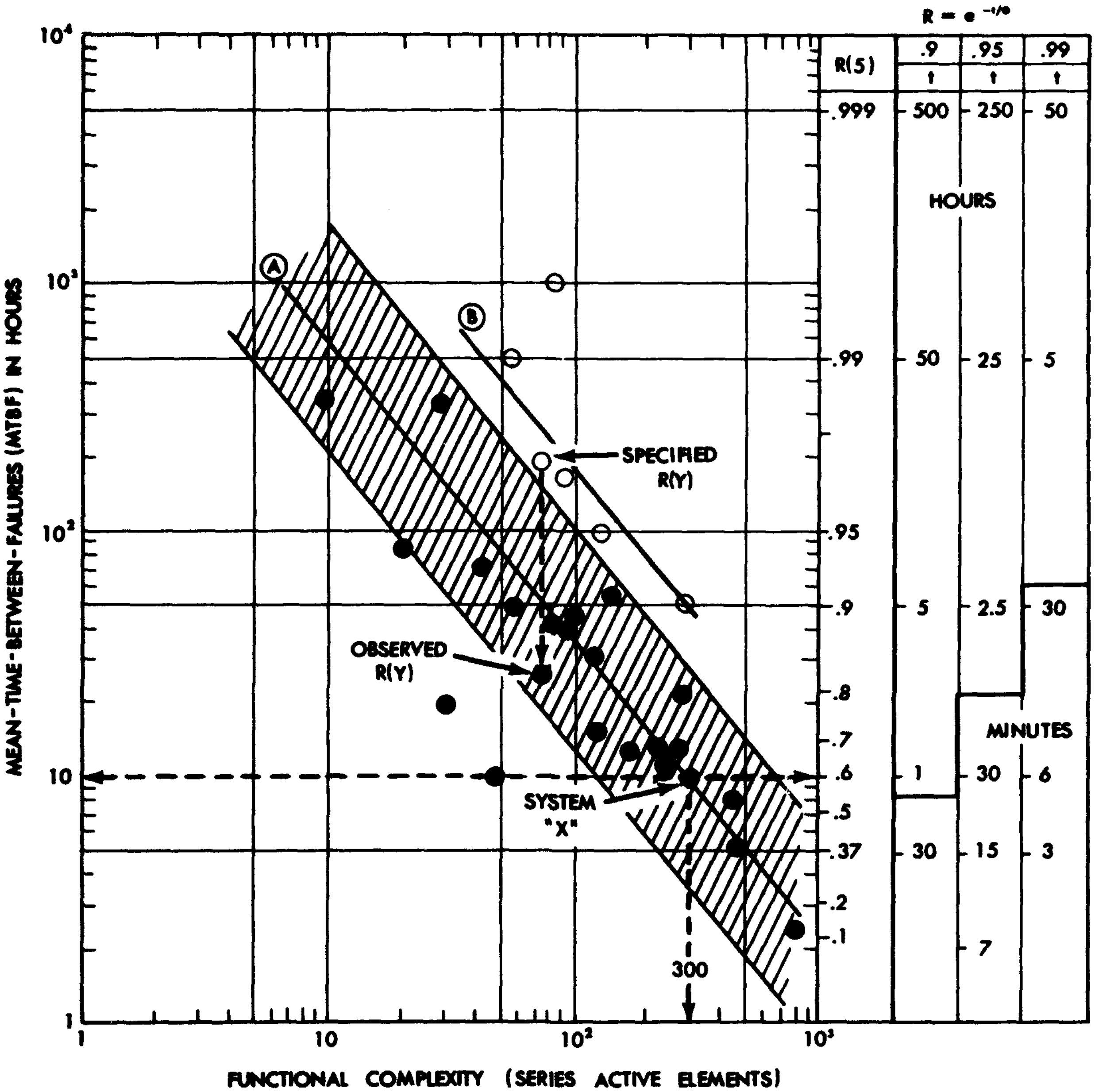


Figure 1-12. Observed Avionics Equipment Reliability (Analog Function)

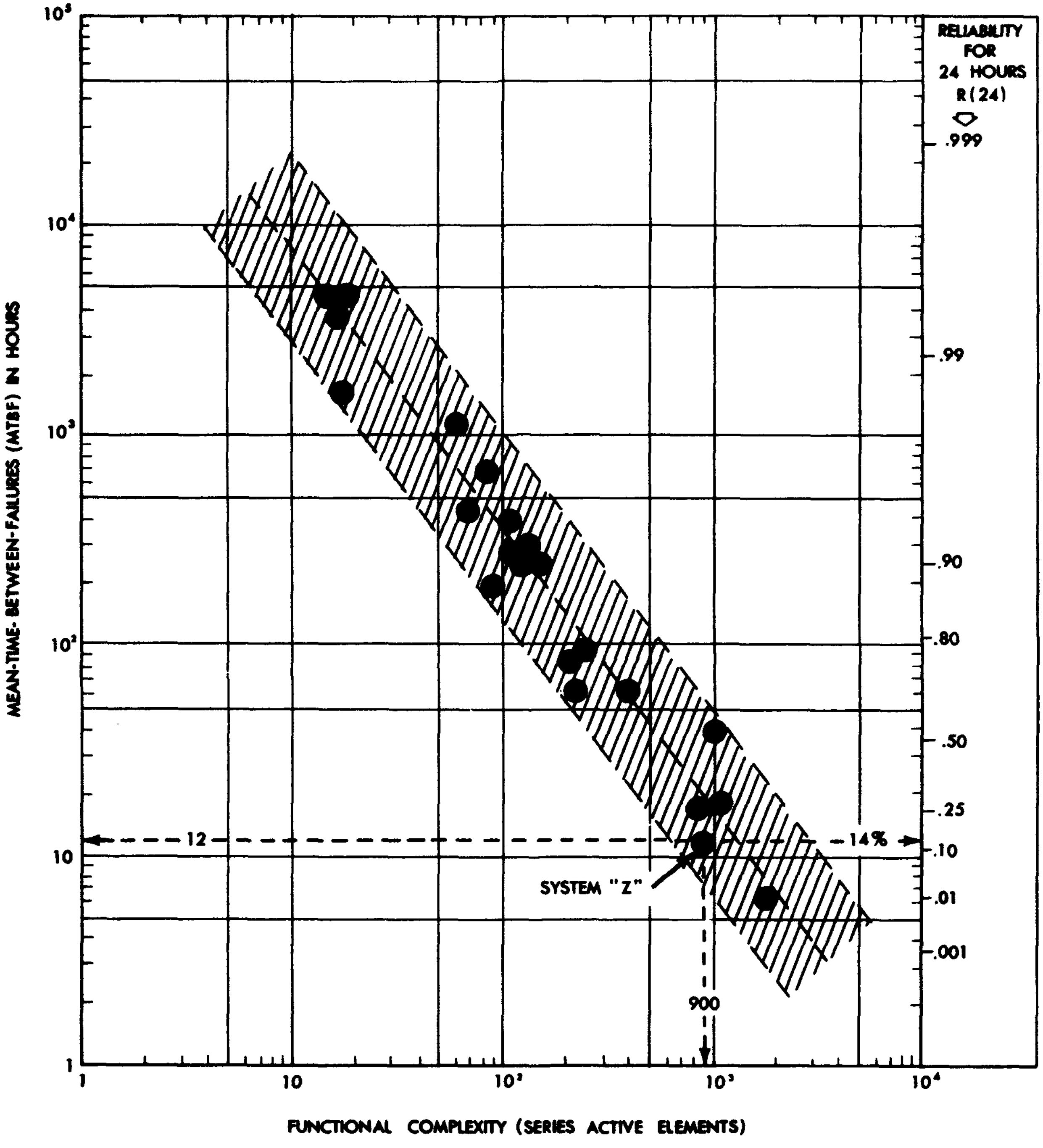


Figure 1-13. Observed Shipboard Electronic System Reliability (Analog Function)

Line A represents, on the average, what can be expected of conventional design; Line B represents an average of several "specified" MTBF requirements corresponding to those subsequently "observed". The difference between what has been "asked for" (defined as a requirement) and what was actually delivered to the Fleet has averaged about 7-to-1; i.e., ask for 700-hour MTBF to get 100 hours.

> *EXAMPLE:* The specification for equipment "Y" called out a requirement for 200 hours MTBF. Current Fleet experience shows MTBF = 25 hours – *one-eighth of* the specified requirement.

1-4-2. Present Shipboard Equipment

Figure 1-12 shows the same type of plot for today's shipboard and shore-based systems – radar and communication. The right-hand scale translates the MTBF measurement to a "probability of survival" for a 24-hour mission or prediction operating period.

> *EXAMPLE:* System "Z" is a shipboard fire control radar system made up of 900 transistors and electron tubes. Its mean-time-between-failures (MTBF) observed in the Fleet is 12 hours. Its reliability (probability of survival without failure) for a 24-hour operating period is about .14.

1-4-3. Future Prospects

New systems can be expected to become even more complex, to satisfy the needs for more integrated functions with increased performance, higher precision, and faster response characteristics. Increased complexity means shorter mean life, longer repair times, reduced availability, and consequently unacceptable levels of effectiveness at higher cost. All these parameters are tied very closely to equipment complexity of *conventional* design, by known factors – based on experience gleaned from past development programs. On the basis of this past experience, it can be predicted with confidence that a vast majority of the systems and equipments now going into development *can never achieve a satisfactory level* of operational effectiveness through conventional design approaches.

This designers' dilemma can be minimized through the following specific steps taken early in the equipment development program:

(1) Quantitative definition of equipment requirements, determined early in project planning:

   Performance
   Reliability
   Availability and Maintainability

(2) Realistic appraisal of design feasibility to satisfy these requirements by conventional approach, within space/weight cost and time limitations.

(3) Resolution of differences between required and feasible attainments by allocation and tradeoff, and by program planning to accommodate and support the required design approach.

(4) Translation of the resolved requirements into *quantitative*

Development Specifications
Demonstration Test Requirements
Acceptance Test Criteria
Program Monitoring Milestones

in order to motivate and *require* adoption of the necessary design approach and reliability assurance measures.

(5) Recognition of the need for R & D in specific areas, in support of new design.

The TDP, the development specification, the RFQ, and the resultant contractual document – all must clearly, completely, and accurately specify *what is required* and how it will be *tested for compliance*.

1-4-4. Project Engineering "MUSTS"

It is always easier to be critical of a state-of-being than it is to be helpful in the improvement of that state-of-being. The Bureau project engineer must be both critical and helpful. On the one hand, he must be reasonably hard to satisfy; yet, on the other hand, he must provide the motivation, guidance, and support required to assure contractor progress and *achievements* that *will* satisfy him. As related to the pursuit and acquisition of reliability objectives, this implies that the project engineer must:

- Know and define the level of reliability he wants;
- Recognize the disparity between what he wants and what he will probably get unless he exercises the required degree of "control" over the reliability growth process;
- Understand the application of certain of the "tools" available to him by which this controlled reliability growth can be *assured* – not merely promised.

The remaining chapters of this handbook outline some of the planning considerations and describe some of the procedures that can be fruitful, both in the achievement of required reliability in specific programs, and in the "self-critique/self-help" control of reliability on a programwide basis throughout the system life cycle.

CHAPTER 2
TECHNICAL REQUIREMENTS ANALYSIS, FEASIBILITY ESTIMATION, AND ALLOCATION

2-1 INTRODUCTION

2-1-1. General

The first and most important phase of the system life cycle is, logically enough, the planning phase, where system requirements are analyzed and translated into *well-defined* technical objectives and where detailed plans are laid to *assure* successful achievement of these objectives – in short, the success of the entire system development program hinges on how well the project engineer does his job *before the first development contract is awarded.*

The system or equipment to be developed is usually part of a larger system complex for which tactical requirements have been defined by CNO through a "Specific Operational Requirement", or SOR.[1] The SOR constitutes a directive to the responsible bureau for the preparation of a "Technical Development Plan" (TDP) to accomplish the CNO objectives expressed by that SOR. It ultimately becomes the task of a project engineer to translate the objectives of the SOR into a detailed technical description of the system to be developed. The description obviously must be expressed in quantitative terms that are amenable to control by prediction and measurement during the design and development phases.

In general, there are three closely related analyses to be made by the project engineer in order to generate the essential descriptive information needed for the preparation of technical development plans, design specifications, requests for proposals, and contractual task statements. These are:

(1) Analysis and definition of the operational requirements – performance, reliability, and maintainability – required for the desired level of system "effectiveness".

(2) Estimation of the feasibility of achieving these requirements by conventional design, in order to assess the practical difficulty of the development job.

(3) Initial allocation of requirements and supporting R & D effort among subsystems, according to an equitable method of apportionment.

[1] An SOR "will state a need for a capability, will outline a system or major component for achieving it, and will state the reasons for the requirement." – OPNAVINST 3910.6

The procedures outlined in this section are intended primarily to assist the project engineer in the analysis of system *reliability* requirements, although stress is also placed on maintainability and performance requirements since all three are equally vital to operational effectiveness of the system. The procedures are in general accord with BuWeps policies expressed or implied in the following applicable documents:

BUWEPINST 3910.2A
"Instructions for Preparing Technical Development Plans (TDP's)"

MIL-STD-756A
"Reliability Prediction"

WR-30
"Integrated Maintenance Management for Aeronautical Weapons, Weapon Systems, and Related Equipment"

WS-3250
"General Specification for Reliability"

MIL-R-22256
"Reliability Requirements for Design of Electronic Equipment or Systems"

2-2 DEFINITION OF SYSTEM OPERATIONAL REQUIREMENTS

2-2-1. General

Every weapon system concept is based on a need to fulfill an anticipated operational requirement. The "effectiveness" with which the system fulfills this need is the ultimate measure of its tactical utility and its value to the Fleet. System effectiveness is a composite of three parameters – performance, reliability, and availability – as depicted by the block diagram of Figure 2-1.

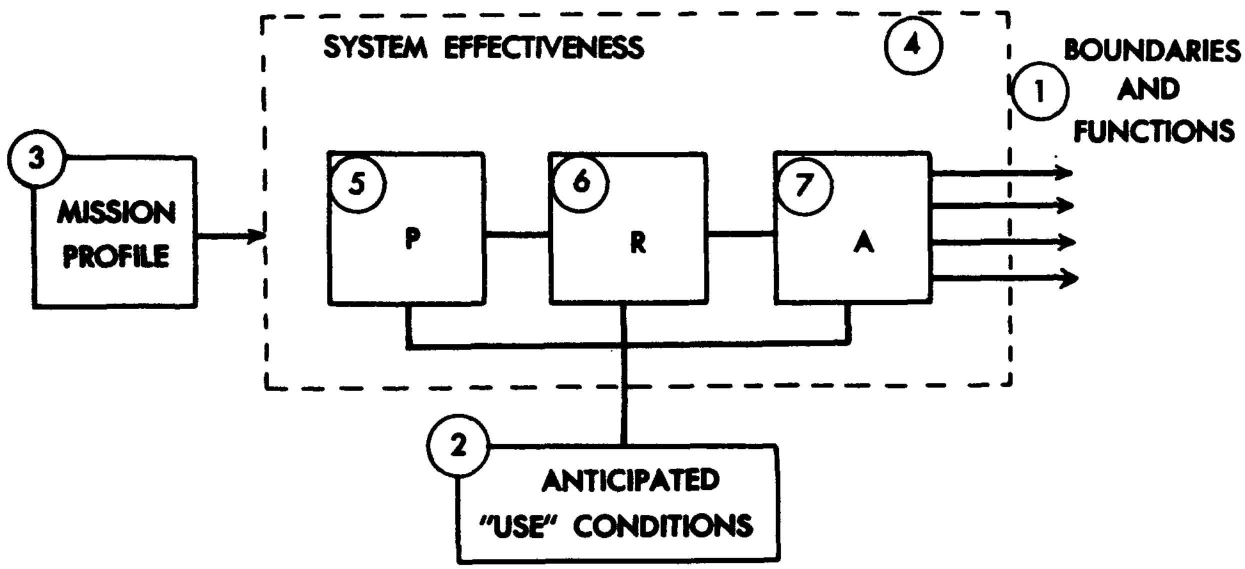


Figure 2-1. System Effectiveness Model for Specified Functions under Stated Conditions

The performance model is the functional or operational (schematic) block diagram used to depict the *functional* relationships of subsystems and components required to fulfill performance requirements of the conceptual system. This diagram is used in the definition of interface characteristics, transfer functions, and tolerance requirements.

The reliability model reorients the blocks of the functional model into a series parallel network to depict the reliability relationships among subsystems and components, for the estimation and allocation of the reliability requirement.

The availability model is an adaptation of the reliability block diagram to reflect proposed maintenance concepts and monitoring provisions which will permit the estimation of failure densities and repair rates. These estimates are then used as a basis for allocating repairability *requirements* and for estimating the maintenance support (personnel, facilities, and spares provisioning) required to achieve specified availability.

In general, effectiveness is the product of performance, reliability, and availability. For a specified level of performance, the effectiveness model simplifies to:

E = Reliability × Availability, for a given level of performance under specified use conditions

2-2-2. Procedural Steps

The following step-by-step procedure relates to the seven points of Figure 2-1:

(1) Describe the functional configuration and boundaries of the system.

(2) Describe the anticipated use conditions.

(3) Describe mission profiles and duty cycles.

(4) Define the operational effectiveness or "kill probability" requirement.

(5) Define performance characteristics and "failure" criteria.

(6) Define reliability requirements.

(7) Define availability/maintainability requirements.

STEP 1 - Describe System Functions and Boundaries.

Describe the "total" system with which the new developmental system is to become integrated. For convenience in visualizing the functional makeup and interface boundaries applicable to the system, construct a functional block diagram, indicating major operating modes and performance functions, including multiple functions and planned redundancy.

Figure 2-2 is an example of a simplified block diagram for a hypothetical weapon system.

The outer boundary of the figure establishes the points of contact and the interfaces between the weapon system and the major system with which it is to become integrated. Within the weapon system boundary the conceptual system is blocked out by major subsystem required for each system performance function. For example:

Blocks 1 and 2 (control console and search radar) are required for system function A – search and detection.

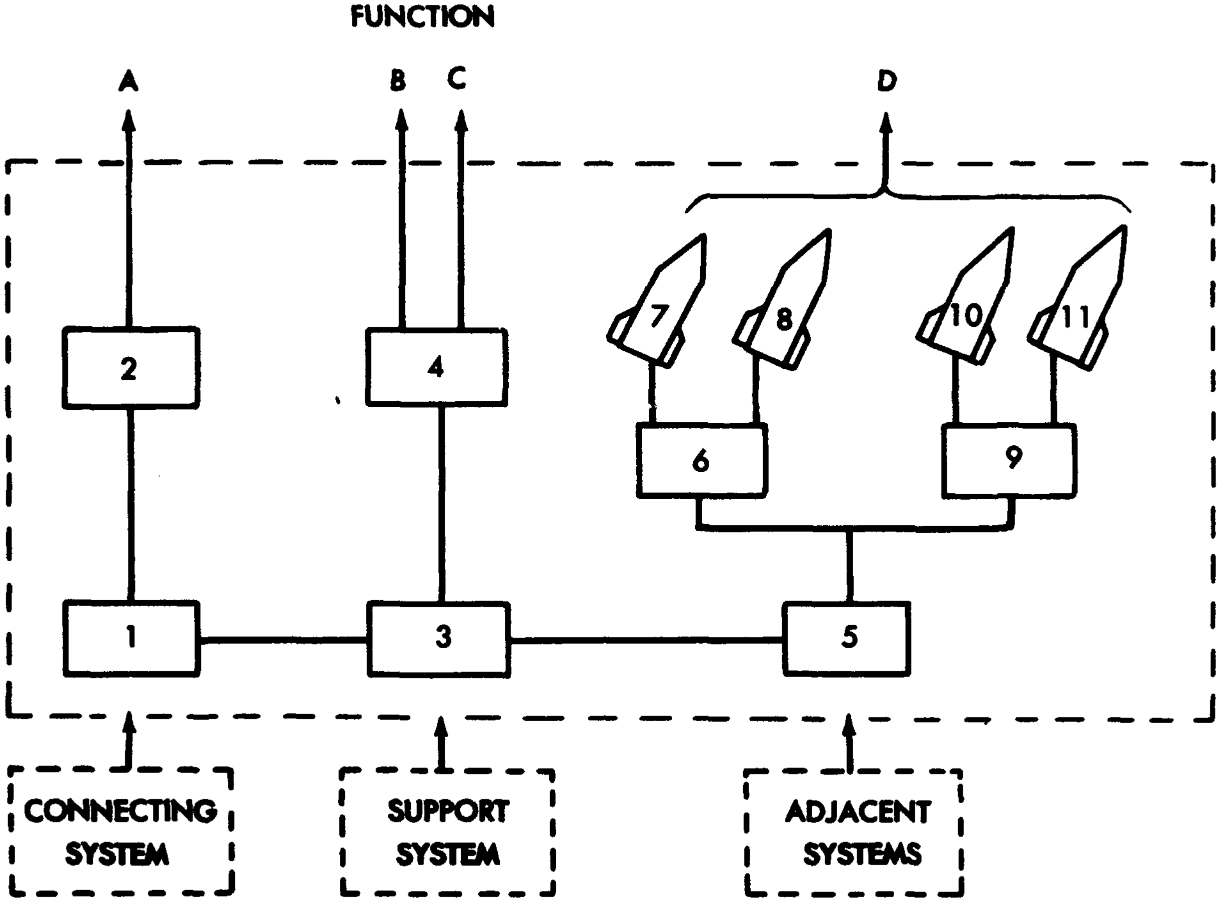


Figure 2-2. Functional Block Diagram of a Weapon System

Block 1, with 3 and 4 (computer and fire control radar), is required for functions B and C – target tracking and missile guidance.

Blocks 1 and 3, with 5 and 6 or 9 (magazine and launcher), are required for successful launch and direction of missiles 7 and 8 or 10 and 11, to perform function D – target intercept.

If enough is known about subsystem configuration at this point, the functional diagram of Figure 2-2 should be expanded to the extent that detailed information is available. Block 4, for example, might be developed as shown in Figure 2-3, to indicate a multiple frequency concept to be employed in the track and guidance radar system.

In this example, it is planned that the computer function will be performed by an existing computer, AN/ASQ-XX, as indicated in the figure by "GFE" (i.e., government-furnished equipment). All GFE and CFE (contractor-furnished equipment) contemplated for use in the new system must be described – input/output "interface" characteristics as well as performance characteristics.

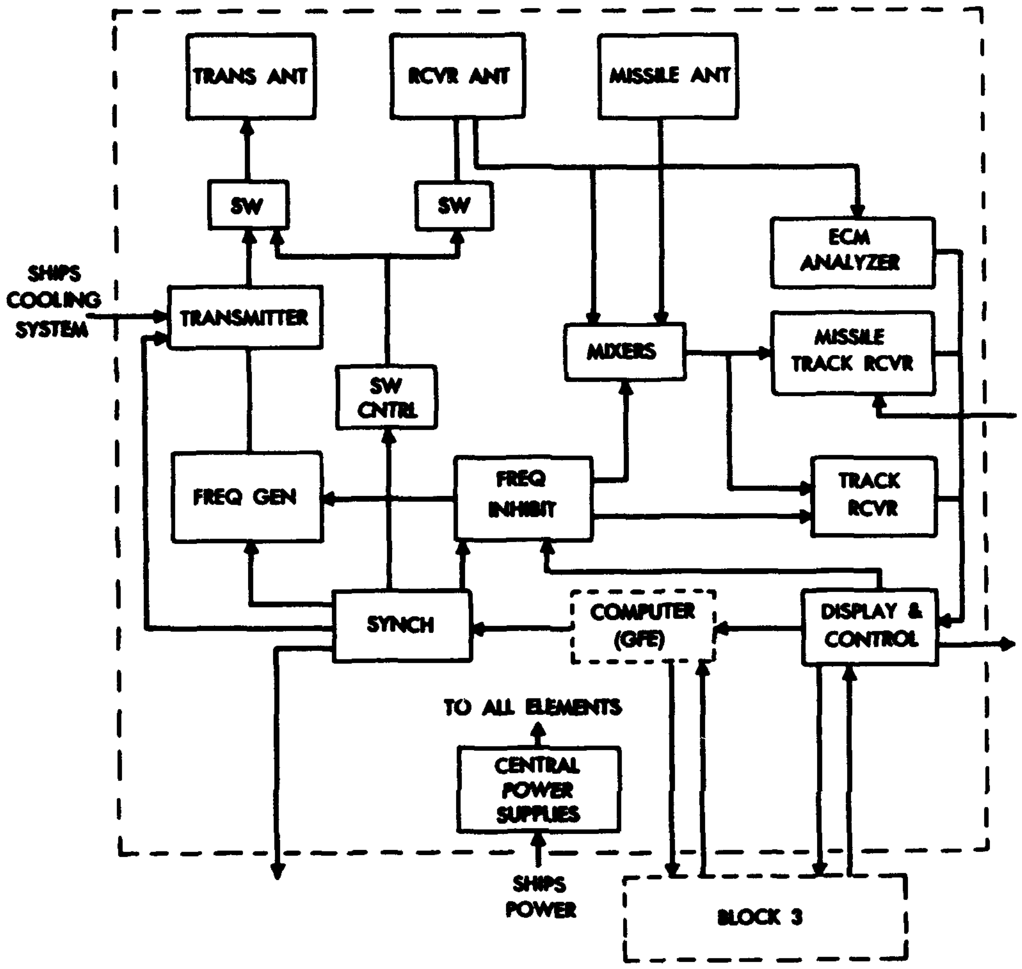


Figure 2-3. Functional Block Diagram for a Typical Weapon Control System

The same requirement applies to any special component – existing or in development – that is an integral part of the new system concept. Assume, for example, that the weapon control radar transmitter is based on a new broadband microwave amplifier concept already in development, although not yet available as a production item. The project engineer for the weapon system, now acting in the role of the "systems integration engineer", must solicit from the transmitter project engineer a complete description of the technical characteristics and requirements peculiar to his particular equipment or component block in the overall diagram. Figure 2-4 presents a hypothetical microwave transmitter block diagram to illustrate the detail that might be known at this point in the planning phase.

Again, the equipment description at this level must include a complete definition

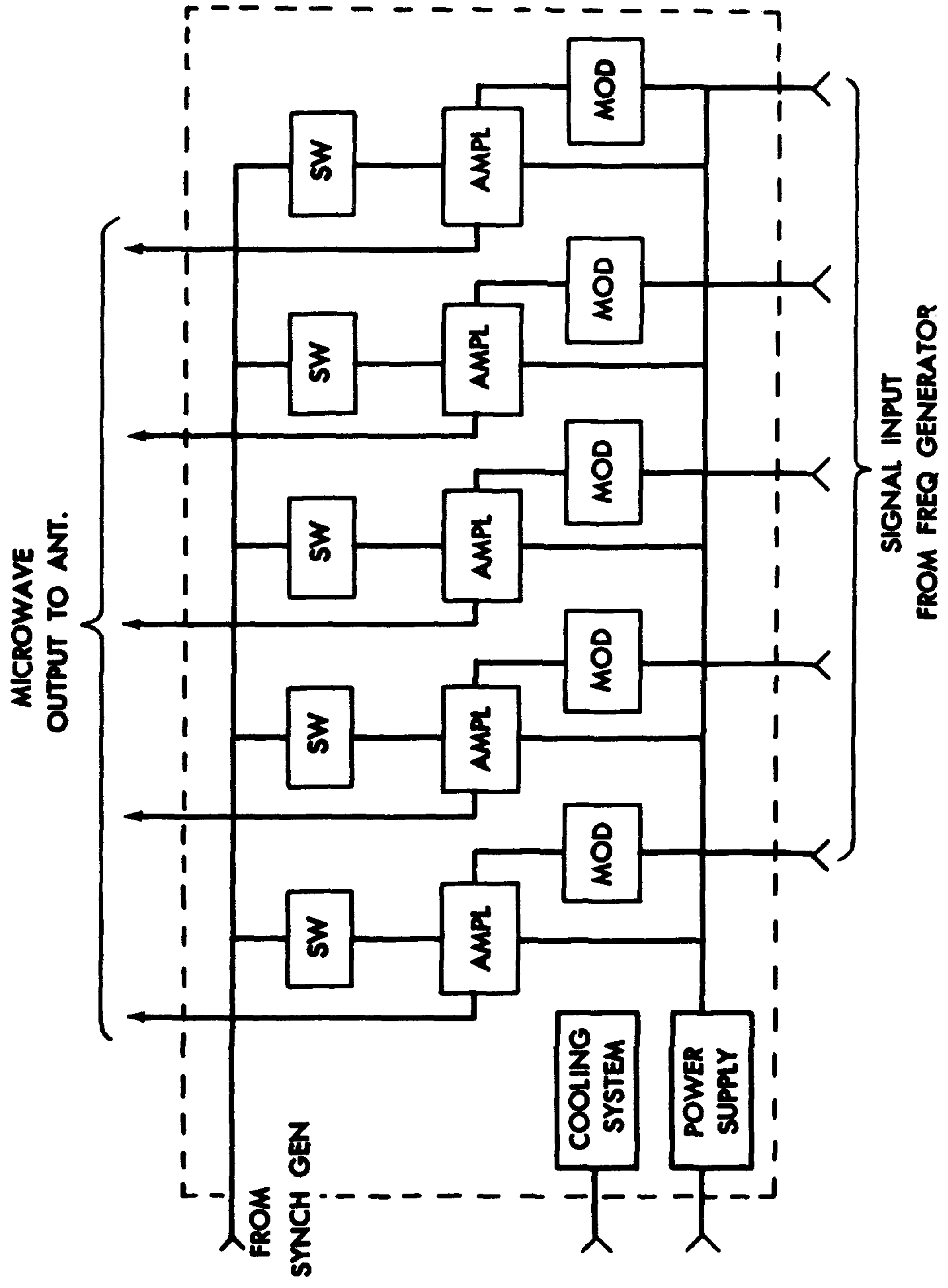


Figure 2-4. Functional Block Diagram of Radar Transmitter

of "boundaries", input/output interfaces, and actual transfer characteristics to be achieved by design. At this level, it is important to describe both the anticipated "application requirement" for the developmental device to be employed in the system and the "application ratings" for the device as recommended by its development contractor.

It is important to point out that the actual equipment design is in fact to be optimized on the basis of a mutual compatibility between fulfillment of system requirements and conservative use of the developmental device, where the latter is a critical factor in the success of the system concept. For this reason, in all instances in which new or unique components are to become integrated into the system design, it is necessary to go one more step in describing the system. Figure 2-5 summarizes a description of recommended "typical application conditions" for the developmental high-power broadband microwave amplifier tube on which the system concept is based.

Characteristics and Conditions	Units	Value		
		Max	Nom	Min
Saturated Power Output	Watts	1500	1200	1000
Gain	DB	35	30	25
Beam Current	MA	2000	1800	1500
Helix Voltage	Volts		8000	
Peak Magnetic Field	Gausses		760	
Period	Inches		1.000	
Transmission	Percent	95	90	85
Wave Guide Coupling	VSWR	2.5		
Frequency Range	MC	12500		8000
Coupler Insertion Loss	DB	0.1		
Temperature (Storage)	°C	+85		−62
Temperature (Operating)*	°C		+55	
Shock, 15 g's 11 milliseconds	Cycles		20	
Vibration, 50 to 500 cps 2 g's (1-minute sweep)	Cycles		30	
*Water cooling.				

Figure 2-5. Characteristics and Typical Operating Conditions for a Critical Component

STEP 2 – Describe the Anticipated "Use" Conditions for the System.

Describe the anticipated installation interfaces, the interference characteristics of adjacent or associated systems, and the general physical environments and use conditions with which the system is to be compatible in its ultimate application. Include storage, handling, maintenance, and checkout, as well as operational conditions. These "exterior" factors, depicted under three broad general categories in Figure 2-2, would include but not be limited to those shown in Figure 2-6.

Interfaces with Connecting Systems	Description
Primary Electrical Power Source:	
Terminal Voltages and Tolerances	115 volts ±10%
Frequency and Tolerances	60 cycles ±1%
Phases and Connection	3-phase delta
Regulation (full load to no load)	2%
Peak and Average Capacity (available to system)	10 kw +0%, -10%
Primary Hydraulic and Pneumatic Power Source:	
Nominal Pressure and Tolerances	
Peak and Average Capacity	
Control Signal Sources (analog and digital):	
Frequency or Bit Rate	
Signal Levels and Tolerances	
Impedances	
Vibration and Shock at Physical (mounting) Interfaces:	
Frequencies	
G-levels	
Duration	
Thermal and Humidity:	
Heat Sink Characteristics (water coolants, etc.)	
Air Conditioning Capacity	

Interactions with Support Systems
Maintenance Policy:
On-Line Maintenance Provisions
Preventive and Marginal Testing Procedures
Level of Technician Qualification
Operating Policy:
Procedures
Qualifications of Personnel
Failure Dependencies:
Isolation Requirements
Protective Features, Inherent
Fail-Safe Protection, Required

Interference from (and to) Adjacent Systems
Radio Frequency Interference (noise and power):
Frequency Spectrum
Modulation Characteristics
Radiation Pattern and Power
Protective Features, Inherent
Isolation (shielding), Required
Radiation (damage) Control Requirements
Physical Interference:
Structural Shadows and Beam Deformation
Induced Vibration, Shock, and Thermal Environments

Figure 2-6. Example of System Use Condition Definitions

STEP 3 – Define Mission Profiles – Operating Time, and Duty Cycles.

Estimate the range of probable mission lengths or engagement periods over which the system should be capable of operating, following an alert. Calculate the operating time or duty cycles of individual system functions and subsystems for particular (typical) mission profiles.

Continuing with the example of Figure 2-2, a hypothetical mission profile is illustrated in Figure 2-7, in which the system must be capable of continuous operation in the general surveillance mode and must be capable of performing all functions continuously throughout a 3-hour engagement period. Note that launcher operation is expressed in number of consecutive launchings or launch cycles, and missiles are divided among 30-day storage time following system checkout, 3-hour launcher time following prelaunch checkout, and 80 seconds of flight time.

STEP 4 – Define Effectiveness Requirements for the System.

The levels of performance, reliability, and availability required in each of the functions listed in the previous step are directly related to the minimum level of effectiveness considered tactically acceptable, where effectiveness is defined as the joint probability of being operationally ready and capable of performing a given function when needed and of surviving the period of the need without failure. This may have been defined by the SOR as the required "kill probability" for the system.

In the missile weapon system example, assume an OPNAV requirement for a kill probability of 50% for an individual missile launched at any time against a particular

Mission or Mode	Function	Level	Subsystems Involved	Operating Time, or Probable Engagement Period, T_m
Surveillance	A	I	1 and 2	Continuous, 24 hours*
Engagement				
Target Acquisition and Track	A and B	ALL	3 and 4	3 hours
Missile Control and Guidance	C		4 and 5	3 hours
Missile Checkout, Load, and Launch	D_1		5 and 6 or 7	60 cycles in 3 hours
Target Intercept	D_2		7 or 8, or 10 or 11	30-day storage; 3 hours in "ready" service; 80 seconds flight.
Warhead and Fuze	Same as D_2 above			

*2-hour daily preventive maintenance period included.

Figure 2-7. Example of Mission Profile and Related Operating Times

class of target, within a *specified* defense perimeter of the weapon system. This requirement implies that the product of performance reliability and availability at the overall weapon system level must be 50%. For a *specified level* of performance, effectiveness might be expressed as

$$E_s = (R_s)(A_s),$$

conditional on performance capability = 1.0 at the specified level

$$= (R_r A_r)(R_{wc} A_{wc})(R_m A_m)(R_L A_L)(E_w)$$

$$= 70\%$$

where (R) (A) are products of reliability and availability for the constituent systems – search radar, weapon control radar, missile, launcher, etc. – and E_w is the known effectiveness of the missile warhead (GFE). Note that these are all "nominal" values based on observed data or on specified nominal design requirements. They are *not* the lower 90% confidence limits. The latter should not be multiplied together unless intentionally to produce a conservative estimate of E_s.

In some cases, the system in question is to become part of an existing system – a radar system within an existing missile weapon system, for example. In such cases, it is necessary to start with the required kill probability for the entire weapon system, and then divide out the known or estimated effectiveness of the other systems with which the new system is to work.

In other cases, the entire weapon system may be a new concept on which there are no past reliability and availability data for any of its subsystems. In such cases, it is necessary to make an arbitrary allocation of effectiveness requirements among subsystems, based on past experience with similar systems of comparable function and complexity. This is permissible until design studies disclose flaws in the extrapolation and appropriately "update" the allocation.

In either case, allocated effectiveness requirements of a subsystem are related to the balance of the major system as shown in the following weapon control radar example.

Solving for E_{wc}, the $R_{wc}A_{wc}$ product for the weapon control radar system yields

$$E_{wc} = R_{wc}A_{wc} = \frac{E_s}{(R_r A_r)(R_m A_m)(R_L A_L)(E_w)},$$

for a specified level of performance

R_{wc} is the reliability requirement for the weapon control radar system for the specified mission period for a specified level of performance. A_{wc} is the availability or operational readiness requirement placed on the radar system, expressed as the probability that the system will be in an operational status at the specified level of performance, *when needed*.

Tentative effectiveness allocations are shown for the hypothetical weapon control system in Figure 2-8.

Assume that the required radar effectiveness derived above is 0.92 for both target track and missile guidance; i.e., the weapon control system must be *operationally ready* to perform at a selected level and must remain in operation at this level throughout an engagement period, T_m = 3 hours, with a probability of 0.92.

The chart also illustrates the assignment of "allocation" of effectiveness requirements to other system elements based on "experience" or stated requirements. All values are *nominal*, i.e., mean observed or mean required.

Mission or Mode	Function i	T_m	Effectiveness Allocation* E_i	Basis
Surveillance	A	3 hours	.96	Req (1)
Target Track	B	3 hours	.95 } .92	Req (1)
Missile Guidance	C	3 hours	.97 }	Req (1)
Missile Launch	D_1	60 cycles	.99	Exp (2)
Missile Ready	D_2	30 days	.93	Exp (2)
Missile on Launcher		3 hours	.95	Exp (2)
Missile Flight		80 seconds	.98	Exp (2)
Target Destruction	Fuze & Warhead	Same as D_2	.92	Exp (2)
			$*\pi E_i = .7$	Req (1)

(1) System requirement derived from SOR.
(2) Estimated on basis of past experience.

Figure 2-8. Example of Effectiveness Allocation Among Major Subsystems

STEP 5 – Define System Performance Requirements and System "Failure" by Operating Mode.

Define system performance requirements within each of the operating modes – the minimum level of performance required for success in each of the several tactical situations described in the SOR. This is frequently difficult because the performance "envelope" of a typical system includes many variables – some independent, many interdependent.

It is usually sufficient, however, to treat the principal system characteristics individually for an approximate solution in the system planning and preliminary design stages.[2] Figure 2-9 illustrates a possible graphical representation of the distribution of one principal performance parameter, X. Discrete points on this curve may correspond to selected lower boundaries of *measurable* performance for particular mission profiles or target classes. These lower boundaries define minimum acceptable performance for several specified levels.

These points also establish the failure definitions for the system in each of the performance levels. Performance characteristics should be tabulated as shown in Figure 2-10.

[2]Later on (in the early design phase) the use of Monté Carlo methods becomes practical with the aid of a computer when enough is known about the distributional forms of individual performance characteristics.

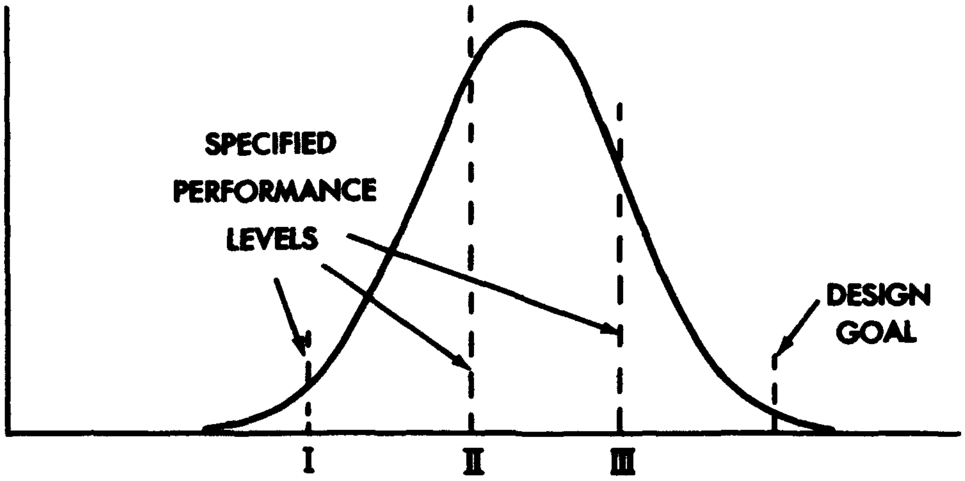


Figure 2-9. Distribution of Performance Parameter

System Function and Performance Characteristic	Units*	Performance Level (Lower Limits) I	II	III
Search (Radar Range for 1 M^2 Target)	*% of* Design Goal	60	80	90
o Transmitter Power Output		(80)	(90)	(95)
o Beam Dimensions				
o Receiver Sensitivity		(80)	(90)	(95)
o Receiver Signal/Noise Ratio		(90)	(95)	(98)
Track (Range and Capacity)				
o				
o				
o				
Guidance (Range, Accuracy, and Capacity)				
o				
o				
o				
Weapon Control and Direction (Automatism, Manual Response, Storage Capacity, Launch Rate, etc.)				
o				
o				
o				
Missile (Range, Maneuverability)				
o				
o				
o				

*Expressed as a percentage of specified design goal to avoid the need for "security" classification.

Figure 2-10. Example of Multiple-Level System Performance Definition

STEP 6 – Define the Reliability Requirement.

Construct a preliminary reliability block diagram from the functional block diagram of Step 1. The reliability diagram should show the series parallel relationships of subsystems required for the performance of individual system functions. Figure 2-11 is an example applied to the weapon control system of Figure 2-2. Translate OPNAV reliability requirements expressed in the SOR into MTBF or probability of mission success definitions depending upon the mission profile and the nature of the time base. For example, reliability of Block 4, the weapon control radar function, should be expressed as a probability of survival of .92 for a three-hour engagement period at Level I (design goal) performance as shown in Figure 2-12.

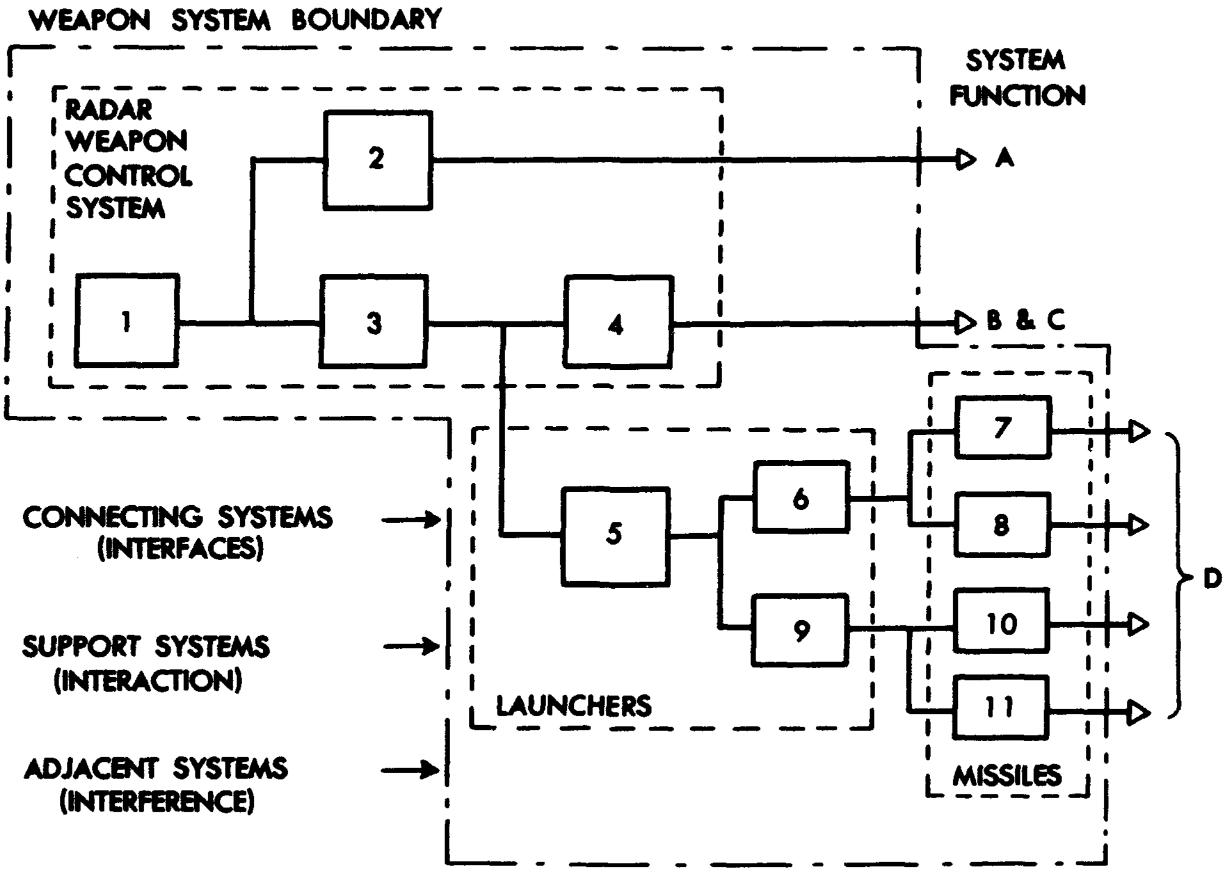


Figure 2-11. Reliability Block Diagram of a Weapon System

Mission	Function or Mode	T_m	Reliability Nominal R_o	Reliability Minimum* R_{min}
Surveillance	A	24 Hours	.82	.65
Tactical Engagement	A	3 Hours	.98	.96
	B and C	3 Hours		
	Level I		.92	.88
	Level II		.96	.93
	Level III		.98	.96
	D_1	60 Cycles	.99	.98
	D_2	30 days	.94	.90
		3 Hours	.97	.94
		80 Seconds	.99	.98
	Fuze & Warhead	Same as D_2	.94	.88

*Usually defined at the lower 90% confidence level (see Appendix 3)

Figure 2-12. Definition of Reliability Requirements for a Typical Weapon System

As in Step 1, it is important that reliability diagramming be extended to the level at which planning information is firm. Figure 2-13 illustrates an expansion of Block 4 of Figure 2-11 corresponding to the *functional* detail developed in Step 1.

Reliability diagrams should show plans for redundancy, "on-line maintenance", and other essential features of design that are prescribed as part of the concept. Figure 2-14 illustrates a reliability block diagram developed for the transmitter subsystem. The project engineer in this case was aware of the potential reliability problems associated with microwave power amplifiers. He prescribed a design configuration employing partial redundancy with "on-line" repair capability, to achieve the desired level of maintainability and "reliability-with-repair"

In this example reliability requirements are related to three levels of performance as shown in Figure 2-15, where Level I is the design goal or "nominal" power output requirement with all five amplifier paths operational. Level III is defined as "minimum" acceptable power output with only three paths operational.

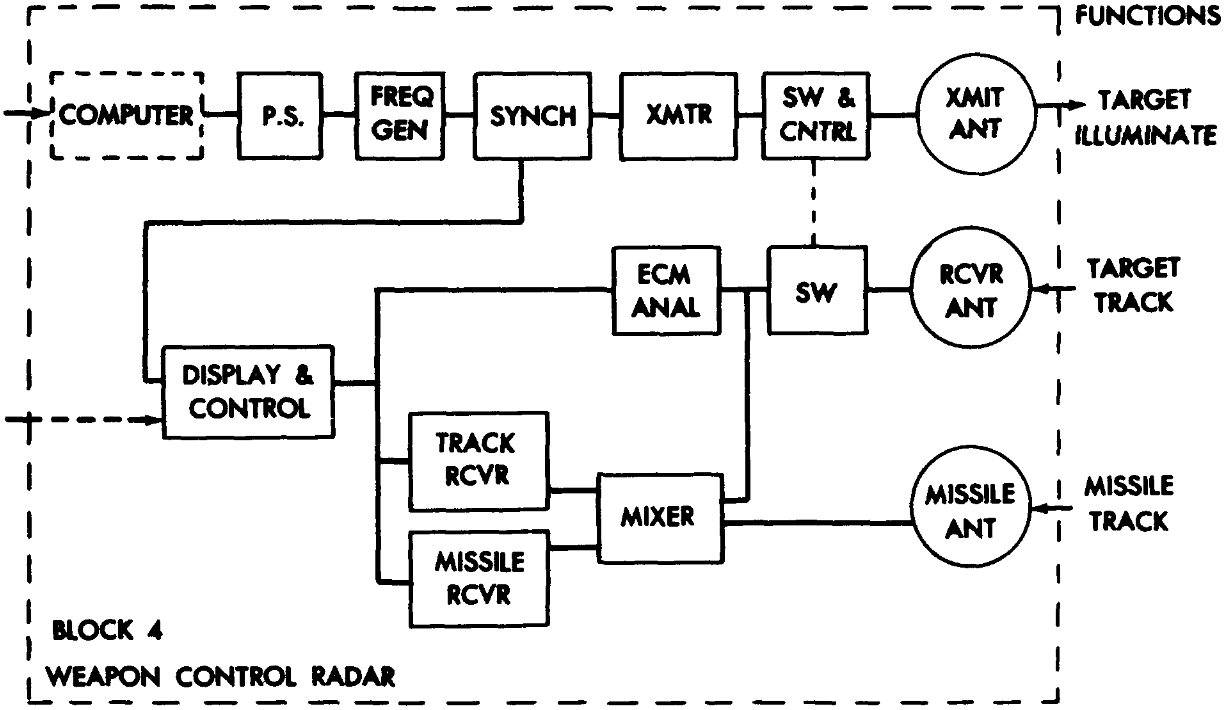


Figure 2-13. Reliability Block Diagram of Block 4 (Figure 2-11)

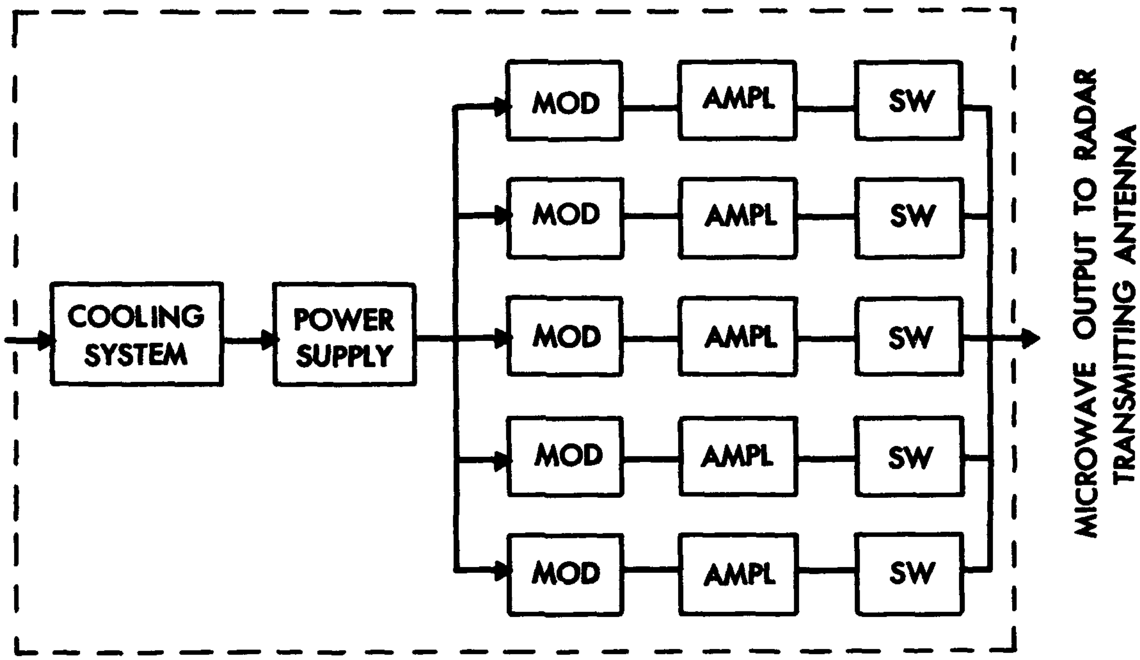


Figure 2-14. Transmitter Reliability Diagram Showing Partial Redundancy as Part of the Design Concept

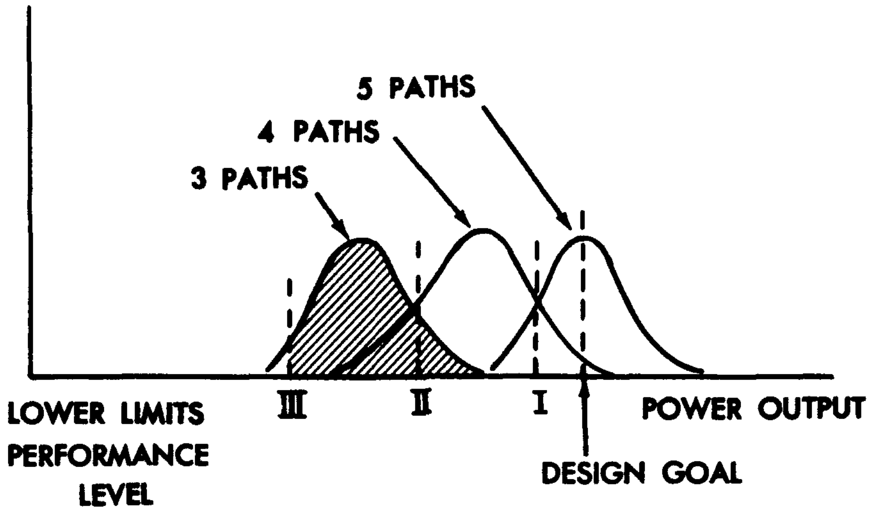


Figure 2-15. Reliability Requirements Related to Performance Levels for the Partially Redundant Case

Reliability requirements that might be derived for the Transmitter Package, are shown in Figure 2-16 for each level of performance. Both nominal and minimum values are shown.

Performance Scale	Power Output Peak KW	$R_{nom.}$ (3 Hrs.)	$R_{min.}$ (3 Hrs.)
I	2000	.98	.96
II	1600	.99	.98
III	1200	.995	.99

Figure 2-16. Reliability Requirements for Microwave Transmitter Package

If it is deduced that the reliability requirement dictated by an effectiveness requirement exceeds the state-of-art benchmark for attainable reliability, redundant design techniques or "on-line" maintenance provisions may be necessary. As a rule of thumb, if the requirement exceeds the "benchmark" by two-to-one or more (in equivalent exponential mean life), redundancy is indicated.

This anticipated design requirement must then be included as part of the system description – to guide program planning and prospective design activities in the study of design feasibility and reliability allocation.

STEP 7 – Define Availability and Maintainability Requirements.

Previous definitions of reliability and effectiveness also establish the value of availability to be specified. From this, the maintainability requirement can be defined as shown in the following equation:

$$\text{Availability, } A = \frac{MTBF}{MTBF + MTR}$$

$$= \frac{1}{1 + \frac{MTR}{MTBF}}$$

where MTBF is the mean time-to-failure; MTR is the average time required to restore the system to operational status following a failure, or initiation of a preventive maintenance task.

Solving for maintainability:

$$MTR = \left(\frac{1}{A} - 1\right) MTBF$$

A trade-off between reliability and maintainability may be permissible. This should be indicated to permit some degree of design flexibility consistent with the effectiveness requirement. Such a trade-off is illustrated in Figure 2-17, showing how an effectiveness requirement of .7 can be

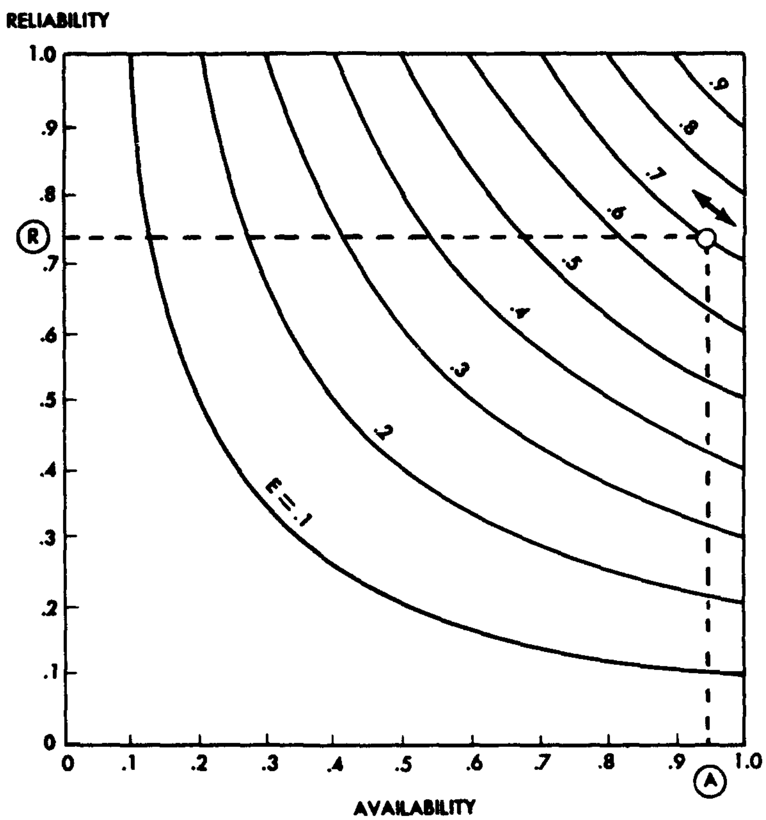


Figure 2-17. Conditional Effectiveness Hyperbola, E, for Specified Levels of Reliability and Availability, Conditional on Performance Capability = 1.0

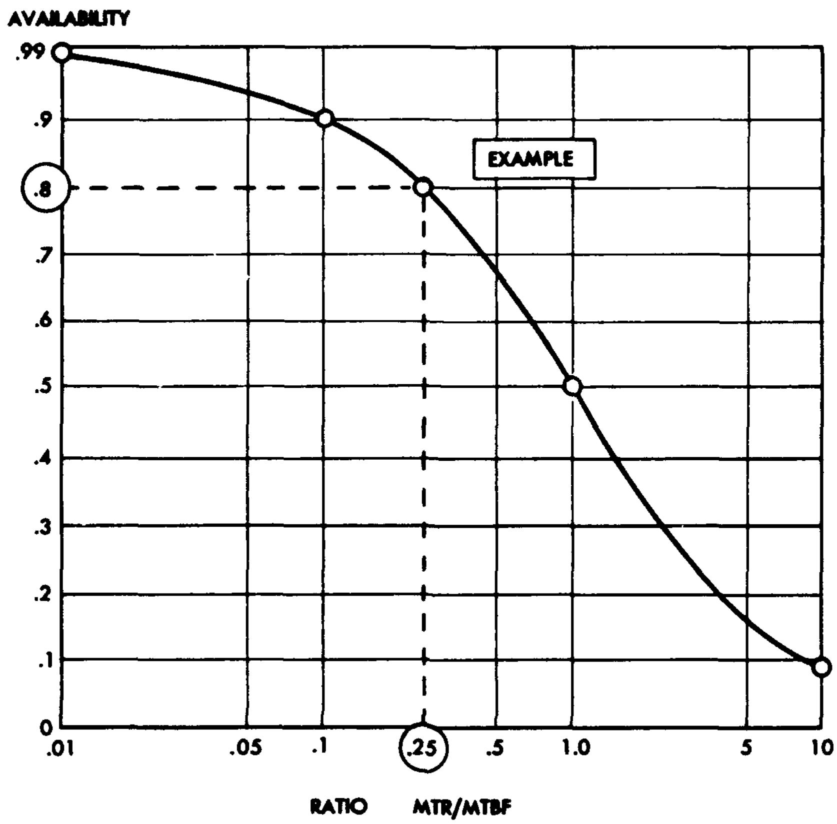


Figure 2-18. Relationship between Availability and MTR/MTBF

satisfied with an infinite number of availability/reliability combinations for a given level of performance.

The relationship between availability, A, and the ratio MTR/MTBF is shown in Figure 2-18.

EXAMPLE: A new system is to be developed to satisfy an availability requirement of .99. An MTR/MTBF ratio of .01 will be necessary to satisfy the requirement. An MTBF requirement of 100 hours had also been specified to satisfy the reliability requirement. From this it can be seen that the system must be designed for a mean-time-to-repair of one hour. This, then, defines the maintainability requirement for this particular combination of effectiveness parameters.

2-3 RELIABILITY FEASIBILITY ESTIMATION AND ALLOCATION

2-3-1. General

The preceding requirements analysis defined "acceptable" levels of system reliability and availability, to satisfy system effectiveness requirements for given levels of performance and modes of operation.

After an acceptable reliability or failure rate for the system has been assigned, it must be apportioned among the various subsystems as design requirements applicable to responsible development contractors. Concurrently, it is necessary to estimate the feasibility of achieving these requirements by conventional design—

(1) To determine the extent of special R & D support required in specific areas.

(2) To determine the advisability of a reliability/maintainability feasibility study prior to award of a design and development contract.

(3) To anticipate the probable tradeoffs in weight, space, power, and performance that will be required to achieve the reliability/availability requirements.

(4) To establish the emphasis for implementation and operation of a formal reliability assurance and monitoring program.

(5) To more accurately predict development costs and time required before an acceptable prototype is likely to be delivered.

The procedures outlined in this section are equally applicable in design, development, and product improvement phases. The precision of estimation, of course, increases in these latter phases as test and field data become available.

Both the feasibility estimate and the allocated requirement must be based on a knowledge of the complexity of the product under consideration, in order to permit an analysis by methods outlined in either MIL-STD-756A or MIL-HDBK-217.

2-3-2. Active Element Group Concept of Complexity

In the concept and planning stage, specific hardware items usually have not been selected or conceived in detail. Thus, allocation of failure rates must be based on prior experience with similar items or functions, although new design philosophies and concepts must not be penalized by being restrictive to existing configurations. This consideration has led to the "active element groups" (AEG) concept of system definition, where the AEG is the smallest practical functional building block which could be economically considered and which would not be specifically related to existing configurations. An active element is defined as a device which controls or converts energy. An active element group consists of one active element and a number of passive elements normally required to perform a specific function. Examples of active elements are electron tubes, relays, pumps, combustion chambers, and rotating machines. A typical electronic AEG might consist of an electron tube or transistor and several resistors and capacitors. A typical relay AEG might consist of the relay, its solenoid, and from two to ten circuit contacts.

Figure 2-19 shows the familiar plot based on *MIL-STD-756A, in which system* mean-time-between-failure (MTBF) is related to system complexity, on the basis of current Fleet experience in airborne and shipboard installations of conventional non-redundant designs, predominantly analog in function.

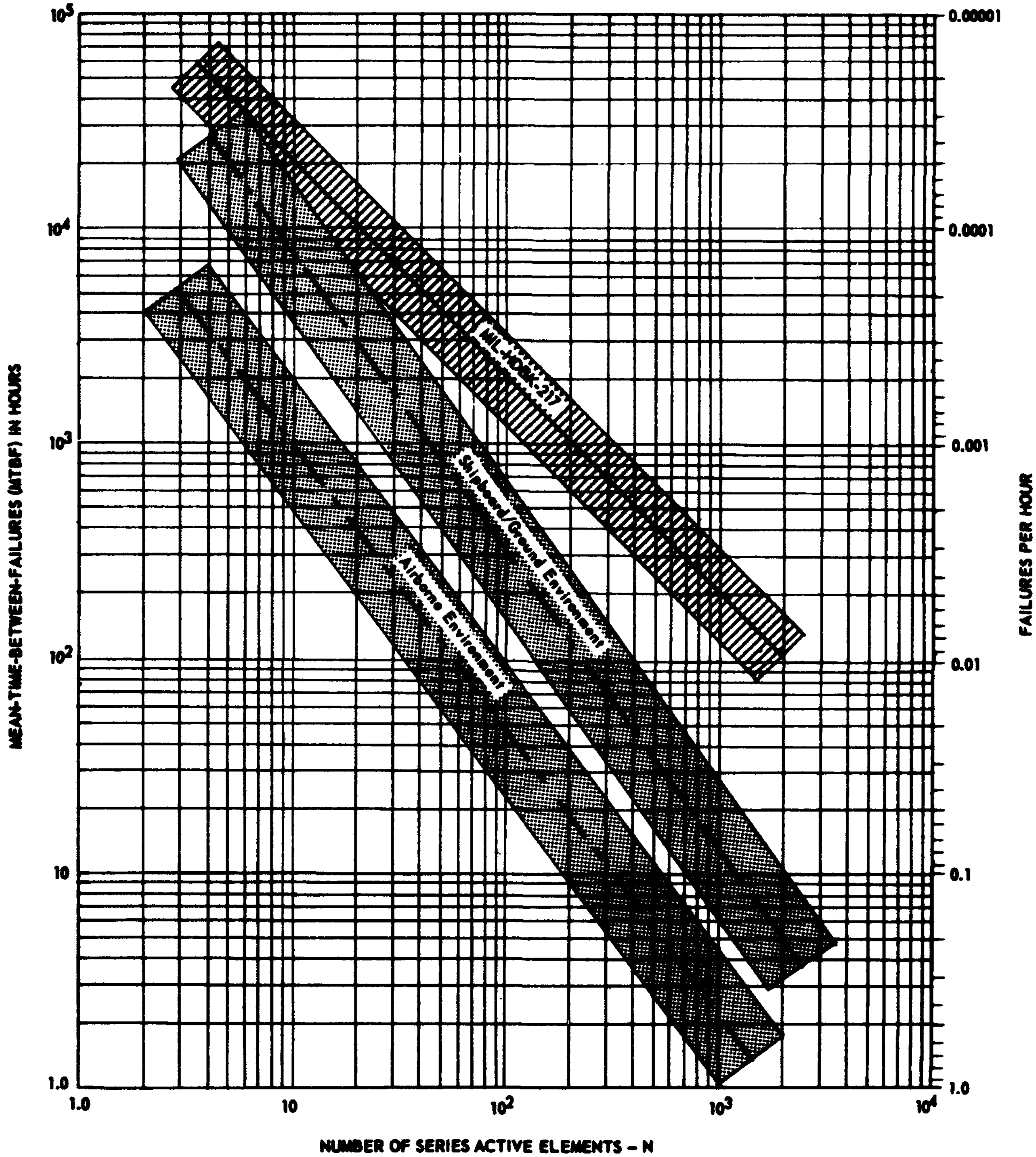


Figure 2-19. Plot of MTBF vs. Complexity Based on Past System Experience (MIL-STD-756A)

2-3-3. Procedural Steps

The following basic steps apply to the use of this past experience in the estimation of reliability feasibility and the allocation of reliability requirements:

1. Develop the Reliability Block Diagram.
2. Derive mathematical models.
3. Estimate complexity and MTBF of the system.
4. Estimate subsystem failure rates.
5. Estimate feasible MTBF and reliability.
6. Allocate failure rate and reliability.
7. Consider redundant configurations.
8. Evaluate feasibility of *allocated* requirements.

STEP 1 – Develop the Reliability Block Diagram.

It is necessary to go within each block of the system block diagram to develop a reasonable approximation of a subsystem diagram containing those units required to perform the subsystem function. To the extent that design information is available at this early stage of system planning, it may be desirable to go further down into the system to block diagram specific design configurations at the subsystem and component levels – especially if *planned* features of the design concept include the application of redundancy or unique devices at these lower levels.

Figure 2-20 shows the evolution of the detailed block diagram – going from the weapon system level down to the part level – as a function of design evolution.

Levels I and II diagrams are usually producible from information available in the system planning phase and are considered adequate for preliminary feasibility estimation and reliability allocation.

The Level III diagram is usually producible in the early design proposal stage.

The Level IV diagram is producible after a period of design formulation and review in which a definitive design has resulted.

Level V represents the failure mode diagram at the part level, where it becomes practicable to perform stress analyses and failure mode studies on individual parts within the system. Such detailed information may be available in the early planning phase on certain critical parts known to be essential to the success of the system concept.

In the development of a block diagram, units that are predominantly electronic in function are classified and symbolized as electronic units, even though mechanical elements may be involved in the performance of an output function. Units that are predominantly mechanical, or otherwise essentially non-electronic in nature, are identified accordingly. Any unit redundancy contemplated at this stage of system planning should be shown, as well as any planned provisions for alternate mode capability. To the extent practicable, the block diagram should be constructed so that each unit can be assumed functionally independent of its neighboring unit so far as its specific transfer function is concerned.

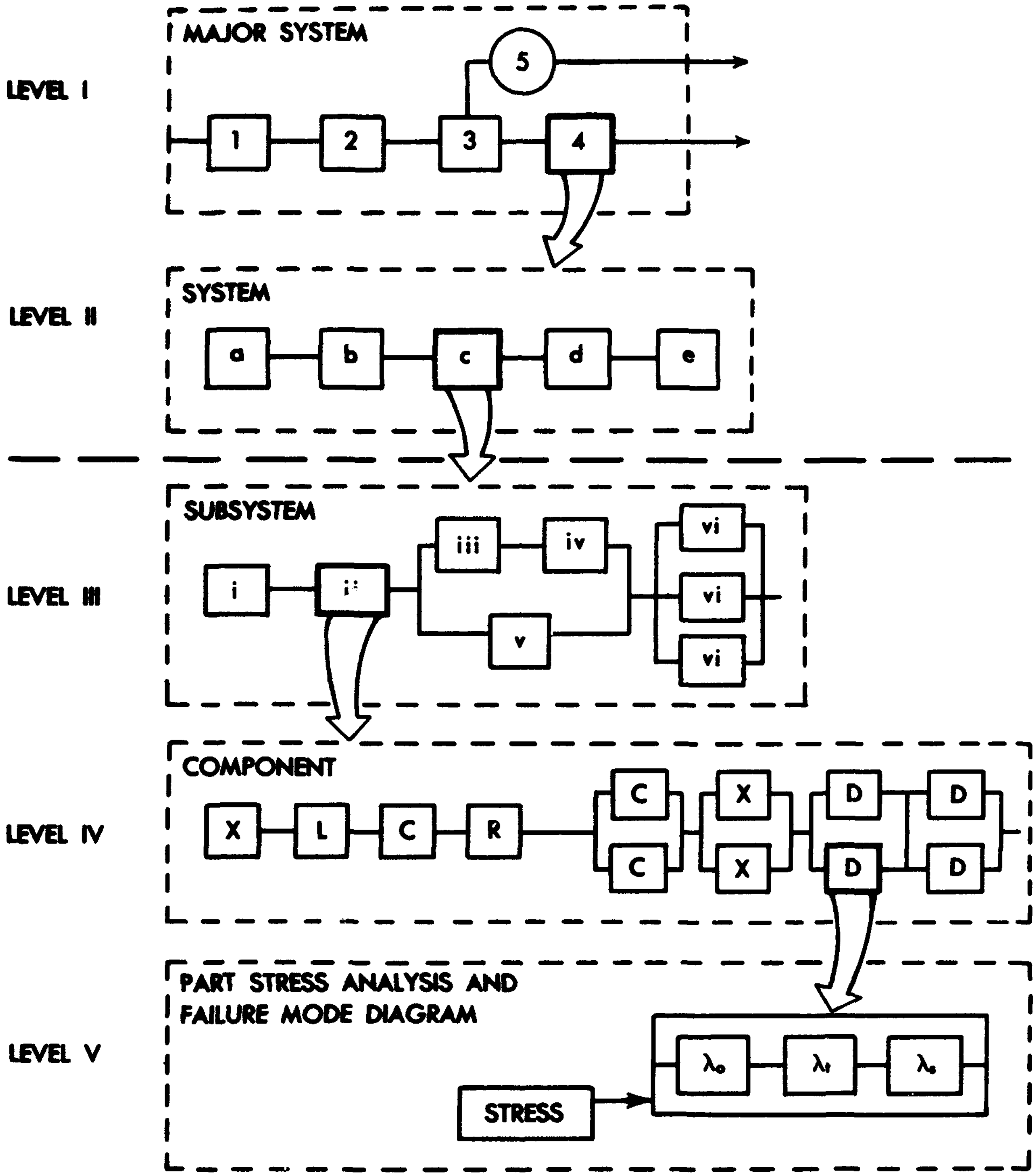


Figure 2-20. Progressive Expansion of Reliability Block Diagram as Design Detail Becomes Known

STEP 2 – Derive Mathematical Models.

The block diagram helps to visualize physical relationships and specific subsystem configurations. The mathematical model relates individual "block" reliability to the reliabilities of its constituent blocks or elements.

Progressing from Level I to Level V, for example,
System Reliability, $R_s = R_1 \times R_2 \times R_3 \times R_4$

where $R_4 = R_a \times R_b \times R_c \times R_d \times R_e$

$$R_c = R_i \times R_{ii} \left[1-(1-R_v)(1-R_{iii}R_{iv})\right]\left[1-(1-R_{vi})^3\right]$$

$$R_{ii} = R_X R_L R_C R_R \left[1-Q_C{}^2\right]\left[1-Q_X{}^2\right]\left[1-Q_D{}^2\right]^2$$

where

$$Q = 1\text{-}R, \text{ e.g., } Q_D = 1\text{-}R_D$$

$$R_D = e^{-\lambda_D t} \text{ for a particular part}$$

$$\lambda_D = \lambda_o + \lambda_t + \lambda_s$$

Subscripts o, t, and s denote open, tolerance, and short modes of failure, respectively

The model can be solved using the simplified notation presented in 2.1.8 of Appendix 2. Applied to the Level III diagram of Figure 2-20, for example, the following notations are appropriate:

Let $R_i = a$; $R_{ii} = b$; $R_{iii} = c$; etc.

and $(1-R_i) = \bar{a}$; $(1-R_{ii}) = \bar{b}$; $(1-R_{iii}) = \bar{c}$; etc.

Then, dividing the Level III diagram into 3 groups, there is the following tabulation for all possible combinations of successful performance:

Group 1: $R_1 = ab$ (a and b required)

Group 2: $R_2 = cde + cd\bar{e} + c\bar{d}e + \bar{c}de + \bar{c}\bar{d}e$ (either c and d, or e, required)

Group 3: $R_3 = fgh + \bar{f}gh + f\bar{g}h + fg\bar{h}$ (2 of 3 required)

Combining: $R_{III} = R_1 \times R_2 \times R_3$

EXAMPLE: Reliability estimates have been derived for all components of Subsystem c – the guidance and control package – of a new surface-to-air missile to be developed for a major weapon system. For a flight time of 80 seconds, the following component reliabilities and corresponding UNreliabilities have been estimated.

Group 1: $a = R_i = .99$ $\qquad \bar{a} = (1-R_i) = .01$

$b = R_{ii} = .98$ $\qquad \bar{b} = (1-R_{ii}) = .02$

Group 2: $c = R_{iii} = .95$ $\qquad \bar{c} = (1-R_{iii}) = .05$

$d = R_{iv} = .95$ $\qquad \bar{d} = (1-R_{iv}) = .05$

$e = R_v = .90$ $\qquad \bar{e} = (1-R_v) = .10$

Group 3: $f = g = h = R_{vi} = .90$ $\qquad \bar{f} = \bar{g} = \bar{h} = (1-R_{vi}) = .10$

$$R_1 = ab = (.99)(.98) = .97$$

$$\begin{aligned} R_2 &= cde + cd\bar{e} + c\bar{d}e + \bar{c}de + \bar{c}\bar{d}e \\ &= (.95)(.95)(.90) + (.95)(.95)(.10) \\ &\quad + (.95)(.05)(.90) + (.05)(.95)(.90) \\ &\quad + (.05)(.05)(.90) \\ &= .99 \end{aligned}$$

$$R_3 = fgh + \bar{f}gh + f\bar{g}h + fg\bar{h} = (.9)^3 + 3(.9)^2(.1) = .97$$

Estimated reliability feasibility for a guidance subsystem of this particular design configuration is then:

$$R_{III} = R_1 \times R_2 \times R_3 = (.97)(.99)(.97) = .93$$

STEP 3 – Estimate the Complexity and Range of Feasible Failure Rates for the System.

Functional complexity of predominantly electronic units can be estimated on the basis of electronic AEG's required to perform the unit function. Non-electronic elements within an electronic unit can be considered uniformly distributed among the electronic AEG's without appreciable error. Non-electronic units, however, must be accounted for separately even though their relative contribution to system unreliability may be small. Non-electronic devices include structural elements, engines and propulsion systems, mechanical drives and linkages, and hydraulic and pneumatic elements.

As an example, assume that Block 4 of Figure 2-20 is the fire control radar of a weapon control system to be developed for airborne use. Depending upon the detail of the available design information and the level at which Block 4 can be diagramed, a range of probable system complexity might be estimated at, say, between 250 and 500 AEG's.

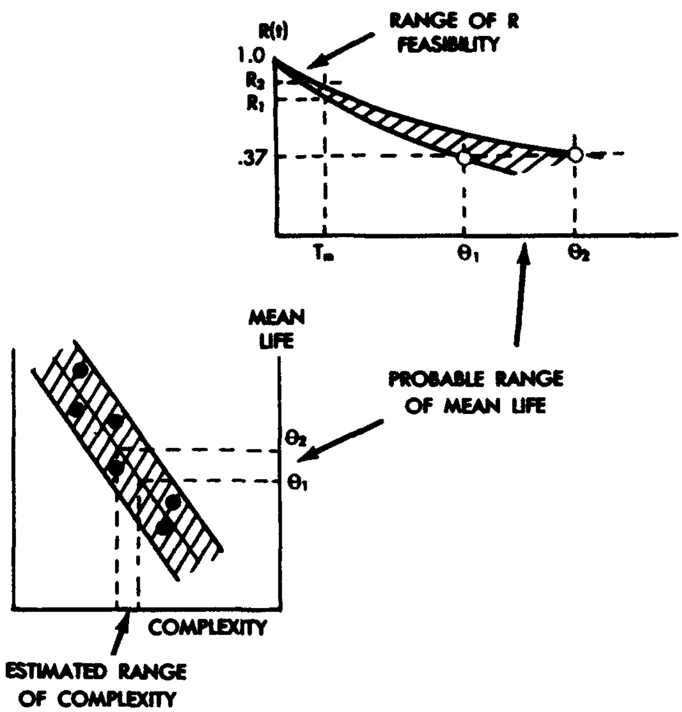


Figure 2-21. Translation of Estimated Complexity Range to Probable Range of Reliability Feasibility

On the basis of Level I data drawn from past experience with systems of comparable function and complexity, a preliminary estimate of MTBF can be made directly from Figure 2-19, as shown in the block below.

Figure 2-21 illustrates the method by which these values of MTBF are derived from Figure 2-19. They are then translated to a pair of reliability functions embracing the feasibility of estimate using either Figure 1-7

Estimated Range of Complexity: 250 to 500 AEG's		
Probable Range of MTBF (θ_1 to θ_2)	=	5.5 to 14 hours
Range of Block Failure Rate ($1/\theta$)	=	.071 to .182 failures per hour
Range of Reliability Feasibility (R_1 to R_2)	=	.40 to .70, for T_m = 5 hours

or the nomographs of Appendix 3. If the stated requirement falls within this range, it can be said to be feasible by conventional design.

As the functional characteristics of the system become better defined, a more precise count of AEG's can be made. Assume for example that Block 4 is to consist of approximately 250 analog and power AEG's and approximately 500 digital AEG's.

Prior experience has shown, for failure-rate estimating purposes, that the digital AEG is equivalent to about one-tenth of an analog AEG – i.e., 1 analog AEG ≈ 10 digital AEG's. This difference is attributable to the fact that the digital AEG is basically an on/off switching device and, consequently is not as susceptible to the variability of parts characteristics.

The equivalent analog complexity of Block 4 is then estimated to be 300 AEG's. Referring now to Figure 2-19 (MIL-STD-756A), the point estimate of system MTBF is 12 hours and system reliability for the 5-hour mission is $R(T_m) = .66$.

STEP 4 – Evaluate Feasibility on Basis of Subsystem Analysis

If the design concept is known in sufficient detail to permit a Level II analysis, Block 4 might be further detailed as follows:

Subsystem	Function	Complexity
a	Power Supply	40 Power AEG's
b	Frequency Generator/Synch	500 Digital AEG's
c	Receiver and Display Group	120 Analog AEG's
d	RF Unit (Transmitter/Modulator)	40 Power AEG's
e	Antenna and Control Group	50 Analog AEG's

The AEG failure-rate chart of Figure 2-22 can then be used to estimate average failure rates as a function of series complexity within subsystems, to account for catastrophic as well as tolerance and interaction failures. However, the following rules and assumptions apply to analyses made at the subsystem level:

(1) It is assumed that interactions among subsystems are negligible and that the tolerance build-up due to complexity is interrupted at the subsystem boundary. This assumption introduces an error in the system-level estimate if subsystem estimates are combined for the system estimate. The error is "optimistic" and is of a magnitude that is proportional to the number of subsystems into which the series system has been divided for analysis. The system-level error can be rectified, however, by reconstituting the subsystems into the single series configuration for a total AEG count and system estimate.

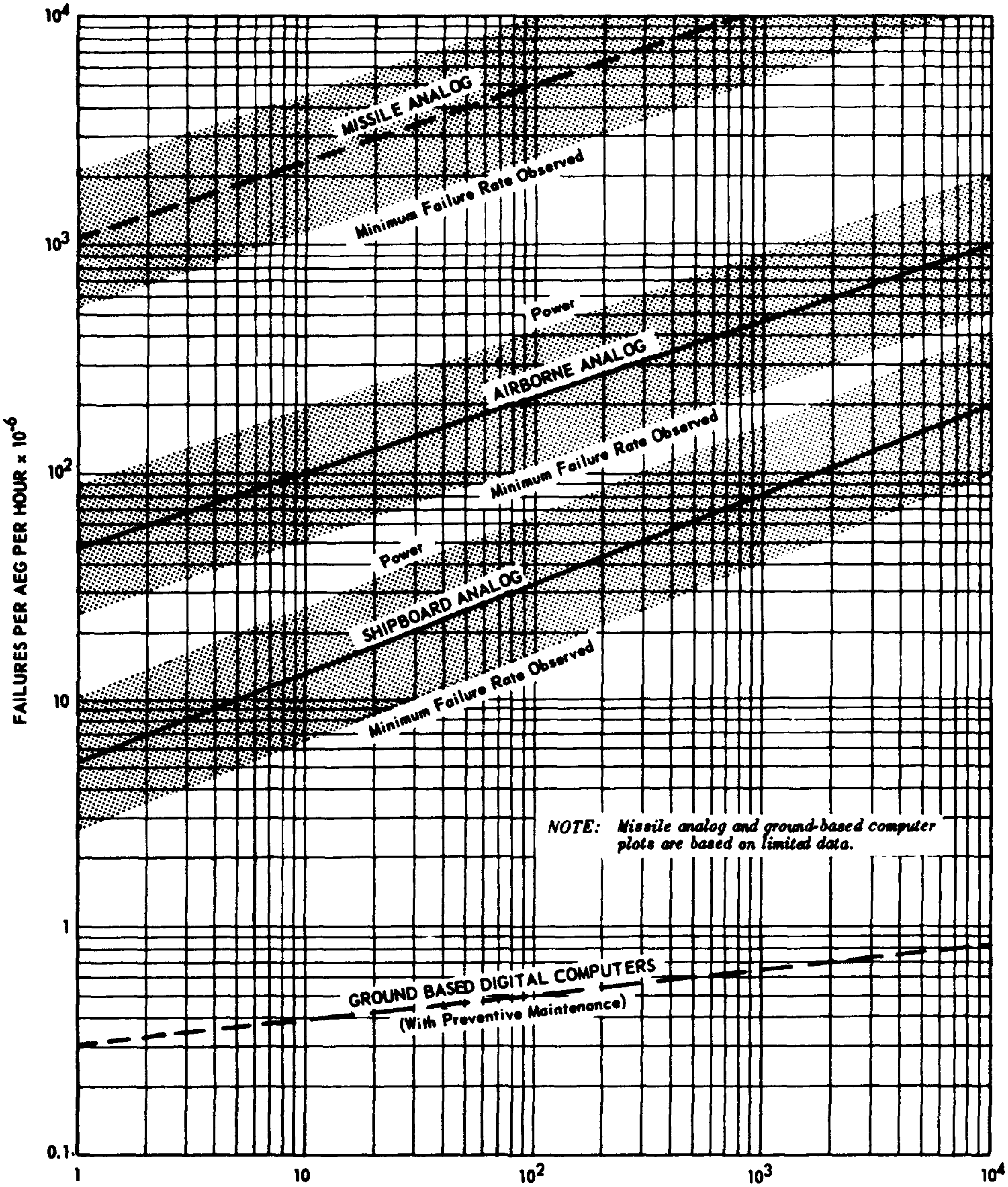


Figure 2-22. AEG Failure Rates for Reliability Estimation When the Number of Active Elements is Known

(2) While power AEG's can be treated as analog AEG's in the series system case because they normally constitute a minority of the total AEG's, they must be treated separately in a subsystem analysis where some of the subsystems are primarily power AEG's. Experience has shown that power AEG's, on the average, experience a failure rate approximately two times the failure rate for analog AEG's – i.e., $\lambda_{power} = 2\lambda_{analog}$. Using this rule of thumb, a power subsystem of a given complexity is assigned an AEG failure rate twice that of an AEG in an analog subsystem of the same complexity.

(3) Digital AEG's, in a system employing both analog and digital circuits, differ from their analog counterparts by a factor of about 10-to-1 in their "equivalent complexity" as measured by their relative insensitivity to characteristic variability among parts within the circuit and their relative freedom from interaction problems – i.e., the number of digital AEG's in a subsystem should be divided by ten when using the analog AEG chart for failure-rate estimation.

The following table is derived from this chart for an estimate of average "expected" failure rate per subsystem, applying the above rules.

Subsystem	AEG Complexity	Failure Rate × 10^{-6} Per AEG	Failure Rate × 10^{-6} Per Subsystem
a	40 Power [3]	340	13,600
b	500 Digital [4]	180	9,000
c	120 Analog	240	28,800
d	40 Power	340	13,600
e	50 Analog	180	9,000
		Total Block 4 Failure Rate	74,000

[3] Power AEG's have double the analog failure rate.

[4] In a system employing both analog and digital AEG's, divide the number of digital AEG's by ten and treat as analog.

STEP 5 – Estimate Feasible MTBF and Reliability Values of the System

As an example, overall reliability of the system would be expected to fall in the following range, on the basis of the tabulation of Step 4:

N = 300 AEG's is the estimated complexity

MTF Range = 6 hours to 30 hours
Most Probable = 12 hours (Avg)
Failure Rate Range = .033 to .166 failures/hour

The three-hour mission reliability is then calculated as follows:

Minimum Likely $R(3) = e^{-3/6} = .61$

Average Expected $R(3) = e^{-3/12} = .78$

Maximum Feasible $R(3) = e^{-3/30} = .90$

STEP 6 – Allocate Required Failure Rates and Reliability Among Subsystems

Allocation of *permissible* failure rates among systems of the major weapon system, and among subsystems within systems, is made on the assumption of equality of improvement feasibility. Allocation is then made by using as proportionality constants the ratios of individual subsystem failure rates to total system failure rate. Thus, if a given subsystem now contributes 10% of the total system failure rate, it is reasoned that it should not be permitted to exceed 10% of the total failure rate allowable under the new requirement.

To allocate failure rates among systems, subsystems, or components, compute the ratio of the smaller block failure rate to the next larger block failure rate; e.g., in the preceding example, the proportionality constant for Subsystem c within System 4 is:

$$k_c = \frac{\lambda_c}{\lambda_4} = \frac{30{,}000 \times 10^{-6}}{82{,}000 \times 10^{-6}} \approx .37 = 37\%$$

Continuing with this example, assume the system reliability requirement for a three-hour mission had been established as:

$R_4(3) = 0.90$, corresponding to a system failure-rate,
$\lambda'_4 = 35{,}300 \times 10^{-6}$ failures per hour

The maximum permissible failure rate for Subsystem c is then:

$$\lambda'_c = k_c \lambda'_4 = (.37)(35{,}300 \times 10^{-6}) = 13{,}200 \times 10^{-6} \text{ failures per hour}$$

The reliability requirement to be apportioned to Subsystem c is then derived from:

$$R_c(3) = e^{-3\lambda'_c} = e^{-3 \times 13,200 \times 10^{-6}} \approx .96$$

The following table shows the allocation procedure applied to other subsystems within System 4:

Subsystem	Expected Failure Rate	Percent of Total	Allocated Failure Rate	Reliability Allocation
a	13,600	18.38	6,850	.98
b	9,000	12.17	4,200	.99
c	28,800	38.90	13,200	.96
d	13,600	18.38	6,850	.98
e	9,000	12.17	4,200	.99
Total	74,000	100.00	35,300	.90

STEP 7 – Allocation Among Redundant Configurations

If redundant elements are known to be part of the system concept, the above allocation method must be modified to account for the planned redundancy.

The following modification is applicable for any type of subsystem and system reliability function. The only necessary statistical or probability assumptions are that failure of any of the subsystems considered will result in system failure and that the failure probability of each subsystem is independent of all other subsystems. This will allow the use of the product formula for system reliability upon which the method is based.

The method of allocation when redundancy is present in the subsystem follows:

(1) Draw a reliability block diagram of the subsystem in question. Also construct an equivalent (functional) non-redundant subsystem. The equivalent non-redundant subsystem would consist of the minimum number of AEG's necessary to perform the subsystem function.

(2) Select the number of hours, T, over which a high system reliability is desired. T would be defined by the mission requirements or the average time interval before corrective maintenance or unit replacement.

(3) Using estimated base failure rates, evaluate R(T) for both the redundant and non-redundant configurations described in (1) above.

(4) The failure rate factor for the redundant subsystem is estimated by:

$$\lambda_r = \frac{R(T)_s}{R(T)_r} \lambda_s$$

where

λ_r is the estimated failure rate for the redundant subsystem

λ_s is the failure rate for the equivalent non-redundant subsystem

$R(T)_s$ is the calculated reliability at time T of the non-redundant subsystem (using AEG failure rates)

$R(T)_r$ is the calculated reliability at time T of the redundant subsystem (using AEG failure rates)

(5) Specify R*(T), the desired system reliability at time T, and compute the total system failure rate,

$$\lambda_o = \sum_i \lambda_i$$

where λ_i is the failure rate of the i^{th} subsystem.

(6) The allocated reliability for Subsystem i is

$$R_i^*(T) = R^*(T)^{\lambda_i/\lambda_o}$$

(NOTE: For non-redundant subsystems, the allocated failure rate is

$$\lambda_i^* = \frac{-LnR_i^*(T)}{T}.)$$

(7) Check the allocation.

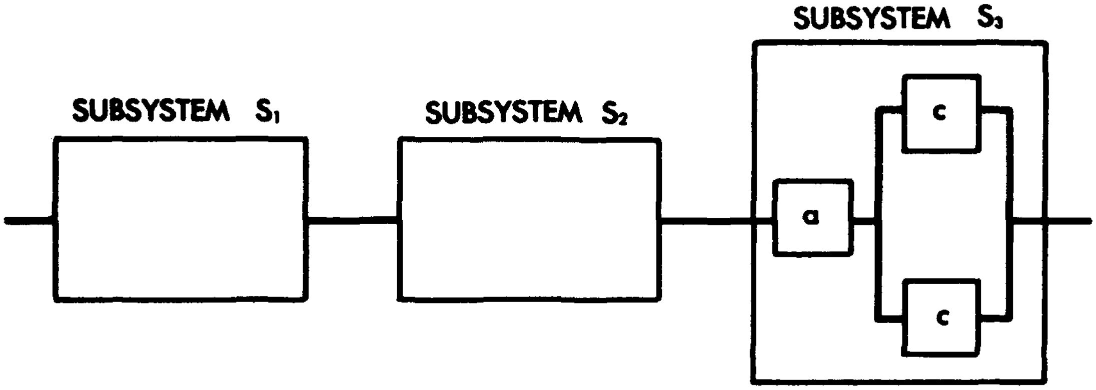


Figure 2-23. Reliability Block Diagram with Redundant Elements

EXAMPLE: Assume the reliability block diagram of a system is as shown in Figure 2-23. Each box represents a complex equipment made of several AEG's. The failure rates and the estimated mean lives are:

Subsystem	Failure Rate $\times 10^{-6}$	Mean Life
S_1	20,000	50 hours
S_2	15,000	67 hours
S_3 (a	10,000	100 hours
S_3 (c	20,000	50 hours

(1) Establish equivalent non-redundant units.

S_1 and S_2 subsystems are non-redundant with all constituent elements in series. S_3 has two parallel elements in series with another element. Since only one of the two parallel elements is necessary for performing the system function, we have S_3', as shown in Figure 2-24.

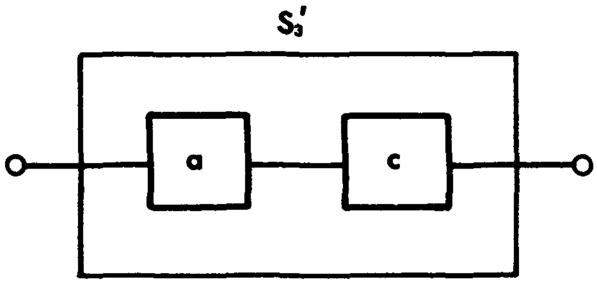


Figure 2-24. Equivalent Non-Redundant Unit

(2) Determine critical time period.

Assume corrective maintenance is performed every 50 hours; hence, T = 50.

(3) Compute $R(T)_s$ for non-redundant units and $R(T)_r$ for redundant units at T = 50 hours.

Non-Redundant Unit:

$$R(T)_s = R(T)_a \times R(T)_c$$

$$= e^{-T/100} \times e^{-T/50}$$

$$= .606 \times .368 = .223$$

Redundant Unit:

$$R(T)_r = R(T)_a \times R(T)_c\,[2 - R(T)_c]$$

$$= .606 \times .368\,[2 - .368] = .364$$

(4) Compute base failure rate factor for redundant unit, with

$$\lambda_s = (10{,}000 + 20{,}000) \times 10^{-6} = 30{,}000 \times 10^{-6}$$

$$R(T)_s = .223$$

$$R(T)_r = .364$$

Then,

$$\lambda_r = \left(\frac{R(T)_s}{R(T)_r}\right)\lambda_s$$

$$= \left(\frac{.223}{.364}\right) 30{,}000 \times 10^{-6}$$

$$= 18{,}300 \times 10^{-6}$$

(5) Convert desired reliability to total system failure rate.

Assume that at 50 hours, the reliability requirement is specified to be .75. Hence, $R^*(T) = .75$.

$$\lambda_1 = 20{,}000 \times 10^{-6}$$

$$\lambda_2 = 15{,}000 \times 10^{-6}$$

$$\lambda_3 = 18{,}300 \times 10^{-6}$$

$$\lambda_o = \sum_i \lambda_i = 53{,}300 \times 10^{-6}$$

(6) Allocate reliability.

$$R_i^*(T) = R^*(T)^{\lambda_i/\lambda_o}$$

$$R_1^*(50) = (.75)^{200/533} = .897$$

$$R_2^*(50) = (.75)^{150/533} = .922$$

$$R_3^*(50) = (.75)^{183/533} = .906$$

(7) Check allocation.

$$R_1^*(50)\, R_2^*(50)\, R_3^*(50) = .7493$$

The allocated system failure rates for non-redundant Subsystems 1 and 2 are:

$$\lambda_1^* = \frac{-\text{Ln } .897}{50} = 2150 \times 10^{-6}$$

$$\lambda_2^* = \frac{-\text{Ln } .922}{50} = 1620 \times 10^{-6}$$

STEP 8 – Evaluate the Feasibility of the Allocated Requirement

In Step 2 the expected failure rate for the proposed new system was determined on the basis of a *conventional* design configuration, represented as a series string of functional subsystems; each subsystem in turn comprised a series of AEG's. This expected failure rate also assumed *normal* design care and attention to tolerances and ratings.

In Step 5 the "permissible" system failure rate – permitted by the reliability *requirement* – was distributed proportionately among the subsystems within the proposed new system. This allocation procedure assumed a uniformity among AEG's of a given class that does not actually obtain in practice. Thus, our allocation at this point can be considered only as a tentative one – for use only as an initial basis for specification of reliability requirements at the system and subsystem levels. This allocation must therefore be reviewed and adjusted early in the design phase, as soon as the detail design study discloses the discrepancies between *allocated* improvement and improvement *feasibility*. It may turn out, for example, that a ten-to-one reduction in failure rate of one unit is entirely feasible, whereas a two-to-one reduction in another unit would be beyond state-of-art capability for some time to come. The reallocation of reliability requirements must therefore ultimately consider improvement feasibility within the constraints of available time and funds.

2-3-4. Classification of Feasibility Levels

For purposes of preliminary feasibility estimation by "rule of thumb", reliability feasibility can be classified according to practicality of failure-rate reduction:

Feasibility Level A – "Practically" Feasible:

The reliability requirement dictates an average reduction in failure rate (or failure probability) of less than two-to-one below that "expected" by conventional design (as determined in Step 2). In this case, the amount of reliability improvement required is generally achievable by conventional design, *if the design is guided* by an effective reliability evaluation and demonstration test program.

Feasibility Level B – "Conditionally" Feasible

The reliability requirement dictates an average reduction in failure rate (or failure probability) of between three- and ten-to-one, over an operating period not exceeding the "expected" mean life determined in Step 2. In this case the reliability requirement is feasible *conditional on* the application of redundancy at critical points in the system, and on the implementation of an effective reliability evaluation and demonstration test program to guide the application and prove the practicality of these techniques.

Feasibility Level C – "Remotely" Feasible:

The reliability requirement dictates a reduction in failure rate (or failure probability) of one or two orders of magnitude below that "expected" over a period of operating time exceeding the "expected" mean life of Step 2. In this case the requirement may become realistically feasible only after an intensive program of basic and applied research, based on the findings of a detailed feasibility study.

Depending on the level of feasibility in which the tentative requirement falls, the following decisions are indicated:

- If the reliability requirement is of Level A feasibility, stand "pat"; i.e., specify the requirement as a firm design requirement.
- If the requirement is of Level B feasibility, stand pat to the extent that weight and space factors will permit the application of redundancy; i.e., specify the requirement as a firm design requirement, but be prepared to trade weight and space.
- If the requirement appears to be of Level C feasibility yet is a tactically realistic requirement, specify the requirement as a design objective in a formal design feasibility study to determine the areas of research and development to be sponsored in support of the system development program.

2-4 ESTIMATION OF MAINTAINABILITY AND AVAILABILITY IN THE DESIGN PHASE

2-4-1. General

The availability of a weapon system is determined principally by its maintainability—the ease with which it can be kept ready to respond to the tactical need *when needed.* The requirement for maintainability (established initially in the requirements analysis phase) must be periodically re-assessed as design progresses, on the basis of a practical analysis of the inherent "repairability" characteristic of the design. Availability is dependent upon the probability of system repair and return to operational status, i.e., the probability that the system can be restored to operational status within a prescribed period of "downtime" for repair or preventive maintenance.

As weapon systems become more complex, the opportunity for failure increases as N^k, where N is a measure of complexity expressed in active elements, and $k = 1.4$ is the value of the exponent in shipboard analog devices ($k = 1.3$ in airborne analog, and approximately 1.1 in most digital applications). As the frequency of failure increases, so does the distribution of failure causes. Thus the maintenance problem can be related to system complexity, for feasibility estimation purposes.

2-4-2. Procedural Steps

The following procedures are useful for the estimation of maintainability, and hence system *availability,* as a feature of design.

STEP 1 – Develop the Maintainability-Availability Model

The basic maintainability-availability model is derived from the following relationships between system "uptime" and "downtime":

$$\text{Availability} = \frac{\text{Uptime}}{\text{Uptime + Maintenance Downtime}}$$

$$= \frac{\text{(Mean-time-between-failures)}}{\text{(Mean-time-between-failures) + (Mean-time-to-restore)}}$$

$$= \frac{\text{MTBF}}{\text{MTBF + MTR}} = \frac{1}{1 + \frac{\text{MTR}}{\text{MTBF}}}$$

From this relationship, maximum permissible average unscheduled maintenance downtime can be derived as a function of the specified availability requirement, i.e.:

$$\text{MTR.} = \left(\frac{1}{A^*} - 1\right)\text{MTBF}^*$$

where A* and MTBF* denote availability and reliability requirement tentatively specified in the requirements analysis phase, and

MTR is the average time required to detect and isolate a malfunction, effect repair, and restore the system to a satisfactory level of performance. MTR can be similarly adjusted to include a permissible period of downtime for preventive maintenance when the definition of availability includes the PM period. See Appendix 4 for more detailed treatment of maintainability in redundant designs.

STEP 2 – Estimate Feasibility by "Conventional" Design

A maintainability model of conventional design provides the basis for estimating "expected" or feasible values if the new concept follows the design approach used in predecessor systems – before the advent of redundancy and reliability-with-repair concepts.

Figure 2-25 is a maintainability estimating chart for a conventional design consisting of series AEG's with no special maintenance provisions. The maintainability "benchmark" or "expected value" of mean-time-to-repair for the conceptual system can be estimated from the chart if conventional design is to be employed.

EXAMPLE: A shipboard fire control system is to be developed. On the basis of past experience with conventional designs, the following "expected" parameters are derived:

N_s = 500 AEG's (Estimated)

MTBF = 40 hours (Derived from midpoint of MTBF band of Figure 2-18)

MTR = 12 hours (Derived from midpoint of MTR band of Figure 2-18)

$$A_B = \frac{1}{1+\frac{12}{40}} = \frac{1}{1+0.3}$$

$$= \frac{1}{1.3} = .77$$

Thus the system would be expected to be ready to operate 77% of the time when needed. This is the availability "benchmark" for the system based on a conventional design approach. By going to the top of the MTBF band and the lower edge of the MTR band, a maximum feasible estimate can be derived for conventional design:

MTBF (max) = 80 hours
MTR (min) = 8 hours
Availability (max) = .9

STEP 3 – Estimation of Maintainability Feasibility by Analysis of the Design Concept

Availability feasibility of the conceptual design can be determined analytically for specific repair actions. The system design concept is analyzed, subsystem by subsystem, to assess the following factors:

- complexity;
- design features for failure indication, switching of redundant units, etc.;
- failure rate and mean life;
- mean-time-to-repair (assuming waiting time = zero);
- mean-waiting-time.

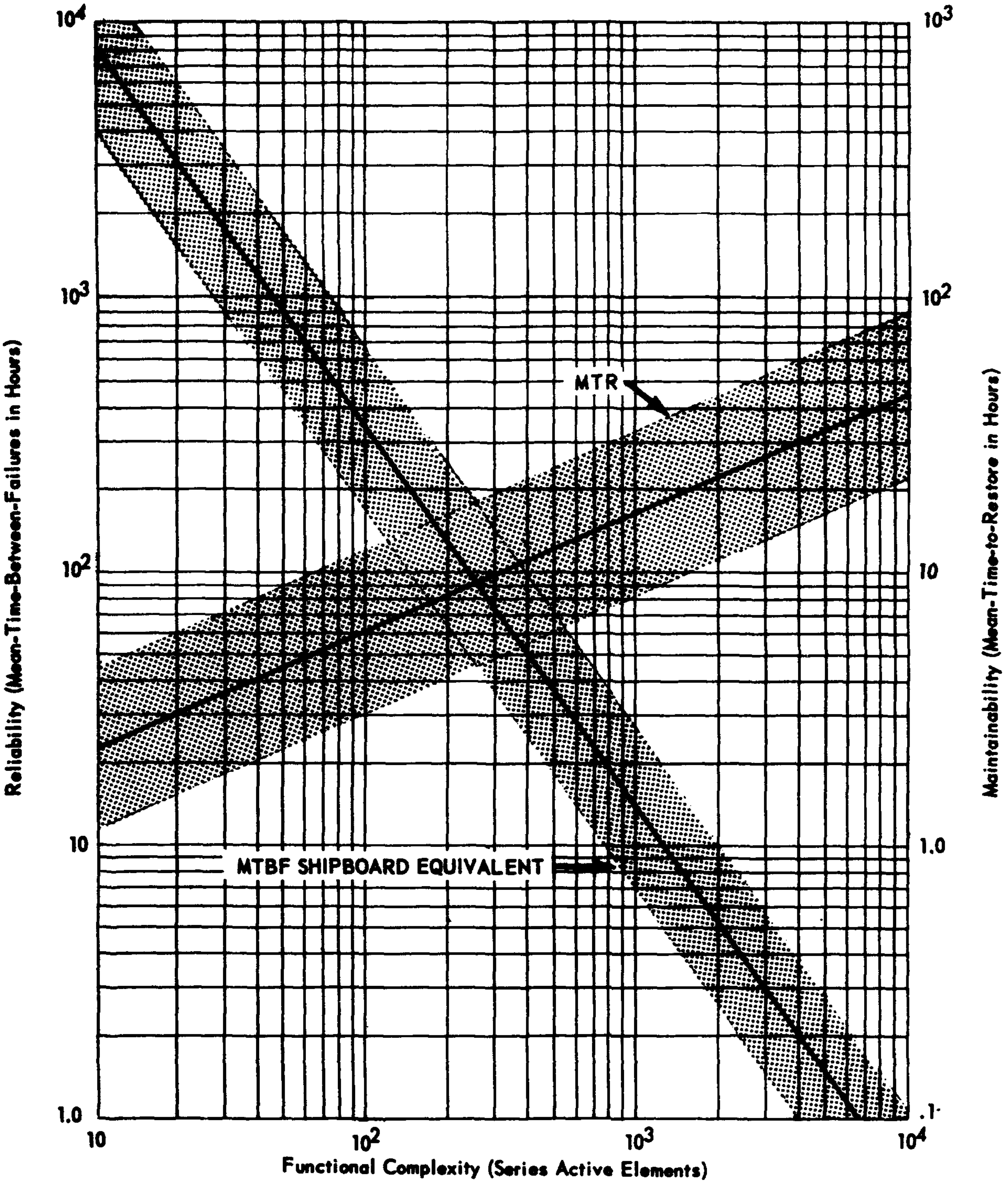


Figure 2-25. Maintainability and Availability Estimating Chart for Conventional Design

Figure 2-26 illustrates the consideration of these parameters and design features.

In this example, Units 1 and 2 are modularized for quick replacement as indicated by (d). Unit 1 is backed up by a standby, Unit 1,' which can be switched into service when scheduled monitoring at test point (a) indicates marginal performance. Unit 2 has been designed with failure indicator monitor 2_m for local indication of the discrepant unit in the event of subsystem failure.

Unit 3 consists of all the nonreplaceable AEG's – i.e., all AEG's whose failure would require removal of the subsystem from service for repair or adjustment. These should be the inherently low failure-rate elements.

Unit 4 consists of three elements considered integral to the physical configuration of the subsystem, failure of which would require replacement of the subsystem as a whole. These are the structural and mechanical features of the packaging design.

The estimation procedure is as follows:

(1) Estimate the complexity and mean life of each of the units in the subsystem.

(2) Estimate the mean time required to repair, replace, or switch units in the event of failure.

(3) Compute the ratio of MTR/MTBF for each of the units, and add, to obtain an estimate for the subsystem.

(4) Solve for availability and compare with the requirement.

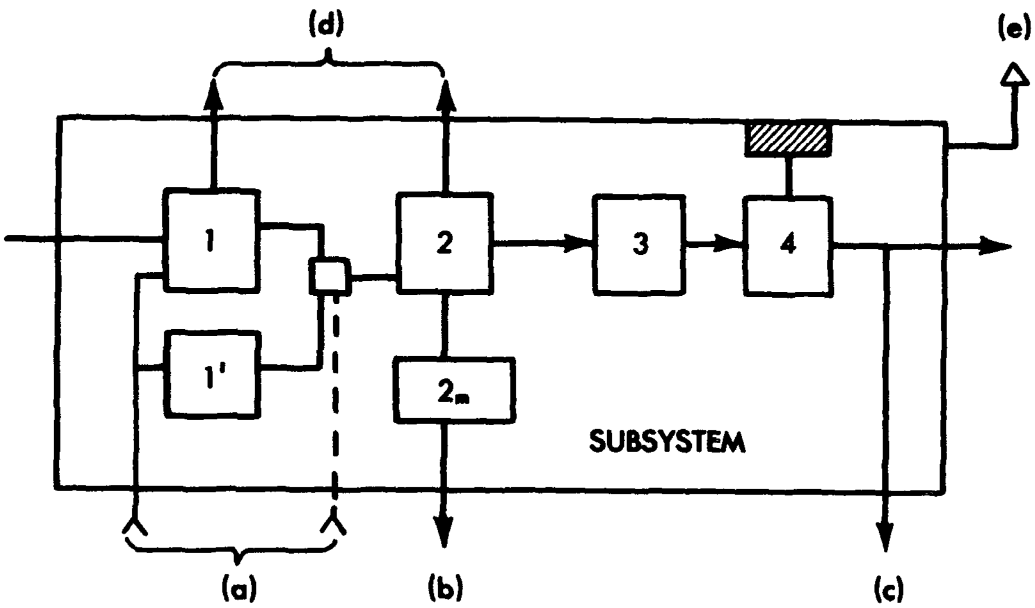


Figure 2-26. Block Diagram Reflecting Design Features for Maintainability

EXAMPLE: Assume the subsystem of Figure 2-26 indicates an "expected" mean-time-to-repair, MTR = 8 hours, on the basis of 200 series AEG's; MTBF = 120 hours; current availability = .93. One of the purposes of the conceptual design is to reduce the maintenance time and the subsystem UNavailability by a factor of ten or more. Determine the feasibility of the design concept as diagramed in the figure to achieve this objective:

(1) **Complexity and mean life by unit:**

Unit 1	50 AEG's Power	50 hours
Unit 2	50 AEG's Analog	100 hours
Unit 3	100 AEG's Digital	1,000 hours
Unit 4	(mechanical)	10,000 hours

(2) **Mean-time-to-repair:**

Unit 1	(Sense & Switch)	.1 hour
Unit 2	(Remove & Replace)	.2 hour
Unit 3	(Isolate & Repair)	1.0 hour*
Unit 4	(Remove, Repair, & Reinstall)	20.0 hours*

*Includes estimated waiting time for spare parts.

(3) **MTR/MTBF ratios:**

Unit 1	.1/50	=	.002
Unit 2	.2/100	=	.002
Unit 3	1.0/1000	=	.001
Unit 4	20.0/10000	=	.002
Total Subsystem MTR/MTBF		=	.007

(4) **Predicted subsystem availability:**

$$A = \frac{1}{1 + .007} = .993$$

The reduction in subsystem UNavailability as a result of the new design concept is calculated as follows:

$$\text{Improvement} = \frac{1 - A(\text{expected})}{1 - A(\text{predicted})}$$

$$= \frac{1 - .93}{1 - .993} = \frac{.07}{.007} \approx \text{10-to-1}$$

The foregoing example illustrates the importance of considering the design aspects of maintainability during the early system planning phase.

STEP 4 – Evaluate Effects of Duty Cycle on Subsystem Availability

It is usually permissible to assume a negligible failure rate for most system elements while in standby, although it is acknowledged that deterioration mechanisms are always at work whether a system is in an operational state or a standby state. When this simplifying assumption is known to produce serious errors in availability estimation, it is necessary to consider relative duty cycles in all modes of operational status in order to more precisely account for *all* the factors which contribute to UNavailability.

Figure 2-27 illustrates a case for a particular equipment in a missile weapon system.

EXAMPLE: The launcher of a hypothetical weapon system has an MTBF = 5 hours under continuous operation, and an MTR = 10 hours. Availability for a 100% duty cycle is given simply as:

$$A = \frac{1}{1 + \frac{MTR}{MTBF}} = \frac{1}{1 + \frac{10}{5}}$$

$$= .33$$

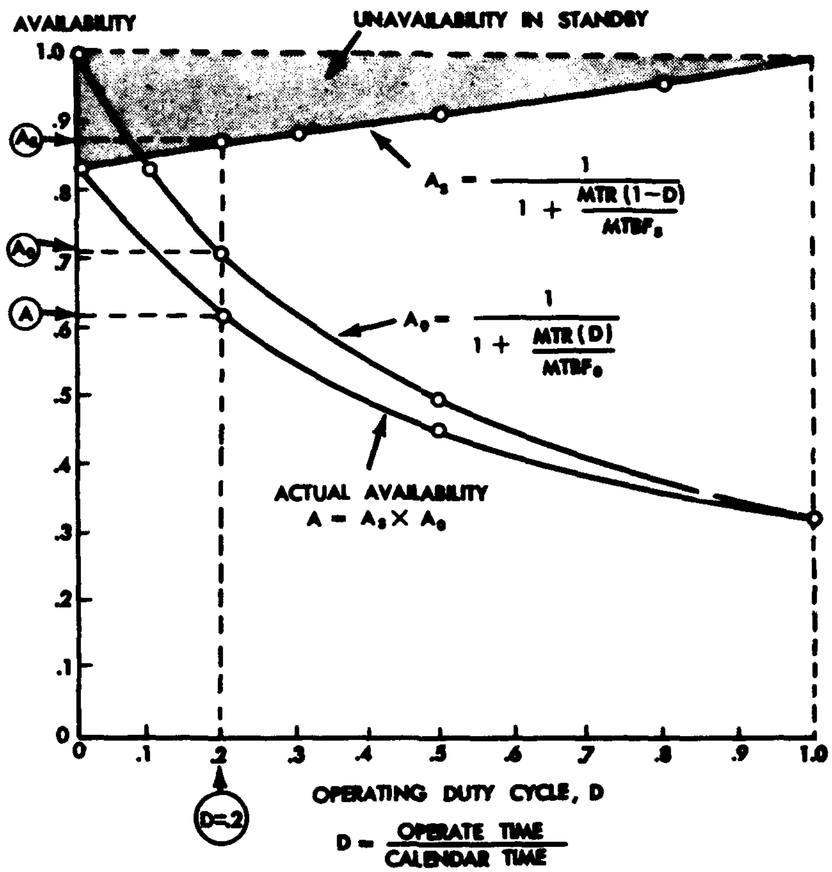


Figure 2-27. Equipment Availability as a Function of Operating Duty Cycle

If the duty cycle of the launcher can be estimated at D = .2 – i.e., will be in use approximately 20% of the time during a normal tactical engagement period – then availability would be calculated as follows, taking account of the effect of duty cycle on "apparent" MTBF:

$$A_o = \frac{1}{1 + \frac{MTR(D)}{MTBF_o}}$$

$$= \frac{1}{1 + \frac{10(.2)}{5}} \approx .72$$

This assumes a launcher failure rate during standby of zero – i.e., $\lambda_s = 0$.

Further analysis discloses, however, *that shipboard environmental* effects will result in a launcher standby mean life, $MTBF_s$, of 50 hours. The probability that the launcher will be available after a period of standby time is given by the following expression:

$$A_s = \frac{1}{1 + \frac{MTR(1-D)}{MTBF_s}}$$

where MTR is the same for both standby and operate failures, and (1-D) is the launcher *standby* duty cycle expressed in terms of its *operate* duty cycle.

In this example, standby availability is actually .87 instead of 1.0 as previously assumed. Actual tactical availability of the launcher for this particular duty cycle is then given by:

$$A = A_s \times A_o = (.87)(.72)$$

$$= .62$$

Availability for other duty cycles can be derived directly from the plot.

STEP 5 – Assess Effectiveness Growth Potential

Values of reliability and availability derived in the preceding sections can now be plotted as a continuous function of time and combined for a time-dependent estimate of system effectiveness, or "kill probability", as illustrated in Figure 2-28. The figure illustrates the combination of reliability, R(t), with availability, A, to produce the effectiveness function E(t) = R(t) × A for a *given level of performance* under specified use conditions.

Both the "benchmark" effectiveness achievable by conventional design and the predicted "feasible" level of effectiveness achievable by the proposed new design concept can be plotted for a graphical assessment of its effectiveness *potential*. The feasibility of a stated effectiveness requirement can then be ascertained by observing its relation to the area bounded by the two curves. This is illustrated in Figure 2-29.

At this point, it may be necessary to adjust the design concept to satisfy either a higher reliability requirement or a higher availability requirement, in order to increase the inherent design potential consistent with the stated requirement.

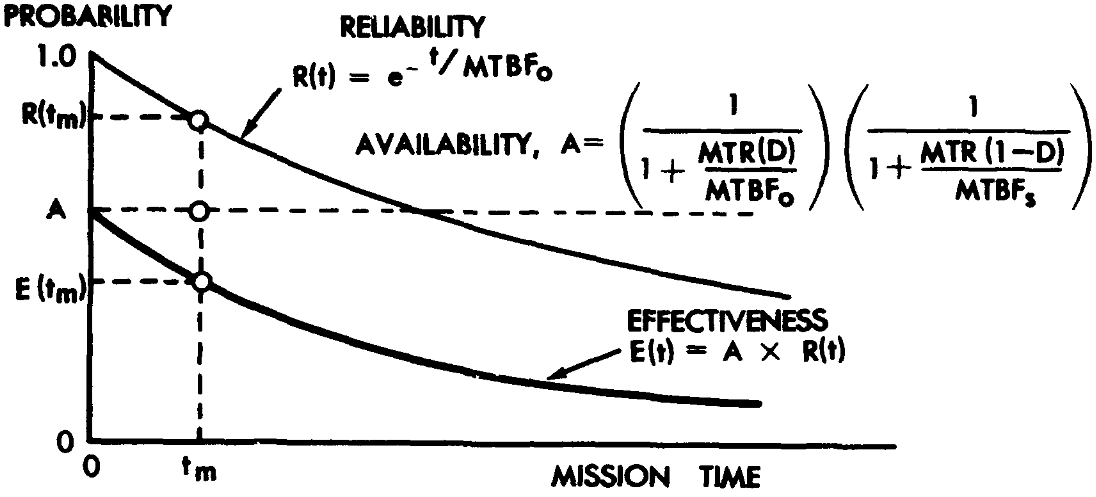


Figure 2-28. System Effectiveness Plotted as a "Time-Dependent" Characteristic

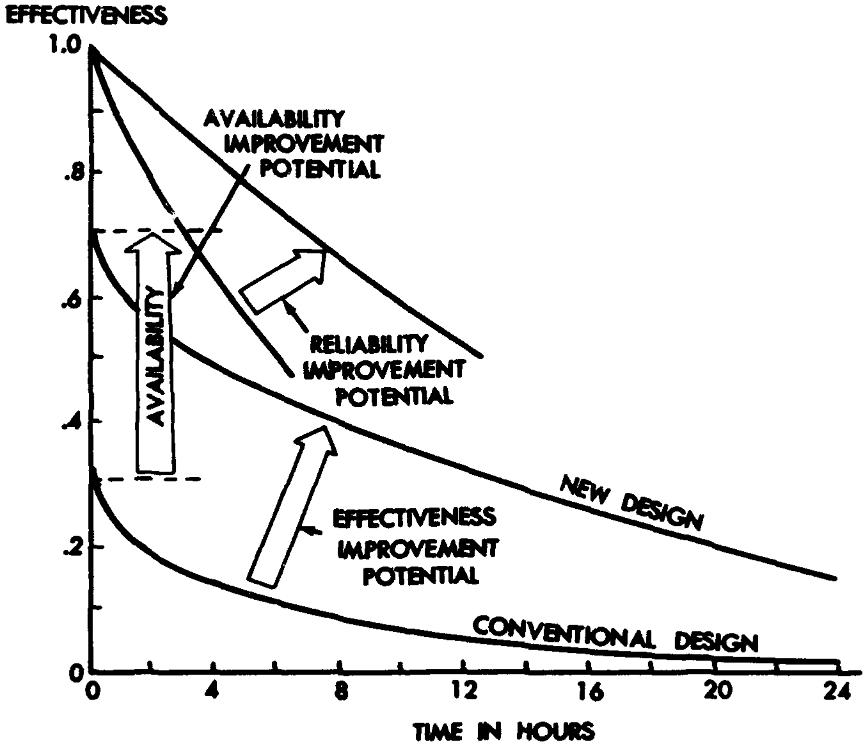


Figure 2-29. Comparison of Effectiveness Potential of New Design with Respect to Conventional Design

CHAPTER 3

RELIABILITY PROGRAM REQUIREMENTS, PLANNING, AND MANAGEMENT GUIDE

3-1 INTRODUCTION

3-1-1. General

Program managers and project engineers are charged with the responsibility for delivering reliable systems to the Fleet. However, most programs today do not provide *either reliability control or monitoring* prior to the evaluation phase, at which time it is usually too late to make modifications for reliability improvement, because:

(1) The equipment is needed now for tactical use (development time has been exhausted); and

(2) The money already spent is too great an investment to be written off because of poor reliability; it is often considered more expedient to add funds in a desperate attempt to make a "product improvement".

This section sets forth the essential *reliability program* activities deemed vital to the success of Bureau of Naval Weapons development programs in general. Emphasis is placed upon reliability program planning, monitoring, and management review procedures.

The primary purposes of a reliability program are:

- To focus engineering and management attention on the reliability requirement;
- To insure that reliability is treated as a design parameter of equal importance with other performance parameters; and
- To alert management, throughout the program, to all reliability discrepancies which may require management decision.

An adequate program must contribute to, and guide in, the orderly and scientific approach to "designing-for-reliability". It must help contractors and individuals overcome their lack of recognition that reliability must be a designed-for parameter, with practical limitations. It must foster the realization that good performance design no longer has the inherent reliability to satisfy Navy requirements. It must change the attitude of many engineers from the negative "no one designs for failures" to the positive "we must design against failures".

A reliability program will not increase the reliability of an equipment, but an effectively-monitored program will not permit an inadequate design to proceed into development, test, production, and Fleet use without specific management approval. It is this effective monitoring that will permit project engineers to assess feasibility of achievement and progress in time to make adjustments equitable to all – the user, the contractor, the Bureau of Naval Weapons.

The concept of a total reliability program, as generally endorsed by DOD, has four major points:

(1) That a quantitative requirement be stated in the contract or design specifications.

(2) That a reliability assurance program be established by the contractor.

(3) That reliability progress be monitored or audited by the Bureau of Naval Weapons.

(4) That a reliability acceptance test be successfully passed prior to acceptance. This applies to prototype or demonstration models prior to production approval, and to production samples prior to Fleet use.

This section deals primarily with the contractor reliability assurance program and the monitoring and audit of progress by Bureau personnel.

3-1-2. Applicable Documents

Bureau of Naval Weapons project personnel should contractually impose reliability and maintainability program requirements in consonance with WS-3250, WR-30, MIL-Q-9858A, MIL-R-22256 or other applicable BuWeps documents outlined in Chapter 1. These documents, in general, include those minimum pertinent contractor program activities which have received general acceptance and widespread industry application. These are *summarized in* 3-2, *following. Paragraph* 3-4 discusses the implementation and monitoring of such programs.

3-2 RECOMMENDED CONTRACTOR PROGRAM

3-2-1. General

Of specific interest is the Reliability Assurance Program required of the contractor. Those activities which experience has shown contribute to an orderly and scientific approach to "designing-for-reliability" are briefly discussed below.

3-2-2. Design Reviews

Engineering design review and evaluation procedures should include reliability as a tangible operational characteristic of the equipment, assembly, or circuit under review. Reliability considerations during the design reviews should include:

(a) Performance requirements and definitions of failure (e.g., tolerances, wear, and parameter shifts).

(b) Environments to which the device, item, or circuits will be subjected in the use configuration, including storage, transport, and production process environments.

(c) The designer's reliability prediction of the current design, supported by detailed calculations and data sources.

(d) Evaluation of tradeoffs between performance, maintainability, weight, space, power, cost, and time factors made for design optimization.

(e) Failure-mode analysis of the design. Particular emphasis should be placed upon the reduction of marginal failure modes, those which are difficult to isolate and repair.

(f) Results of all tests conducted to date.

(g) Plans for reliability improvement and problem solutions.

3-2-3. Production Control and Monitoring

Production Control and Monitoring in accordance with MIL-Q-9858A are required to assure that the reliability achieved in design is maintained during production. Detailed consideration should be given to:

(a) Integration of reliability requirements into production process and production control specifications.

(b) Production environments induced by handling, transporting, storage, processing, and human factors.

(c) Quality standards from incoming piece-part inspections through production acceptance on the basis of time-dependent parameter variations occurring during application, storage, and transportation.

(d) Calibration and tolerance controls for production instrumentation and tooling.

(e) Integration of reliability requirements and acceptance tests into specifications for the purchase of materials and parts to be used in production of the system.

(f) Determination of failure modes related to production process and production control discrepancies and evaluation of corrective action taken on production process discrepancies.

(g) Design and production processing change orders for compliance with reliability requirements.

(h) Life tests of production samples to verify quality standards and inspection techniques.

3-2-4. Subcontractor and Vendor Reliability Control

Provisions should be established to insure subcontractor and vendor selection and performance consistent with the reliability requirements of the contract. Subcontractors and vendors must look to the prime contractor for a clear definition of reliability required in subcontracted items. Once these requirements have been adequately defined, the prime contractor must extend the scope of his reliability assurance program to the monitoring and control of his subcontractors and vendors. Such monitoring and control should include:

(a) Incorporation of reliability requirements in subcontractor and vendor procurement documents.

(b) Provision for assessment of reliability progress, including reliability qualification and acceptance testing of incoming products.

(c) Adequate liaison to insure compatibility among vendor products to be integrated into the end item.

(d) Initial selection procedures for subcontractors and vendors which consider, in relation to the requirement: past performance, willingness to test and share test data, interest and response on feedback of deficiency information, test philosophy, and realism of cost and delivery schedules.

3-2-5. Reliability Development Test Program

Reliability demonstration tests are, in general, statistically-designed experiments in which due consideration is given to confidence levels and sampling errors. Unless proof of adequacy can be substantiated by other available data acceptable to the procuring activity, all items of equipment of higher-order designations should be tested in order to verify that reliability is achievable with the proposed design. If it is not achievable, the problem areas which prevent its attainment should be isolated and defined. The test program should include:

(1) Tests of questionable areas where reliability experience is not available, particularly new concepts and materials.

(2) Tests to determine the effects of unique environments or combinations of environments.

The extent of the test program is determined by weighing the cost of testing against the degree of assurance required that the product will have a given level of reliability.

In addition to those tests performed specifically for reliability demonstration *all* formally planned and documented tests which are performed throughout the contract period should be evaluated from a reliability viewpoint to maximize the data return per test dollar. Data which are obtained should facilitate:

(a) Estimation of reliability on the basis of individual and accumulated test results.

(b) Determination of performance variabilities and instabilities that are induced by time and stress.

(c) Evaluation of maintenance accessibility and operator-adjustment requirements.

3-2-6. Reliability Analyses

Periodic analyses of reliability achievement should be included as a normal part of technical progress evaluations. These analyses should be scheduled to coincide with quarterly, semi-annual, or other technical progress reporting requirements established by the contract. These analyses should consider:

(a) Reliability estimates based on predictions and test data.

(b) The relationship between present reliability status and scheduled progress.

(c) The changes in concepts and approaches that are necessary to accomplish the contract objective.

(d) The effects of changes made in design and manufacturing methods since the previous analysis.

(e) Changes in operational requirements, including environmental conditions, operator and maintenance personnel qualifications, logistic support requirements, and interface conditions.

(f) Criteria of success and failure, including partial successes (degraded operation) and alternative modes of operation.

(g) Production tolerances and techniques, including assembly test and inspection criteria and test equipment accuracies.

(h) Specific problem areas and recommended alternative approaches.

3-2-7. Failure Reporting, Analysis, and Feedback

A formalized system for recording and analyzing all failures should be established. Analyses should be fed back to engineering, management, and production activities on a timely basis. Complete reporting provides chronological data on operating times, on-off cycling, adjustments, replacements, and repairs related to each system, subsystem, component, and "critical" part. Through the analysis of these reports, reliability is measured and improved on a continuing basis. Reports should be complete and accurate in recording:

(a) System, subsystem, component identification.

(b) Total accumulated operating time on system and component in which failure occurred.

(c) Performance behavior or malfunction symptom which accompanied the failure.

(d) Test or "use" conditions at time of failure.

(e) Identification, classification, and apparent cause of failure.

(f) Repair action taken to restore next higher assembly to operational status.

(g) Time required for fault detection and correction (maintainability evaluations).

(h) Identification of test activity or organization, and the individual operator or technician making report.

(i) Report serial number, date and time.

(j) Failure-diagnosis summary and recommended recurrence-control measures.

Timely analysis of all discrepancy or failure reports by an analysis team formally constituted by management determines the basic or underlying causes of failure in parts, materials, processes, and procedures. The analysis includes failures in design, manufacture, procurement, quality control, maintenance, and operation. Results of failure analyses should be fed back to design, production, and management personnel for assignment of corrective action and follow-up responsibilities as appropriate.

3-2-8. Reliability Monitoring

The contractor should establish a monitoring activity to insure the adequate development of reliability. The monitoring activity performs three basic functions: analysis of reliability status relative to requirements; determination of corrective action needs; and follow-up on the corrective action. Documentation of the reliability-assurance and monitoring procedures developed for this activity, including checklists and instructional material normally used by the contractor, should be maintained in a form clearly delineating the approach used and the results obtained. Such documentation of the procedures and objective evidence of reliability conformance should be available for review by the procuring activity as required. The results of monitoring should be made available to responsible management through the following types of reports:

(a) *Reliability-assessment reports:* Periodic objective assessments of reliability status relative to contract requirements and schedules. These assessments should be performed by personnel who are not directly responsible for the design, development, or production of the procurement item.

(b) *Reports of major discrepancies and corrective action taken:* Methods for alerting contractor and procuring activity managements to all major reliability discrepancies which may require management decisions with respect to changes in schedules or requirements and the like, and methods for reporting the results of corrective action.

3-3 SPECIFICATION OF THE PROGRAM

The requirement for a reliability assurance program should be included as a subparagraph of the "Requirements" section of the design specifications or other contractual documents. This subparagraph may specify the entire required program, or it may reference a standard document such as WS-3250, with specific additions and deletions, as necessary. Coverage, not method, is important. Figure 3-1 lists those requirements which should be specified.

Program Activities	Reliability Reporting Requirements
Program Activities: – Design Reviews – Production Control – Vendor Reliability Control – Development Test Program – Reliability Analyses – Failure Reporting and Analyses – Specific Activities or Tests Peculiar to a Given Class of Equipment **Reliability Monitoring Functions:** – Independent Assessments (MIL-HDBK-217) – Major Discrepancy Reporting – Program Documentation	**Reliability Reporting Requirements:** – Coverage in Technical Progress Reports – Specific Reliability Design Analysis Reports (MIL-HDBK-217; MIL-STD-756A) – Acceptance Test Report **Maintainability Requirements:** – Program Plan and Activities (WR-30) – Reporting and Monitoring (WS-3099) **Quality Assurance:** – Specific QA Plan (MIL-Q-9858A) – Progress Evaluations by Procuring Activity – Acceptance Tests Required

Figure 3-1. Reliability Assurance Program Requirements (Presently Reflected in WS-3250)

3-4 PROGRAM MANAGEMENT AND MONITORING

It is one thing to reference a specification (or requirement) in procurement documents, and quite another to determine "how to comply" and "what is compliance?" Effective implementation requires that both the Bureau project engineer and the contractor fulfill their obligations and responsibilities in a spirit of teamwork toward the common objective – reliable equipment in the Fleet. The following sequence of steps is presented as a guide in this implementation.

STEP 1 – Specify Reliability Requirements

The project engineer should state the reliability requirements in design specifications or procurement documents (including requests for proposals.) Figure 3-2 is a checklist to assist in preparation or review of reliability specifications.

Section 1. Scope

- Clear, concise abstract of specification coverage.
- Description of the item (or items) in sufficient detail to preclude misinterpretation of the extent of coverage intended by the specification.

Section 2. Applicable Documents

- Reference only to those specifications and documents that are referenced in the body of the specification. (Additional references not directly pertinent tend to cloud the basic specification requirements.) Documents referenced must be available in approved form at time of specification issue.

Section 3. Requirements

- Clearly expressed quantitative requirements which reflect minimum acceptable operational demands.
- Definition of satisfactory performance and the dividing line between satisfactory and unsatisfactory performance (success or failure). More than one definition may be included to correspond to different modes, functions, and degrees of failure in large, complex systems.
- The time period of interest in the form of mission sequence (or profile), duty cycles, and the like.
- Environmental and other use conditions under which the system will be expected to achieve the quantitative requirements.
- Program requirements specifically applicable to the system and phase of development or production.
- Reference to appropriate general specifications.
- Reporting requirements as a part of total program reporting.
- Submission dates for special reports required by general specifications and other referenced documents.
- Date of submission of detailed acceptance test plans for approval.

Section 4. Quality Assurance Provisions

- Scheduled periodic progress monitoring by the procuring activity.
- Acceptance test plan(s) outline, including:
  1. General test or inspection conditions, or duty cycles.
  2. Description of item(s) to be accepted under the tests (if different from the total system as defined under "Scope").
  3. Number and sampling plan for selection of items to be tested.
  4. Estimated test duration.
  5. Success and failure criteria related to test conditions.
  6. Accept/reject criteria of the test plan.
  7. Statement of consumer's risk (a measure of the adequacy of the test plan in discriminating between acceptable and unacceptable product).

Section 5. Preparation For Delivery

- Disposition of test items.

Section 6. Notes

- Unique or changed definition of terms.
- Explanatory information as required to aid in clarity of previous sections.

Figure 3-2. Specification Checklist

STEP 2 – Establish Schedules

The project engineer should establish schedules for reliability reporting and monitoring.

- **Reliability Design Analysis Report(s).** Delivery dates for such reports may be specified on either a calendar or a program-phase basis. It is usually preferable to have a report submitted quarterly, with each succeeding report refining the analysis.

- **Acceptance Test Plan – Submission for Approval.** The detailed test plan should be submitted 30 to 60 days prior to test initiation, in order to allow sufficient time for Bureau review and approval.

- **Progress Evaluation Schedule.** Progress evaluations, as visualized for effective monitoring, are made by a team of Bureau personnel or their independent consultants who perform the evaluation by detailed personal review at the contractor's facilities. These reviews are best scheduled to correspond with major milestones, rather than at fixed time intervals.

STEP 3 – Prepare RFP

The project engineer should include desired proposal coverage of reliability in the Request for Proposal. The following may be inserted in the RFP:

Proposals responsive to this RFP shall contain the following:

1. *Understanding of the requirements.*

2. *Proposed technical and management approach toward achievement within the stated or implied limitations (if the bidder deems the requirement unrealistic, that which he considers realistic and achievable should be stated).*

3. *Supporting evidence for 1 and 2 above, including: reliability estimates of the proposed concept and approach (refer to MIL-STD-756A); source and applicability of data; experience of bidder with similar programs; specific ways and means of attainment (e.g., redundancy, improved parts, or new techniques); assumptions and non-controllable dependencies upon which the approach is based.*

4. *Description of the proposed Reliability Assurance Program, including:*

   a. *Description of proposed program in relation to overall contract effort.*

   b. *Specific technical activities, where appropriate.*

   c. *Magnitude of effort by activity.*

   d. *Responsibilities and authorities within the proposed organizational structure (including list of key personnel, together with background and experience).*

   e. *Proposed schedule of reliability activities.*

   f. *Recommended monitoring points and major milestones.*

   g. *Proposed reliability development test program.*

STEP 4 – Prepare Proposal

The prospective contractor should prepare proposal in response to RFP and the requirements of Step 3. Specifically, the proposing contractor must:

- Analyze the reliability requirement and make a preliminary prediction to determine the feasibility of the requirement for a given time and cost. This forces the bidder to establish cost, development time, and reliability tradeoffs as a part of his proposal effort.

- Establish and cost his reliability activities and integrate them into the total program. For contractors whose reliability awareness is reflected in supporting staff activities, this task is routine. For contractors who previously have ignored or have merely given lip service to reliability, it can be a difficult task, at times requiring major reorganizations within the company. Contractors must firmly commit themselves to a future course of action. They must give evidence of adequate experience, competence, facilities, and data.

- Schedule in-house reliability accomplishments and monitoring which become part of the master schedule. Where a PERT program is required, the schedule must include the intended accomplishments of significant reliability activities.

- Plan development reliability tests. The proposing contractor must evaluate the design approach and planned developments to determine which assemblies and components will require test demonstration. This determination affects proposed development cost and time estimates.

STEP 5 – Evaluate Proposals

The project engineer should evaluate proposals for their response to the previous steps. The proposals should be evaluated in terms of their applicability to the specific task at hand. Although apparently well-organized, well-staffed, and well-documented reliability organizations and procedures indicate a wealth of experience, the willingness of a contractor to pursue the specific reliability program required by the proposal is of prime importance.

The proposal review should give particular attention to the reliability activities proposed by the contractor rather than stress the contractor's organizational structure per se. The reliability activities will inevitably reflect the policies and procedures of management upon whom the line organization depends for effective guidance, support, and continuity; and in the final analysis the strength of the organization will be determined by its effectiveness in contributing to the acceptability of the end product.

Figure 3-3 is a guide for use in evaluating proposals.

Requirements Analysis:

- Is the reliability requirement treated as a design parameter?
- Has the requirement been analyzed in relation to the proposed design approach?
- Is there a specific statement that the requirement is, or is not, feasible within the time and costs quoted? If not feasible, is an alternative recommended?
- Is there evidence in the proposal that the reliability requirement influenced the cost and time estimates?
- Is an initial prediction included in sufficient detail (data souices, complexity, reliability block diagram, etc.) to permit Bureau evaluation of its realism?
- Are potential problem areas and unknown areas discussed; or, if none are anticipated, is this so stated?
- If the requirement is beyond that which presently can be achieved through conventional design (MIL-STD-756A), does the proposal describe "how" and "where" improvements will be accomplished?

Reliability Program and Monitoring:

- Is the proposed program in accord with the procurement request?
- If the contractor has indicated that certain of the reliability activities requested are not acceptable to him, has he suggested satisfactory alternatives?
- Is the program specifically oriented to the anticipated needs of the proposed equipment?
- Are program activities defined in terms of functions and accomplishments relating to the proposed equipment?
- Does the proposal include planned assignment of responsibilities for reliability program accomplishments?
- Is it clear by what means the reliability program may influence development of the proposed equipment?
- Have internal "independent" reliability assessments been scheduled?
- Does the reliability demonstration test program designate which equipments, assemblies, or components will be tested, and to what extent?

Figure 3-3. Proposal Evaluation Guide (Sheet 1)

- Does the proposal provide justification (data derived from testing or other experience) for the exclusion of specified items from demonstration testing?
- Is the proposed documentation of activities, events, and analysis designed for ease of monitoring, ease of data retrieval, and use on future programs?
- Are planned accomplishments (events) scheduled and included in PERT, if applicable?

Acceptance Testing:

- Has the bidder agreed to perform acceptance tests and included the costs and time within his proposal?
- If acceptance test plans were not included in the request for proposal, has the bidder recommended any?
- Does the proposal contain a positive statement concerning the bidder's liability in the event of rejection by the acceptance test?

Background Organization and Experience:

- Does the bidder have documented reliability experience on previously developed equipments, components, etc.?
- Does the bidder have an established system whereby past experience is made available to engineers and designers?
- Does the bidder have a designated group (or individual) to whom designers can turn for reliability assistance, including part ratings, selection, and test design?
- Does the assignment of responsibilities indicate that reliability is treated as a line function rather than a staff function?
- Is overall responsibility for reliability assurance vested in top management?
- Do (or will) company standards manuals or other documents set forth standard reliability operating procedures?
- Does the bidder have in being a formal reliability training program for management, engineering, and technical personnel?
- Does the bidder implement and conduct planned research programs in support of line activities, seeking new materials, new techniques, or improved analytical methods?

Figure 3-3. Proposal Evaluation Guide (Sheet 2)

STEP 6 – Review Contractual Documents

The project engineer should review contractual documents prior to contract negotiation. Changes in the reliability requirements, the reliability program, or the acceptance test that are recommended in the proposal submitted by the successful bidder, if accepted, must be reflected in the design specifications, references, or contractual documents. When the recommendations are not accepted, the prospective contractor should be notified early in the negotiation period in order that his cost and time estimates may be adjusted prior to final negotiation.

STEP 7 – Implement Reliability Program in Design

Both contractor and project engineer should implement and monitor the reliability program during design. The contractor is committed to perform in accordance with the referenced specifications and items covered in the contractual documents. (Unless the proposal is a referenced document in the contract, the contractor is not obligated by its statements.) The project engineer's primary avenue of monitoring is the review of reports, as follows:

A. **Initial Technical Report** – The following major items of the initial report should be promptly reviewed and judged for adequacy:

- Contractor's understanding and interpretation of the reliability requirements specified in the contract, with a description of the engineering approaches contemplated.
- Description of the reliability assurance program and monitoring procedures to be used throughout the contract period.
- Progressive reliability milestones and monitoring schedule.

B. **Progress Reports** – Follow-up technical reports during the design phase should be reviewed for:

- Status of design reviews and pertinent results.
- Trade-offs and reliability considerations in the selection of parts, circuits, and configurations.
- Reliability allocations and requirements included in subcontractor or vendor supplied items.
- Reliability predictions:
  - (1) Check model consistency and accuracy;
  - (2) Insure that all parts and units are included;
  - (3) Insure that failure rate data from report to report remain constant unless change is justified by data.
- Summary of test results.
- Summary of reliability problem areas and planned solutions.
- Adherence to reliability program schedule.
- Analysis of the effect that schedule changes, design problems, and procurement delays will have upon the reliability program.
- Status of reliability program activities in relation to the program plan submitted in the initial report.

C. **Separate Reliability Design Analysis and Prediction Report** – The separate reliability design analysis and prediction report(s) should contain a thorough analysis of the design. Design analysis should include:

- Reliability predictions in accordance with MIL-STD-756A and MIL-HDBK-217.
- Comparison of present design status of the equipment with the contractual requirements.
- Analysis of failure modes, summary of findings, and plan to design changes.
- Results of tolerance, stability, and life tests.
- The effects of total program problems upon reliability status and achievements.
- Actions, if required, that are planned in order to improve the reliability of the design.

D. **Progress Evaluation** – Effective monitoring requires periodic progress evaluations as one of the Quality Assurance provisions in assessing reliability attainment. Progress evaluations scheduled during design phases should, in general, verify that the reliability assurance program approved in the initial technical report is in fact being implemented and that the progress reports are complete and factual with respect to progress and problems reported. The overall effectiveness of the reliability program can be partially assessed (final assessment comes with the acceptance test and subsequent Fleet use) by determining the degree to which the program has influenced the following:

(a) Simplicity and conservatism in design.

(b) Recurrence control of failures and reduction in the effects of failure.

(c) Safety factors and derating policy in component and part applications.

(d) Provision for functional independence among major subsystems.

(e) Due consideration of reliability in trade-off decisions.

(f) Documentation of reliability in specifications, operating instructions, and handbooks.

(g) Reliability requirements considered in test planning and design.

(h) Analysis and use of data in problem solving.

(i) Dissemination of reliability techniques and training in their use.

(j) Awareness and routine consideration of reliability as a system parameter within the various personnel skill groups.

It is suggested that the most efficient way of conducting these progress evaluations is by personal discussions with engineers, test technicians, specification writers. Comparisons

• Engineers' predictions or estimates	versus	Published predictions
• Data feedback from tests	versus	Test log and test technicians' observations
• Engineers' description of design reviews	versus	Program plans
• Personnel from whom designer obtains reliability assistance	versus	Organizational structure
• Actual company-sponsored reliability training of technical personnel	versus	Documented company training program
• Actual availability of data on standard parts from past experience	versus	Stated or implied availability
• Designer's knowledge of reliability requirements, including environments and performance limits	versus	Actual requirements
• Procurement personnel's considerations in vendor selection	versus	Program plan
• Part counts and stress analysis, from working drawings	versus	Those presented in prediction report

Figure 3-4. Progress Evaluation Guidelines

such as those listed in Figure 3-4 will prove fruitful.

The project engineer and evaluation team should prepare a detailed report on the results of the evaluation, pointing out areas of compliance and progress as well as omissions. Specifically, the report should state that progress appears satisfactory, or not, in relation to the time in development and the contractual requirements. This will aid Bureau management in deciding on the future course of the development program.

STEP 8 – Implement Reliability Program in Development

Both contractor and project engineer should implement and monitor the reliability program during the prototype-development phase. Again, reports are the principal monitoring tool, as follows:

A. **Progress Reports** – Each progress report must update its predecessor in each area of coverage. The additional coverage that must be reviewed is as follows:

- Subject of design changes to the same design reviews and reliability analyses as were performed on the original design, as a prerequisite for incorporation into the product.
- Summarized results of circuit temperature, vibration, and other environmental tests.
- Demonstration test plans and results.
- Supporting data and justification for reliability confidence in those items not subjected to reliability demonstration tests (cases in which a test waiver is requested should be approved).
- Summaries of failure analysis and major discrepancies, and proposed solutions for the latter.
- Approach to packaging designs and methods of environmental analysis.
- Progress in the procurement and use of end-item parts.

B. **Reliability Design Analysis and Prediction Reports** – This series of reports should become successively refined as the development progresses. The major additional points to review are:

- Completion of stress and environmental analysis of all applications.
- Confirmation or rejection of predicted results on the basis of test data.

C. **Progress Evaluation** – Progress evaluations performed during the development period should concentrate on:

- Continued adherence to the program plan.
- Subcontractor and vendor success in meeting requirements.
- Development test program.
- Review and approval of data which provide confidence in the reliability of items not tested.
- Degree of analysis and feedback in the failure-reporting activity.
- Deviations, waivers, and modifications of the prototype models from the design initially conceived or still planned for production.

STEP 9 – Monitor Acceptance Test

The project engineer should monitor the reliability acceptance test and approve the test report. The reliability acceptance test plan should include the following information:

- A complete description of the item or items to be submitted for acceptance under the test requirements.
- The test conditions to be applied, including operational and environmental duty cycles and test levels.
- Number of items to be tested.
- Estimated test duration.
- Success and failure criteria.
- Accept/reject criteria of the test plan.
- Consumer's risk (a measure of the adequacy of the test plan in discriminating between acceptable and unacceptable products).

(In complex system development programs where it is not feasible to perform a complete systems acceptance test, individual acceptance tests for various sublevels may be specified, together with methods to be employed for synthesis of reliability at the system level.)

The reliability test report should summarize the test plan and the procedures employed. It should note any deviations from the initial planning document with their probable effect upon the test results, and it should include the applicable reliability requirements, acceptance criteria, test results, and conclusions.

If a design is rejected by the test, the test report should contain a detailed analysis of failures and the contractor's plan to overcome the deficiencies in the design.

STEP 10 – Implement Reliability Program in Production

Both contractor and project engineer should implement and monitor the reliability program during production of the equipment. Throughout production, periodic progress reports should be reviewed for:

- Design changes, in order to insure that each production engineering and design change is given the same reliability assurance considerations and approvals that the original design received.
- Procurement of parts and assemblies in accordance with the appropriate reliability requirements.
- Evidence that each step in the production process has been evaluated for its possible detrimental effect upon reliability.
- Effectiveness of production inspections and collection, analysis, and feedback of test data in maintaining design quality.
- Summary of qualification, environmental, and other test data.
- Compliance with the production acceptance tests.

STEP 11 – Monitor Service Test

The project engineer should monitor the service test, evaluation, and Fleet performance of the delivered equipment. The life cycle is not complete until the reliability in the Fleet has been evaluated and the results have been recorded, analyzed, and fed back into new equipment programs. Monitoring of these phases in the equipment life cycle is largely by review of routine reports. Specifically, the following should be reviewed and analyzed:

- Reports by service test and evaluation units, such as the VX squadron. (Where some measure of control can be maintained, required data should cover performance, operating time, unit and part removals, and failure symptoms.)
- Routine failure reports (DD-787's, EFR's, and FUR's).
- Specific operating unit reports detailing equipment casualties, problems, etc.
- Operator or pilot logs, maintenance shop logs, and overhaul/repair facility records.
- Logistics and experience with the issuance of spare parts.

3-5 RELIABILITY AND MAINTAINABILITY TRAINING

3-5-1. General

The concepts of reliability and maintainability in weapon system development are not new, nor are they too complicated to understand if they are clearly and simply described. Only a few of the fundamental principles need be understood by project management and engineering in order to put quantitative measurements on these two system parameters for which they already have an intuitive feel. It is true that the complexities of redundancy, statistical test design and sampling, and many other aspects of reliability assessment are difficult to understand. They are also difficult to teach. On the other hand, these same aspects usually require the help of a specialist anyway, so at most a training course for project engineers need only make them aware of the methods and appreciative of the need for this specialized help from other Bureau offices whose functions are to provide such services.

The problem, then is to prepare and present a highly practical course in the fundamentals of reliability and maintainability, tailored to fit the needs of individual groups within the Bureau. Thus, the course must be dynamic in its flexibility and adaptability. It must be well documented with examples and "tools" of the trade.

3-5-2. Guidelines

One of the best concepts of teaching follows the well-known, long-established, "on-the-job" training approach wherein the instructional material is related to the specific jobs for which a particular group is responsible. The following guidelines may be helpful in planning the training course:

- The course should be a conference-type presentation, with prepared notes and figures distributed to students at least one week in advance.
- Maximum use should be made of case histories – hypothetical, if not real – to demonstrate the application of reliability concepts and existing documentation[1] to specific areas of responsibility.

[1] Specifications such as MIL-R-22973 (Wep), MIL-STD-756A, and MIL-HDBK-217.

- Course material must be acceptable to people with mixed backgrounds – administrative as well as technical. Where an understanding of technical and mathematical details is vital for the application of a concept, these details should be covered in appendices to the conference notes.

- Scope of course content should range from management practices to engineering methods, from the conceptual stage of system development through delivery of hardware to the Fleet.

- Depth of content (oral presentation) should be adjusted to fit within a total of eight to ten sessions of one-and-a-half hours each.

3-5-3. Course Outline

The following suggested course outline can be adapted to specific needs drawing on appropriate sections of this handbook.

- What you should know about basic concepts of reliability, availability, and maintainability as measurable product characteristics:
  - How to define these characteristics for specific equipments;
  - How to graphically and mathematically "visualize" these characteristics;
  - How to measure reliability and availability with known confidence.

- What you should know about specifications pertaining to reliability and maintainability:
  - How to determine requirements for parts, equipments, systems;
  - How to specify the requirements;
  - How to specify tests for compliance with given confidence.

- What you should know about reliability as an engineering function:
  - How to estimate reliability feasibility of new design concepts;
  - How to predict reliability achievement during the design and development phase;
  - How to evaluate the described reliability problem areas, for correction early in design.

- What you should know about reliability as a reliability-assurance function:
  - How to "control" reliability;
  - How to demonstrate reliability achievement.

- How to review and develop specific equipment and system program plans and specifications:
  - Requirements;
  - Quality assurance provisions for reliability and maintainability.

- How to review development status of specific systems:
  - Reliability assessment.
  - Problem areas.

- What you should know about contractor reliability programs:
  - How to evaluate a program;

- How to specify program requirements;
- *How to monitor contractor programs for compliance.*

- What you should know about reliability monitoring and failure diagnosis:
  - In design, development, production, and field use;
  - To assure earliest practicable correction.
- What specific steps you can take today to assure higher reliability and maintainability in systems for tomorrow:
  - Requirements analysis and specifications,
  - Demonstration and acceptance;
  - Procurement documentation;
  - Monitoring and follow-up.

3-5-4. Planning Considerations

The proposed outline should be coordinated with designated staff members of a particular branch in order to more exactly fit the needs of that branch – to more comprehensively satisfy all the needs within the student group. However, it is difficult to achieve universal acceptance without sacrificing details in certain restrictive yet vitally important areas of interest, and a "play-by-ear" approach is the best method for keeping the course dynamically in tune to the needs of the group.

CHAPTER 4

DOCUMENTATION OF RELIABILITY AND MAINTAINABILITY REQUIREMENTS

4-1 INTRODUCTION

4-1-1. Purpose and Scope

It is now generally recognized that early system development plans are not complete if they do not *quantitatively* define the required characteristics of the product or system proposed for development. While in the past the characteristics of a new equipment or system have been adequate to guide development effort toward full realization of *performance* requirements, they have not been sufficiently descriptive of the reliability and maintainability characteristics required for system success under Fleet use conditions.

It is also a generally accepted fact that these important "success" characteristics must be *planned for* and *designed in* – they cannot be later added to the system as an afterthought. One need only to review field reports from present systems to become acutely aware of the disparity between what was *needed* (but not specified) and what was delivered (but not wanted).

This chapter of the handbook outlines step-by-step procedures for the definition and documentation of reliability and maintainability requirements in essential planning documents, specifications, and contractual task statements.

The problem is one of first determining system requirements for reliability and maintainability from the Specific Operational Requirement (SOR) which constitutes the directive for preparation of the Technical Development Plan (TDP)[1], then defining the requirements, and finally *documenting* these requirements in the TDP and the design specification, in order to give the system concept a clean entry into its development cycle – to assure years hence that an operationally suitable weapon system evolves as a result of *good planning now*, followed by *effective pursuit* of planned objectives.

4-1-2. Documentation Checklist

Figure 4-1 presents a checklist of the technical and operational points which should have been defined during the requirements analysis stage discussed in Chapter 2. The chart serves as a checklist to evaluate completeness of information about the system whose requirements are about to be fully documented in the Technical Development Plan and the design specification.

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[1] Paragraph 2 of OPNAV 3910.61 see also SOR, Appendix 1.

Requirements Data Must Describe:

- Planned installation, storage, and tactical deployment of the system – type of vehicle, type of storage, and geographical areas of use.
- Reaction time required – time between "command" to operate and "operate".
- Mission duration requirement for each type of mission or operating mode.
- Turnaround time required – elapsed time between successive missions.
- ★ Overall mission reliability for each type of mission, operating mode, and specified level of performance, allocated down to the subsystem level.
- Availability or combat-ready rate (operational readiness) – percent of the total number of systems that are to be "ready" at any point in time, or percent of times a system must successfully respond to an "operate" command.
- Maintenance and operating environmental conditions – climatic, facilities, and support.
- Planned utilization rate – number of missions expected per unit of time under combat conditions.
- Minimum allowable time between scheduled maintenance.
- Test and checkout philosophy – extent of automatism, complexity or test equipment, degree of fault isolation and indication at the local level, degree of remote failure monitoring.
- Echelons of maintenance or maintenance concept to be used – replaceable modular packaging, etc.
- Maintenance and crew personnel requirements – numbers and skills, level of training.
- ★ Mean-time-to-repair requirement, and specified level of "intrinsic" system availability.

Starred items are directly related to the reliability and maintainability requirements to be documented.

Figure 4-1. Checklist for Evaluation of Documentary Source Data

4-2 DOCUMENTATION OF RELIABILITY AND MAINTAINABILITY REQUIREMENTS IN TECHNICAL DEVELOPMENT PLANS (TDPs)

4-2-1. Role of the TDP

> The Technical Development Plan (TDP) . . .
>
> "comprises the plan for the fulfillment of an Advanced Development Objective or a Specific Operational Requirement. It serves as a basic decision-making document at the Bureau management level, and above. Approval by CNO constitutes the authority to commence development commensurate with funds that are provided by separate action. When funded, the TDP becomes the primary management control and reporting document for the life of the development. It is essential that it be kept up to date on a continuing basis."
>
> —OPNAVINST 3910.4A, 11 December 1962

Thus the important role of the TDP is established.

4-2-2. TDP Format

There are two ways in which reliability and maintainability requirements are documented in the TDP:

(1) As integral requirements of the system and the system development program, in which reliability and maintainability requirements are integrated into the overall system description along with performance and other requirements; and

(2) As supplemental requirements presented in separate sections (or appended as separate documents)

The first method (integrated requirements) is consistent with the argument that *effectiveness* is what we must really attempt to define; and effectiveness is *jointly dependent* upon the three major system characteristics: performance, reliability, and maintainability (availability). The second method arises from the proven need for special emphasis on the effectiveness problem in complex systems. In either case, reliability and maintainability should be treated jointly in requirements and planning documentation, since both must be simultaneously considered by the designer in effectiveness optimization tradeoff studies, and neither can be separately achieved as a system requirement without due consideration of the other *during the design phase*.

The following steps are related to specific sections of the TDP, consistent with TDP format outlined in BUWEPINST 3910.2A, as shown in Figure 4-2.

Section	Contents
1.	Cover Sheet and Table of Contents
2.	TDP Summary
	Block 1 – Identification and Picture
	Block 2 – Descriptive Highlights
	Block 3 – Major Subsystems
	Block 4 – RDT & E Funding
	Block 5 – Lead Bureau
	Block 6 – Technical Director
	Block 7 – Principal Contractors
	Block 8 – Major Milestones
	Block 9 – Fiscal Year Milestones
3.	Index of Effective Pages
4.	Narrative Requirement and Brief Development Plan
5.	Management Plan
6.	Financial Plan
7.	Block Diagram
8.	Subsystem Characteristics
9.	Associated System Characteristics
10.	Dependability Plan
11.	Operability and Supportability Plan
12.	Test and Evaluation
13.	Personnel and Training Plan
14.	Production Delivery and Installation Plan

Figure 4-2. The Technical Development Plan Format

4-2-3. Procedural Steps for Documentation of Reliability and Maintainability in TDP's

STEP 1 – Summarize the Reliability Requirement in the TDP Summary

Block 2 -- Descriptive Highlights

State the reliability and maintainability requirement, the effectiveness or "kill probability" requirement, along with other performance characteristics.

EXAMPLE:

Kill Probability (per mission)	.90
Reliability (2-hour mission)	.92
Availability	.98
Maintainability (30 minutes)	.90
Performance:	
Mach	3.7
Range NM	175.0

Blocks 8 and 9 -- Major Milestones

Show as milestones the completion of reliability maintainability prediction analyses, final design review, prototype evaluation, and acceptance tests for reliability and maintainability.

STEP 2 – Prepare Narrative of Requirement (Section 4 of TDP)

State the system operational reliability/maintainability requirements in narrative form:

EXAMPLE: The XYZ system shall operate without malfunction and release its payload within the prescribed tolerance of target position with a minimum probability of .8, after check-out with operational test equipment. Or, more simply, the system shall be capable, after checkout with operational test equipment, of releasing its payload within the prescribed area and returning to the ship 8 out of every 10 attempts. The system shall maintain a minimum operational readiness of .92 while the ship is deployed; or, more simply, the system shall either be operating or in a state of readiness to operate, on the average, 22 out of each 24 hours.

STEP 3 – Describe Subsystem Characteristics (Section 8 of TDP)

Include for each subsystem that portion of the overall requirement assigned to the subsystem.

EXAMPLE (for defined mission):

Total System Reliability Requirement	= .92
Subsystem:	
Guidance & Command	= .96
Engine & Air Frame	= .999
Recovery	= .99
Autopilot & Stabilization	= .97

New programs – those that have not had a completed feasibility or detailed requirements analysis performed to date – may state that certain requirements have not yet been determined, provided a fixed date and program phase for their determination are shown as one of the milestones in Step 1.

Indicate whether specific components are available "on the shelf" or require development.

Indicate the degree of technical risk involved, problems anticipated, plans for solving, and adequately describe the technical "how".

Indicate anticipated interface problems between components of the system and the plans for solving and testing. Indicate the human engineering needed to assure optimum performance of men as components of the system.

STEP 4 – Describe Associated System Characteristics (Section 9 of TDP).

Describe the expected interface problems – compatibility of tolerances, interactions among components, induced environmental hazards, radio interference problems, etc., and describe plans for their solution and test verification.

EXAMPLE: System must be capable of operation in a nuclear radiation field of as yet undisclosed magnitude. Nature of this radiation and its exact effects on system performance will be evaluated and precisely determined by ___*(date)*___, as reflected in Milestone No. 6 of the TDP summary.

STEP 5 – Describe the "Dependability" Plan (Section 10 of TDP)

The reliability/maintainability programs and specific activities which are *planned and scheduled* should be covered in this section of the TDP. A recommended outline for the reliability and maintainability plan is shown in Figure 4-3. Outline the reliability program plan for the period covered by the TDP as related to the overall development effort, progress, schedule, and major milestones. Use proposed BuWeps Instruction, subject "Reliability and Maintainability, Instructions Concerning", for general guidance and wording for the appropriate major development phases. Fill in details peculiar to specific programs. Specific reliability/maintainability activities which will be required for each major subsystem development should be listed. The activities given in WS-3250 – BuWeps "General Reliability Specification"–may be used as a guide or check list.

A. Technical Requirements

(1) Nominal (design goal) requirements for reliability, availability, and maintainability.

(2) Minimum acceptable requirements and acceptance test conditions.

(3) Definition of failure, environmental factors, use conditions, and time bases applicable to (1) and (2).

(4) Engineering and statistical criteria for acceptance test design.

(5) Feasibility estimate and special design considerations, for achievement of (1) and (2).

B. Program Plan

(1) Applicable documents and specific basis for the program plan.

(2) Organization, management, and monitoring plan for "dependability" control by the Project Team.

(3) Reliability and maintainability analyses and design review schedule.

(4) Reliability specification review schedule.

(5) Review of parts qualification status.

C. Reliability and Maintainability Status Summary to Date

(1) Program status and progress summary.

(2) Summary of prediction analyses and development test results.

(3) Results of requirements review, reallocation, and tradeoff studies.

(4) Definition of the ten most critical problem areas and action requirements.

(5) Reassessment of reliability and maintainability growth potential and program realism.

D. Future Plans and Actions

(1) Correction of program deficiencies.

(2) Resolution of technical incompatibilities.

E. Schedule of Major Milestones and Monitoring Points

Figure 4-3. Suggested Format for an Integrated Reliability and Maintainability Assurance Plan for Section 10 of TDP

STEP 6 – Establish Reliability and Maintainability Schedule of Major Milestones and Program Review Points

A checklist of the major milestones to be included under Item E of Figure 4-3 is shown in Figure 4-4. Responsibility for accomplishment of the specific events represented by each milestone should be indicated and tentative dates for accomplishment should be established. The checklist represents a *minimum* of points deemed necessary for controlled growth of reliability and maintainability in development. Others should be added as required.

MILESTONES	DATE	RESPONSIBILITY
(1) Technical requirements and program plan documented in TDP.		
(2) Requirements documented in RFP.		
(3) Requirements and acceptance test criteria included in preliminary detail specifications.		
(4) Technical proposal evaluated; proposed contractor program reviewed.		
(5) Requirements and acceptance tests spelled out in definitive contract.		
(6) Detailed contractor program plan reviewed, modified, and approved.		
(7) Detailed technical monitoring plan developed and implemented by responsible office.		
(8) Critical points of contractor activity schedule incorporated into TDP milestone schedule.		
(9) Preliminary reliability and maintainability analysis, allocation, and feasibility study completed by contractor.		
(10) Specifications reviewed and updated on basis of (9).		
(11) Formal design review procedures documented and scheduled.		
(12) First design review; reliability stress analysis and maintainability assessment evaluated.		
(13) Reliability and maintainability requirements documented in subcontractor specifications.		
(14) Contractor failure reporting and analysis "feedback loop" implemented.		
(15) Integrated test plan formalized for reliability and maintainability evaluation, control, and acceptance.		
(16) Critical problem areas defined and reported; corrective action status recommended.		
(17) Reliability evaluation tests conducted.		
(18) Maintainability evaluation tests conducted.		
(19) Dependability assessment, based on test data from (17) and (18).		
(20) Dependability acceptance tests of prototype models begun.		
(21) Prototype accept/reject decision reached on basis of (20).		
(22) Plans reviewed and formalized for production.		
(23) Dependability requirements and acceptance tests defined in production specifications.		

Figure 4-4. Checklist of Major Reliability and Maintainability Milestones

STEP 7 – Describe a Maintenance Philosophy for the Supportability Plan and the Personnel and Training Plan

•

- Echelons or levels of maintenance, including maintenance tasks and skills required for each level.
- Planned use of built-in maintenance aids, such as self-test features, malfunction indicators, specialized or standard test equipment, etc.
- Planned use of job aids such as trouble-shooting logic charts, system technical Manuals, audio-visual presentation of maintenance tasks, etc.
- Other design features which may affect spare parts and repairs such as use of standard circuits from specific handbooks, disposable modules, etc.
- *Unique knowledge of skills required by the system.*

STEP 8 – Describe the Reliability/Maintainability Monitoring Plan

Identify by BuWeps code personnel who are designated the responsibility for monitoring reliability/maintainability progress. Describe briefly methods and frequency of monitoring (i.e., monitoring teams, independent assessments, review of progress, test results, contractor reports, etc.).

STEP 9 – Verify the Development Cost and Time (Sections 5 and 6 of the TDP)

Estimate the development cost by the method given in Chapter 9. Use these estimates to verify the time allotted in the schedule, and the funds budgeted for the development phase of the program. In the absence of other inputs, these estimates may be used in the management and financial plans of the TDP (Sections 5 and 6, respectively).

STEP 10 – Describe the Test and Evaluation Plan (Section 12 of TDP)

Outline the planned reliability/maintainability test and evaluation program and schedule (related to the overall test and evaluation schedules). State which tests are acceptance and which are evaluation in nature. Indicate the desired degree of assurance (confidence) in the test results.

Prepare detailed description of reliability and maintainability measurements tests, demonstration tests, and acceptance tests. Define accept/reject criteria, test design parameters, and *decision alternatives* in the event of a reject.

(1) Indicate here the plans for tests, investigations appraisals, and evaluations, including assignment of responsibility for *who does what, when,* and *where.* Show the objectives or goals of the technical evaluation on which acceptance or rejection will be based. Indicate any unique facilities,

equipment, or personnel capabilities which may be required.

(2) Indicate here the recommended tests and evaluations which should be conducted in order to determine operational suitability under service operating conditions. Indicate anticipated requirements for Fleet services. *Performance, reliability, maintainability, operability,* and *supportability* must be verified in the operational environment.

STEP 11 – Describe Personnel and Training Requirements (Section 13 of TDP)

Describe levels of personnel training and qualifications of operator/maintenance personnel for which the system must be designed; and, conversely, describe special training requirements (including schedules, programs, equipment, facilities) required for full exploitation of "inherent" reliability and "intrinsic" availability planned as features of the proposed system.

4-3 DOCUMENTATION OF RELIABILITY AND MAINTAINABILITY REQUIREMENTS IN PROCUREMENT DOCUMENTS AND SPECIFICATIONS

4-3-1. General

> The specification is . . .
>
> "a document intended primarily for use in procurement, which clearly and accurately describes the essential and technical requirements for items, materials or services including the procedures by which it will be determined that the requirements have been met".
>
> –Defense Standardization Manual M200A

Manual M200A further sets forth the following general policies relative to the preparation of specifications:

> "Specifications should establish requirements, insofar as is practicable, in terms of performance . . . . however, in order to control those features of design which pertain to interchangeability, compatibility, reliability, it is necessary, in most instances, for specifications used by the Government to include design requirements which achieve these essential controls."

While the specification is often referred to as the "communication media" between buyer and seller, it cannot in itself assure the buyer that the seller has been communicated with, except by a contractual stipulation of its applicability as the acceptance specification. Accordingly, the implementation of a specification ideally begins with the initial RFP, in order to assure that prospective contractors are fully aware of the *detailed quantitative* requirements of the procurement and the quality assurance criteria by which its acceptability is to be measured. To be responsive to the RFP, then, the prospective contractor must present his proposed technical and management approach toward the fulfillment of requirements as defined by the specification. The contract which is negotiated on the basis of the successful proposal can then make both the *specification* and the *proposal* contractually binding. This is the ideal implementation cycle.

Frequently, however, the equipment to be developed is one whose concept originates with the industry, resulting in a design proposal before the design specification has been developed. In these instances, the project engineer may require the submission of a proposed design specification in M200A format, [2]for review and revision as required to meet the needs of the equipment class. Now, as before, the specification should become contractually binding as the legal description of the product to be developed.

In other cases, the very nature of the procurement may indicate the impracticability of a firm specification requirement at the time of a contract award. Here, an "objective" design specification is prepared. One of the contractual tasks can then require an evaluation of specification realism – to determine the feasibility of the objective requirement. In this case, the specification is adjusted for realism, consistent with the proposed design approach, before design is permitted to proceed.

Reliability specification requirements consist of three distinct but related areas of coverage:

1. Detailed quantitative requirements.
2. General program requirements.
3. Quality assurance provisions.

These three areas may be included in the overall design specification for a product (Method A) or covered under a separate reliability specification (Method B).

Method A – Integrated Specifications: Reliability as a design parameter is logically specified in Section 3 of the design specification (both detailed and general coverage) and the quality assurance provisions integrated into the overall provisions of Section 4.

Method B – Separate Specifications: This alternative, although commonly used today, is recommended only when clarity and simplicity can be greatly enhanced. A reliability specification must follow approved specification format, consisting of the following:

[2] Bureau of Naval Weapons Specification XAV-1000 provides a recommended format to guide the preparation of development specifications for Avionics equipment.

1. Scope
2. Applicable Documents
3. Requirements
4. Quality Assurance Provisions
5. Preparation for Delivery
6. Notes

4-3-2. Procedural Steps

Whether Method A or B is used, certain basic information must be included in each section. In either case, an equipment or system description should ultimately include the information itemized under the following steps. While the procedural steps relate to specific sections of M200A specification format, they are equally applicable to design documentation in requests for proposals (RFP's) and contract task statements.

STEP 1 – Define Scope and Purpose of the Specification (Section 1)

Present a clear, concise abstract of the total coverage embraced by the specification, with a short but comprehensive description of the item to be developed, the system with which it is to work, and the functional role to be performed. Define the specific objectives of the specification – to establish requirements for reliability and maintainability and to prescribe acceptance test requirements by which compliance is to be assured.

STEP 2 – Specify Other Applicable Documents (Section 2)

Reference only those specifications and documents that are referenced in the body of the specification. (Additional references not directly pertinent tend to cloud the basic specification requirements.) Documents referenced must be available in approved form at time of specification issue.

STEP 3 – Describe System Operational Requirements (Section 3)

Reliability and maintainability are system characteristics in the same sense that speed, range, and maneuverability are system characteristics. To be placed in proper perspective, however, other operational requirements must be described to insure full understanding of the R&M requirement. Include the information outlined in Figure 4-1 for a full definition of system requirements.

The dividing line between a satisfactory and unsatisfactory system is seldom clearly defined in present-day system specifications; yet this is a necessity for a complete quantitative reliability statement. Current practice in design and development specifications is to specify "design goals" while nevertheless being willing to accept somewhat less. The inclusion of a quantitative reliability requirement thus requires at the outset that the "somewhat less" be explicitly defined.

> *EXAMPLE:* Present radar design specification calls for the system to "detect 1 sq. meter targets at 300,000 yards." Inclusion of a quantitative requirement necessitated the following change: "The design objective shall be to detect 1 sq. meter targets at 300,000 yards. The system shall be considered unacceptable if 1 sq. meter targets are not detected to at least 176,000 yards and marginal out to 225,000 yards."

The preferred method is to include both design objectives and minimum acceptable values as a lower tolerance limit on the performance parameter.

STEP 4 – Describe "Use" Conditions (Section 3)

Establish in standard terminology the conditions under which the item must provide the above performance. "Use" conditions refer to all known use conditions under which the specified reliability is to be obtained, including the following:

Temperature	Pressure	Weather (wind, rain, snow)
Humidity	Penetration/Abrasion	Sea State
Shock	Ambient Light	Operator Skills
Vibration	Mounting Position	

and other conditions covered in MIL-STD-210A, "Climatic Extremes for Military Equipment".

The "Use" conditions are presented in two ways:

Narrative:
Brief description of the anticipated operational conditions under which the system will be used.

EXAMPLE:
(1) The MK 000 Computer will be installed in temperature-controlled spaces aboard ships of the DD and DLG classes.

(2) The TOY missile must be capable of withstanding exposed shipboard environments encountered while suspended from the launcher arm for periods up to two hours. This includes possible ice-loading conditions in subzero weather.

Specific:
Itemized list of known or anticipated ranges of environments and conditions. When changes of environment are expected throughout an operating period, as in an aircraft flight, an environmental profile should be included.

EXAMPLE:
(1) MK 000 Computer shall operate as specified under the following environments, either singly or combined:

Vibration:	10–25 cps at 2.5g
Ship Motion:	
Roll:	47°
Pitch:	10°
Yaw:	20°
Temperature:	65°F. to 80°F.
Humidity:	to 95%
Input Power:	Nominal 440 cps at 110 v. ± 20%

(2) The AN/ARC-000 shall meet its performance requirements when subjected to the mission temperature profile, as illustrated in Figure 4-5.

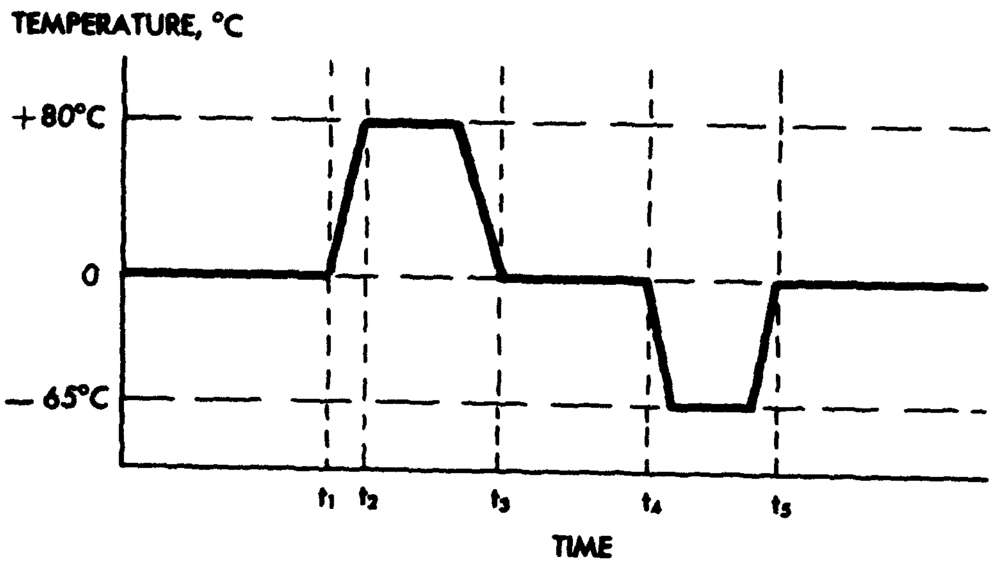


Figure 4-5. Temperature Profile

MIL-STD-210A provides comprehensive, worldwide environmental coverage. Many individual specifications for specific categories of systems provide environemental classifications which may be referenced *providing* the standard environments *adequately cover* the specific system's planned "use" conditions. The practice of stating extreme environmental ranges for systems which will be used under controlled or limited conditions leads to undue costs, both in development and production.

EXAMPLE: A general purpose digital computer for shipboard fire control systems will be installed in air-conditioned ship's spaces (65°F. to 80°F.). With planned forced air cooling, the system can be compactly built. Cabinets, doors, and drawers do not need insulation or weatherproofing. The specification of temperature requirements of −55°C. to +55°C. would increase the size and weight. The cabinet would require insulation and an elaborate temperature control system installed to provide both heat and cooling; or major circuit development would be required to render the device insensitive to temperature changes – both approaches are unwarranted.

STEP 5 – Define the Time Measure or Mission Profile

Time is vital to the quantitative description of reliability. It is the independent variable in the reliability function. The system usage, from a time standpoint, in large measure determines the form of the reliability expression of which time is an integral part. The types of mission times commonly encountered are given in Figure 4-8. For those cases where a system is not designed for continuous operation, total anticipated time profile or time sequences of operation should be defined either in terms of duty cycles or profile charts.

> *EXAMPLE:* The mission reliability for the "x" missile fire control system shall be at least .9 for a 6-hour mission having the typical operational sequence illustrated in Figure 4-6.

From the example it can be seen that a large portion of the time was standby-time rather than full-power-on-time.

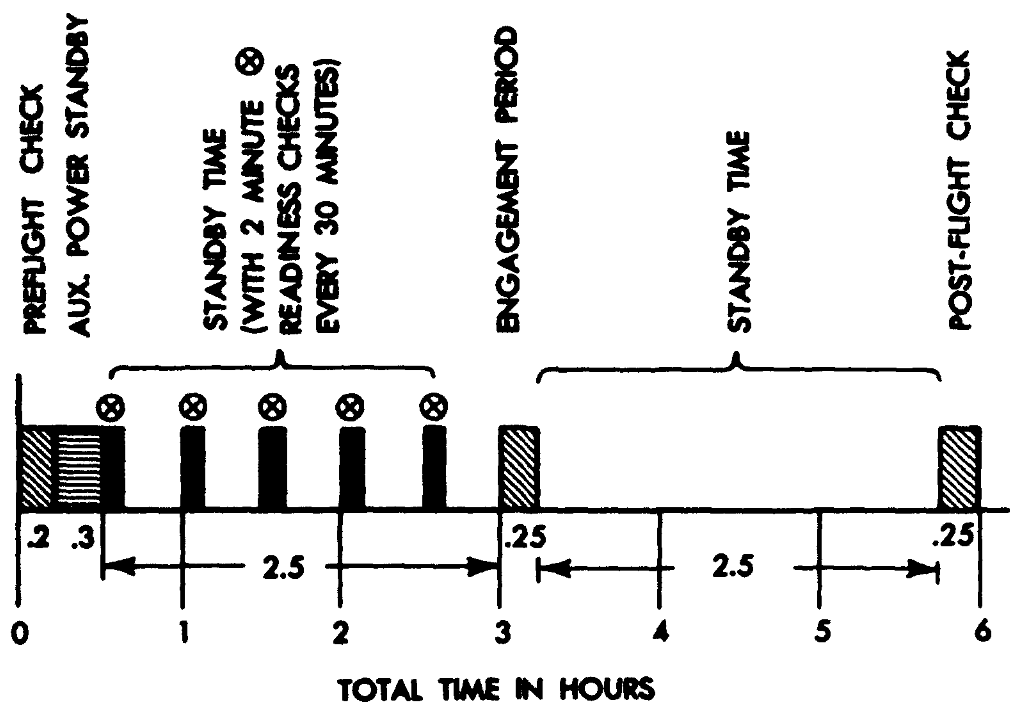


Figure 4-6. Typical Operational Sequence for Airborne Fire Control System

STEP 6 – Specify Reliability Design Objectives and Requirements (Section 3).

The intent of this subparagraph is to int out the specific functions in which liability improvement is sought. It is suggested that both the specific *functions* to be improved and the *nature* of the improvement be described in enough detail that prospective designers will have advantage of the earlier feasibility analysis.

> *EXAMPLE:* "A 10-to-1 improvement in servo amplifier module reliability is sought as a design objective. Specifically, it shall be the objective to reduce tolerance and instability failures, as defined in the description of performance characteristics elsewhere in this specification, by the application of inverse feedback at the circuit and servo loop level, and to reduce the catastrophic failure rate by the application of redundancy to those critical elements in which further derating is ineffectual."

When the need for unconventional or unique design approaches can be determined in the predesign phase, they should be described in sufficient detail to aid the designer who must ultimately adopt these or equivalent concepts to overcome the indicated limitations of conventional design. Such specific design requirements would include:

- Redundancy planned in the system concept as a means of overcoming anticipated limitations of part and component reliability. The level of complexity at which redundancy is needed to achieve the overall reliability requirement should be indicated, and whether standby or active redundancy is contemplated.
- Special parts, components, or items of GFE on which the system concept is based, together with the estimated reliability and references to supporting data that justify the specification of such particulars.
- Special packaging, modular construction, or potting methods required to conform to maintenance and logistics plans.
- Particular maintenance features contemplated by the system concept for the achievement of the specified effectiveness requirement. These would include design features for scheduled or continuous performance monitoring, failure indication, and failure sense-switch devices, and should prescribe the system levels at which these features were considered to be applied in the conceptual determination of maintainability feasibility.

STEP 7 – Specify the Quantitative Reliability Requirements (Section 3).

Specify the value of *inherent* reliability on which the success of the conceptual system is based. This should be quantitatively defined at one or more points to establish the desired reliability and maintainability characteristics.

Figure 4-7 illustrates four basic ways in which a reliability requirement can be defined:

(1) As a "mean life" or mean-time-between-failure, MTBF. This definition is useful for long-life systems in which the form of the reliability distribution is not too critical, or where the planned mission lengths are always short relative to the specified mean life. Although this definition is adequate for specifying life, it gives no positive assurance of a specified level of reliability in early life, except as the assumption of an exponential distribution can be proven to be valid.

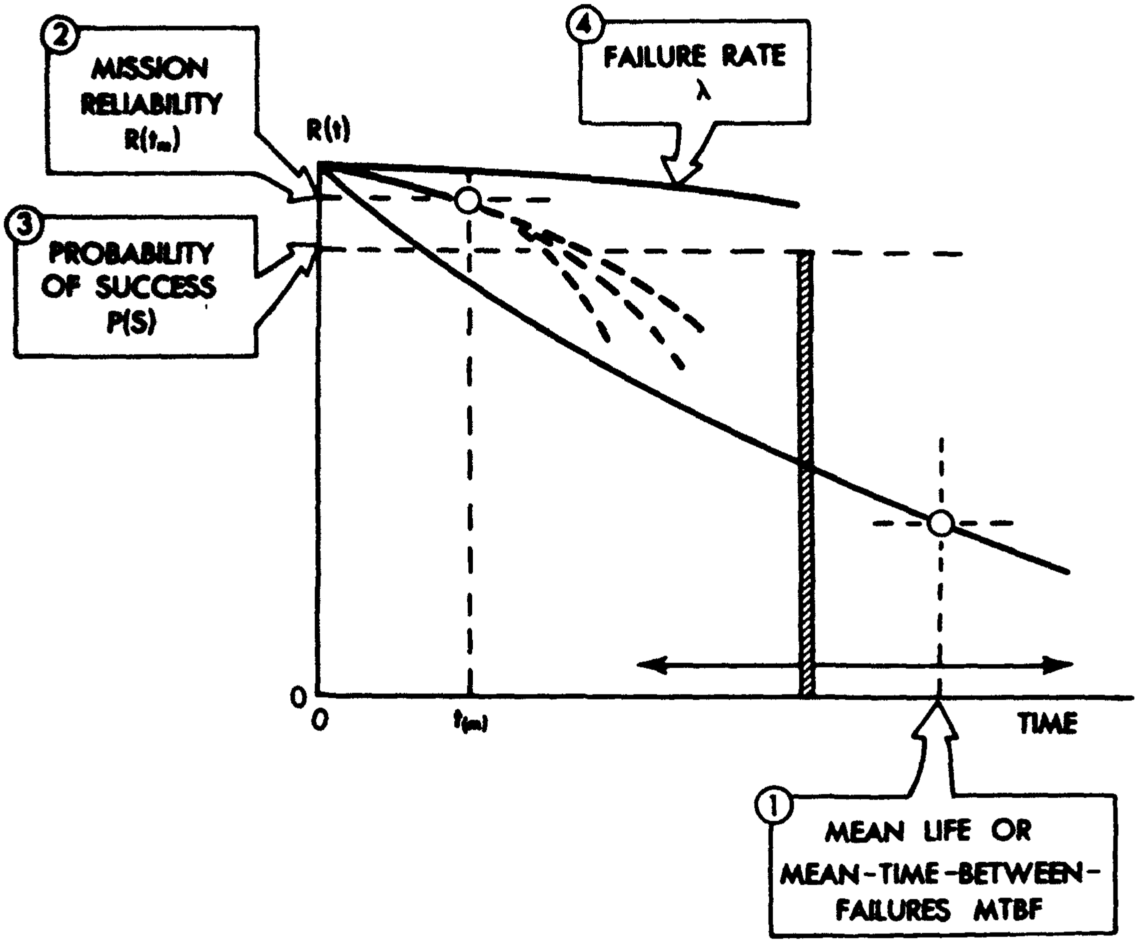


Figure 4-7. Four Definitions of Reliability

(2) As a probability of survival for a specified period of time, t. This definition is useful for defining reliability when a high reliability is required during the mission period, but mean-time-to-failure beyond the mission period is of little tactical consequence except as it influences availability.

(3) As a probability of success, independent of time. This definition is useful for specifying the reliability of one-shot devices and those which are cyclic, such as the flight reliability of missiles, the launch reliability of launchers, the detonation reliability of warheads, etc.

(4) As a "failure rate" over a specified period of time. This definition is useful for specifying the reliability of parts, components, and modules whose mean lives are too long to be meaningful, or whose reliability for the time period of interest approaches unity.

Figure 4-8 summarizes appropriate methods of stating the reliability requirements for various functions, usage, and maintenance conditions.

EXAMPLE: A complex radar has both search and track functions. It is also possible to operate the search function in both a low and high power mode. The reliability requirement for this system could be expressed as:

"The reliability of the system shall be at least:

Case I - High power search – 28 hours MTBF

Case II - Low power search – 40 hours MTBF

Case III - Track – .98 probability of satisfactory performance for ½ hour"

The definition of satisfactory performance must include limits for each case. This can be conveniently tabulated for inclusion in the specification. A portion of the Satisfactory Performance Table for the radar is shown in Figure 4-9.

STEP 8 – Define the Specified Reliability Requirement in Terms of Nominal or Minimum Values (Section 3)

The reliability requirement may be specified in either of two ways:

- As a NOMINAL value with which the Fleet would be satisfied, *on the average*; or
- As a MINIMUM value below which the Fleet would find the system totally unacceptable.

LEVEL OF COMPLEXITY \ CONDITIONS OF USE	Continuous Duty Long Life (Repairable)	Intermittent Duty Short Missions (Repairable)	Continuous or Intermittent (Non-Repairable)	One-Shot (Time-Independent)
Complex Systems (Larger than 500 AEG's)	R(t)	R(t)	R(t)	P(S)
Systems Subsystems Equipments (Less than 500 AEG's)	R(t) or MTBF	R(t) or MTBF	R(t) or λ	P(S) or P(F)
Modules Components Parts (10 AEG's or less)	λ	λ	λ	P(F)

Code:

R(t) - Reliability for specified mission, or period of time, t.
MTBF - Mean-time-between-failures, or mean life.
P(S) - Probability of success.
P(F) - Probability of failure.
 - Failure rate.

Figure 4-8. Methods of Specifying Reliability According to Levels of Complexity and Conditions of Use

System Characteristic	Units	Performance Limits		
		Case 1	Case 2	Case 3
Range	Yards	300,000	120,000	120,000
Resolution – Range	Yards	±50	±50	±10
– Velocity	Ft./Sec.	±100	±100	±25
Bandwidth	M			

Figure 4-9. Satisfactory Performance Limits

Whichever value is chosen as the specified requirement, the following rules should be applied:

- When a nominal value is specified as a requirement, *always* specify a *minimum* value which the system must exceed.

- When a minimum value alone is used to specify the requirement, always insure that it is clearly defined as minimum.

Of the two methods, the first is by far the best, since it automatically establishes the design goal at or above a known nominal.

Figure 4-10 shows the relationship between "minimum" and "nominal" values of specified mean life, as they would appear on the operating characteristic (OC) curve for a reliability acceptance test. This relationship is discussed in considerable detail in Chapter 7 on acceptance test design.

As an illustration of the first method consider points A and B_3. The specification requirement may be stated as follows:

"*MTBF requirement*. The nominal and minimum MTBF requirements for System X shall be met when tested in accordance with Section 4 of this specification.

"*Nominal MTBF*. The nominal MTBF shall be 300 operate hours.

"*Minimum MTBF*. The minimum MTBF shall be at least 100 operate hours demonstrated at the 90% level of statistical confidence."

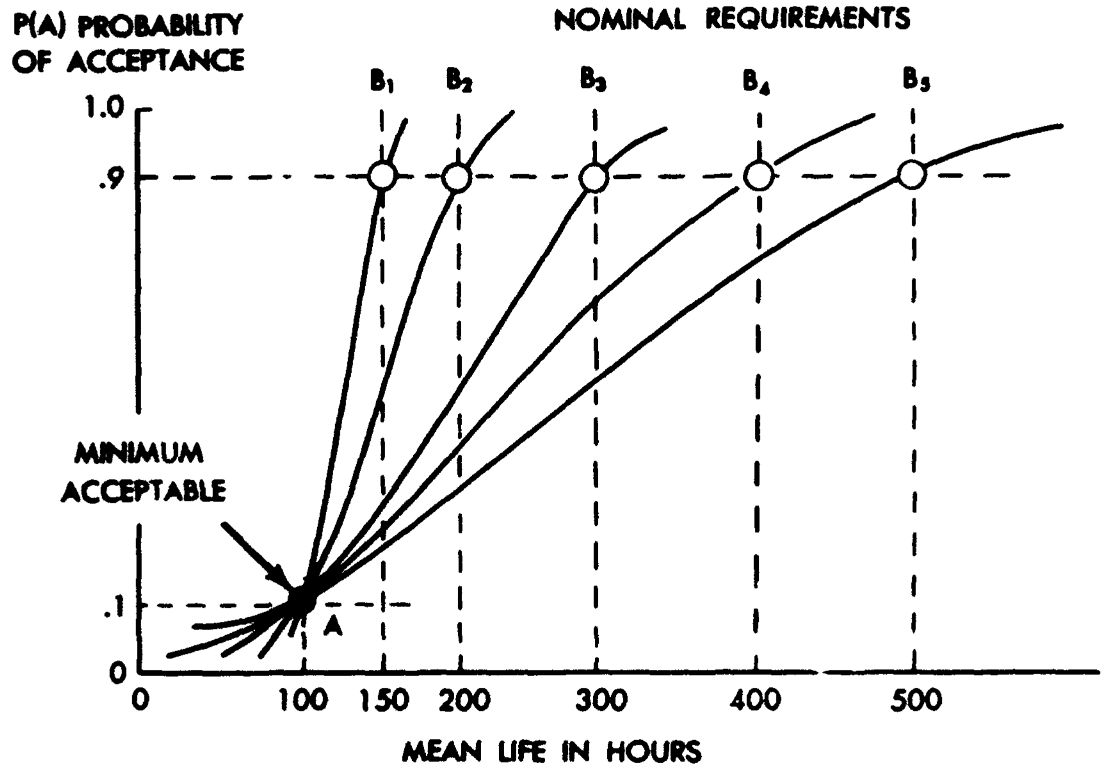


Figure 4-10. Relationship of "Nominal" Reliability Requirement to "Minimum" Acceptable Shown on Operating Characteristic (OC) Curves for Reliability Acceptance Tests

STEP 9 – Specify the Maintainability Requirement (Section 3)

Figure 4-11 illustrates a typical maintainability function, with two basic methods for defining the maintainability requirement:

(1) As a mean-time-to-restore requirement. This definition does not control the distribution of maintenance task times. The definition is useful for specifying maintainability of long-life systems.

(2) As a probability, "y"%, of restoration within a specified period of maintenance time, t_r. This definition is useful for systems to be designed for high maintainability, employing reliability-with-repair or module maintenance concepts.

The following are examples of paragraphs that might be included in a design specification to cover the availability (maintainability) requirement:

Effectiveness considerations.
The equipment shall be planned, designed, analyzed, and reported as outlined in the following subparagraphs:

Effectiveness requirement.
The effectiveness requirement, when determined by the product of the service use reliability and the availability goals specified in the following subparagraphs, shall be at least 99%. Trade-off adjustments between the reliability and availability goals may be initiated by the contractor and shall be subject to procuring activity approval.

Availability requirement.
The availability, or "operational readiness" goal, expressed as a percentage of the number of times (at the start of a mission) that equipment operation is successfully initiated, divided by the number of times equipment operation is demanded, shall be at least 99.58%.

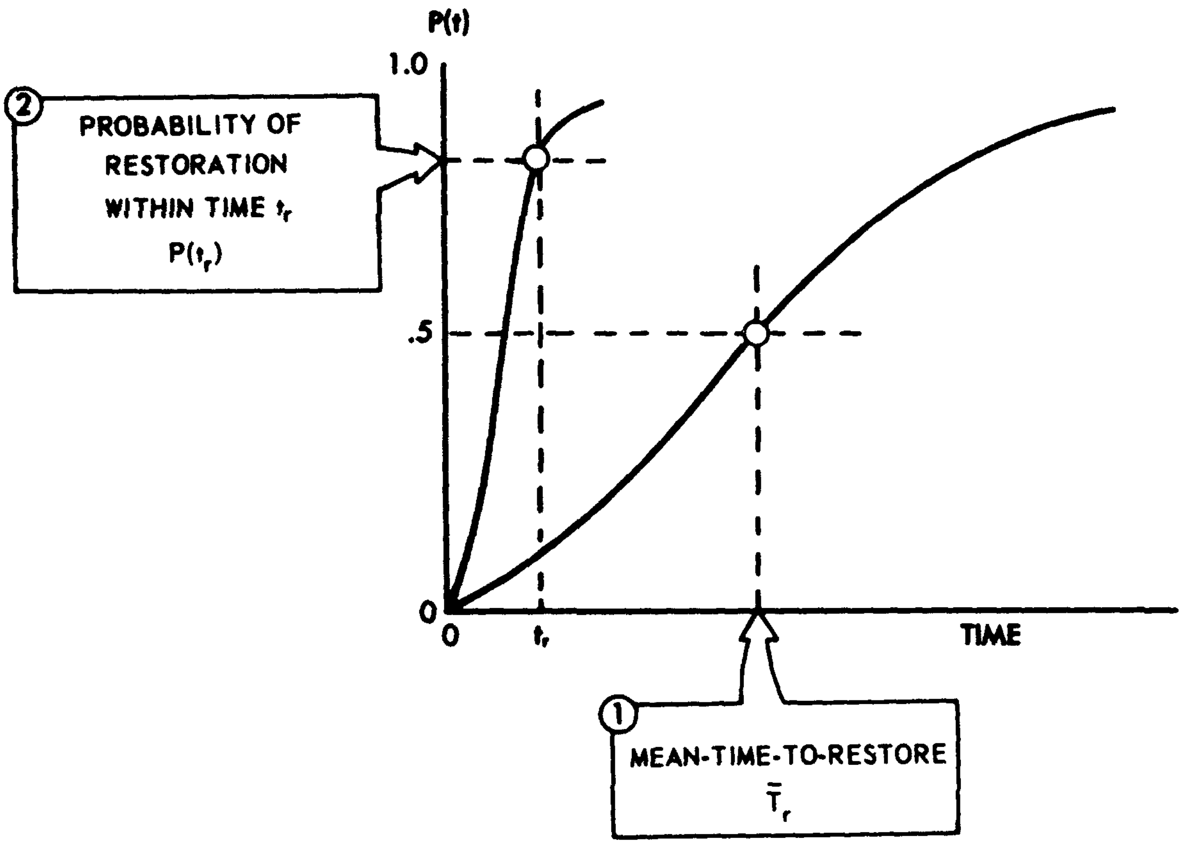


Figure 4-11. Two Definitions of Maintainability

Maintainability requirement.

The maintainability goal expressed as a mean-time-to-restore shall be not greater than 1.7 hours when determined by procedures approved by the procuring activity.

STEP 10 – Define Reliability/Maintainability Assurance Program Requirements (Section 3)

It is the general policy of the Bureau to describe the characteristics of the system it proposes to develop in sufficient detail that *when* the end product satisfies the requirements of the description the development program will have fulfilled the purpose for which it was implemented. Responsibility for satisfying the requirements of the system description must rest with the development contractor, although the Bureau will provide management and engineering guidance for support of the contractor's program toward fulfillment of the system requirement.

The Bureau does not propose to dictate the specific methods by which the contractor is to achieve specified requirements, but does intend to evaluate the contractor's "output" from several important reliability/maintainability assurance program monitoring points, to assess development status and progress with respect to "milestone" goals. Thus both the Bureau and the contractor can forecast impending trouble and take preventive action long before the panic stage. The outcome of the development program can thus be guided, controlled, and predicted long before hardware is delivered to the Fleet.

Development program plans, specifications, requests for proposals, and contractual documents should therefore define the specific program activities and assurance provisions by which success of the system development program is to be monitored, guided, and assured. Planned requirements for the following principal program activities should be defined:

(1) **Test and Evaluation:**
Development and demonstration test program plan; prototype evaluation and preproduction acceptance test plan (acceptance criteria and sampling risks); service evaluation.

(2) **Reliability and Maintainability Analysis:**
Prediction and apportionment studies; design review and stress analysis; maintenance task and skill analysis; failure mode and consequence analysis; tolerance and interaction regression analysis; maintenance task time and motion studies; reliability-with-repair and redundancy studies.

(3) **Reliability and Failure Reporting:**
Failure analysis; corrective action assignment and follow-up.

(4) **Quality Assurance:**
Vendor and subcontractor selection and control; parts and materials specification, qualifications, and acceptance testing; process controls.

(5) **Reliability/Maintainability Monitoring:**
Monitoring program plan and schedule of monitoring points; tentative assignment of "milestone" goals; schedule of planned requirements and trade-off reviews.

(6) **Documentation:**
Reporting requirements for contractor program plans, procedures, specifications, design proposals, failure analyses; schedule of planned review of TDP and specification documentation.

Cautionary Notes

- *Do not* expect a reliability assurance program to provide unlimited reliability. On the contrary, expect the program to provide realistic appraisals of progress, status, and potential of the overall program.
- Avoid specifying, as part of the reliability assurance program, organizational or internal (contractor) responsibilities which would limit or constrain the contractor's individual approach.
- Reliability analyses or assessments are primarily design guides and monitoring techniques and should not be used as acceptance criteria or in lieu of acceptance testing.

STEP 11 – Specify the Quality Assurance Provisions (Section 4)

The specification must now set forth the methods by which product acceptability will be determined. This step involves many detailed determinations of approach and methods which are based not only upon technical considerations but also upon considerations of cost and time. A partial list of the items which must be determined prior to establishment of quality assurance provisions follows:

(a) A complete description of the item or items to be accepted under the test requirements.

(b) The test conditions to be applied, including operational and environmental duty cycles (acceleration factors permissible, if known).

(c) Number of items to be tested.

(d) Estimated test duration.

(e) Success and failure criteria.

(f) Accept/reject criteria of the test plan.

(g) Consumer's risk (a measure of the adequacy of the test plan in discriminating between acceptable and unacceptable products).

In complex system development programs where it is not feasible to perform a complete systems acceptance test, individual acceptance tests for various sub-levels may be specified, together with methods to be employed for synthesis of reliability at the system level. Detailed test design procedures presented in Chapter 7 of this handbook will assist the project engineer in the selection of the most appropriate test method and the most suitable test design within the selected method. Step 11 therefore suggests the use of Chapter 7 in the development of specific quality assurance measures.

STEP 12 – Follow Through

Even though the reliability and maintainability requirements for the proposed new development program have been determined, defined, and documented in the Technical Development Plan, there remains the task of effective implementation of the planning document as a binding contractual requirement on those in whose hands the destiny of the entire program will ultimately rest. The sequence of events leading to successful implementation is straightforward:

(1) Integrate the requirements defined in the Technical Development Plan into the detailed design and development program specifications for the proposed system.

(2) Reference these specifications in requests for proposals, emphasizing their importance by integrating principal requirements in the statement of work on which the bid is to be based.

(3) Review design proposals, using the TDP and the design specification as a check list to evaluate the responsiveness of proposals to the initiating RFP.

(4) Evaluate bidder's understanding and demonstrated capability to implement and successfully execute the program on which he is bidding.

(5) Reference the design and program specifications as applicable documents in the contract which results from (4) above.

(6) Critically read, analyze, and evaluate program planning and status reports as part of the planned monitoring program.

(7) Evaluate program effectiveness by comparing *measured* progress in achievement of technical goals, with planned progress established as milestone goals. Do not evaluate on the basis of report volume alone.

CHAPTER 5

RELIABILITY DESIGN AND DESIGN ASSESSMENT

5-1 INTRODUCTION

5-1-1. Principles of "Estimation"

The "design phase" is defined as the period immediately following the award of a development contract. In this phase of the equipment life cycle, a design is formulated to meet the quantitative requirements stated in the design specification. The contractor is required by specification and committed by contract to demonstrate that these requirements are being met, or *will be met* in the course of the contract period. The only known way to demonstrate that a particular design approach will satisfy a specified requirement while the design is still in the formative "blueprint" stage is by *estimation* – estimation of expected results on the basis of past experience with other designs.

Designers have always been able to estimate or "predict" quantitative performance characteristics of their designs with good accuracy, because the operational equations they used for predicting performance were the very ones used for deriving the design in the first place. Until recently, however, it was not feasible to predict accurately the quantitative reliability characteristics of a new design, because mathematical "modeling" techniques had not yet been developed for expressing the reliability characteristics of different design configurations and the failure characteristics of parts used in these designs were still largely unknown.

5-1-2. Basis for Standardization

MIL-STD-756A and MIL-HDBK-217 represent the culmination of several years of effort by the three services to overcome this lack of knowledge. These documents now provide standard mathematical modeling procedures and standard failure data to use with the procedures, supplying guidance which will permit two or more estimators to come up with the same realistic prediction for the same design – an obviously important requirement if prediction procedures are to be used initially for evaluation of competitive designs and are to be used thereafter for "measuring" design progress toward established goals.

5-1-3. Applicability of Estimating Procedures

The procedures described in this section follow those prescribed by MIL-STD-756A, and demonstrate the use of data presented in MIL-HDBK-217. The procedures are useful in the following applications:

- As a planning tool for the initial establishment of reliability requirements.
- As a design tool to guide the contractor's designer in the choice of parts and circuit configurations to meet the specified reliability requirement.
- As a design review tool by contractor management, for the evaluation of design adequacy to meet the reliability requirement, and to point up potential reliability problem areas for design correction.
- As a monitoring tool for the assessment of development program progress toward established goals, to predict and circumvent oncoming problems before the hardware stage.

5-2 STEP-BY-STEP PROCEDURE

5-2-1. Basic Considerations

Design reliability assessments can be divided into two phases:

- The conceptual or design proposal phase – in which a prediction is based on the design "concept" as reflected in development specifications and early design documentation.
- The design or development phase – in which predictions are based on the actual "implemented" design.

In either case the procedure for estimating design reliability is the same. Application of the procedure will vary only to the extent of increasing availability of detailed design information as the program advances from phase to phase.

5-2-2. Specific Procedural Steps

STEP 1 – Define the System or Equipment.

Develop functional block diagrams for the complete system to the depth that design information is firm. Define:

- *Boundary conditions* – input/output and interface characteristics.
- Environmental and "use" factors.
- Operating modes and functions, mission profiles, and duty cycles.
- Success and failure criteria – performance tolerances and degradation limits.
- Physical constraints – space, weight, and configuration.

As an example, to illustrate this and succeeding steps, consider a design proposal for an airborne integrated electronic central (comparable to AN/ASQ-19). The equipment is to provide complete communication-navigation-identification (CNI) functions for the aircraft in which it is to be installed.

Five functions are to be performed:

(1) Communications (COMM)
(2) Direction Finding (ADF)
(3) Intercommunication (AIC)
(4) TACAN (TACAN)
(5) Identification (IFF)

Equipment boundaries for reliability assessment purposes will be at the following points:

Aircraft primary power terminals;
Antenna terminals, except ADF;
Compass synchro transmitter terminals;
Push-to-talk microphones and headsets (part of flight suit);
Cooling air supply output duct;
Equipment mounting bases.

Physical characteristics will be:

Power: Not to exceed 400W average drain on aircraft primary supply.

Weight: Not to exceed 200 lbs., with individual component less than 50 lbs.

Space: Not to exceed 3.5 cu. ft., within specified dimensions.

Performance characteristics will be:

Communications:
Receiver Transmitter:
Power output:
20W average, 16W minimum

Modulation:
AM

Frequency coverage:
1750 channels, 225 to 400 MC (19 preset channels)

Guard channel (preset):
243 MC

Auxiliary Receiver:
20 preset channels, 265 to 284.9 MC, including guard channel

Navigation:
TACAN:
126 preset channels in range 962 to 1213 MC
Bearing accuracy ± 0.7 degree
Range 0 to 196 nautical miles
Range accuracy ± 0.1 mile ± .1% of distance reading

ADF:
Receive over range of 225 to 400 MC

Identification:
IFF:
Power output:
+27 (±3) dbw on 1090 MC

Receiver trigger level:
-79 dbv on 1030 MC

Modes:
Mark X and SIF

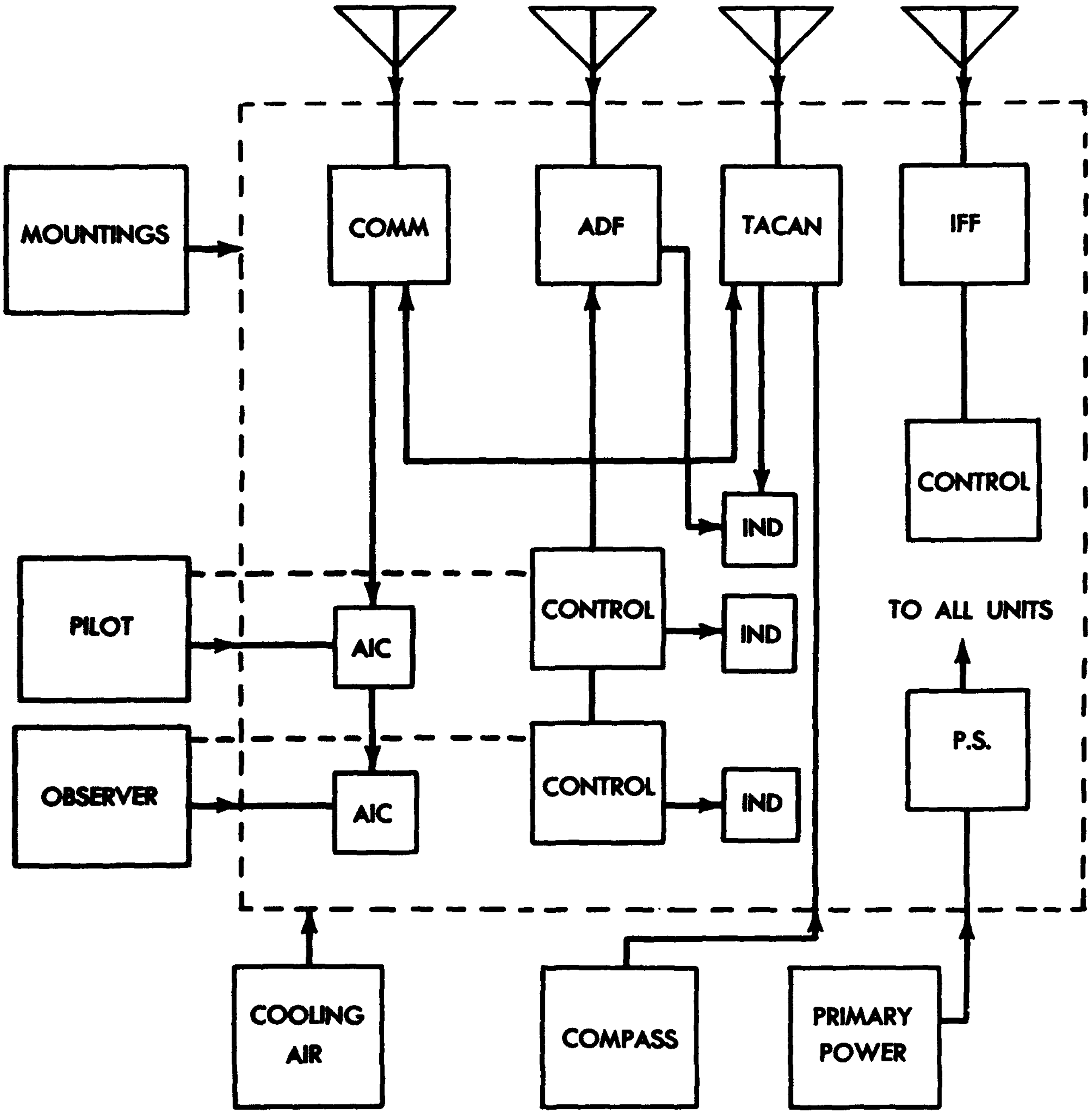


Figure 5-1. Simplified Block Diagram of Integrated CNI System

Mission profile will be:

> The complete equipment must be capable of operating without failure throughout a 3-hour mission, with a probability of .9. During this 3-hour period, the transmitter duty cycle will be 1/3 (5 minutes on/10 minutes off). All other equipment must be operational 100% of the time. Failure of any of the functions will be classed as an equipment failure in the primary mode. Alternate mode capability shall be provided for guard channel monitoring, and for navigation by ADF in event of TACAN failure. Centralized CNI controls shall be provided for pilot and radar observer. These can be considered redundant so long as the AIC function is operational.

A simplified functional block diagram fulfilling the requirements of the above performance description is shown in Figure 5-1.

STEP 2 - Develop the Reliability Block Diagram.

Continuing with the CNI example, the navigation function will be used to illustrate a design assessment of reliability status. Figure 5-2 is the overall reliability block diagram with the navigation function highlighted.

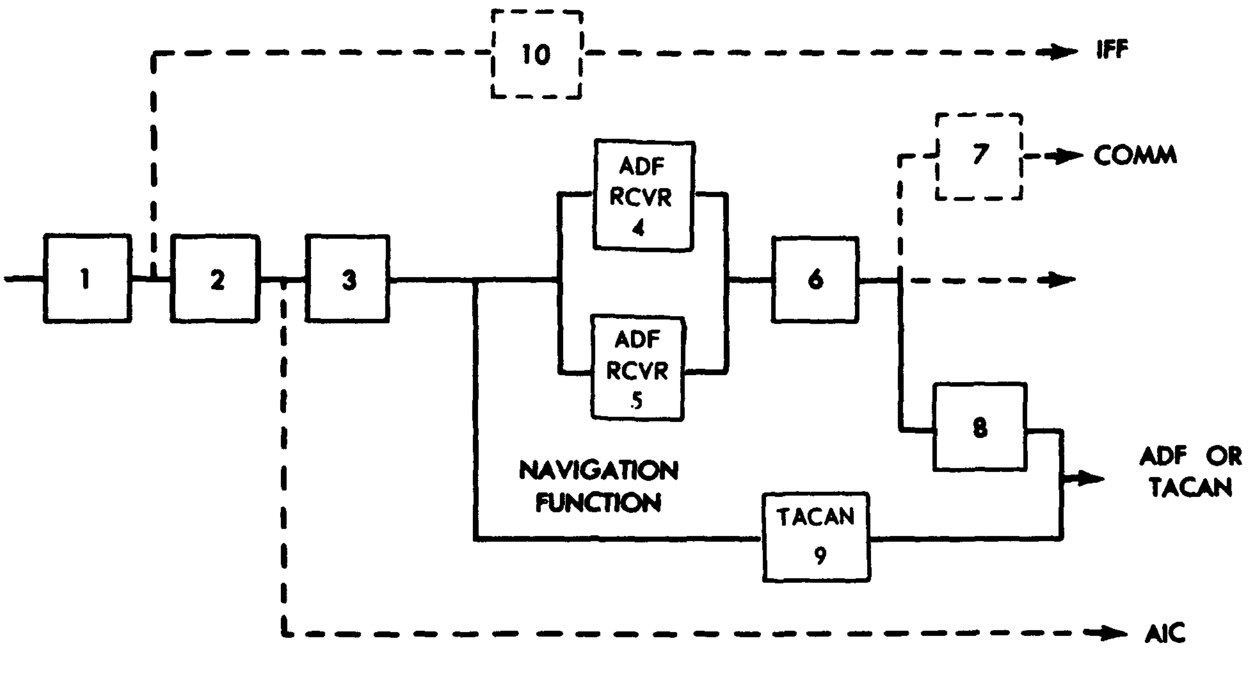


Figure 5-2. Reliability Block Diagram for the Navigation Function (CNI)

The reliability model for Figure 5-2 is derived as follows:

$$R_{NAV} = R_1 R_2 R_3 (1 - Q_{ADF} Q_{TACAN})$$

where $Q_{ADF} = (1 - R_{ADF})$
is the probability of ADF failure; and

$Q_{TACAN} = (1 - R_{TACAN})$
is the probability of TACAN failure.

But $R_{ADF} = R_6 (1 - Q_4 Q_5) R_8$

$= R_6 (R_4 + R_5 - R_4 R_5) R_8$

where Q_4 and Q_5 are, respectively, the probabilities of receiver failure, i.e.:

$Q_4 = (1 - R_4)$

$Q_5 = (1 - R_5)$

Thus,

$$R_{NAV} = R_1 R_2 R_3 \left(1 - [1 - R_6 R_8 (R_4 + R_5 - R_4 R_5)][1 - R_9]\right)$$

STEP 3 – Determine Parts Population for Each Block.

Compile a list of parts, by circuit symbol, for each of the blocks in the navigation reliability model of Figure 5-2. As an example, consider Block 5, the ADF and auxiliary receiver.

It is convenient to go one step further in block diagramming, if the proposed design configuration lends itself to further subdivision. Assume, for example, that it is proposed to design the receiver using four replaceable modules for ease of maintenance, as shown in Figure 5-3.

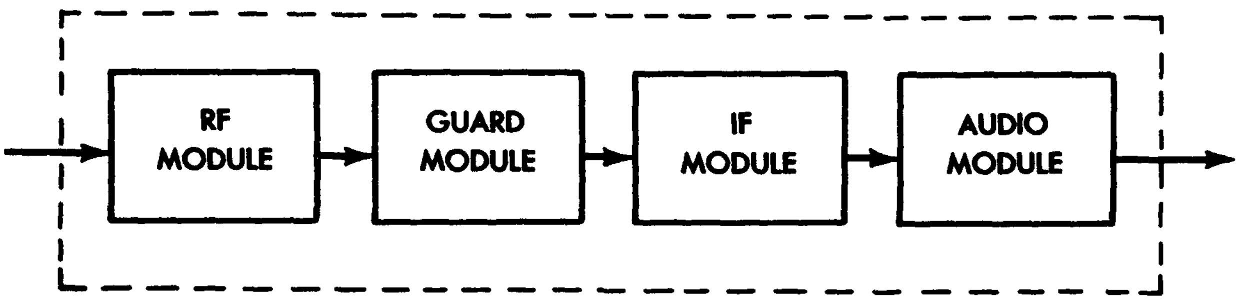


Figure 5-3. Reliability Diagram of ADF Receiver

Within the RF module, the AEG's proposed by the designer are summarized in Figure 5-4.

PARTS \ AEG	1st RF Amplifier	2nd RF Amplifier	1st Mixer	OSC and Tripler	Injection OSC	Freq. Tripler	TOTAL Module
Transistors (or Tubes)	1	1	1	1	1	1	6
Resistors	3	3	3	7	3	4	23
Capacitors: Fixed	4	6	4	4	3	5	26
Variable	2	2	2	2	1	1	10
Inductors	4	2	2	2	2	1	13
Xtals & Holders					20		20
Switch (Rotary)					2		2

Figure 5-4. Parts Count within the RF Module

PART CLASS \ MODULE	RF	Guard	IF	Audio	TOTAL Receiver
Transistors (or Tubes)	6	5	4	4	19
Resistors: Fixed	23	22	37	28	110
Variable				4	4
Capacitors: Fixed	27	35	52	13	127
Variable	10				10
Inductors	13	14	15	3	45
Xtals & Holders	20	2			22
Switch: Wafer	2				2
SPDT				6	6
Diodes			3	2	5
Relay DPDT				1	1

Figure 5-5. Parts Count for the Receiver

The same tabulation would be extended to other modules of the receiver, resulting in a tabulation of parts for the receiver as shown in Figure 5-5.

The same procedure would be applied to other equipment and subsystems used in the performance of the ADF and TACAN navigation functions, to produce a table of equipment parts populations in the ADF/TACAN Loop.

STEP 4 – Determine Appropriate Stress Factors and "Base" Failure Rate for Each Part.

As pointed out in MIL-STD-756A, it may be necessary in early design assessments to

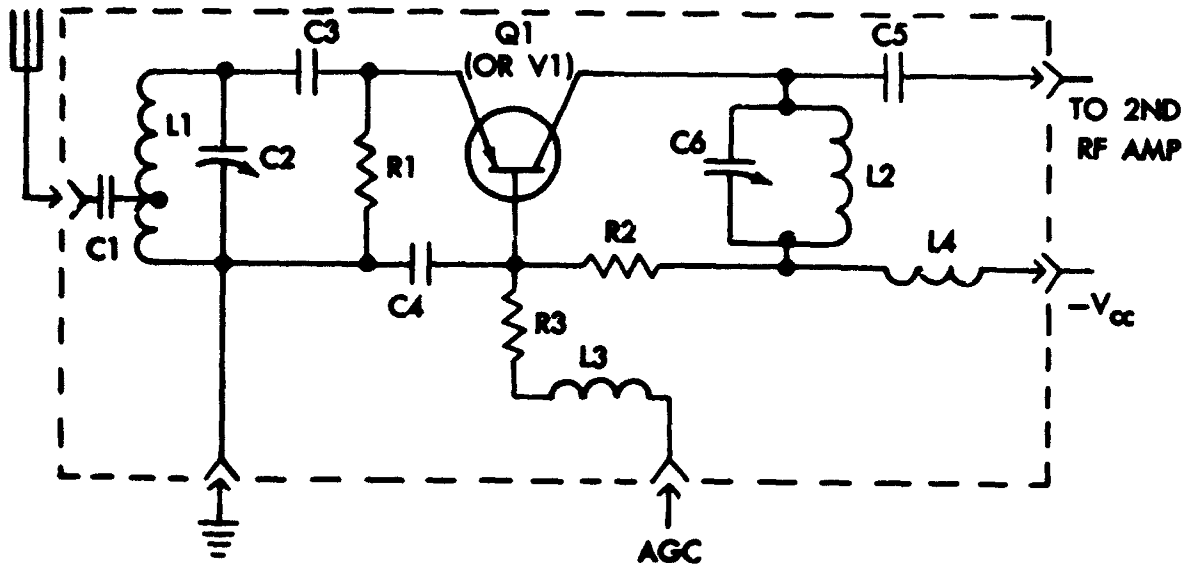


Figure 5-6. Proposed Circuit Diagram of 1st RF Stage

apply an average derating or stress factor to each class of parts on the basis of *planned* derating. Later, as design becomes more definitive, each critical part should be individually evaluated for the derating factor actually applied. In the example shown in the schematic of Figure 5-6, the 1st RF amplifier of a proposed ADF receiver RF module will be analyzed by an average stress level applied to all parts classes except critical parts which carry heavy currents or dissipate a relatively large amount of power with respect to ratings – e.g., tubes, transistors, relays, and motors.

MIL-HDBK-217 will be used to determine failure rates under the stress conditions anticipated in each application. The example chosen to illustrate the procedure assumes design using either transistors or tubes as active elements. Micromodules are discussed separately in another subsection.

Detailed procedures used in the stress analysis are presented here to illustrate the general method:

Transistor Q1

(1) Determine Circuit Operating Conditions for Transistor Application, Q1.

Silicon Transistor Type 2NXXXX:

Ambient Temperature	= 75°C
Average I_C	= 5mA
Average V_{CE}	= 9V
Power Dissipation	= .005 × 9
	= .045 watts
Specification Ratings:	
Rated Dissipation	= 150 mW
Derating Interval	= 25°C to 150°C

(2) Determine Normalized Temperature Ratio from the Following Equation

$$T_n = \frac{T_{actual} - T_{rated}}{T_{max} - T_{rated}}$$

$$= \frac{75 - 25}{150 - 25} = \frac{50}{125} = .4$$

The normalized temperature ratio represents that proportion of maximum rated dissipation used by the excess of the particular ambient temperature over the temperature at which derating starts. This relationship is shown in Figure 5-7.

(3) Determine Normalized Stress Ratio.

$$\frac{\text{Applied Dissipation}}{\text{Rated Dissipation}} = \frac{45}{150} = .3$$

Entering the chart shown in Figure 5-8 (Figure 14B of MIL-HDBK-217) at a normalized temperature, $T_n = .4$, proceed to the stress/failure rate curve marked .3, corresponding to the wattage ratio. The estimated average catastrophic failure rate for transistors in an application of Q1 severity is indicated as $.52 \times 10^{-6}$ failures per hour.

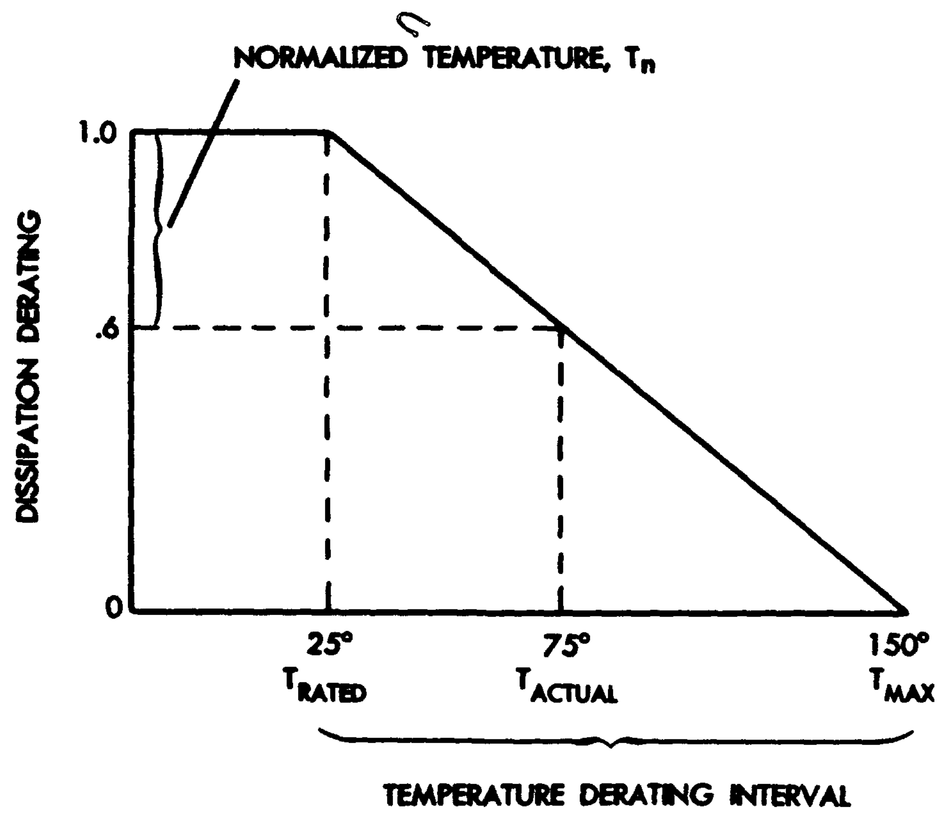


Figure 5-7. Dissipation Derating Curve as a Function of Part Ambient Temperature, for Transistor Q1

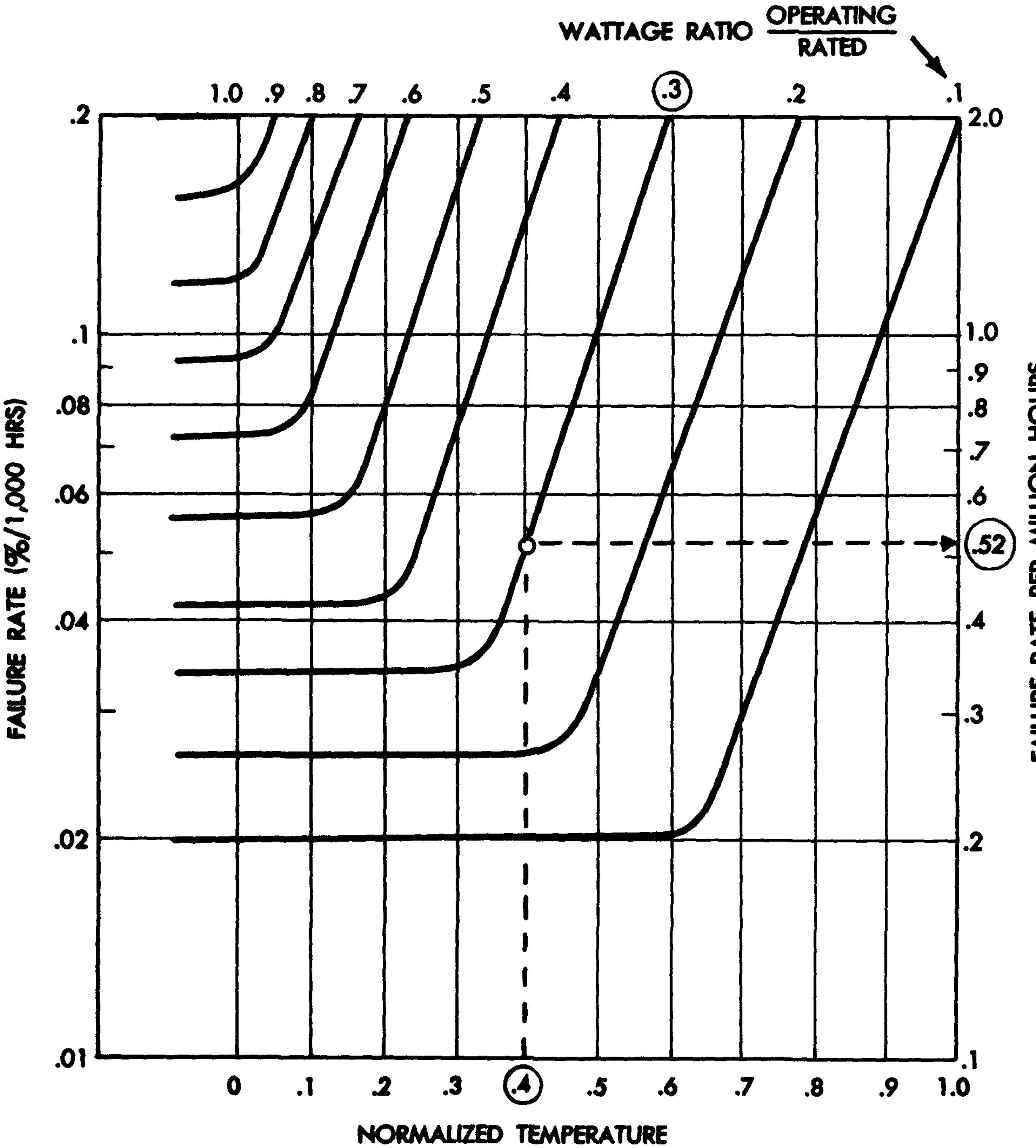


Figure 5-8. Transistor Failure Rates

Electron Tubes

(1) Determine Circuit Operating Conditions for the Tube as Shown in Figure 5-9.

Total Dissipation Derating:

Plate Dissipation	= 8 mA × 130V =	1.04 watts
Screen "	= 2 mA × 130V =	.26 watts
Heater "	= 175 mA × 6.3 V =	1.10 watts
Total Dissipation		2.40 watts
Rated Dissipation (MIL-E-1B)		3.3 watts

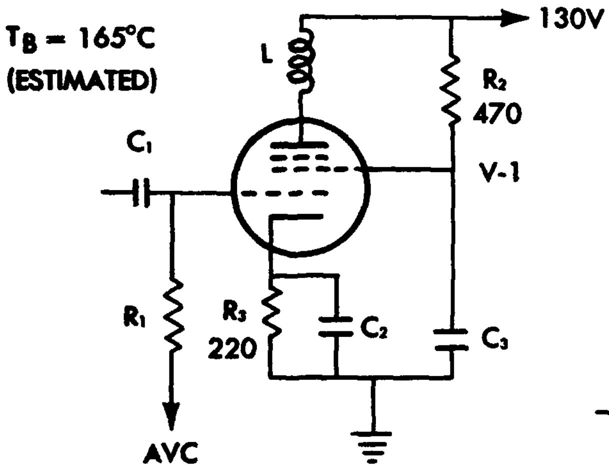
$$\text{Stress Derating Factor} = \frac{2.4}{3.3} = .72$$

Heater Voltage Derating:

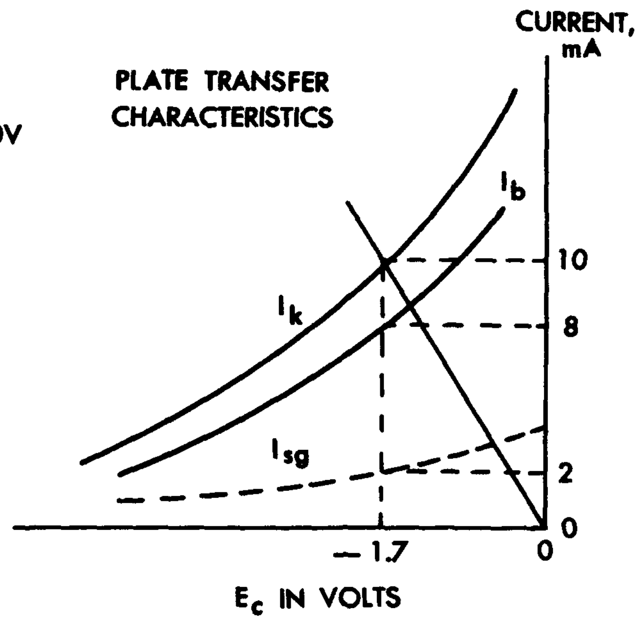
Circuit E_f	= 6.3
Rated E_f	= 6.3
Heater Derating Factor k_f	= 1.0

Temperature Derating:

Estimated Bulb Temp.	= 165°C.
Rated Bulb Temp.	= 165°C.
Temperature Derating k_t	= 1.0



FROM PROPOSED DESIGN OF FIGURE 5-6



FROM MIL-HDBK-211 1/

Figure 5-9. Modifications Necessary for Electron Tube AEG's

1/ "Techniques for Application of Electron Tubes in Military Equipment", Government Printing Office.

(2) Determine Base Failure Rate.

From Table IV, MIL-HDBK-217,

Receiving type Pentodes:
"Base" Failure Rate, B = 0.3% per 1000 hours
$= 3.0 \times 10^{-6}$ failures/hour

(3) Determine Adjustment Factors.

From Figure 8B of MIL-HDBK-217, as shown in Figure 5-10, determine the failure rate adjustment factor for a dissipation ratio of .72 and a temperature ratio of 1.0 (at 165° C). The dissipation adjustment factor, $k_d = .82$.

(4) Compute Adjusted Base Failure Rate for the Particular Application.

$$\lambda_B = k_t k_f k_d B = (1.0)(1.0)(.82)(3.0 \times 10^{-6})$$

$$= 2.46 \times 10^{-6} \text{ failures per hour for Tube V-1}$$

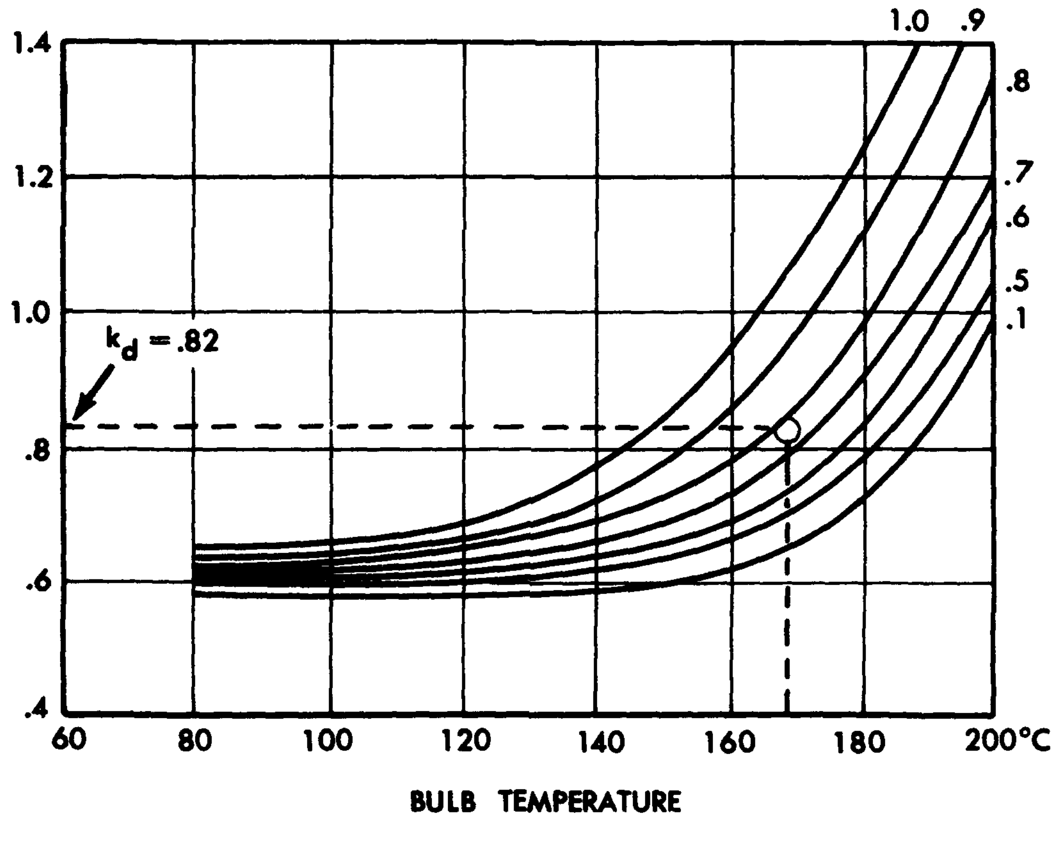


Figure 5-10. Temperature Derating Curves for Electron Tubes

Resistors, Capacitors, and Other "Passive" Parts

The ... me procedure is illustrated now for the resistor population in the circuit, using Figure 20B of MIL-HDBK-217 shown in Figure 5-11. Resistors are found to be operating at 50% of dissipation rating from the stress analysis, using Ohms Law: $P = E^2/R$. Part ambient temperature is estimated to be 80°C. Base failure rate from the chart is then .01% per 1000 hours, or $.1 \times 10^{-6}$ failures per hour, for each resistor, *on the average*. Total catastrophic failure rate of resistors in the circuit is then $3 \times .1 \times 10^{-6} = .3 \times 10^{-6}$ failures per hour. Other passive parts in the circuit would be treated similarly.

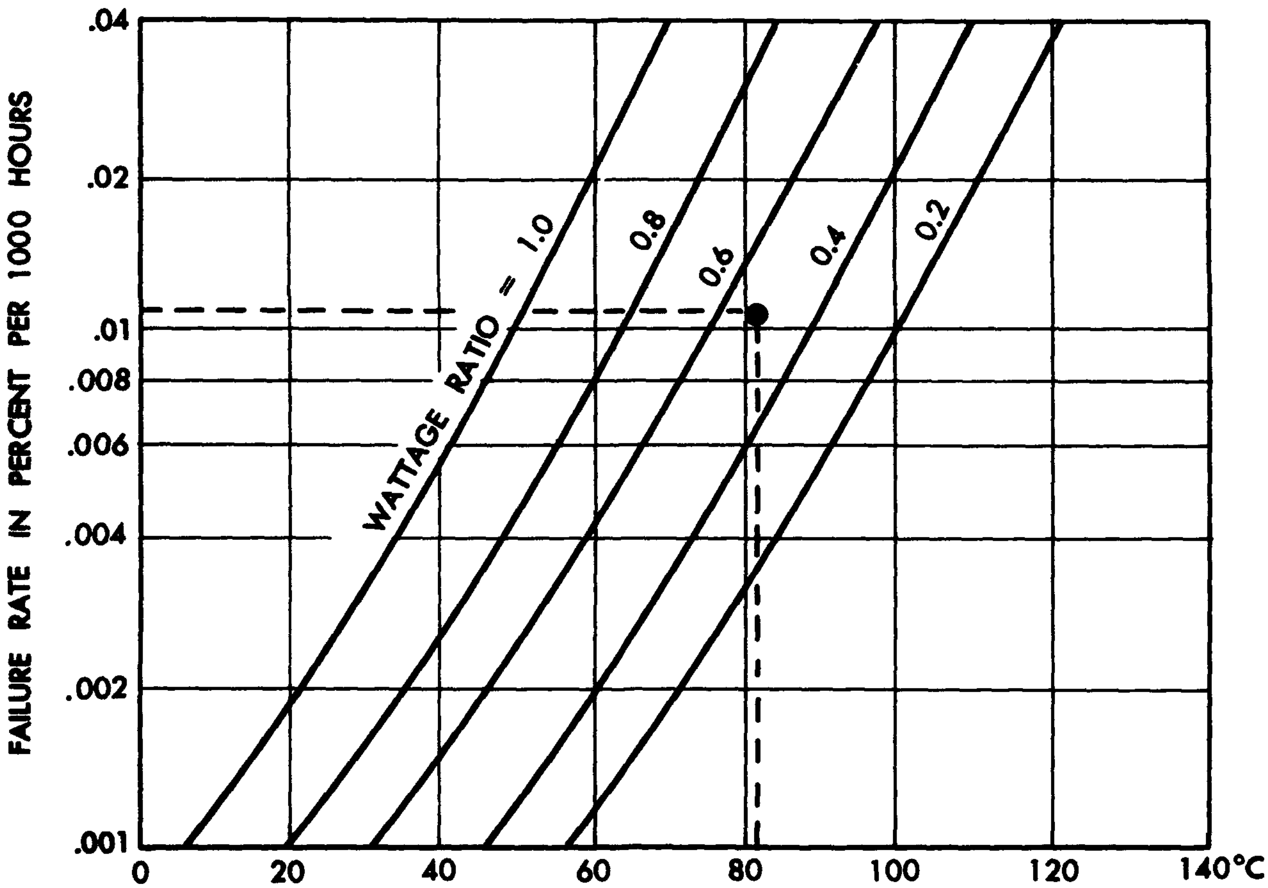


Figure 5-11. Temperature/Wattage Derating of Resistors

STEP 5 – Computation of Module or Unit Failure Rate.

Figure 5-12 summarizes the stress and failure-rate analysis just conducted for the 1st RF amplifier stage. For the transistorized version, a predicted base failure rate of 1.66 × 10^{-6} failures per hour is shown. If the designer proposes the use of subminiature electron tubes, the predicted failure rate for the 1st RF stage is 3.66 × 10^{-6} failures per hour.

The same procedure would now be applied to all other AEG's within each module to yield an estimated module failure rate.

Part	Total Dissipation		Use Stress Factor Actual/	Temp. °C		Temp. Stress	FR × 10^{-6} Per	No.	F.R. Per	Page Reference MIL-
	Rated	Actual	Rated	Rated	Actual	Factor	Part	Parts	Class	HDBK 217
Transistor Q1 (or Tube V-1)	150mW (3.3W)	45 (2.4W)	.3 (.7)	150 (165)	75 (165 est)	.4 (1.0)	.52 (2.46)	1 (1)	.52 (2.46)	49 (15)
Resistors: MIL-R-11C			.5 avg.		80		.1	3	.3	79
Capacitors Fixed: MIL-C-5B		Voltage	.5		80		.06	4 (5)	.24 (.3)	109
Variable: JAN-C-92A					80		.1	2	.2	183
Inductors: MIL-C-15305A (Class C, grade 2)					80		.1	4	.4	133
TOTAL BASIC FAILURE RATE, 1st RF STAGE								TRANSISTOR AEG	1.66	
								TUBE AEG	(3.66)	

Note: Numbers shown parenthetically apply to RF amplifier AEG using electron tubes instead of transistors.

Figure 5-12. Stress Analysis of Parts in 1st RF Amplifier AEG

STEP 6 – *Determine Subsystem or Equipment Failure Rate.*

At this point, the "base" AEG failure rates are combined within modules, for an estimated module base failure rate. Failure rates of modules are then added for an equipment base failure rate. For example, if the procedure of Step 4 were extended to all other AEG's and modules within the ADF receiver, the following table might result:

Module	Base Failure Rate × 10^{-6}
RF	25
Guard	20
IF	20
Audio	16
Total Receiver	81 × 10^{-6}

(1) Correct for "Use" Environment.

It is now necessary to correct for gross "use" environment using $k_u \approx 6.5$ from MIL-STD-756A.

Airborne base failure rate for the receiver is then:

$$\lambda_R = 6.5 \times 81 \times 10^{-6}$$

$$= 526.5 \times 10^{-6}$$

This is the failure rate to be expected of the airborne ADF receiver due to catastrophic part failures.

(2) Correct for Tolerances and Interactions.

It is next necessary to adjust the estimated parts failure rate by a complexity factor that relates part failures to equipment failures. This factor is derived from Figure 5-13. Entering the figure at N = 19 (the estimated complexity of the proposed receiver design), read off $K_c = 2.6$.

Predicted failure rate of the receiver due to all causes, catastrophic and tolerance, is then given by

$$\lambda_o = 526.5 \times 10^{-6} \times 2.6$$

$$= 1370 \times 10^{-6} \text{ failures/hour}$$

of which the catastrophic failures account for 38%.

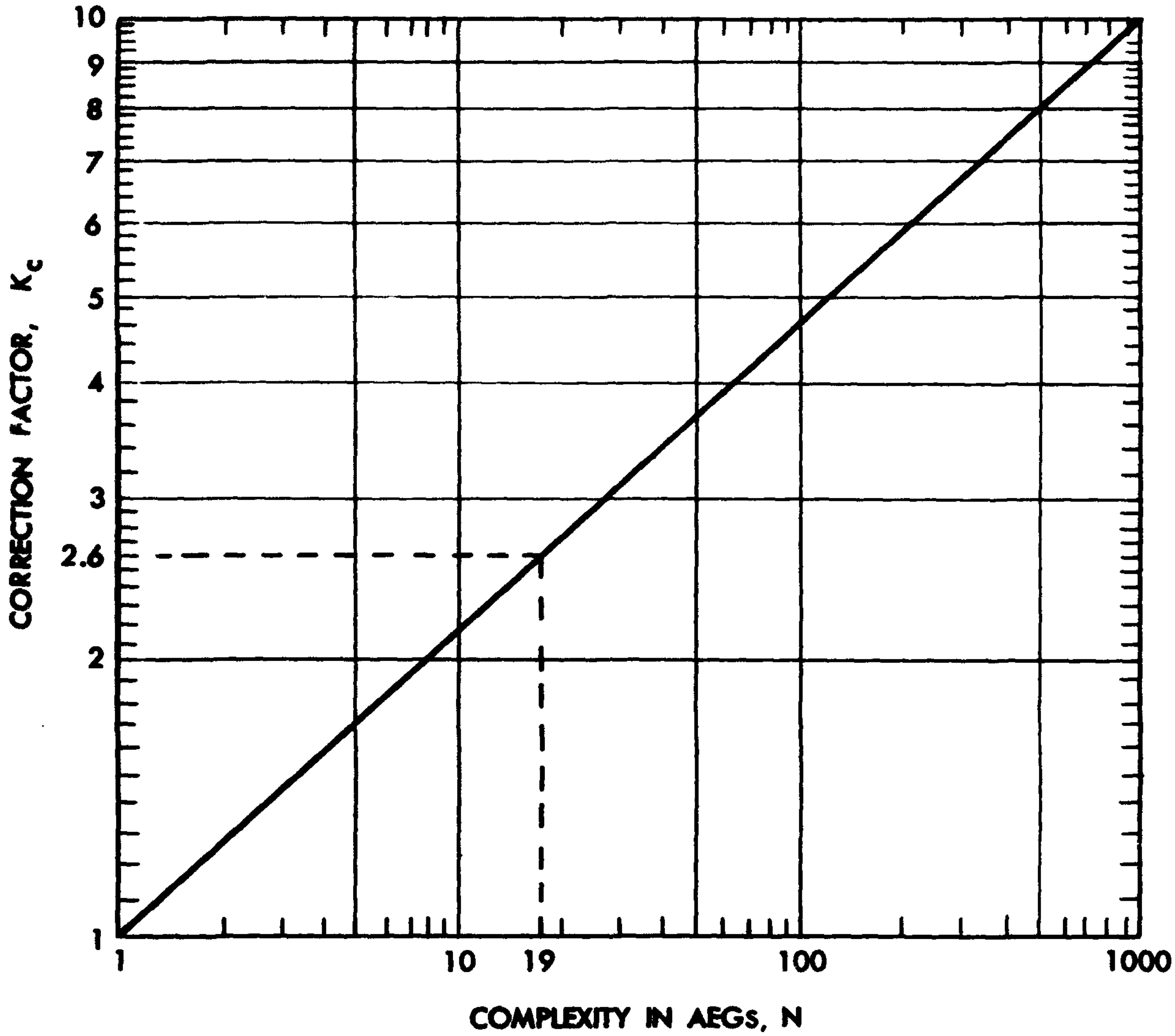


Figure 5-13. Correction Factor for Tolerance Failures in Avionics Equipment

$$K_c = (N)^{.33}$$

STEP 7 – Determine Subsystem or Equipment MTBF and Reliability.

The ADF receiver should have the following mean life:

$$\text{MTBF} = \frac{1}{\text{Failure Rate}} = \frac{1}{1370 \times 10^{-6}}$$

$$= 730 \text{ hours}$$

Reliability for a 3-hour mission from the nomographs shown in Appendix 3, or from the expression $R = e^{-t/MTBF}$, is:

$$R = e^{-3/730} = e^{-.004} = .996$$

STEP 8 – Combine Equipments or Subsystems of the System or its Individual Functions.

Continuing with the navigation function of Figure 5-2, the following table might result from the completed parts stress analyses and reliability estimates given in the preceding steps:

Equipment	Reliability (3 hours)
1	.980
2	.999
3	.999
4	.992
5	.996
6	.999
8	.990
9	.900

Substituting in the reliability model previously derived in Step 2,

$$R_{NAV} = R_1R_2R_3\left(1 - [1 - R_6R_8(R_4+R_5-R_4R_5)][1 - R_9]\right)$$

$$= (.98)(.999)(.999)\left(1 - [1 - (.999)(.99)(.9999)][1 - .9]\right)$$

$$= .977$$

This is the probability that *either* the ADF or TACAN will remain operational throughout the 3-hour mission.

Primary mode navigation reliability (probability that both ADF *and* TACAN will remain operational throughout the 3-hour mission) is simply the product of all reliability values in the table, i.e.:

$$R_{NAV} = R_1R_2R_3(R_4+R_5-R_4R_5)R_6R_8R_9$$

$$= .871$$

The reliability of each of the other functions is estimated by the same procedure as that outlined in the preceding steps, using estimates for the following additional blocks:

$$R_7 = .95$$

$$R_{10} = .93$$

For example, IFF reliability for three hours is given by

$$R_{IFF} = (R_1)\cdot(R_{10}) = (.98)(.93) = .91$$

Overall equipment reliability – all CNI functions operational – is now the product of the reliabilities of all functions, including R_7 and R_{10}, yielding:

$$R_{CNI} = .769$$

The nomograph indicates that the mean-time-between failures in the CNI system will be:

$$MTBF = 11.7 \text{ hours}$$

Stress analysis thus indicates to the designer that an improvement of somewhat better than 2-to-1 will be needed to meet the specified requirement of $R_{CNI} = .9$ which corresponds to an MTF ≈ 30 hours.

Further derating of parts and possible use of redundancy in critical elements may be necessary.

5-3 CONSIDERATIONS IN THE USE OF REDUNDANCY AND MICRO-ELECTRONICS FOR RELIABILITY IMPROVEMENT

5-3-1. "Micro" Characteristics

The preceding step-by-step procedure is applicable to designs employing micro-electronic modules, to the extent that failure data sources are now adequate. This is a limitation which will, of course, disappear as data from life-tests and field service are accumulated. In the interim, it is well to be conservative in estimating micro-module AEG failure rate, in order to insure that the design adequacy of an equipment is closely related to the probable reliability growth characteristics of the micro-electronics program, rather than to be contingent on long-range objectives that may not be realized for some time. If a conservative approach is used, the equipment design can be expected to exhibit an inherent reliability "safety" margin proportional to the difference between the conservative estimate based on current data and the finally achieved long-range goals.

While micro-electronic modules can be expected ultimately to surpass their conventional circuit counterparts in *consistently* exhibiting high inherent reliability, two micro-module characteristics other than reliability will probably have the greater effect in increasing equipment reliability in the immediate future. These characteristics are the small volume and low power consumption of the micro-module, which make it readily adaptable to multiple redundant design configurations.

When redundancy is involved, the reliability-assessment procedure outlined above should be expanded to provide a deeper *failure-mode analysis at the part and element levels*, as well as at the module level, in order to permit a proper evaluation of the feasibility of operational redundancy as *against standby redundancy*.

The following analytical steps would be taken in micro-electronic design formulation, and in assessing the reliability achieved in a given design configuration.

5-3-2. Specific Procedural Steps

STEP 1 – Evaluate Failure Modes and Effects.

Determine the effects on circuit performance of module failure in each of the module's possible failure modes. Failures can be grouped into three broadly defined modes (illustrated in Figure 5-14) having certain general failure effects at the circuit function level:

Short Mode—
usually resulting in catastrophic loss of circuit function.

Tolerance Mode—
resulting in circuit's failure to stay within tolerance limits.

Open Mode—
generally resulting in catastrophic loss, extreme degradation, or "run-away" of circuit function.

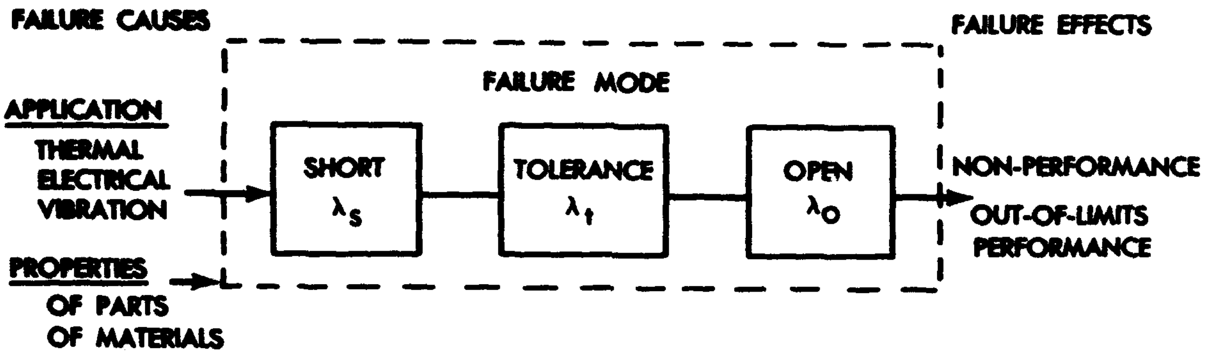


Figure 5-14. Failure Modes in a Micro-Module

STEP 2 – Evaluate Failure Cause and Frequency, by Mode.

Determine the relative frequency of each failure mode with respect to known failure causes and anticipated conditions of use. Assume that the ADF receiver of the proposed CNI system is to be "micro-modularized" to the fullest practicable extent. Consider the 1st RF Amplifier stage analyzed in Step 4 of Section 5.2. Under the proposed conditions of use (compartment temperature ambients, vibration levels, etc.), it has been determined by laboratory evaluation of RF modules that the relative frequency of failure in these general modes is as shown in Column V of Figure 5-15.

USE LEVEL / MODE	I	II	III	IV	V
Open	10	15	20	20	25
Short	10	20	25	30	40
Tolerance	80	65	55	50	35
All Modes	100	100	100	100	100
Relative by Stress Level 1 to 10	2	3	5	8	10

NOTE: All figures in this table are hypothetical, to illustrate methods. Such information should ultimately become available in the form of module application notes as the program progresses.

Figure 5-15. Failure Mode Analysis

STEP 3 – Evaluate Design Configuration Requirements for Protection Against Predominant Failure Modes.

In general, the following protective measures become practicable in micro-module design configurations:

- To protect against predominantly "open" failure modes, use parallel redundant modules, with suitable fusing and decoupling circuitry to further protect against the eventuality of failure in the short mode.
- To protect against predominantly "short" modes, use series redundant modules, with suitable bypass protection against possible open modes (if warranted).
- To protect against predominantly "tolerance" modes, use parallel configuration if the characteristics of importance vary in a random fashion – i.e., if the mean of these variabilities is approximately zero. If the characteristic of importance (e.g., voltage gain of the 1st RF stage) always varies in one direction – i.e., deteriorates with time-use series redundant modules with negative feedback stabilization.

These configuration possibilities are shown in Figure 5-16.

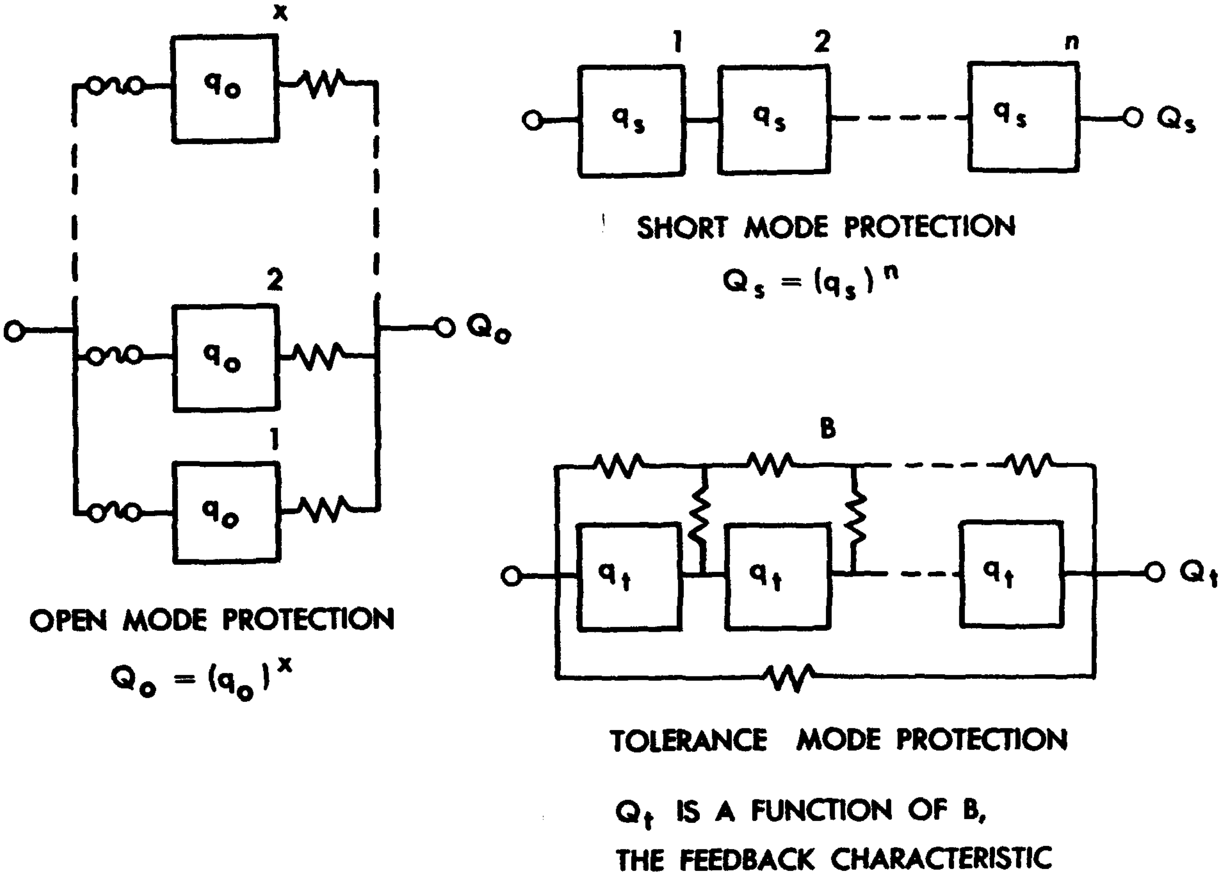


Figure 5-16. Failure Mode Block Diagram for Possible Micro-Electronic Modules

STEP 4 – Evaluate Circuit Configuration for Overall Reliability.

As an example, consider the 1st RF amplifier as a micro-module quad in which inverse feedback has been applied to reduce the conditional probability of tolerance failure to essentially zero during the mission period. Under this assumption, only an open-mode or a short-mode failure is likely to occur. Figure 5-17 is a simplified block diagram to illustrate the derivation of the basic reliability model for the circuit:

q_x is the probability of micro-module failure in an open or short mode.

$$q_x = q_x \text{ open} + q_x \text{ short}$$

$$= 1 - R_x(t)$$

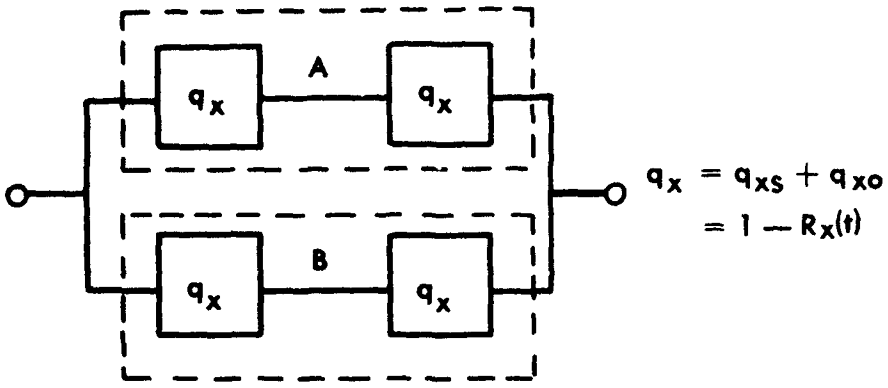


Figure 5-17. Reliability Model for 1st RF Amplifier Quad

where

$R_x(t)$ is the reliability of a micro-module for period of time t.

Probability of short in Leg A

$= q_{xs}^2$

Probability of short in both legs

$= P_s = 1 - [1 - q_{xs}^2]$

Probability of open in Leg A

$= q_{xo}(2 - q_{xo})$

Probability of open in both legs

$= P_o = [q_{xo}(2 - q_{xo})]^2$

Reliability of the quad is then

$R = 1 - P_s - P_o$

$= (1 - q_{xs}^2)^2 - [q_{xo}(2 - q_{xo})]^2$

Assume, for example, that $q_{xo} = q_{xs} = .001$ for each of the four modules based on a reliability $R_x(t) = .998$ at $t = 1000$ hours.

Then,

R(t), for the quad

$= [1 - (.001)^2]^2 - [.001(2 - .001)]^2$

$= .999994$

This is equivalent to approximately a 300-to-1 improvement in failure rate over the 1000-hour period.

A full treatment of redundancy becomes quite complex, but can be evaluated graphically, if MIL-HDBK-217 is used as a guide. The analytical results are still to be considered theoretical, however, until they are verified in design testing. It is the purpose here simply to indicate the potential gains to be achieved in equipment reliability when the techniques of redundancy are applied to designs in which micro-electronic modules are used as the basic building blocks.

The remaining steps of the assessment procedure are the same as those given at the beginning of this section, the goal being to build up estimates, block by block, until the entire equipment estimate is developed.

CHAPTER 6

DEVELOPMENT TESTING AND TEST DESIGN

6-1 INTRODUCTION

6-1-1. An Empirical Design Technique

Achievement of high "inherent reliability" is a growth process dependent on the degree to which design has been *guided* and *verified* by reliability testing – the extent to which testing has been used as a means of "designing in" reliability during the early formative stages of development.

Development testing can be defined generally as an empirical technique used to generate information that is not otherwise readily obtainable because of the inadequacy of applicable theory or the relative difficulty of achieving a theoretical solution. As it applies to the evolutionary growth of a weapon system, the definition of development testing must also embrace the need for "proof" of a theoretical solution – even when the adequacy of the applicable theory is not in question. The need for proof stems from a very practical management need for a *measure of confidence* in the results being achieved in the development program, long before hardware items are produced for delivery to the Fleet. At later stages in the production cycle, development tests are supplemented by qualification and acceptance tests in order to strengthen confidence in the design and manufacture of the product.

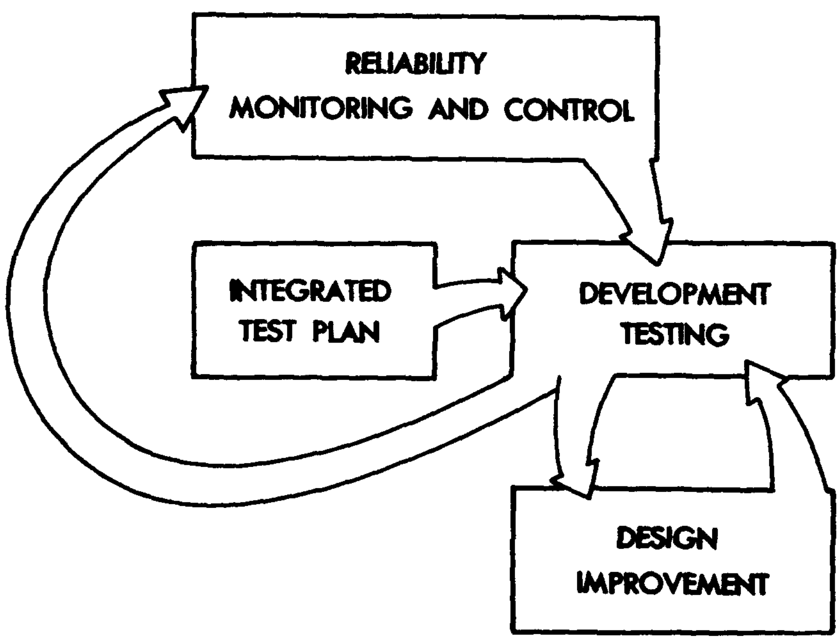


Figure 6-1. The Development Test "Feedback" Cycle

Development tests are employed by the designer to evaluate the adequacy of his design and to point out its weaknesses. As indicated above, such tests may be thought of as design techniques, in that test results are applied directly to the problem of design refinement. At the same time, development tests provide management with a finger-on-pulse awareness of design status with respect to program requirements. Thus, the "outputs" of a well-planned development test program become important "inputs" to the management monitoring program. This development test feedback cycle is illustrated in Figure 6-1.

Qualification tests are employed to provide a formal evaluation of development progress as well as assurance that specified requirements for one phase of the development program have been satisfied before the next phase is embarked upon. For example, such tests are used to qualify the design for prototype development; to qualify the prototype for pilot production; or to qualify the preproduction model for full-scale production.

The reliability achieved during development is monitored through acceptance tests, which are employed to assure continued *controlled* compliance with specification requirements. Acceptance tests are discussed in detail in Chapter 7.

6-1-2. Reliability Test Objectives in Development

The foregoing discussion introduced the general concept of development testing as an empirical design technique, a tool used by the designer to assure that the basic design has an inherent capability for meeting all the requirements of the design specification, including performance, maintainability, safety, reliability, and other factors.

When properly planned for, much of the development testing conducted for performance information can be made to yield simultaneously the desired amount of reliability information with very little alteration in test plans. In other instances, there is no alternative but to design and conduct a test purely for reliability investigation or verification purposes. Certain fundamentals of test design must be understood and translated into "reliability test" design criteria, regardless of whether the reliability test is an investigative or exploratory test (test of inquiry); or a verification or comparison test (test of hypothesis).

A. The Investigative Test

In the investigative test, an experiment is designed to *formulate a hypothesis* about the reliability of a product or process; e.g., to determine the failure mode of a new part being considered for use in the design, under stress conditions anticipated in the design, or to determine the interactions among several variables in a proposed design configuration. On the basis of test results, a hypothesis concerning cause/effect relationships is formulated for design guidance, pending verification by the second type of test.

B. The Verification Test

A verification test, on the other hand, is designed to *verify a hypothesis* concerning the reliability behavior of a product or process. A verification test is frequently used to compare a measurement of MTBF achieved in development against an earlier MTBF predicted in design, to verify the prediction hypothesis.

6-1-3. Test Procedures

Reliability test methods applicable in the development phase are, in general, "designed experiments" – test conditions and methods of data analysis are preplanned on the basis of engineering requirements and statistical considerations. Whether the test is designed for investigation or for verification, the following cycle must be completed if effective and unbiased results are to be expected:

- Define the Problem
- State Test Objectives
- Establish Test Requirements
- Design Test
- Implement Test
- Analyze Results

6-1-4. Test Design Consideration

The design and implementation of a development test program must weigh and balance the seldom compatible engineering, statistical, and administrative requirements deemed essential for satisfying the test objectives.

- Engineering requirements dictate the environmental stress levels, duty cycles, range of applications, and performance limits used to define success or failure of the item under test. In general, test conditions must be representative of those anticipated in use if an acceptable basis for decision is to result.
- Statistical requirements relate to the desired accuracy of results, the order and manner of selection and testing, and the confidence which can be placed in the decisions made.
- Administrative requirements pertain to practical limitations on time, funds, and facilities which may necessitate compromises in engineering and statistical criteria.

To optimize test design with respect to these often conflicting requirements, consideration may be given to the relative advantage of smaller sample sizes and longer test times over larger sample sizes and shorter test times. The feasibility of accelerating test conditions in order to induce more failures may be considered when the effects of such accelerated conditions on failure modes are *known*. Although the effects of accelerated test conditions on parts failure modes and rates are fairly well known in certain instances, it is generally not feasible to translate these effects into predictions of component and equipment failure behavior.

Consideration may also be given to a sacrifice in the required test confidence, thus permitting a reduction in sample size or time requirements, in order to conform to existing administrative limitations. This is always permissible on the grounds that a test designed around a relatively low level of confidence is always better than no test at all!

In the following paragraphs, procedures are outlined for design and application of those test methods which have wide application in the solution of reliability problems during design and development. The nature of the problem will determine which of the general test categories will apply.

6-2 TESTS OF INQUIRY

6-2-1. Basic Types

Tests of inquiry applied to reliability problems are, in general, divided into two categories:

(1) **Measurements tests** –
those designed to measure the reliability of an item.

(2) **Evaluation tests** –
those designed to evaluate relationships between environments or stresses and parameters which influence reliability (or failure rate) of the item.

Examples of the design and application of each of these are given in the following paragraphs.

6-2-2. Measurement of Reliability (Application of Confidence Limits)

Reliability measurement tests should be conducted under known operating conditions – ideally closely simulating use conditions to be expected in the Fleet. The operating times accumulated and the number of failures observed provide the basis for measuring reliability. Confidence in test results is directly related to the number of failures which are observed during the test.

A test of inquiry does not presuppose or hypothesize a desired reliability, but rather depends upon the analysis of test data to obtain the observed value. The following example illustrates the procedure which may be used to design and implement a typical measurement test.

6-2-3. Procedural Steps

STEP 1 – Define the Problem

A new traveling wave tube is developed, and prototype models are available for evaluation by prospective users. Characteristics of the tube suggest its use in an unmanned ECM application. However, no data are available on the reliability of these tubes. Therefore, the problem is to measure the reliability of the traveling wave tubes, under the proposed operating conditions, by a planned test program.

STEP 2 – Establish Test Requirements

Test requirements are defined as follows:

"The test facilities shall duplicate the estimated operating environments, electrical stresses, and duty cycles. Tube characteristics of phase shift, cathode current, and helix current shall be monitored. A tube shall be considered to have failed if performance varies outside performance limits. Momentary surges such as 'arcing', which are *self-sustaining and* discontinuance of which requires removal of high voltage, shall be considered as failures; however, if the tube involved is not damaged, it may continue in test. In order to assure reasonable confidence in test results, the test shall provide sufficient operating time to permit the accumulation of at least five failures."

STEP 3 – Design the Test

It is possible to accumulate controlled test time on the TWT's by either of the following two methods:

(1) Fabrication of one test position and testing a single tube at a time until five have failed; or

(2) Fabrication of several test positions and simultaneous accumulation of test time.

It is determined that the use of five test positions is economically feasible. Eight tubes are procured for test purposes so that a tube failing in any one of the five positions can be replaced by one of the three spares. This approach, known as a "replacement test", is chosen in order to optimize the number of test hours which can be utilized in any given calendar time. Administrative restrictions limit the test to a maximum of three months, operating 24 hours a day, five days a week.

Preparation of detailed test procedures, data recording, and data analysis are assigned to the reliability engineering group; conduct of the test is assigned to the test department.

STEP 4 – Implement the Test

Figure 6-2 graphically portrays the test period.

STEP 5 – Analyze Data

The following equation is used to determine the mean-time-between-failures (MTBF):

$$\text{MTBF} = \frac{\text{Operate Hours}}{\text{Number of Failures}}$$

The number of operate hours and failures accumulated during the TWT test is shown in Figure 6-3.

$$\text{Observed MTBF} = \frac{6420}{7} = 917 \text{ hours}$$

STEP 6 – Establish Confidence Limits on the MTBF

The observed MTBF of 917 hours represents the best estimate of TWT mean life, based on the 8-tube sample. Since the 917 hours is derived from a small sample, the true MTBF for the population of tubes could lie either somewhat above or below this estimate. A range of values, within which it can be stated with 90% confidence [1] that the true value will fall, is established by placing the 90% confidence limits (upper and lower estimates) about the test value of 917 hours. These limits are obtained from Table 3-1 of Appendix 3 of this handbook. By entering the table at 7 failures:

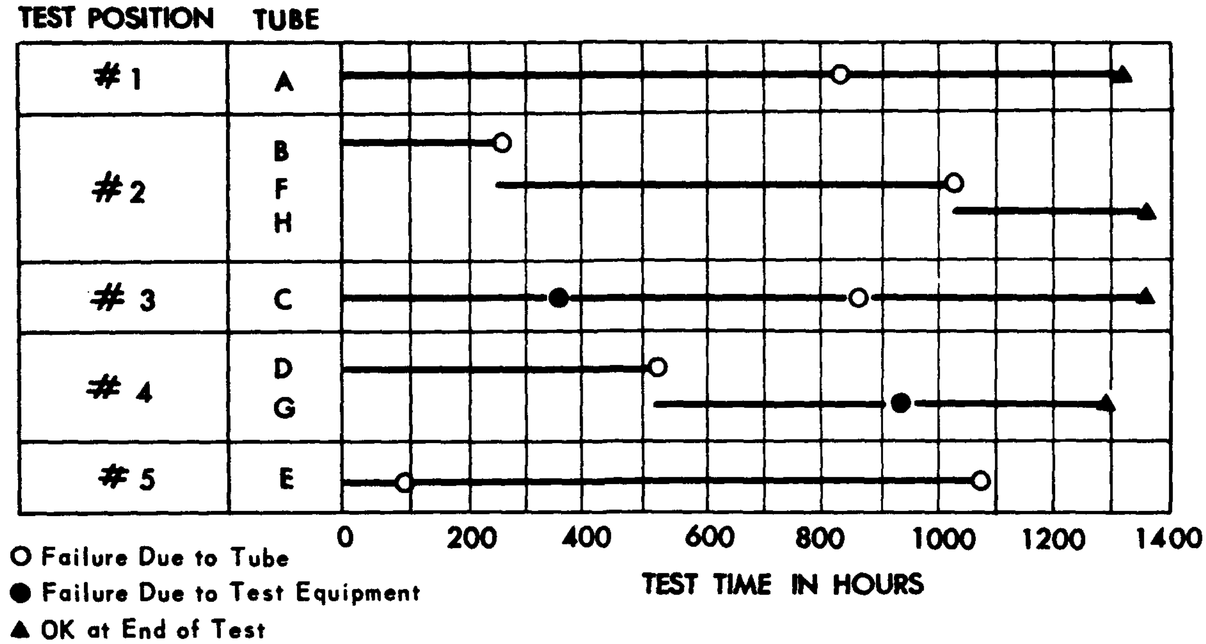
Lower 90% confidence limit
$= 917 \times .591$
$= 542$ hours

Observed MTBF $= 917$ hours

Upper 90% confidence limit
$= 917 \times 2.130$
$= 1953$ hours

These computations are plotted on Figure 6-4.

[1] Any desired degree of confidence may be chosen and corresponding confidence limits derived; however, 90% is the most widely used level for reliability estimation.



FAILURE SUMMARY:

TUBE	HOURS AT FAILURE	REMARKS
A	838	Arcing–Tube test OK, continued in test.
B	264	Open filament.
C	375	Low cathode current (test equipment induced failure) – Tube test OK, continued in test.
C	860	Arcing – Tube test OK, continued in test.
D	555	Arcing–Tube burned out.
E	90	Arcing–Tube test OK, continued in test.
E	1070	Phase shift out of tolerance.
F	766	Arcing–Tube burned out.
G	405	Phase shift out of tolerance (test equipment error)– Tube test OK, continued in test.
H	–	No failures.

Figure 6-2. Example Test Summary

	Total Operate Hours	Number Failures
Tube A	1320	1
Tube B	264	1
Tube C*	1380	1
Tube D	555	1
Tube E	1070	2
Tube F	766	1
Tube G	735	0
Tube H	330	0
	6420	7

* Test equipment failure occurring at 375 hours is not counted.

Figure 6-3. TWT Test Results

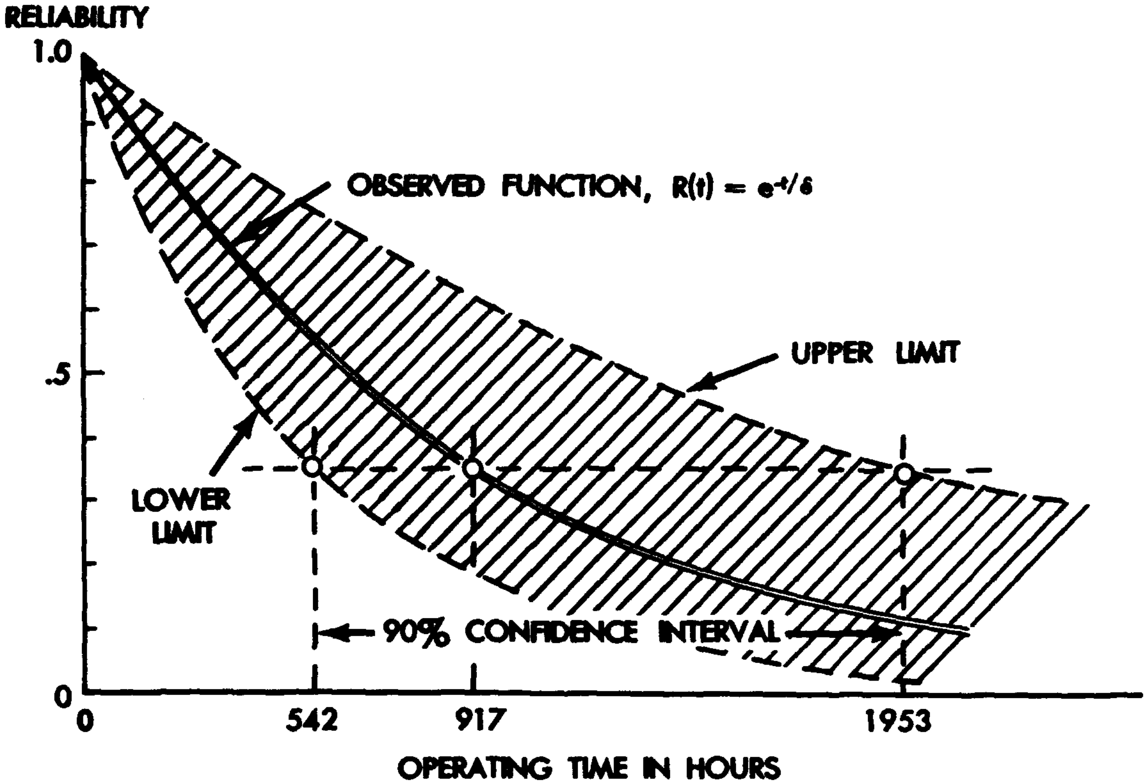


Figure 6-4. TWT Reliability Function, Showing the 90% Confidence Interval

6-2-4. Evaluation Tests (Regression Analysis)

Part-to-part variations and changes in environmental conditions cause corresponding changes in circuit or equipment parameters, such as output power or voltage, frequency stabilization, and failure rate. Knowledge of the relationship which exists between two variables can often be obtained through a planned regression analysis test program.

Regression analysis is a statistical technique which quantitatively defines the best fit of a line through a set of data points, as shown in Figure 6-5. The usual "by-eye" engineering procedure is improved upon by the use of regression analysis in the determination of the constants a and b in the regression equation of Figure 6-5. (Statistical regression analysis may be extended to many variables and to nonlinear relationships. However, it is recommended that this be attempted only under the guidance of experienced statisticians.)

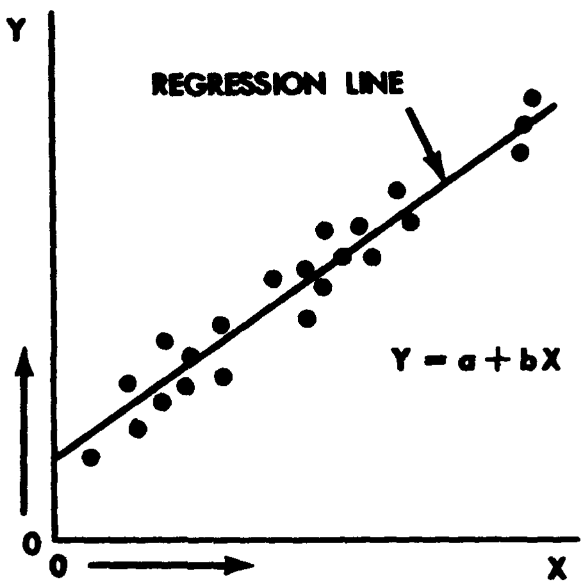


Figure 6-5. Regression Line Through Observed Values of Y for Given Values of X

6-2-5. Procedural Steps

The step-by-step procedure for the design and analysis of a simple (2-variable) regression application follows:

STEP 1 – Define the Problem

A turbo jet engine is experiencing blade-fatigue failures due to prolonged vibrations at resonance. It is assumed that resonance occurs at steady-state engine RPM. The problem is to determine if a relationship exists between bench-measured blade resonance points and the actual RPM at which the blade reaches resonance. If such a relationship is established, it might be possible, by bench measurement, to determine the acceptability of blades for actual engine use, thereby reducing or eliminating this engine failure mode.

STEP 2 – Establish Test Requirements

To prevent bias in the test results due to the use of a given production lot, the blades chosen for test are selected at random from several production lots. It is considered desirable, from a statistical viewpoint, to test at least 30 blades under the proposed bench-measurement conditions, followed by test in a production turbo jet engine with the turbine inlet temperature maintained at 1500°F.

STEP 3 – Design the Test

Only one turbo jet engine is available for use as a test bed. Consideration of the time involved and other commitments limit the test to the minimum 30 blades. The following test sequence is established:

(1) Selection of 30 blades, at random, from the past three months' production.

(2) Identification and bench-test of each blade to determine resonance frequency.

(3) Installation of blades in successive build-up of the turbo jet engine, with engine RPM varied to determine RPM at which blade resonance occurs.

STEP 4 – Order the Test Data

Figure 6-6 gives data accumulated on the 30 blades, ranked in order of bench-resonance frequency.

STEP 5 – Plot the Data as a Scattergram

The test data summarized in Figure 6-6 are plotted as a scattergram in Figure 6-7.

Blade No.	Resonance		Blade No.	Resonance	
	Frequency	RPM		Frequency	RPM
7	960	9420	27	1062	10550
18	969	9400	3	1069	10700
2	986	9550	11	1078	10550
28	988	9750	17	1085	10800
1	998	9650	5	1090	10650
16	998	9850	22	1130	11000
21	1011	9800	26	1149	11400
9	1012	10100	29	1169	11900
8	1025	10000	30	1180	11750
12	1035	10300	4	1181	11600
23	1042	10000	13	1190	11900
25	1043	10200	19	1215	11950
10	1047	10300	24	1217	12200
20	1055	10500	14	1240	12350
15	1058	10300	6	1271	13800

Figure 6-6. Test Results: Blade Resonance Frequency and RPM Resonance

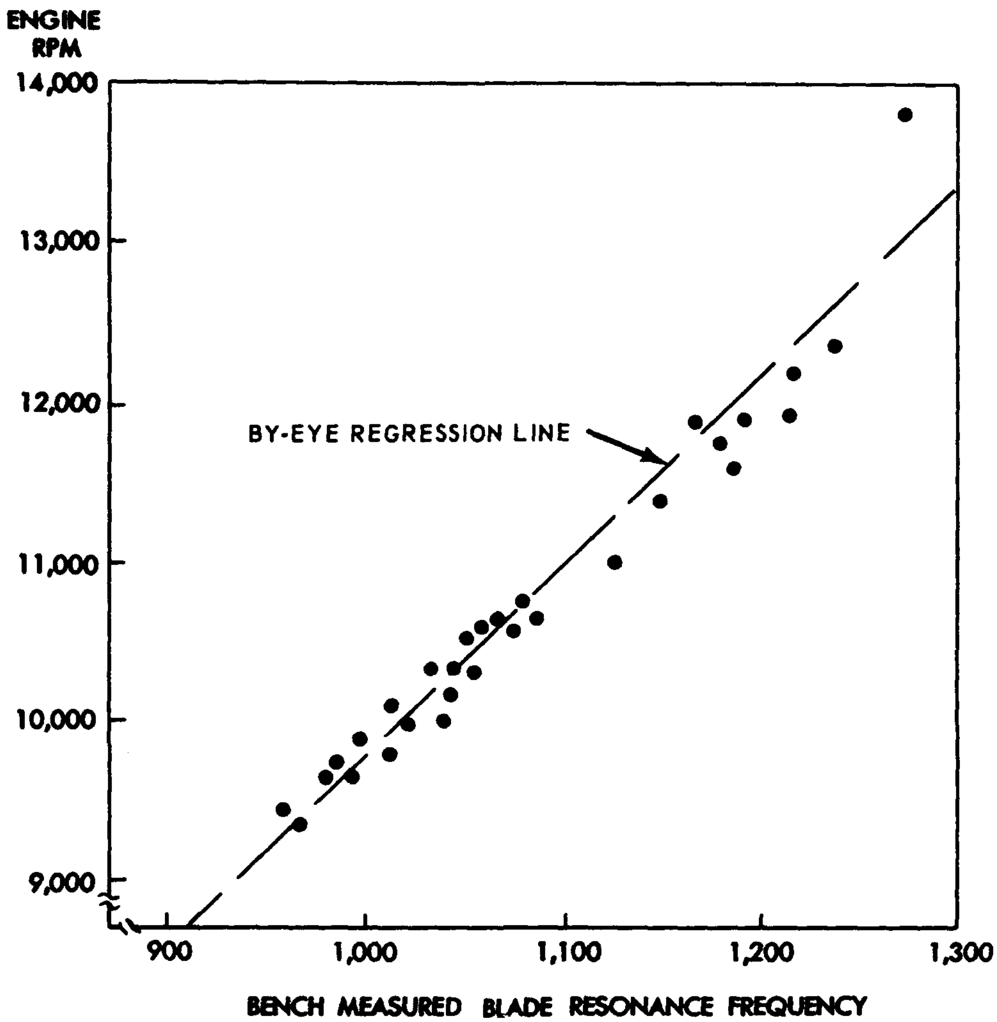


Figure 6-7. Scattergram of Test Data

STEP 6 – Determine the Line Which Best Represents the Relationship Between X and Y

This may be accomplished "by-eye" when the data are closely grouped and obviously linear, as shown in Figure 6-7. However, it is often desirable to use computational techniques to obtain a more accurate equation than can be approximated by eye. Accomplishment of this is illustrated in Figure 6-8.

EQUATION: $Y = a + bX$

X = Bench-Measured Blade Frequency

Y = Resonant Engine RPM

Blade No.	X	X²	Y	Y²	XY
1	998	996,004	9,650	93,122,500	9,630,700
2	986	972,196	9,550	91,202,500	9,416,300
3	1,069	1,142,761	10,700	114,490,000	11,438,300
.					
.					
.					
29	1 169	1,366,561	11,900	141,610,000	13,911,100
30	1,180	1,392,400	11,750	138,062,500	13,865,000
TOTAL	32,553	35,546,527	332,220	3,492,681,400	352,246,750

$n = 30$

$\Sigma X_i = 32{,}553$ $\qquad \Sigma Y_i = 322{,}220$

$\Sigma X_i^2 = 35{,}546{,}527$ $\qquad \Sigma Y_i^2 = 3{,}492{,}681{,}400$

$(\Sigma X_i)^2 = 1{,}059{,}697{,}809$ $\qquad (\Sigma Y_i)^2 = 103{,}825{,}728{,}400$

$\Sigma X_i Y_i = 352{,}246{,}750$ $\qquad \overline{X} = \frac{\Sigma X_i}{n} = \frac{32{,}553}{30} = 1085.1$

$(\Sigma X_i)(\Sigma Y_i) = 10{,}489{,}227{,}660$ $\qquad \overline{Y} = \frac{\Sigma Y_i}{n} = \frac{322{,}220}{30} = 10740.67$

$$b = \frac{n(\Sigma X_i Y_i) - (\Sigma Y_i)(\Sigma X_i)}{n(\Sigma X_i^2) - (\Sigma X_i)^2} = \frac{(30)(352{,}246{,}750) - 10{,}489{,}227{,}660}{(30)(35{,}546{,}527) - 1{,}059{,}697{,}809}$$

$$= \frac{78{,}174{,}840}{6{,}698{,}001} = 11.671$$

$$a = \overline{Y} - b\overline{X} = 10740.67 - (11.67)(1085.1) = -1922.446$$

$$\boxed{Y = -1922.446 + 11.671X}$$

Figure 6-8. Computations in Regression Technique

STEP 7 – Apply the Analysis

The steady-state turbo jet rotational speed is a nominal 10,000 RPM, controllable by the fuel system to within ±200 RPM. Thus, it is possible to plot the range of expected RPM, as shown in Figure 6-9. Turbine blades whose bench-measured resonance frequencies fall in the range of X_1 to X_2 (as obtained either from the plot or from the regression equation) could possibly become resonant at steady-state engine RPM.

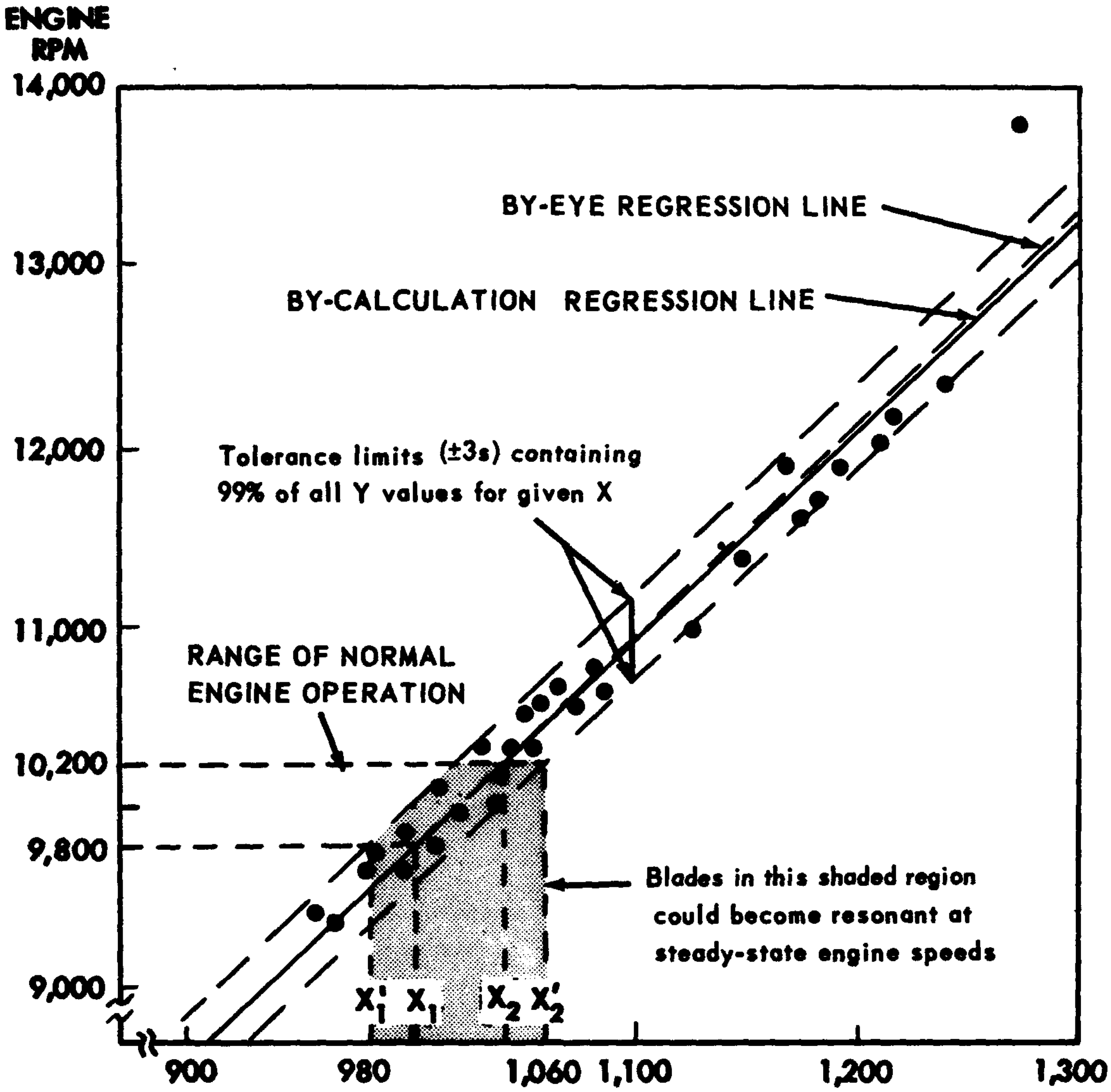


Figure 6-9. Application of Regression Analysis in Relating Bench Measurements to Application Conditions

The regression of Y on X is not perfect – i.e., some variation of Y exists for a given value of X. It is possible to obtain an estimate of the extent of this variability by assuming that the variation in Y is normally distributed, by solving the following equation, and by plotting the vertical ±3s tolerance limits[2/] about the the regression line for any selected value of Y.

$$s = \sqrt{\frac{n\Sigma Y_i^2 - (\Sigma Y_i)^2 - b[n\Sigma X_i Y_i - (\Sigma X_i)(\Sigma Y_i)]}{n(n-2)}}$$

Using the data in Figure 6-8,

$s = 224.5$

The ±3s tolerance limits for a blade resonant frequency of 1100 cps are shown on Figure 6-9. Two lines drawn parallel to the regression line and passing through the ±3s limits will, in general, encompass 99% of the expected variability of Y for any given value of X. These lines may also be approximated "by-eye", if drawn to encompass all observed data points. The range of resonant blade frequencies which would indicate a potential engine resonance condition, is thus broadened to the area bounded by X_1' and X_2'.

If blade resonance frequencies are kept either below 980 cps or above 1060 cps, less than 1 in 100 should become resonant over the acceptable range of steady-state engine operation. This knowledge is used in the improved design of blades and in the establishment of acceptance test limits for bench measurements.

6-3 TESTS OF HYPOTHESIS (DECISION MAKING)

6-3-1. Categories of Decision

Tests of this type are designed to assist in decision making. Design decisions, based on reliability parameters, fall into two broad categories:

(1) Verification that an item meets a prescribed minimum reliability, and

(2) Selection of the more reliable item or approach from two or more alternatives.

Through the testing of developmental samples, inferences (decisions) may be drawn about the total projected population. A statement or hypothesis is first made concerning the item(s) to be tested. Decision criteria are established prior to the test such that a simple inspection of results will determine whether to accept or reject the hypothesis. The test design, in terms of sample size, is also adjusted to provide a given level of confidence in the decision or, conversely, a given risk of making an incorrect decision. Two types of risk are are associated with decisions based on test results:

2/ Tolerance limits are used here in the sense that a given proportion of the Y values will lie between the limits as follows:

±1s = 68.3%
±2s = 95.4%
±3s = 99.7%

This is based upon the standard deviation s of the normal distribution as discussed in Paragraph 2.2.3 of Appendix 2.

(1) Rejection of the hypothesis when in fact it should have been accepted – commonly referred to as a Type I error and denoted by α; and

(2) Acceptance of the hypothesis when in fact it should have been rejected – commonly referred to as a Type II error and denoted by β.

In general, these risks are inversely related to the sample size of the test.

The following examples of decision-making through test provide the step-by-step procedures required in the establishment of the decision criteria and the associated risks.

6-3-2. Tests of Verification

Tests of verification are employed to verify that a desired result has (or has not) been obtained. This type of test is usually employed to provide the development team "proof" (with a known confidence in their test answer) that the design has in fact achieved the specified reliability. The following example illustrates the design of a test to verify a reliability prediction.

STEP 1 – Define the Problem

A reliability requirement has been allocated to a static inverter on the basis of its importance and complexity relative to other components of the new system. This requirement is stated as a reliability of (at least) 0.9 for 1,000 hours of operation. Past experience indicates that for the preliminary design a reliability of 0.9 cannot be expected beyond 100 hours of operation. However, it has been *predicted* that a redesigned circuit, employing extensive derating of parts and redundancy at the parts level in critical areas, would considerably exceed the required 10-to-1 increase in the specified period of operation, as illustrated in Figure 6-10.

The problem is to determine whether or not the proposed new design would yield the required 0.9 reliability for 1,000 hours of operation under simulated field conditions, as predicted. Since redundancy is employed in the design, it cannot be assumed that the exponential reliability function represents the inverter's time-to-failure characteristics. This eliminates an MTBF test.

STEP 2 – Determine Test Objectives

Primary objective of the test is to verify the design prediction or, statistically, the hypothesis that the reliability of the inverter is equal to or greater than .9 for 1,000 hours of operation (the predicted reliability is .972 for 1,000 hours). This is expressed mathematically as

$$H_o: \quad R \geq .9$$

Secondary objectives of the test include:

- Estimation of the actual reliability observed during the test.
- Investigation of the effects of redundancy on the reliability function.
- Analysis of failures to determine causes and possible corrective actions or design improvement.

STEP 3 – Establish Test Requirements

The conditions under which the inverters are tested correspond to those specified for anticipated installation environments and load requirements. Acceptable test performance tolerance limits are based on those which are required to maintain successful operation in the intended application(s).

Statistical requirements which must be established are those associated with the risks involved in the decision to accept or reject the hypothesis. Since the primary purpose of the test is to determine whether the inverters have achieved or exceed a .9

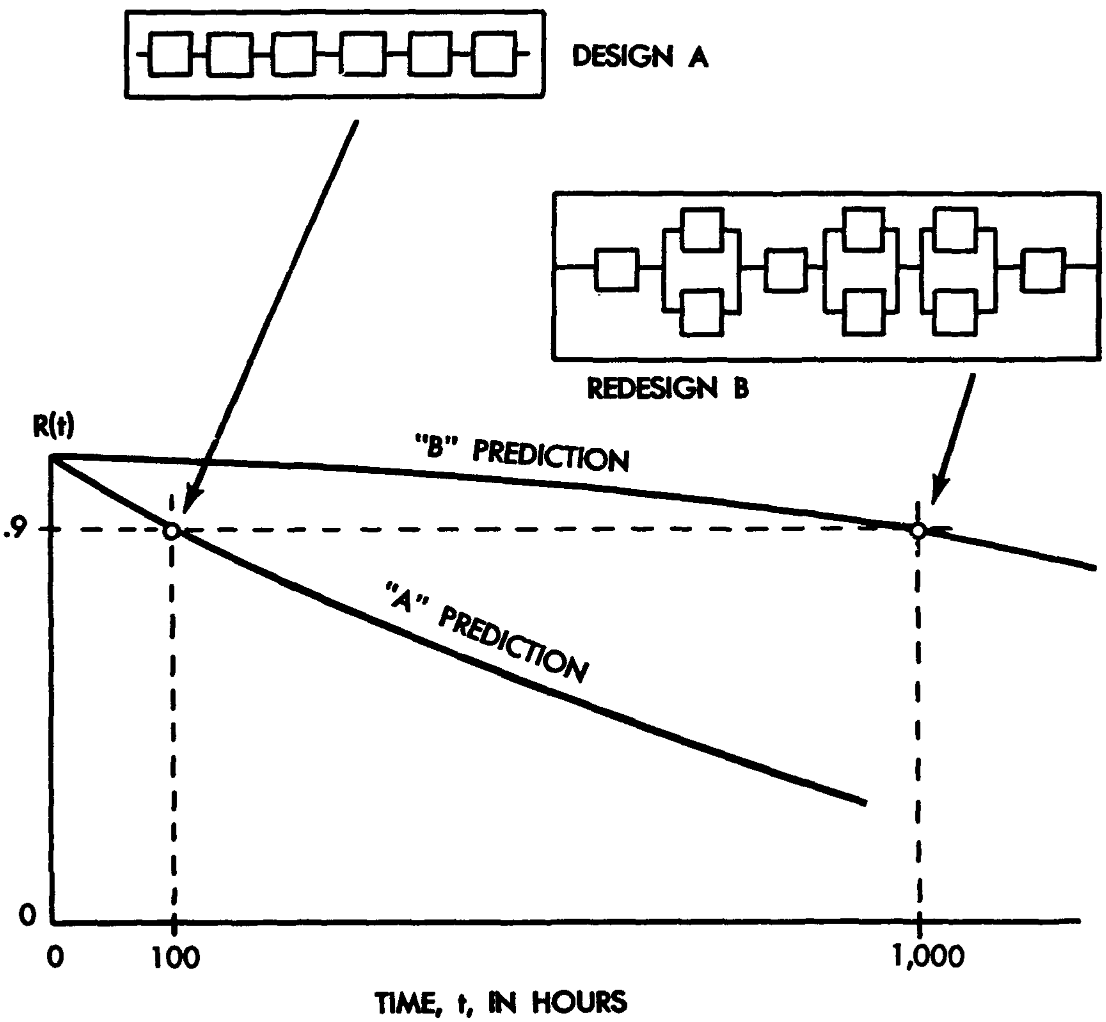


Figure 6-10. Predicted Reliability Functions

reliability, a confidence of 90% in a decision to accept the hypothesis is established; or, conversely, a .10 risk of making a wrong accept decision can be tolerated.[3]

It is also desirable to establish a maximum risk of .10 against a reject decision if the inverters are in fact as good as their predicted reliability of .97.

STEP 4 – Test Plan

The basic plan for the design of the inverter test is indicated by the specified reliability allocation and the predicted non-exponential reliability function to be investigated. It is desirable that the test be planned for at least the specified 1,000 hours of operation of each item in order to escape the dangers involved in extrapolation of the results if shorter times are used. In order to investigate the theoretical non-exponential reliability function, it is also advisable to plan for continuation of the test after the primary objective is met, thus furnishing more failure information to satisfy the secondary objectives. Since the equipment employs redundancy at the parts level, the replacement or repair of test items during the test complicates the test design. Consequently, a simple non-replacement test plan is chosen.

The sample of inverters is placed on test under simulated conditions. Items that fail during the test are removed and are not replaced.[4] After 1,000 hours of testing, a decision based on the observed number of failures is made concerning the test hypothesis. Data on all failures (including the time-to-failure) are recorded and the test is continued for an arbitrary additional period of 1,000 hours in order to gain further information on the reliability characteristics of the design.

STEP 5 – Test Design

Test design involves deriving a set of decision criteria based upon the maximum number of failures (c) which may occur during test of a sample of (N) units prior to a reject decision (e.g., if c or less failures occur in N sample units, accept the hypothesis; if c plus one or more failures occur in the N sample units, reject the hypothesis). The size of the sample and acceptable number of failures are chosen to provide the desired risks of rejecting a true .972 reliability and of accepting a true reliability lower than the minimum .9.

Ideally, a test should provide perfect discrimination (i.e., an equal risk of rejecting the inverters if their true reliability is slightly less than .9 and of accepting them if their reliability is .9 or above). However, sampling tests cannot provide this ideal discrimination; therefore it becomes necessary to establish an acceptable discrimination ratio which is determined as follows:

$$\frac{(1 - \text{Minimum Reliability})}{(1 - \text{Design Reliability})} = \frac{\text{Maximum Proportion Defective}}{\text{Minimum Proportion Defective}} = \text{Discrimination Ratio } (k)$$

The minimum inverter reliability was established by specification as .9. Design reliability is that value greater than .9 which the design group expects to achieve. It has been predicted that the nominal

[3] The risks associated with development test decisions are normally limited to either .10 or .20 for both the Type I and Type II errors.

[4] All items which fail are to be subjected to a failure analysis as discussed in Chapter 8.

design reliability of the inverters will be .972. Therefore, the discrimination ratio for the test should not exceed

$$\frac{1-.9}{1-.972} = \frac{.1}{.028} = 3.6 = k$$

The discrimination ratio plays a vital role in the determination of sample sizes. The inverse relationship between sample size and k requires that a compromise be reached between the test cost in terms of sample size and the natural desire of the design group for a discrimination ratio which approaches unity.

ɔmbination of sample size and c ıʊr a given minimum reliability and β error is uniquely defined by its operating characteristic (OC) curve. The OC curve for a given test design relates the probability of reaching an accept decision to the true reliability of the product.

Figure 6-11 presents an OC curve for

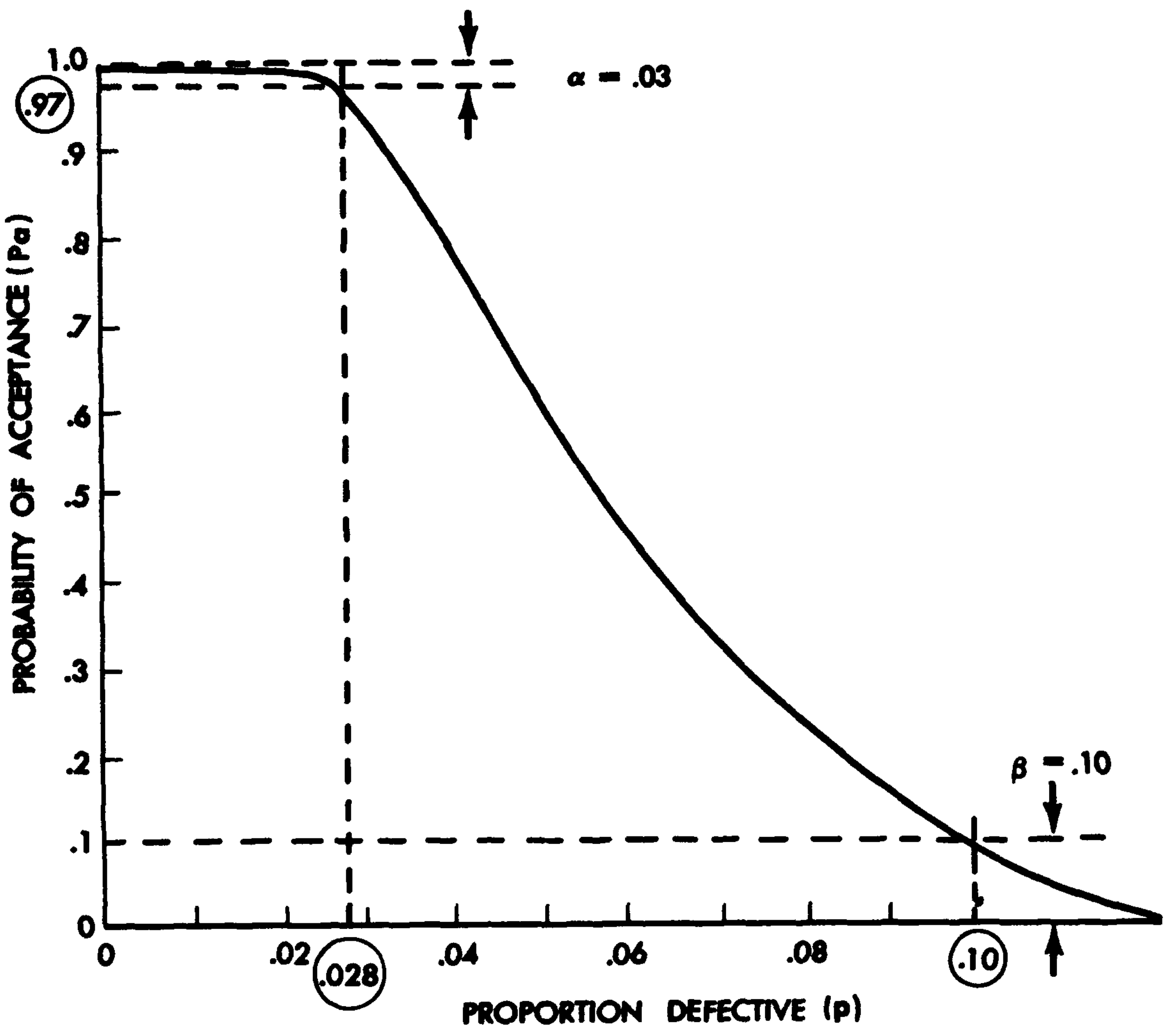


Figure 6-11. OC Curve for Test Design, N = 93; c = 5

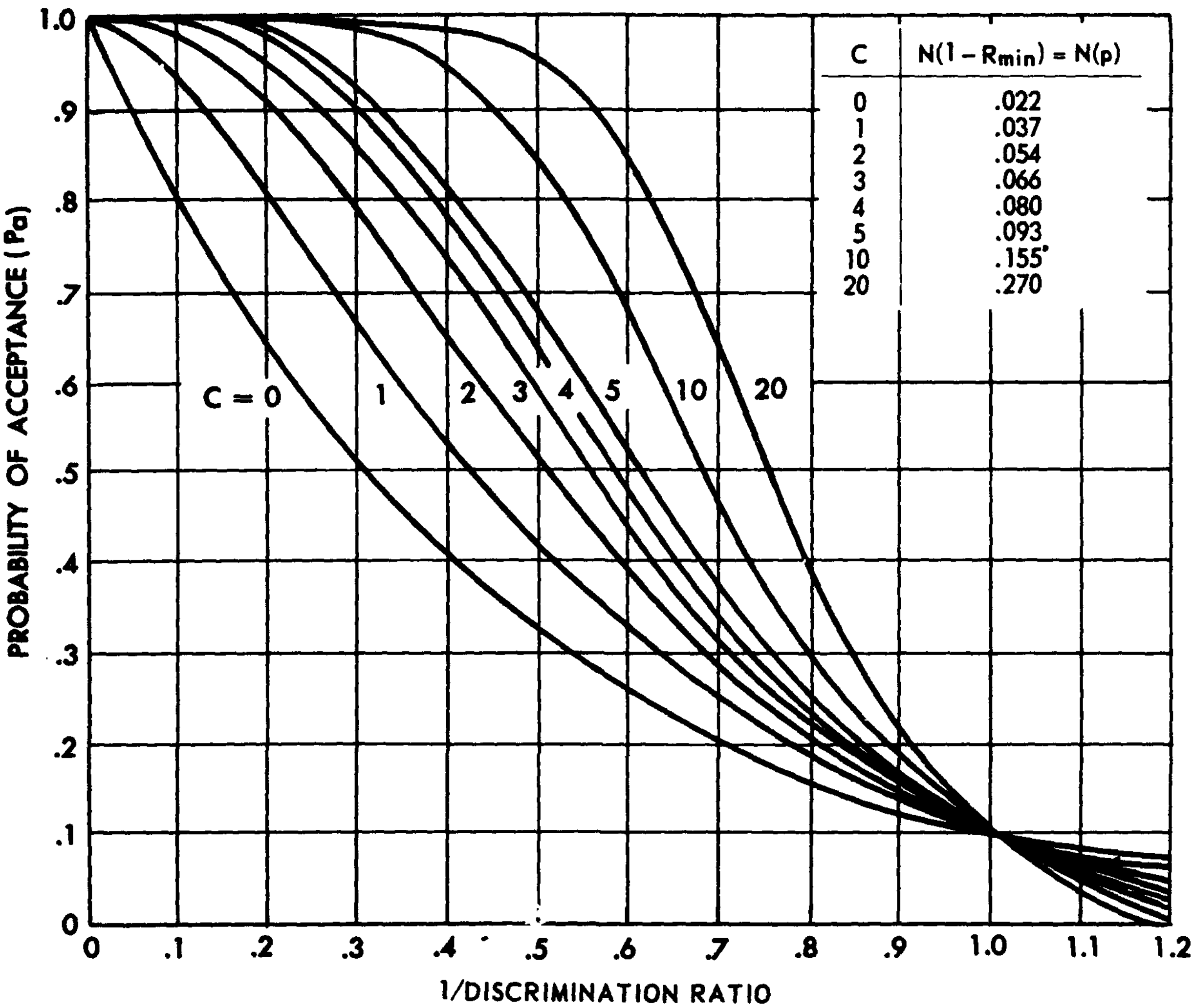


Figure 6-12. OC Curves for Selected Test Plans with a Fixed β Risk of .10

a sample size of 93 and a c number of 5 (5 failures allowed to occur in the sample) for a β risk of .10 in accepting the hypothesis that the inverters have at least .9 reliability. Using the k of 3.6 determined above, the risk of rejecting the inverters if their true reliability is .972 (proportion defective = .028) is only .03; or, conversely, the probability of accepting $(1 - \alpha)$ the hypothesis, and thus the inverter, is .97.

This test design exceeds the requirements and thus does not minimize the number of samples. Figures 6-12 and 6-13 are used to aid in selecting test designs and establishing the tradeoffs between sample size, discrimination ratios, and α/β risks.

The test requirement of a β risk of .10 and a minimum reliability of .9 (maximum of 10% defective) dictates the use of Figure 6-12. The minimum test plan which fulfills the requirement is that plan which falls nearest to the intersection of the .10 α risk line and the .028 design-predicted proportion defective. This results in a c of 3 and a corresponding N of 66. Therefore, the recommended test design tests 66 inverters for 1,000 hours and accepts the hypothesis if 3 or less defectives are observed.

If the sample size is too high, it is possible to reduce the number by either or both of the following methods:

- **Increase the k value.** This assumes that the design reliability is better than the prediction and, in effect, increases the risk of rejecting inverters which in fact exceed .9 reliability. For example,

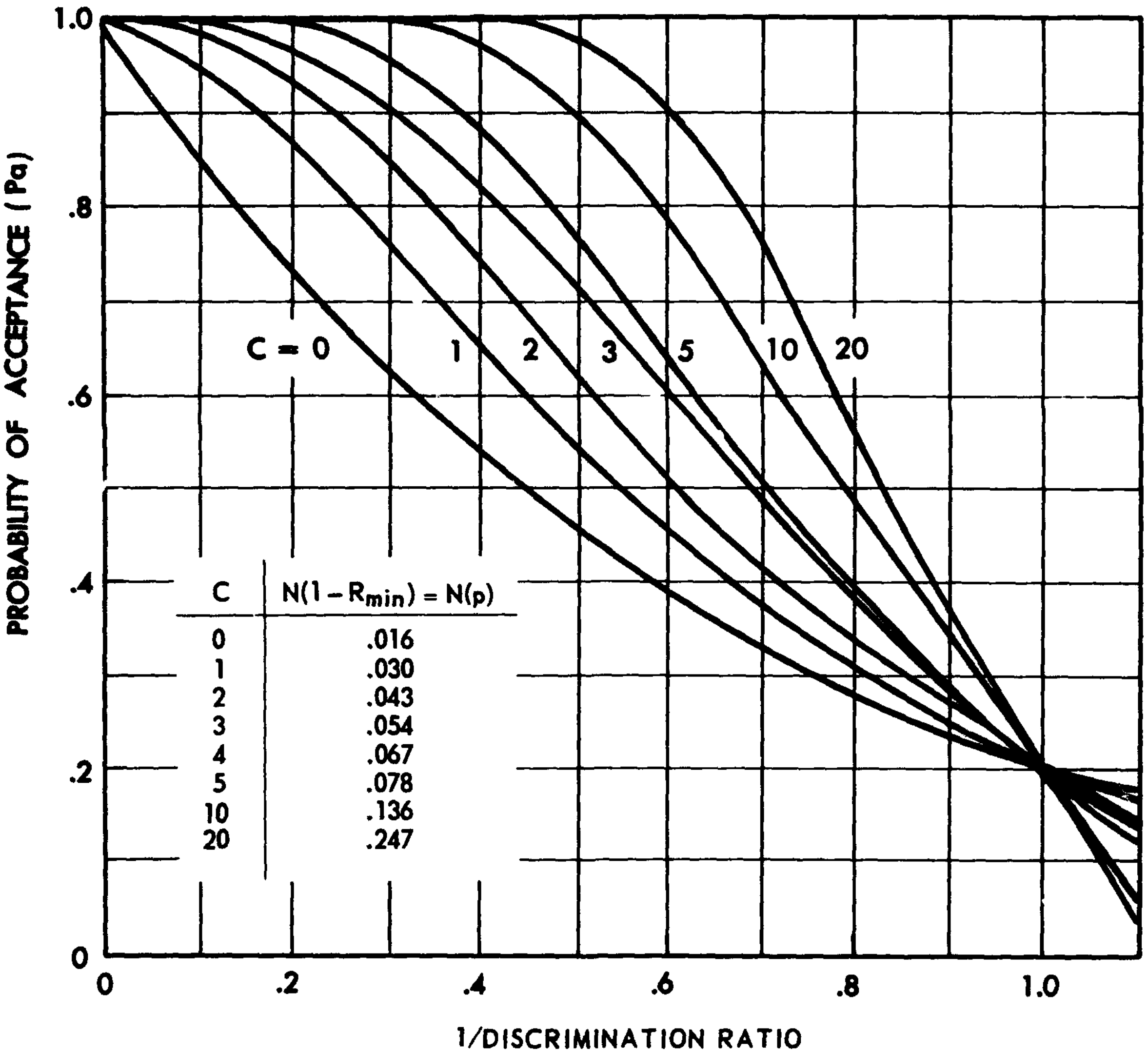


Figure 6-13. OC Curves for Selected Test Plans with a Fixed β Risk of .20

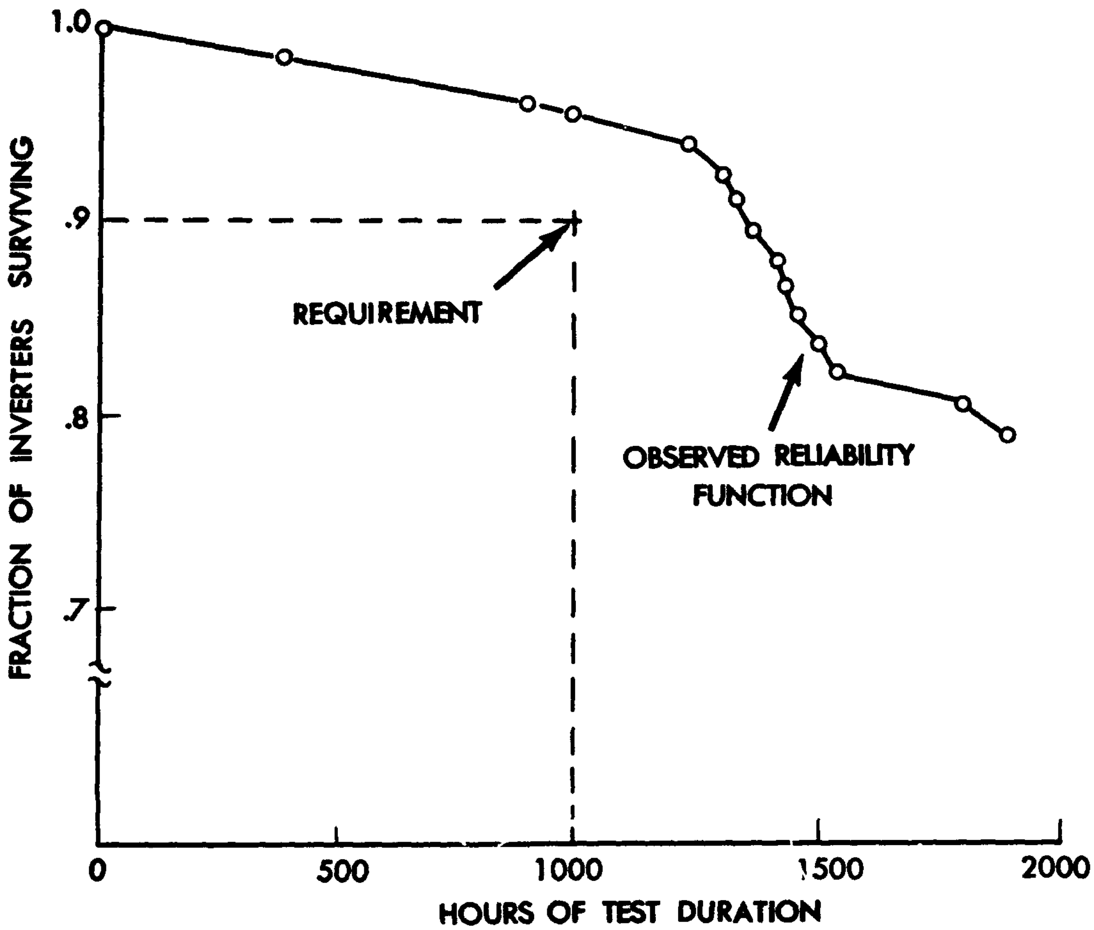


Figure 6-14. Observed Reliability Function for 2000-Hour Test with 66 Inverters

increasing k to 5 would permit a c value of 1 and would result in a sample size reduction to 37. However. the inverters would require a least a .986 reliability if a .10 risk of rejection is maintained; or the original risk of .10 for a .972 reliability would increase to a risk of .30.

- **Increase the β risk.** Figure 6-13 presents test designs for a β risk established at .20. If the original k value of 3.6 is held constant and the α risk is also adjusted to .20, the sample size could be reduced to 16 (c = 1). This approach, however would necessitate a change in the basic test requirements.

STEP 6 – Implement the Test

The test of 66 inverters in accordance with the test plan yields the following results: 52 inverters operate the full 2,000 hours without failure; 14 inverters fail after accumulating the test times shown below:

Inverter No.	Operate Time at Failure (Hours)
2	1320
4	1510
7	1430
12	1246
17	852
22	1369
25	1440
31	1900
32	1540
39	1321
45	1447
54	1823
56	402
64	1006

STEP 7 – Data Analysis

Inspection of the data reveals the failure of only 2 of the 66 inverters during the 1,000-hour test period. Thus, the hypothesis Ho: $R_{1000} \geq .9$ is accepted it can be stated (with 90% confidence) that the true reliability is at least .9.

The observed 1,000-hour reliability is 64/66 = .97. Continuation of the test for the additional 1,000 hours permits a plot of the observed reliability function as shown in Figure 6-14 and analysis of the individual failures as recommended in Chapter 8.

6-3-3 Tests of Comparison

A problem which frequently confronts the designer is the choice between two or more possible items for use in the design, the choice between approaches, or perhaps the choice between processes. When sufficient test data are not available for a decision, relatively simple, straightforward tests of comparison can be performed to aid in the decision process. The following example outlines the design and conduct of such a test.

STEP 1 – Define the Problem

A system designed with conventional circuitry could be redesigned using microcircuitry, with a significant reduction in weight and space. However, both the redesign cost and the per/equipment cost would be inflated. A 5-to-1 increase in reliability will offset higher initial costs. Therefore, the problem is to determine if a microcircuit design will yield a minimum of 5-to-1 increase in reliability or a 5-to-1 decrease in failure rate.

STEP 2 – State Test Objectives

The primary objective of a comparative test is to determine whether a pre-established hypothesis can be accepted or rejected. For tests of this type, the basic hypothesis is that "no difference exists":

Ho: $\lambda_c = \lambda_m$

where λ_m = Failure rate of microcircuitry

λ_c = Failure rate of conventional circuitry

An alternative hypothesis (Ha) is also established:

Ha: $\lambda_c \geq 5\lambda_m$

This hypothesis will be accepted in the event Ho is not supported by test data.

STEP 3 – Establish Test Requirements

The primary environmental conditions under which the comparison must be made include input voltage variations and transients typical of those seen by airborne electronic equipments and an ambient temperature cycle varying from −55°C to +60°C.

The comparison, by necessity, is based upon several individual circuit functions rather than upon complete equipment designs. The IF strip is chosen as the group of circuit functions upon which the decision will be based. For the purposes of the test, the definition of IF strip failure is to be based upon the "design-required" output signal tolerances, with the minimum expected input signal (output from RF stages) fed into the IF strip. The acceptable risks of making a wrong decision are both set at .05; i.e.:

- α, the probability of rejecting Ho when it should be accepted, = .05
- β, the probability of accepting Ho when it should be rejected, = .05

STEP 4 – Test Design

The basic design of comparison tests is illustrated in Figure 6-15. Samples of each item to be compared are selected and randomly subjected to the same test conditions. Data analysis and decision criteria are based upon the proportion of defectives

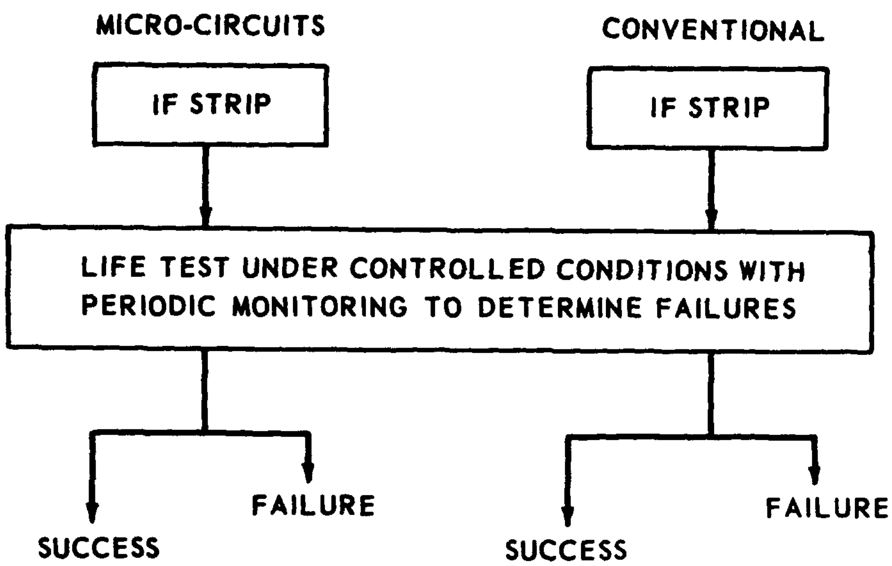


Figure 6-15. Comparative Test Plan Schematic

observed for each type item. A sample, N, of each type of IF strip is subjected to a 1,000-hour life test under anticipated use conditions. At least once a day (24 hours), the signal outputs of each IF strip are measured. Any measurement which is outside the preset tolerance limits is cause for rejection of the IF strip (it is optional whether the rejected item is continued in test or removed). The initial sample sizes are determined by a combination of three factors:

(1) The acceptable decision risks ($\alpha = \beta = .05$).

(2) The desired discrimination ratio or differences to be detected (5-to-1).

(3) The "expected" minimum proportion defective to be observed.

Figure 6-16 is a graph relating sample size to the minimum proportion defective for several discrimination ratios at $\alpha = \beta = .05$.

The expected minimum proportion defective is estimated by assuming that micro-circuit IF strips have a failure rate one-fifth of that predicted for the conventional strip:

$$\text{Expected } \lambda_m = \frac{1}{5}\lambda_c = \frac{1}{5}(.0002)$$

$$= .00004 \text{ failures per hour}$$

A 1,000-hour test would thus be expected to yield 4% defectives (a proportion defective of .04).

A total sample size of approximately 100 IF strips (50 micro-circuit and 50 conventional) is obtained from Figure 6-16, under the assumption of 4% defective and a 5-to-1 discrimination ratio.

In summary, the test plan consists of:

- Fabrication of 50 IF strips of each type.
- Initial performance test to assure all good at t = 0.
- Conduct of 1,000-hour life test with once-a-day performance measurements.
- Classification of the test data into one of four groups, organized as illustrated in Figure 6-17.

STEP 5 – Implement the Test

Results of the test are tabulated in Figure 6-17.

STEP 6 – Analyze the Results

The data classified in Figure 6-18 are analyzed by a process labeled the "Chi-Square Test of Independence".[5/] First it is necessary to construct a table of expected values (Figure 6-18) corresponding to the observed data table. "Expected" values are derived by multiplying row totals by column totals and then dividing this product by the overall sample size, as shown in Figure 6-18(a). The Chi-Square (χ^2) Test compares the observed values against the expected values in the tables.

Expressed mathematically,

$$\chi^2 = \sum_{1}^{4} \frac{(|O - E| - .5)^2}{E}$$

where O is the observed data, E is the

5/ Duncan, Acheson J., "Chi-Square Tests of Independence and Comparison of Percentages", *Industrial Quality Control*, American Society for Quality Control, New York, June 1955, Vol. XI, No. 9.

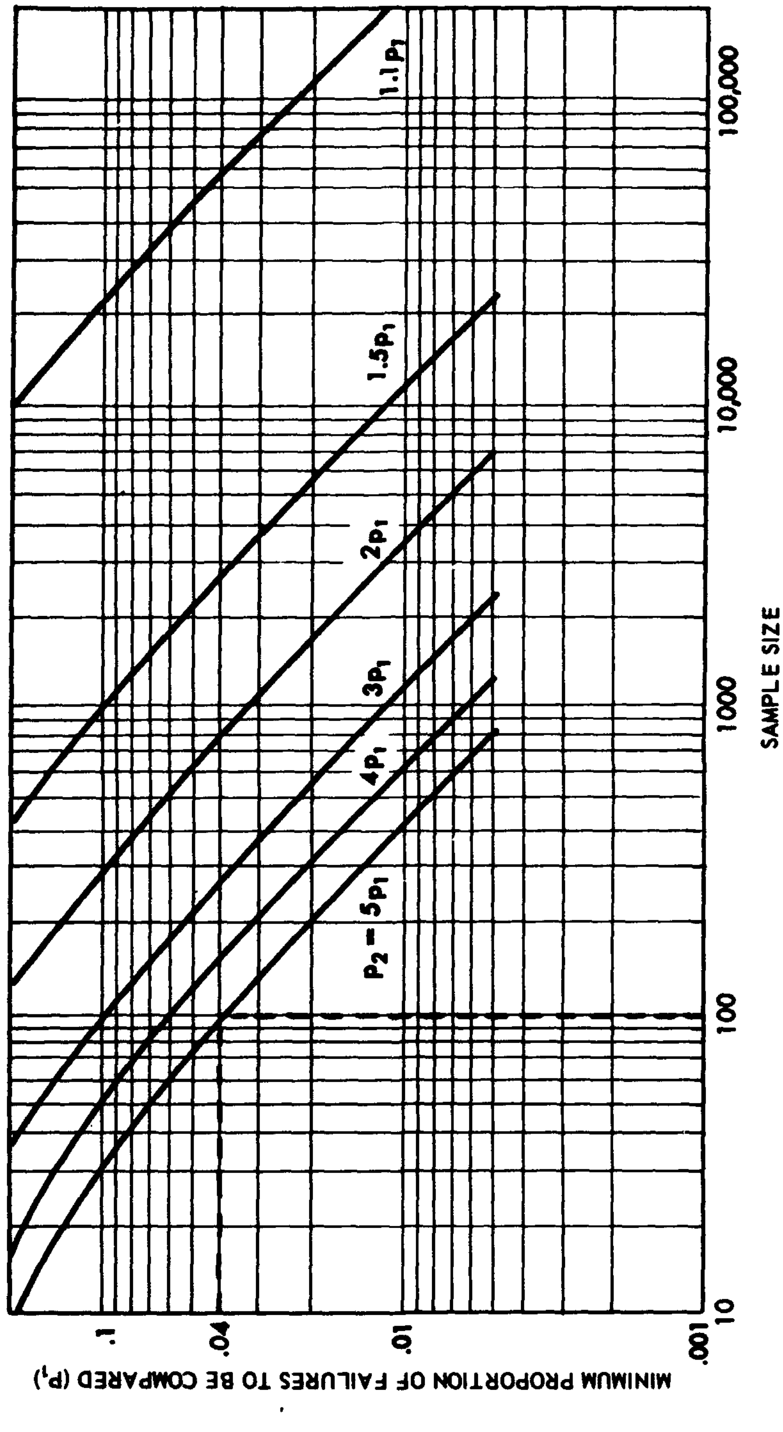


Figure 6-16. Sample Sizes Required to Detect Given Differences Between Two Proportions of Failures with 0.95 Probability, with $\alpha = \beta = .05$

	Micro-Circuit	Conventional	Total
Success	46	38	84
Failure	4	12	16
TOTAL	50	50	100

Figure 6-17. Results of Comparison Test Between Micro-Circuit and Conventional IF Strips

expected value for the same cell, and $|O - E|$ is the absolute value of the difference.[6] The summation is the sum of the four data cells.

The hypothesis that $\lambda_c = \lambda_m$ (or that the percent defective throughout 1,000 hours of test are equal) is rejected if the above summation is greater than 3.84.[7]

The critical χ^2 for the micro-circuit test is computed as

$$\chi^2 = \frac{(|46-42|-.5)^2}{42} + \frac{(|38-42|-.5)^2}{42} + \frac{(|4-8|-.5)^2}{8} + \frac{(|12-8|-.5)^2}{8}$$

$$= \frac{(4-.5)^2}{42} + \frac{(4-.5)^2}{42} + \frac{(4-.5)^2}{8} + \frac{(4-.5)^2}{8}$$

$$= .292 + .292 + 1.53 + 1.53$$

$$= 3.64$$

Therefore, the hypothesis that the failure rates of the micro-circuit IF strip are equal to those of conventional design is not rejected. On the basis of the test results, the gain in reliability by a redesign to include micro-circuitry would not provide the minimum 5-to-1 improvement established as the economical break-even point.

Secondary data analysis include analysis of failures and plots of the observed reliability functions in accordance with Chapter 8.

6/ The value of .5 is subtracted from the absolute difference between the observed and expected before squaring if the total of all the cells is greater than 40 and the smallest expected value is less than 500.

7/ The value of 3.84 is that value of the Chi-Square (χ2) obtained from Tables of the Chi Square for one degree of freedom and the .05 level of significance (i.e., the probability of rejecting the hypothesis when it should have been accepted.

(a) Method of Calculation

	Micro-Circuit	Conventional	Total
Success	$\frac{50(46 + 38)}{100}$	$\frac{50(46 + 38)}{100}$	84
Failure	$\frac{50(4 + 12)}{100}$	$\frac{50(4 + 12)}{100}$	16
TOTAL	50	50	100

(b) "Expected" Data Table

	Micro-Circuit	Conventional	Total
Success	42	42	84
Failure	8	8	16
TOTAL	50	50	100

Figure 6-18. Table of Expected Data Under the Assumption that the Percent Defective is Equal for Each Type of IF Strip

CHAPTER 7
RELIABILITY ACCEPTANCE TESTS

7-1 INTRODUCTION

Acceptance testing of a product involves the evaluation of product characteristics under *specified* conditions. If the evaluation discloses that these characteristics fall within "acceptable" limits as defined in the product specification, the product is deemed acceptable. Thus the acceptance test need not produce a measurement of the characteristics, but only show that they are "good enough" to meet minimum acceptance requirements. It is not necessary to know how much better a product is than its specified minimum in order to make an accept decision. What is necessary, however, is a knowledge of the *risk* involved in making the decision to accept a product on the basis of the test results. In general, when more test time is used (more failures are expected during the test), less risk is involved in making a decision or, conversely, there is more confidence in the test results. Two types of risks are involved in any acceptance test plan – the risk of rejecting an acceptable product, and the risk of accepting an unacceptable product. These will be discussed further in the step-by-step test design procedure.

Because of the high costs of product testing at low risks, sequential test plans have been developed[1] to more effectively utilize the test results for decision making. Two types of sequential plans are applicable:

- MTBF or failure-rate tests based on the exponential distribution which are applicable to most systems and equipments of conventional design that do not make extensive use of redundancy.

- Probability of survival tests, based on the inclusion of operating time (or cycles) as a test condition. These tests are generally applicable to all products irrespective of their time-to-failure distribution.

Procedures for each of these types of sequential tests are outlined in this chapter.

The major advantage of sequential testing plans is that, on the average, they require less testing than attributes or variables plans, especially when the product is either very poor or very good. The major disadvantage is that the exact number of items needed cannot be determined before the test is run. However, it is possible to compute the average number of items required.

In general, a good product will be accepted quickly and a poor product will be rejected quickly, while a questionable product will usually require a longer testing time (although a smaller number of failures) than is required by other sampling plans. Another feature of sequential plans is that they can be used either for testing one item at a time or for testing many items simultaneously.

[1] See for example, OASD Handbook H108, "Sampling Procedures and Tables for Life and Reliability Testing", 29 April 1960.

7-2 SEQUENTIAL TEST DESIGN FOR MTBF ACCEPTANCE

7-2-1. General

Sequential test for MTBF acceptance may be used when the exponential distribution may be assumed for the MTBF or failure rate. When the assumption of exponentiality is not valid, a different test design should be used (see 7-3).

7-2-2. Procedural Steps

A method for designing a sequential MTBF acceptance test is presented and demonstrated by example in the following steps.

STEP 1 – Define "Acceptable" and "Unacceptable" MTBF

The nominal MTBF expressed as the design requirement is the acceptable MTBF, usually denoted by θ_0. The unacceptable MTBF corresponds to the *minimum* acceptable originally defined in the design specification, usually denoted by θ_1. Figure 7-1 illustrates the concept of θ_0 as the nominal MTBF, with θ_1 as the lower tolerance limit as discussed in Chapter 6. For purposes of illustration, a normal distribution of MTBF's is depicted about the mean value.

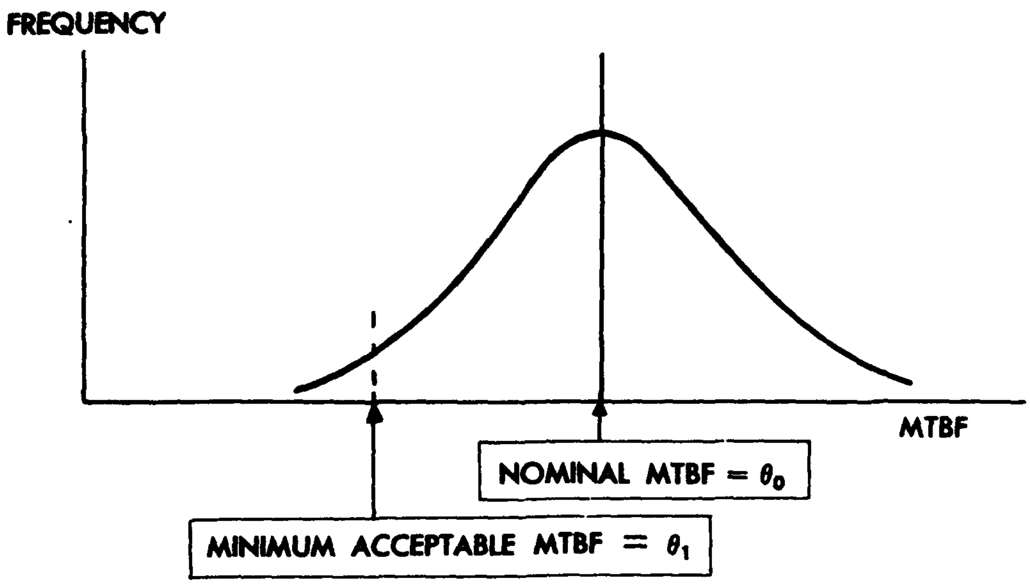


Figure 7-1. Distribution of MTBF's Centered at a Nominal Value θ_0, Showing a Minimum Acceptable Value at θ_1 as a Lower Tolerance Limit on Acceptable MTBF

Where reliability has been expressed as a failure rate, the requirement is easily translated into MTBF in the exponential case by taking the reciprocal of failure rate; i.e.:

$$MTBF = \frac{1}{\text{Failure Rate}}$$

EXAMPLE: An equipment has been designed to meet a specified nominal MTBF of 200 hours that is based on a TDP definition of nominal (200 hours) and minimum acceptable (100 hours). Design predictions followed by development (evaluation) tests have verified design achievement of the 200-hour requirement under the specified test conditions. The basic design is thus qualified for prototype development. An acceptance test is now to be designed as a means of deciding whether to accept the prototype for production or to reject the prototype and require that the design be further refined.

On the basis of development test data, a hypothesis can be formulated concerning the probable MTBF of the prototype: it is hypothesized that MTBF = θ_0 = 200 hours. This hypothesis is termed the "Null" hypothesis, shown as

H_0: $\theta_0 \geq 200$ hours

An alternative hypothesis is stated on the basis of the specified minimum acceptable MTBF (100 hours) as follows:

H_1: $\theta_1 \leq 100$ hours

STEP 2 – Define the Allowable "Risks".

Two risks are involved, at least one of which – "the consumer's risk" – will have been stated in the specification description of the reliability requirement:

- The "producer's risk", denoted by α (alpha), is the chance or risk of rejecting a product that in actuality is acceptable. This is the risk taken by the development contractor or equipment manufacturer when he submits his product to the acceptance test. Most tests are designed for a 5% or 10% producer's risk.

- The "consumer's risk", denoted by β (beta), is the chance or risk of accepting a product that is in actuality below the minimum acceptable level. This is the risk taken by the Bureau when it determines product acceptability on the basis of an acceptance test. Most tests are designed for a 10% or 20% consumer's risk.

In the preceding example, assume that the minimum acceptable level of reliability was defined for a consumer's risk of β = 10% (i.e., the Bureau wants 90% confidence (1 - β) that the product has an MTBF of at least 100 hours). Assume also that the development contractor had agreed to a producer's risk of α = 10% (i.e., the producer wants 90% confidence that the prototype will be accepted by the test if its MTBF is in fact 200 hours). Thus,

$$\alpha = 10\%$$
$$\beta = 10\%$$

STEP 3 – Determine Accept/Reject Boundaries.

With θ_0 and θ_1 both defined, and the two risks α and β established, all parameters needed for the choice or design of the sequential test are known. Two methods of test design are available to the project engineer (or to the contractor, if the test design requirement has been given him as a task):

- **Handbook Method** -- This requires the use of OASD Handbook H108 or a comparable document in which sequential test plans are already available for different combinations of α, β, θ_1, and θ_0.
- **Mathematical Method** -- This requires the derivation of the test plan from the basic equations.

The mathematical method will be illustrated here, to acquaint the engineer with the formulae that underlie the plans presented in handbooks. Figure 7-2 is a graphic representation of a sequential sampling plan, showing the equations for lines of acceptance and rejection.

Accept Line: $T(t) = h_0 + rs$

Reject Line: $T(t) = -h_1 + rs$

where

$$h_0 = \frac{Ln\left(\frac{1-\alpha}{\beta}\right)}{\frac{1}{\theta_1} - \frac{1}{\theta_0}}$$

$$h_1 = \frac{Ln\left(\frac{1-\beta}{\alpha}\right)}{\frac{1}{\theta_1} - \frac{1}{\theta_0}}$$

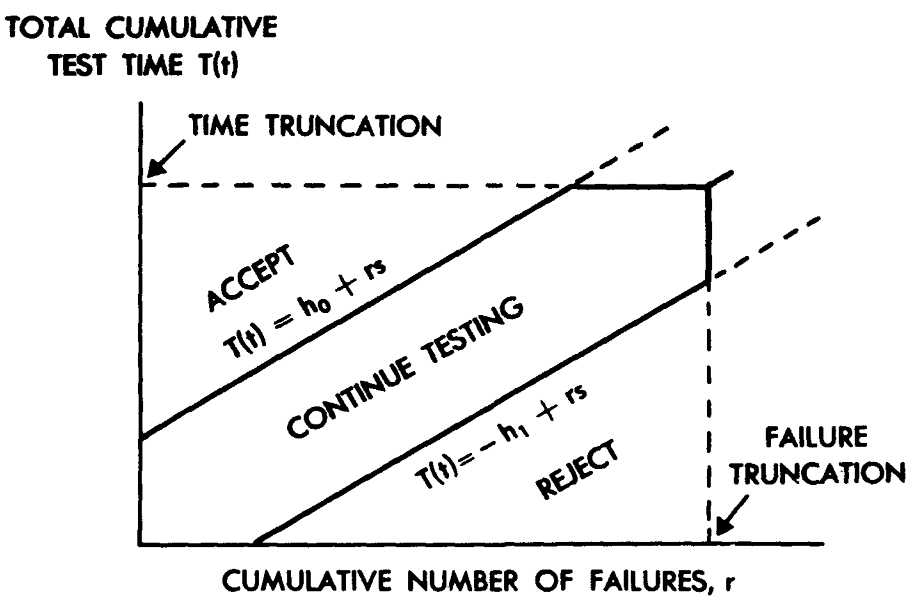


Figure 7-2. Graphic Representation of Sequential Test Plan

$$s = \frac{Ln\left(\frac{\theta_0}{\theta_1}\right)}{\frac{1}{\theta_1} - \frac{1}{\theta_0}}$$

Ln = Logarithm to base e

r = Number of failures observed by time t

and T(t) = Total number of hours accumulated by all items up to time t

$= \sum_{i=1}^{r} t_i + (n - r)t$, in the non-replacement case

In the replacement case, T(t) = nt, where n is the number of items initially placed on test. The replacement type of test is usually employed in equipment and system testing; i.e., each equipment that fails is either replaced in the test or is repaired and reinstalled in the test.

EXAMPLE: In the preceding example, $\alpha = \beta = .10$, $\theta_0 = 200$ hours and $\theta_1 = 100$ hours. Accept/reject equations would be derived as follows:

$$h_0 = \frac{Ln\left(\frac{1-.1}{.1}\right)}{\frac{1}{100} - \frac{1}{200}} = \frac{Ln\ 9}{.01 - .005}$$

$$= \frac{2.2}{.005} = 440$$

$$h_1 = \frac{Ln\left(\frac{1-.1}{.1}\right)}{\frac{1}{100} - \frac{1}{200}} = \frac{2.2}{.005} = 440$$

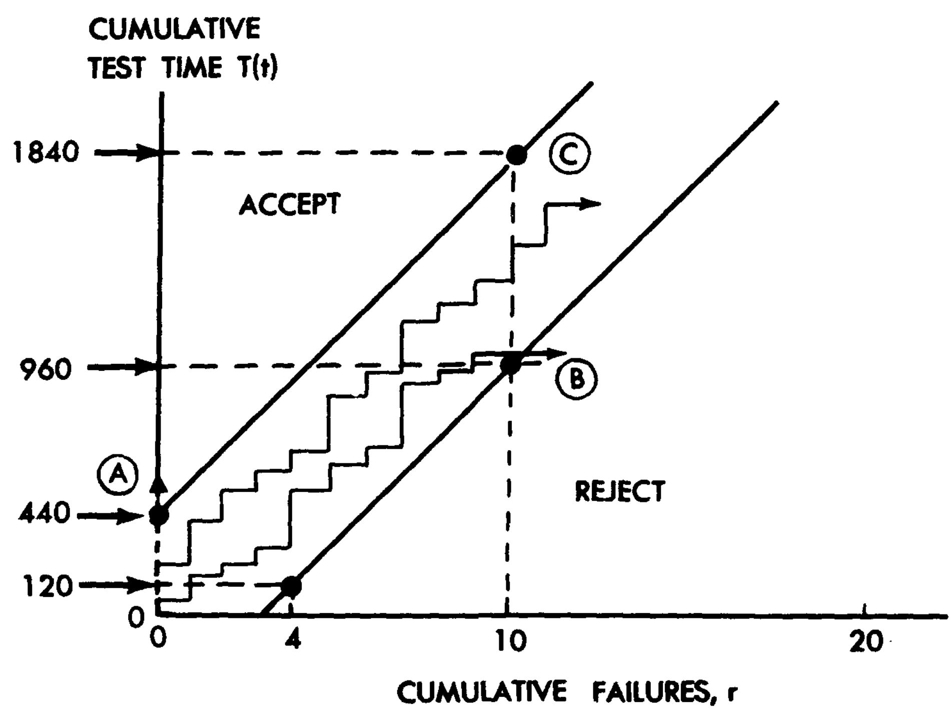


Figure 7-3. Sequential Test Plan for θ_0 = 200 hours; θ_1 = 100 hours; $\alpha = \beta$ = 10%

$$s = \frac{Ln\left(\frac{200}{100}\right)}{\frac{1}{100} - \frac{1}{200}} = \frac{Ln\ 2}{.005} = \frac{.693}{.005}$$

$$= 138.6 \approx 140$$

The accept line then is defined by

$$T(t) = T^*_A = 440 + 140r$$

The reject line is defined by

$$T(t) = T^*_R = -440 + 140r$$

These two lines are plotted in Figure 7-3 for several values of T(t) and r, as shown in Figure 7-4.

To illustrate the use of the test plan, three possible outcomes are plotted in Figure 7-3:

Case A: Equipment is accepted because the accept decision line is crossed after 440 hours of test time, before the first failure occurs.

Case B: Equipment is rejected because the reject decision line is crossed at 960 hours with the 10th failure.

Case C: Equipment on test without either decision boundary being crossed; the 10th failure after 960 hours, but before 1840 hours. (Truncation methods for this case are discussed in 7-4.)

The test plan derived above could have been approximated from the Master Table of Sequential Life Tests (Table 2D-1) of H108. For example, plan C-11 of H108, having $\alpha = \beta = .1$ and $\theta_1/\theta_0 = .512$, most nearly fits the criteria derived above.

Number of Failures (r)	Minimum Time to Accept (T^*_A)	Maximum Time to Reject (T^*_R)
0	440	--
1	580	--
2	720	--
3	860	--
4	1000	120
5	1140	260
6	1280	400
7	1420	540
8	1560	680
9	1700	820
10	1840	960
.	.	.
.	.	.
15	2540	1560
.	.	.
.	.	.
20	3240	2360

Figure 7-4. Accept/Reject Numbers (Failures) as a Function of Total Test Time T(t) for a Replacement Test

An excerpt from Table 2D-1 of H108 is shown below:

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Code	r_0	h_0/θ_0	h_1/θ_0	s/θ_0	$E_0(r)$	$E\theta_1(r)$	$E_s(r)$	$E\theta_0(r)$
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
C-11	45	2.3053	-2.3053	.7024	3.3	9.7	10.8	6.2
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.

Sequential Life Test Plan for $\theta_1/\theta_0 = .512$, $\alpha = \beta = .10$

h_0/θ_0, h_1/θ_0, and s/θ_0 are normalized constants for the accept/reject equations. To determine the equation, simply multiply these constants by $\theta_0 = 200$. The following equations result:

Accept Line: $T(t) = 461 + 140r$

Reject Line: $T(t) = -461 + 140r$

STEP 4 – Develop the OC Curve for the Sequential Test Plan.

The operating characteristic (OC) curve, denoted by $L(\theta)$, for the sequential plan is given approximately by

$$L(\theta) = \frac{A^h - 1}{A^h - B^h}$$

where

$$A = \frac{(1 - \beta)}{\alpha}$$

$$B = \frac{\beta}{(1 - \alpha)}$$

and

$$\theta = \frac{\left(\frac{\theta_0}{\theta_1}\right)^h - 1}{h\left(\frac{1}{\theta_1} - \frac{1}{\theta_0}\right)}$$

The curve is determined by assigning values to h and solving for θ and $L(\theta)$. Five points on the OC curve are shown in Figure 7-5. From these points it is possible to make a rough sketch of the OC curve and to determine what further points are needed for more accurate detail.

h	θ	$L(\theta)$
$-\infty$	0	0
-1	θ_1	β
0	s	$h_1/h_0 + h_1$
1	θ_0	$1 - \alpha$
∞	∞	1

Figure 7-5. Five Points on the OC Curve

Graphically, the OC curve will take roughly the shape shown in Figure 7-6.

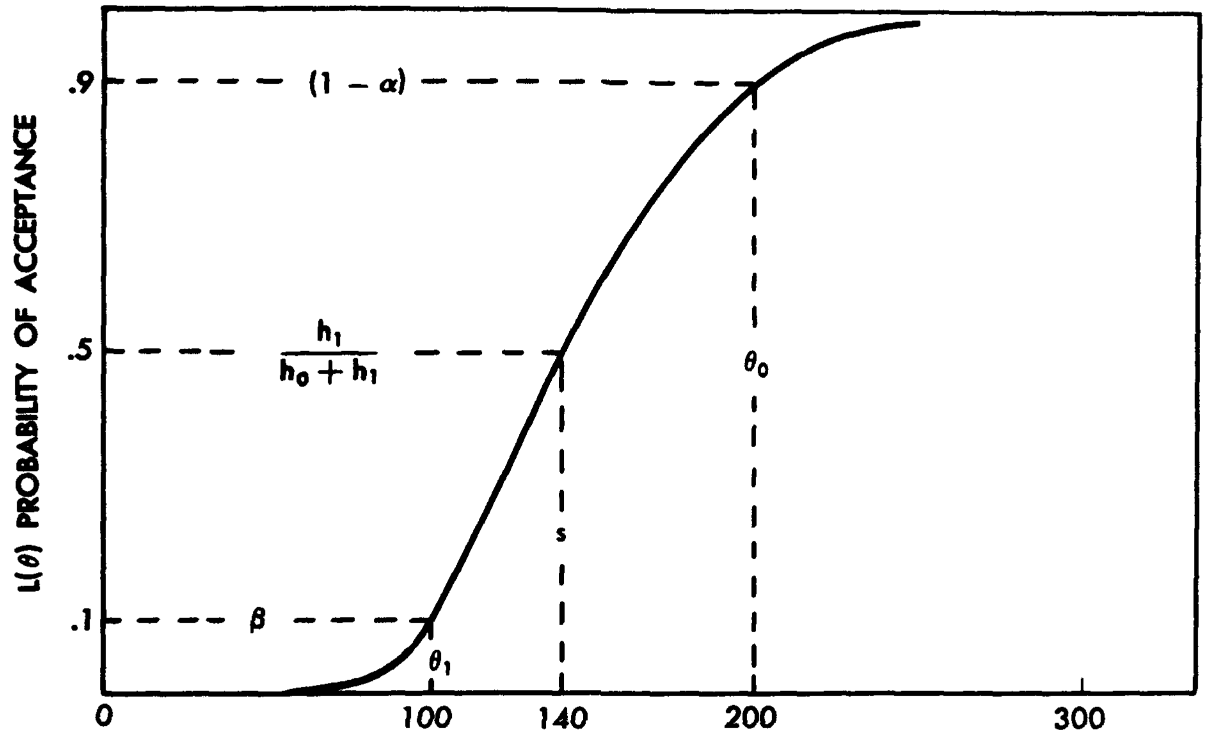


Figure 7-6. OC Curve for Sequential Sampling Plan

STEP 5 – Estimate Expected Length of Test Required for Decision

The average length of test (test operate hours/unit) to reach a decision is given by:

$$E_\theta(t) = \frac{\theta}{n} E_\theta(r)$$

in the replacement case, and

$$E_\theta(t) = \theta \, \text{Ln} \frac{n}{n - E_\theta(r)}$$

in the non-replacement case, where n is the number of units on test and θ is the actual (or true) MTBF.

$E_\theta(r)$, the expected number of failures required to reach a decision, is given by:

$$E_\theta(r) = \frac{h_1 - L(\theta)(h_0 + h_1)}{s - \theta}$$

for any finite value of the actual (or true) MTBF (θ) except when $\theta = s$, in which case:

$$E_s(r) = \frac{h_0 h_1}{s^2}$$

The curve of $E_\theta(t)$ versus θ (average length of test curve) is determined by choosing values of θ (together with corresponding values from the test plan and OC curve), the number of units which will be tested at any one time, and whether a replacement or non-replacement test procedure will be used.

Five points on the average length of test curve are tabulated in Figure 7-7 for the example test design which is implemented by testing ten units at a time in the replacement case.

From the points in Figure 7-7, a sketch of the average length of test curve may be made as shown in Figure 7-8.

A sketch such as that shown in Figure 7-8 may often be sufficient for test estimating purposes.[2] However, additional points of particular interest may be calculated.

[2] Requests for proposals should instruct bidders to base cost estimates on the expected length of test when the actual *MTBF equals a "nominal"* requirement.

θ	$E_\theta(r)$	$E_\theta(t)$ 10 Units (Replacement Case)
0	3.2	0 (a minimum)
θ_1 (100)	9.1	91
s (138.6)	10.1	140 (maximum)
θ_2 (200)	5.7	114
L	0*	44*(a minimum)

* Determined by engineering inference

Figure 7-7. Five Points on the $E_\theta(t)$ Versus θ Curve

While the curve of Figure 7-8 presents the average length of test to reach a decision for a given MTBF, the actual test length in any one test may be either significantly lower or up to three times the average test length. Furthermore, the decision whether to accept or reject depends on the OC curve and not on the length of test.

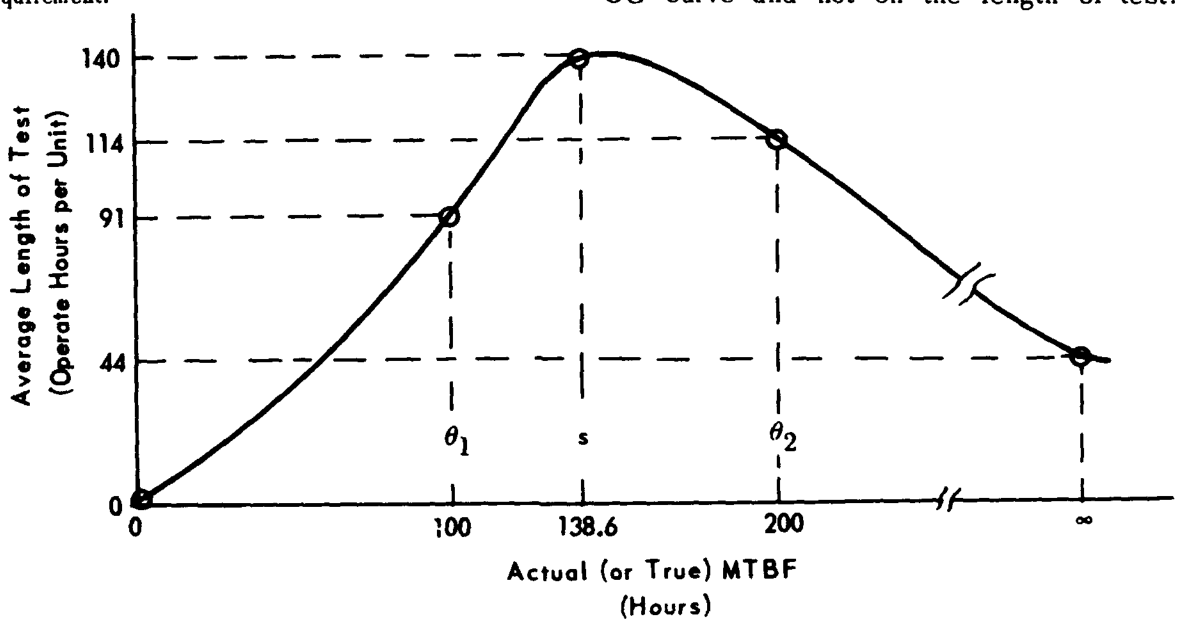


Figure 7-8. Average Length of Test Curve for Sequential Sampling Plan

7-3 SEQUENTIAL TEST DESIGN FOR RELIABILITY ACCEPTANCE

7-3-1. General

The procedures for MTBF acceptance outlined in the previous section are applicable when the equipment under test is known to follow the exponential law. However, in cases where the assumption of exponentiality is not valid,[3/] it is necessary to express the reliability requirement as a probability of survival for the prescribed mission time. A sequential test can then be designed to accept or reject the product on the basis of its "unreliability" or "proportion defective" during a prescribed mission time.

7-3-2. Procedural Steps

The same procedure is applicable to one-shot devices and cycle-dependent equipments that are not directly dependent on time. System availability or operational readiness can also be evaluated for acceptability by sequential test methods. Application of sequential test design procedures to these latter cases is also illustrated.

STEP 1 – Define "Acceptable" and "Unacceptable" Proportion Defective (p)

The nominal proportion defective, designated as p_0, is defined as $(1 - R_0)$, where R_0 is the design requirement for acceptable reliability throughout the specified mission period, i.e., $R_0 = R(t_m)$nom. (In the exponential case, θ_0 was used instead of R_0.) The unacceptable proportion defective, designated as p_1, is defined as $(1 - R_1)$, where R_1 corresponds to the *minimum* acceptable reliability originally defined in the design specification for the specified mission period. (In the exponential case, θ_1 was calculated from R_1.)

EXAMPLE: An equipment has been designed to have a nominal reliability of .97 for a 6-hour mission. A minimum acceptable reliability of .94 has been specified for the same period. Redundancy has been used in design; therefore, the assumption of exponentiality does not hold. Design predictions and development tests indicate that basic design is qualified for prototype development. An acceptance test is now to be designed as a means of determining whether to accept the prototype for production or to reject it and require further design refinement. On the basis of development test data, a hypothesis may be formulated concerning the probable proportion defective of the prototype, i.e., it is hypothesized that p = .03, or (1 - .97). This hypothesis is termed the "Null" hypothesis and is shown symbolically as:

$$H_0: \quad p_0 \leq .03 \text{ at } t = t_m$$

An alternative hypothesis that p = .06, or (1 - .94), is formulated from the specified minimum acceptable reliability of .94, as follows:

$$H_1: \quad p_1 \geq .06 \text{ at } t = t_m$$

[3/] Equipment designs that employ redundancy at the part or module level usually void the exponential assumption, as do those equipments which are judged on an attributes basis, i.e., number of successes in a given number of trials and tests.

STEP 2 – Define the Allowable "Risks"

The two risks involved have meanings equivalent to those discussed for MTBF acceptance tests. The "producer's risk", α, is the contractor's chance or risk that a product with an acceptable proportion defective p_0 will be rejected. The "consumer's risk", β, is the Bureau's chance or risk of accepting a product proportion defective which is worse than the minimum acceptable level, p_1.

To illustrate the test design, assume that the minimum acceptable level of reliability, represented by p_1, was defined for a consumer's risk of $\beta = 10\%$ – i.e., the Bureau wants 90% confidence $(1 - \beta)$ that the product has a proportion defective (unreliability) of not more than .06. Assume also that the development contractor had agreed to a producer's risk of $\alpha = 10\%$ – i.e., the producer wants 90% confidence that the prototype will be accepted by the test if its proportion defective is in fact .03 or less. Thus $\alpha = \beta = 10\%$.

STEP 3 – Determine Accept/Reject Decision Boundaries.

With p_0 and p_1 both defined and the two risks α and β established, all the

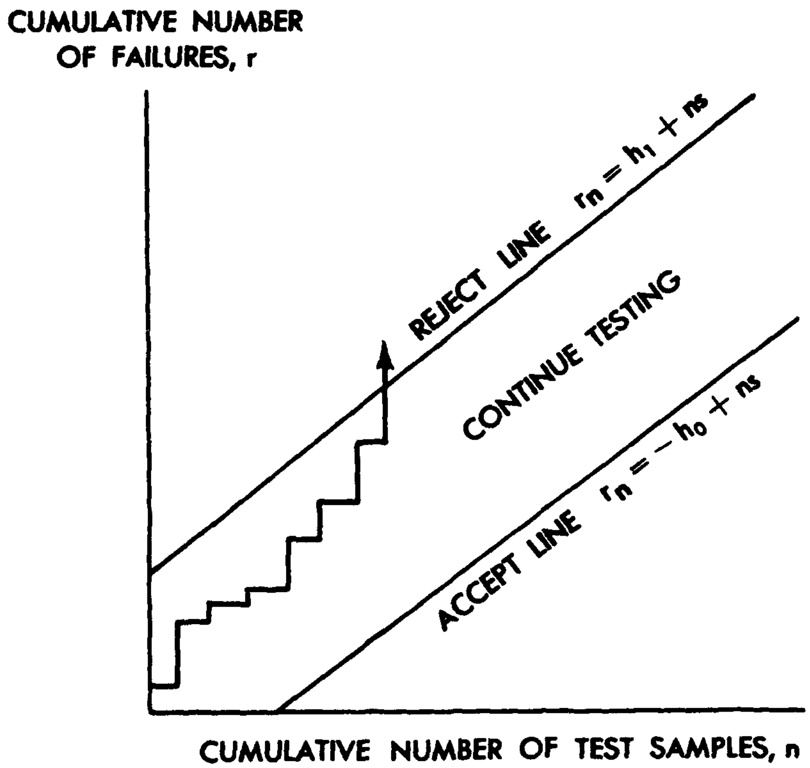


Figure 7-9. Graphic Representation of Sequential Test Plan for Proportion Defective

necessary parameters for the choice or design of the sequential test are known. Until such time as a handbook of test plans is published, the project engineer or contractor must derive the plan from the basic equations.

Figure 7-9 is a graphic representation of a sequential sampling plan showing the equations for lines of acceptance and rejection.

The two equations expressing r as a function of n are:

Accept Line $r_n = -h_0 + ns$

Reject Line $r_n = h_1 + ns$

where

$$h_0 = \frac{Ln\left(\frac{1-\alpha}{\beta}\right)}{Ln\left[\frac{p_1(1-p_0)}{p_0(1-p_1)}\right]}$$

$$h_1 = \frac{Ln\left(\frac{1-\beta}{\alpha}\right)}{Ln\left[\frac{p_1(1-p_0)}{p_0(1-p_1)}\right]}$$

$$s = \frac{Ln\left[\frac{(1-p_0)}{(1-p_1)}\right]}{Ln\left[\frac{p_1(1-p_0)}{p_0(1-p_1)}\right]}$$

n = Sample size (number of events or tests attempted) when r failures are observed.

r_n = Total number of failures observed in sample size n

It is frequently desirable to express the accept/reject criteria as the number of tests, n, required for decision for a given number of failures, or n as a function of r. In this event, the preceding line equations are easily solved for n as follows:

Accept Line $n_A = \frac{h_0}{s} + \frac{r_n}{s}$

Reject Line $n_R = \frac{-h_1}{s} + \frac{r_n}{s}$

EXAMPLE: Reliability Acceptance Test. A sequential test for MTBF acceptance, designed for $\theta_0 = 200$ hours, $\theta_1 = 100$ hours, and $\alpha = \beta = .10$, is based on an assumption of random equipment failure, which is often experienced by equipments of mature conventional design. Let us assume that θ_0 and θ_1 were calculated from SOR reliability requirements, which called for values of .97 (nominal) and .94 (minimum acceptable), respectively, for a 6-hour mission, as represented by Figure 7-10.

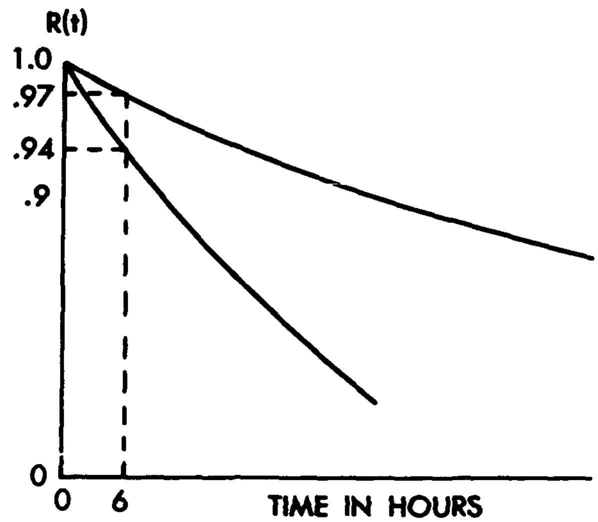


Figure 7-10. Reliability Function

Assume further that a design feasibility study has revealed the reliability requirement cannot be met by conventional design. As a result, a new design incorporating redundancy has been established, and it is now proposed that models of this new design be tested to determine that required reliability has been achieved. Since the rate of failure may no longer follow the random pattern because of the redundancy employed, the MTBF test previously designed may not be suitable. Therefore, with reference to the equations of Step 3, the accept/reject equations for the non-exponential situation would be derived as follows:

$$h_0 = \frac{\mathrm{Ln}\left(\frac{1-.1}{.1}\right)}{\mathrm{Ln}\left[\frac{.06(1-.03)}{.03(1-.06)}\right]} = \frac{\mathrm{Ln}\ 9}{\mathrm{Ln}\ 2.06}$$

$$= \frac{2.20}{0.723} = 3.04$$

$$h_1 = \frac{\mathrm{Ln}\left(\frac{1-.1}{.1}\right)}{\mathrm{Ln}\left[\frac{.06(1-.03)}{.03(1-.06)}\right]} = \frac{\mathrm{Ln}\ 9}{\mathrm{Ln}\ 2.06}$$

$$= \frac{2.20}{0.723} = 3.04$$

$$s = \frac{\mathrm{Ln}\left[\frac{(1-.03)}{(1-.06)}\right]}{\mathrm{Ln}\left[\frac{.06(1-.03)}{.03(1-.06)}\right]} = \frac{\mathrm{Ln}\ 1.03}{\mathrm{Ln}\ 2.06}$$

$$= \frac{.0295}{0.723} = 0.0408$$

The accept line is defined by

$$r_n = -3.04 + .0408n$$

The reject line is defined by

$$r_n = 3.04 + .0408n$$

For comparison on a basis consistent with the MTBF test, the line equations are solved to express n as a function of r, and they result in the following accept/reject criteria:

Accept when $n \geq 74.5 + 24.5\ r_n$

Reject when $n \leq -74.5 + 24.5\ r_n$

These two lines are plotted in Figure 7-11, with several values of r_n and n substituted in the equations, as shown in the Figure 7-12.

To illustrate use of the test plan, four possible results of testing are plotted:

(1) Four failures occurred prior to completion of 23 tests, causing a reject decision.

(2) Completion of 75 tests occurred prior to the first failure, causing an accept decision.

(3) Ten failures occurred prior to completion of 170 tests, causing a reject decision.

(4) Completion of 320 tests occurred prior to the 11th failure, causing an accept decision.

Each "test" or "sample" in this example refers to an attempt to operate an equipment for a period of 6 hours (t_m) between inspection and repair or replacement, if needed, of the redundant item.

EXAMPLE: Availability Acceptance Test. For the equipment used in the previous example, let us assume that an availability requirement has been specified in addition to the reliability requirement (A_0) is .97, and that for

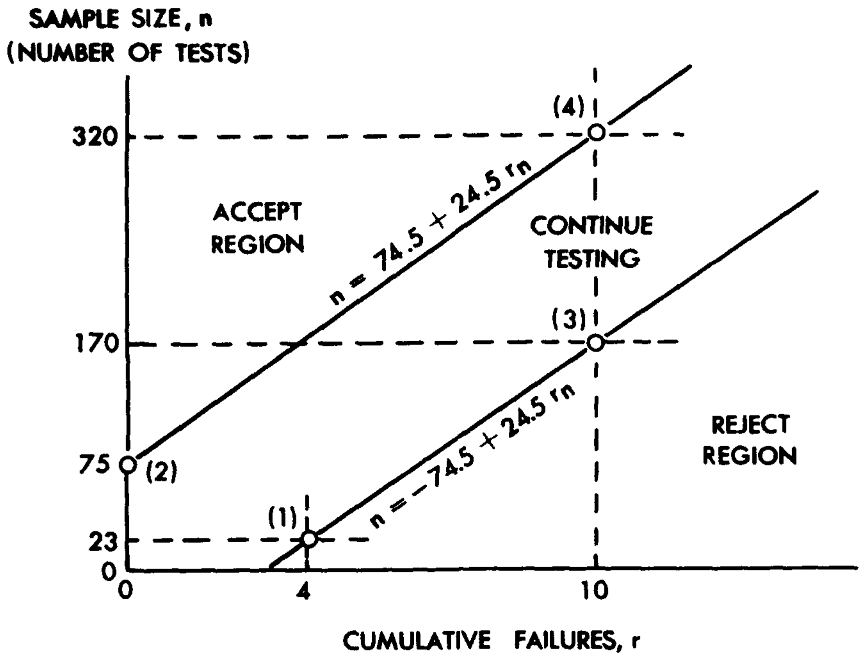


Figure 7-11. Sequential Test Plan (Non-Exponential) for p_0 = .97; p_1 = .94; $\alpha = \beta$ = 10%

Number of Failures (r)	Minimum Sample to Accept (n_A)	Maximum Sample to Reject (n_R)
0	75	N/A
1	99	N/A
2	124	N/A
3	148	N/A
4	173	23
5	197	48
6	222	72
7	246	97
8	271	121
9	295	146
10	320	170
11	344	195
12	369	219
13	393	244
14	418	268

Figure 7-12. Accept/Reject Numbers (Failures) as a Function of Number of Tests or Sample Size (n)

β = .1 the minimum acceptable availability requirement (A_1) is .94. (Although A_0 and A_1 often exceed R_0 and R_1', respectively, the same values are used here to facilitate using the test plan designed in the previous example.) Since availability is a measure of system "readiness" to operate (on demand) at the start of a mission, the availability test will assess whether the equipment is operational or can be made operational within a prescribed period of warning time. From a practical standpoint, the warning time is necessitated by normal equipment turn-on, warm-up, and operational checkout required to transfer a system from standby to "full-on" condition.

Assume here that it takes 15 minutes to get the system into operational condition. The availability "sample" test would then consist of one attempt to turn on, warm up, and check out the system within 15 minutes. The test plan of the previous example may be used directly for the availability test, except that the time period of interest is 15 minutes of operation instead of 6 hours as used for the reliability test. The failure criteria for availability tests are likely the same as those used for the reliability test. Test "samples" are selected at random points in time.

STEP 4 – Develop the OC Curve for the Sequential Reliability Test Plan

The operating characteristic (OC) curve for the sequential plan (i.e., the probability of accepting H_0 when p is the true proportion defective of the items being tested) is given approximately by:

$$L(p) = \frac{A^h - 1}{A^h - B^h}$$

where

$$A = \frac{1 - \beta}{\alpha}$$

$$B = \frac{\beta}{1 - \alpha}$$

and

$$p = \frac{1 - \left(\frac{1 - p_1}{1 - p_0}\right)^h}{\left(\frac{p_1}{p_0}\right)^h - \left(\frac{1 - p_1}{1 - p_0}\right)^h}$$

The OC curve is determined by assigning values to h and solving for L(p) and p. Five points on the OC curve are shown in Figure 7-13. From these points, a rough sketch of the OC curve (Figure 7-14) can be made to determine whether additional points are needed for the desired accuracy.

n	p	L(p)
$-\infty$	1	0
-1	p_1	β
0	s	$h_1/(h_0+h_1)$
1	p_0	$1 - \alpha$
∞	0	1

Figure 7-13. Five Points on the OC Curve

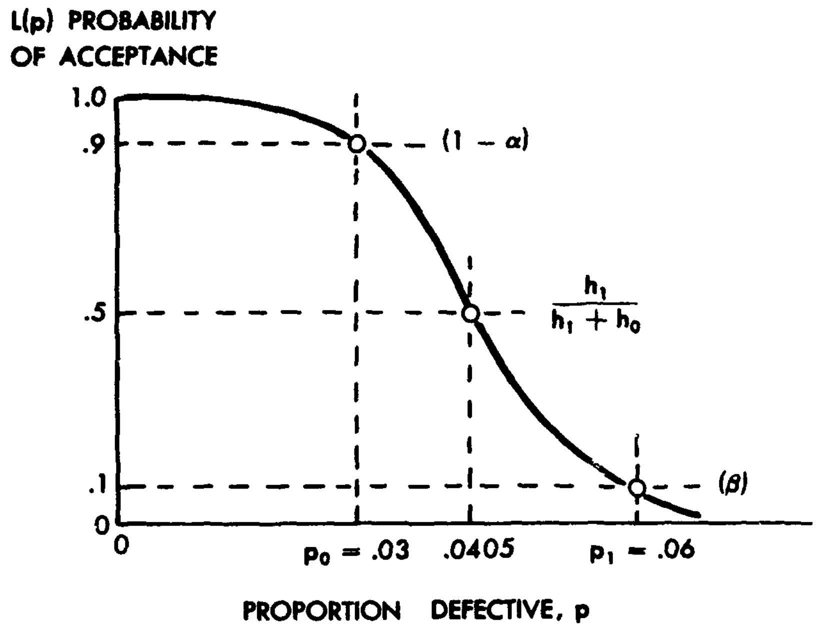


Figure 7-14. Sketch of the OC Curve

STEP 5 – Estimate Expected Number of Tests Required for Decision

With a sequential reliability test plan the expected (average) number of tests required to reach a decision is a function of the actual (or true) proportion defective (p) of the items tested and may be calculated approximately as follows:

$$E_p(r) = \frac{L(p)\ \mathrm{Ln}\ B + (1 - L(p))\ \mathrm{Ln}\ A}{p\left[\mathrm{Ln}\left(\frac{p_0}{p_0}\right)\right] + (1 - p)\left[\mathrm{Ln}\left(\frac{1 - p_1}{1 - p_0}\right)\right]}$$

where

$$A = \frac{(1 - \beta)}{\alpha}$$

and

$$B = \frac{\beta}{(1 - \alpha)}$$

The curve of $E_p(r)$ versus p is determined by first choosing values of proportion defective (p) together with the corresponding values from the OC curve and test design, and solving for $E_p(r)$. Five values of $E_p(r)$ may be calculated more easily than other points and may be used to sketch the curve. Figure 7-15 presents these values for the example test design.

p	$E_p(r)$
$= 1$	$E_1(r) = \frac{h_1}{(1 - s)} \approx 4$ (a minimum)
$= p_1 = .06$	$E_{p_1}(r) = \frac{(1 - \beta)h_1 - \beta h_0}{p_1 - s} = 127$
$= s = .0405$	$E_s(r) = \frac{h_0 h_1}{s(1 - s)} = 236$ (maximum)
$= p_0 = .03$	$E_{p_0}(r) = \frac{(1 - \alpha)h_0 - \alpha h_1}{s - p_0} = 225$
$= 0$	$E_0(r) = \frac{h_0}{s} = 75$ (a minimum)

Figure 7-15. Five Points on the $E_p(r)$ Curve

From the points in Figure 7-15, a sketch of the curve may be made as shown in Figure 7-16. Such a sketch may often be sufficient for test estimating purposes.[4] However, additional points of particular interest may be calculated.

While the curve of Figure 7-16 presents the average number of tests to reach a decision for a given proportion defective, the actual number of tests in any one test situation may be either significantly lower or up to three times the average number of tests. Furthermore, the decision whether to accept or reject depends on the OC curve and not on the number of tests.

[4] Requests for proposals should instruct bidders to base cost estimates on the expected number of tests when the actual proportion defective corresponds to the "nominal" reliability requirement.

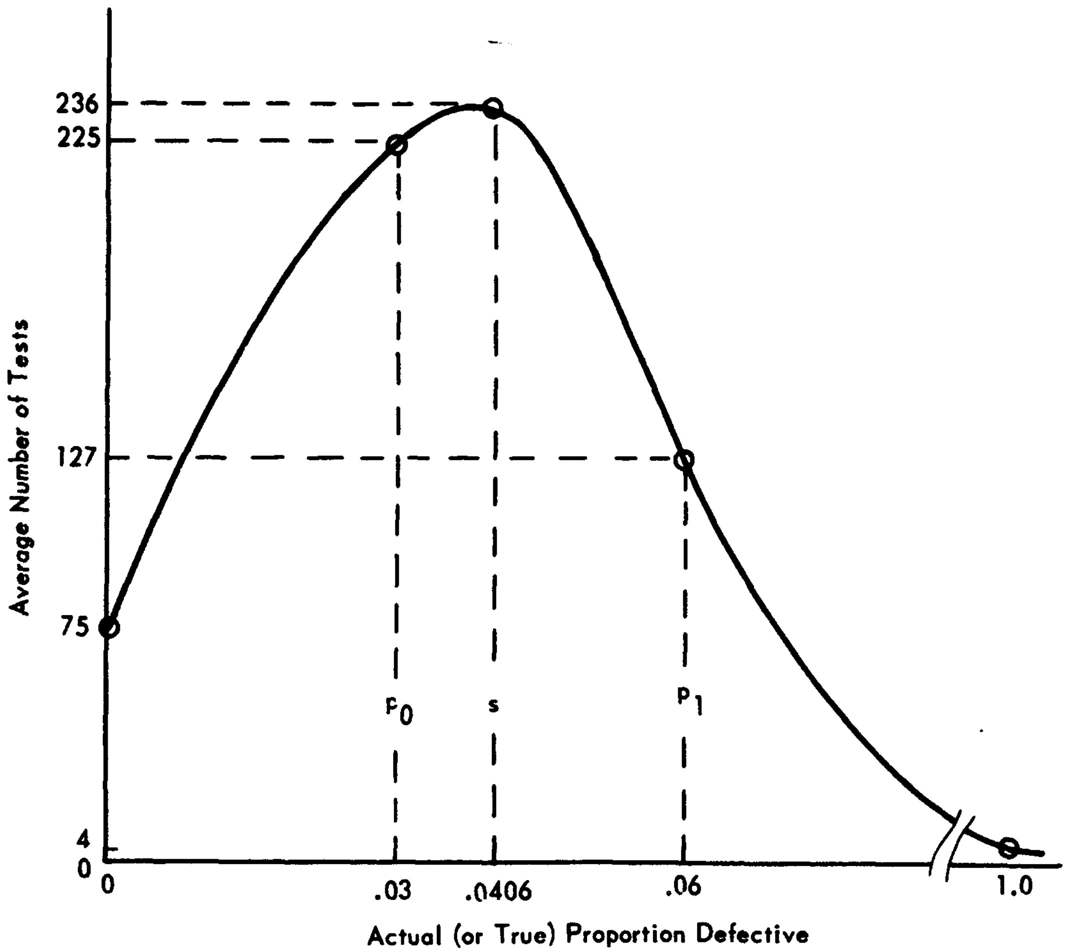


Figure 7-16. Curve of $E_p(r)$ Versus p for Sequential Sampling Plan

7-4 TRUNCATION OF ACCEPTANCE TEST PLANS

7-4-1. General

When a sequential test plan is used, there are rare occasions in which, because of marginally acceptable MTBF, reliability, or availability, a decision boundary may not be crossed even after continuous testing. In order to avoid this contingency, decision rules to truncate the test should be established in advance of the test. Truncation, or termination of the test prior to crossing of a decision boundary, usually occurs when either a maximum number of failure, a maximum number of tests or hours of test, or a combination of both of these is reached. The α and β risks at truncation are affected by where the truncation occurs. In general, early truncation results in a substantial increase in either α or β or both, and truncation after a substantial amount of testing causes an increase in risk that is of no practical engineering significance.

7-4-2. Procedural Steps

A truncation method based on a maximum number of failures is shown below for a *single* sampling plan. It is believed that this method of truncation often will be practical for sequential acceptance test plans. However, if the test program will allow additional testing to avoid significant increase in risk at truncation, the truncation may be established at three times the number of failures required for truncation of the single sampling plan (the latter method of truncation is used for H108 sequential plans).

The method for truncating to be described requires inputs of values for α, β, and θ_1/θ_0 or p_0/p_1, as applicable, and makes use of a "Thorndike Chart", which is presented in Chart 2-II of Appendix 2. To illustrate the method, a test plan having $\alpha = \beta = .10$ and θ_1/θ_0 or $p_0/p_1 = .20$ will be truncated.

STEP 1 – Define "probability of r or fewer failures" ordinates for Chart 2-II corresponding to $(1 - \alpha)$ and β. For this test, the ordinates will be .9 and .1, respectively.

STEP 2 – Determine from Chart 2-II abscissa values of np corresponding to the ordinates of Step 1 and values of r (number of failures). The values of r to be used are determined first by starting with r = 0 and then by using for guidance the results of Step 3.

STEP 3 – Calculate the ratio of $np_{.9}/np_{.1}$ for each value of r used in Step 2 until you find the smallest r value that will give a ratio of $np_{.9}/np_{.1}$ that is greater than θ_1/θ_0 or p_0/p_1 for your test plan. The tabulation of Step 4 indicates the desired value of r is 2 since r=2 yields the ratio $np_{.9}/np_{.1}=.209$ which exceeds our θ_1/θ_0 value of .2 and no smaller value of r yields a ratio greater than .2.

STEP 4 – Tabulate the results as follows:

r	$np_{(1-\alpha)} = np_{.9}$	$np_\beta = np_{.1}$	$np_{.9}/np_{.1}$
0	.11	2.30	.048
1	.53	3.89	.136
2	1.11	5.32	.209
3	1.75	6.68	.262

STEP 5 – Truncate the test at the number of failures (r_0) that is equal to one plus the number of failures identified in Step 4 -- in this case, at three failures.

STEP 6 – Truncate the test at the number of hours or number of tests that corresponds to r_0 and is determined by the slope of the selected test plan.

For the MTBF test,

$$T = r_0 s$$

For the reliability test,

$$T = \frac{r_0}{s}$$

For test plans commonly considered, the number of failures required for truncation is given for various discrimination ratios and risk values in Figure 7-17.

θ_1/θ_0 or p_0/p_1	$\alpha = .05$ $\beta = .05$	$\alpha = .05$ $\beta = .10$	$\alpha = .10$ $\beta = .05$	$\alpha = .10$ $\beta = .10$	$\alpha = .20$ $\beta = .20$
1/10	3	3	2	2	1
1/5	5	4	4	3	2
1/3	10	8	8	6	3
1/2	23	19	18	15	7
2/3	67	55	52	41	18

Figure 7-17. Failures Required for Truncation, Based on Single Sampling Plan

Truncation at other ration of θ_1/θ_0 between 1/10 and 2/3 for the risk pairs in the table shown in Figure 7-17 may be determined with practical accuracy by graphical interpolation.

For truncation beyond the range of the table and Chart 2-II, consult tables of the "Summation of Terms of Poisson's Exponential Binomial Limit".

7-5 COMPARISON OF MTBF (EXPONENTIAL) AND PROPORTION UNRELIABLE (NON-EXPONENTIAL) SEQUENTIAL ACCEPTANCE TEST PLANS

The sequential test plans used as examples in this section may be used to provide a limited comparison of the exponential and non-exponential tests.

In the MTBF test, total test time is plotted on the ordinate. In order to compare the MTBF test to the reliability test, which has "number of 6-hour tests" as an ordinate, simply divide the total test time of the MTBF test by six, the length of test in the non-exponential test plan. When the plans are compared for a given number of failures, the relative test length may then be seen.

For the example plans of this chapter, three points are tabulated in Figure 7-18 to illustrate the comparison.

In general, decisions in the MTBF test can be made in a shorter time than the time required for decision in the non-exponential test.

Number of Failures	EXPONENTIAL		NON-EXPONENTIAL	
	TA/6	TR/6	Minimum Sample to Accept	Maximum Sample to Reject
0	73	--	75	--
4	167	20	173	23
10	306	160	320	170

Figure 7-18. Comparison of Exponential and Non-Exponential Sequential Tests for K = ½; $\alpha = \beta = .1$

CHAPTER 8

RELIABILITY EVALUATION, FAILURE ANALYSIS, AND CORRECTION – THE "FEEDBACK" LOOP

8-1 INTRODUCTION

8-1-1. General

Successful or satisfactory operation – the goal of all design efforts – yields little information on which to base improvements. Failures, on the other hand, contribute a wealth of data of "what to improve" or "what to design against" in subsequent efforts. The *feedback* of information obtained from the analysis of failures is one of the principal stepping stones of progress.

Failure data are recorded, reported, and controlled in many ways – from the sophisticated "controlled surveillance" approach where personnel are specifically assigned to record all occurrences of failure accurately and in detail, to the "uncontrolled" approach where maintenance personnel are relied upon to record failure events on standard forms and to forward the forms to central collection agencies on a routine basis.

8-1-2. Data Forms

Data forms currently employed for routine failure reporting by Fleet personnel are:

- DD 787, "Electronic Failure Report". This report is the forerunner of most failure reporting systems. It is currently being replaced by several of the newer forms listed below.
- NAVORD 2214, "Ordnance Equipment Casualty Report". This form is used to report failures on non-electronic naval ordnance equipments. It is being replaced by NAVWEPS Form 8000/13.
- NAVWEPS Form 8000/13, "Weapon Systems Component Failure Report". This form supersedes both DD 787 and NAVORD 2214 as the basic reporting form for ordnance equipment, both electronic and mechanical. It requires recording of maintenance as well as failure data.
- NAVAER-3067 (FUR), "Failure, Unsatisfactory or Removal Report". This is a failure reporting form in wide use for airborne equipment.
- 4ND-NATSF-13070/6, "Electronic Equipment Failure, Removal, Repair Report". This form is being introduced with new avionics equipments. It supersedes DD 787 and NAVAER-3067 for this category of equipment.
- BuShips 10551-1, "Electronic Equipment Failure/Replacement Report". This form replaces DD 787 for equipment under cognizance of the Bureau of Ships.

Other data forms in use by specific field activities and contractors differ, in general, only in format, entry coding, and degree of detail recorded. The following entries may be considered standard on all forms.

- Parts repaired or replaced – by name and reference designator.
- Technician's analysis of part failure and cause of the trouble.
- Date of report or trouble, and report number.
- Identification of higher level assemblies and equipments.
- Effect of failure on performance.
- Status of equipment and type of maintenance action when failure was discovered and repaired.
- Time-meter readings of the higher level assemblies.
- Maintenance man-hours and calendar time to repair trouble.
- Space for remarks.
- Individual and activity filing report.

These basic data will permit the isolation, identification, and ranking of reliability (and maintainability) problem areas without requiring detail, accuracy, or coverage in excess of that presently attained by the routine or "uncontrolled" data systems in common use.

8-1-3. The Feedback Loop

A comprehensive failure analysis and corrective action feedback loop must determine:

What failed.
How it failed.
Why it failed.

Failure data provide information to determine the first two factors. The third, essential to corrective action, usually requires information which can be obtained only by laboratory study of the problem areas uncovered by failure analysis.

This chapter of the handbook is devoted primarily to the analysis of failure data. Although emphasis is placed on reliability, the procedures are applicable to the analysis of maintainability problems and estimates. An example of the detailed laboratory analysis of "why it failed" is presented in Paragraph 8-5, using the test techniques presented in Chapter 6 of the handbook.

Maximum utilization of a failure reporting system occurs only when the results are analyzed and disseminated in a form which will have wide application to new system designs. Several methods and data sources have been established to facilitate the exchange and interchange of failure experience within the military services and industry. Paragraph 8-6 summarizes those sources which are considered most useful to the Navy and its contractors.

8-2 ANALYSIS OF FAILURE DATA

Of the many questions which may be asked of a failure reporting system, the most useful and most readily answered is:

> What, within an equipment, contributes most to its unreliability?

The following paragraphs present a step-by-step method of analyzing present failure reports, whether originating in the Fleet, at a test facility, or at a contractor's plant, in order to answer the above question.

STEP 1 – Organize the Data.

Arrange the data first by identifiable units or subassemblies within the subject equipment, then by circuit or part reference designation within each unit or module, and finally by cause of failure within each part reference designation. (This step is easily accomplished by machine sorting of data transcribed to punch card or tape data systems.)

Four example equipments are used extensively throughout this section to illustrate the step-by-step procedures for

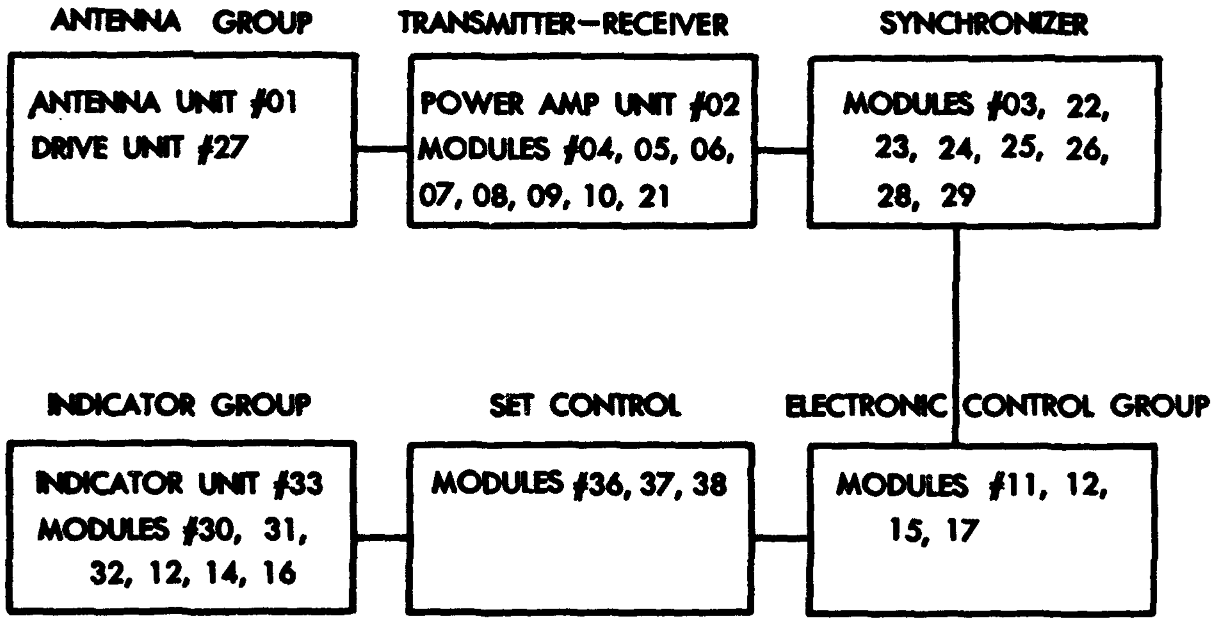


Figure 8-1. Example System Block Diagram

data analysis. Failure data are recorded on equipments of a new type during service evaluation (repeated flights in test aircraft to demonstrate performance capability). The equipments, packaged into modular units, are individually identified by module numbers running consecutively from 1 through 38. The final configuration is represented in Figure 8-1.

The failure data, ordered by module, appeared as follows:

Equipment	Equipment Serial No.	Unit or Module	Part Ref. Designation		Part Type
			Symbol	Location	
AN/XN-000	-	01	-	-	–
	1	02	V	201	Tube
	2	02	V	202	Tube
	2	02	V	202	Tube
	3	02	V	202	Tube
	1	03			Tolerance
	4	03			Adjust
	2	03	C	304	Capacitor
	3	03	CR	313	Diode
	1	03	Q	302	Transistor
	1	03			

STEP 2 – Plot Failure Data by Unit Versus Number of Failures per Unit.

On the basis of knowledge of the equipment derived from operator handbooks and maintenance manuals, obtain a list of all units within the equipments and plot the failure data as shown in Figure 8-2 for the example equipments. (The number of units exhibiting zero failures can be determined only if the total number of units in the equipment is known.)

Inspection of Figure 8-2 reveals that 3 modules out of the 38 contributed more failures than all their companion modules – i.e., Modules #03, #27, and #38 contributed 60% of the total failures reported. Each of these modules should now be analyzed relative to complexity and circuit function, to determine if in fact it does represent a "maverick" problem (i.e., indicate a failure rate considerably higher than should be expected for its level of complexity and function). The module may exhibit a relatively large number of failures because it is more complex in total circuits and parts than others, or because it contains parts which, due to state-of-the-art limitations, are relatively short-lived in the particular application.

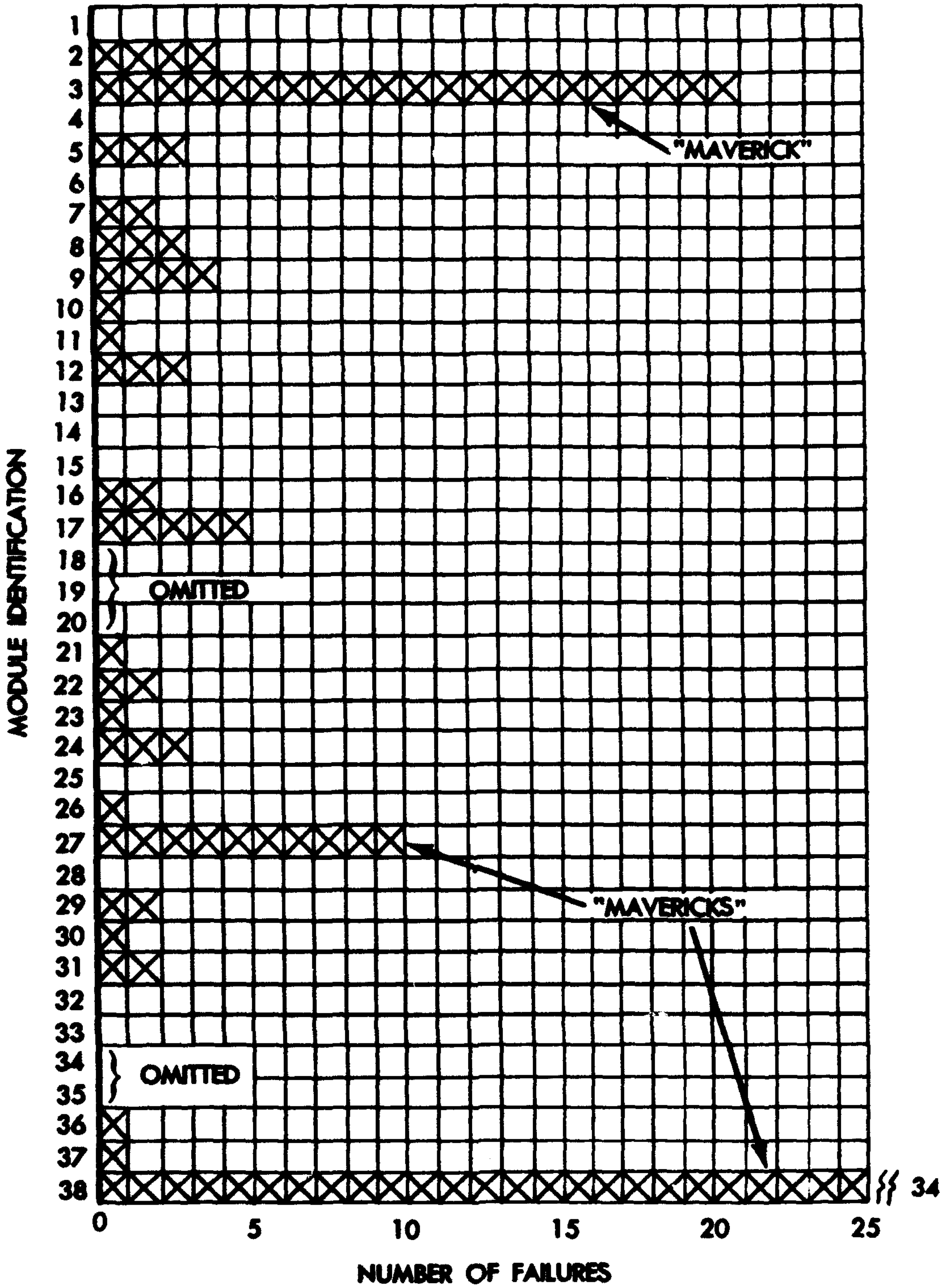


Figure 8-2. Module Failure Distribution for Example Equipment

STEP 3 – Designate the "Maverick" Units.

Some complex items may appear erroneously as mavericks. On the other hand, a very simple unit may exhibit a high number of failures relative to complexity, yet not appear as a "unit" problem. It is therefore necessary to "normalize" the observed unit failure data with respect to functional complexity. If all units are approximately equal in complexity, then it can usually be assumed that no hidden mavericks exist; if, on the other hand, a wide variation in complexity exists among units, it is important that unit failures be normalized on a *per-active-element* basis (transistor, tube, relay, motor) before mavericks can be legitimately designated.

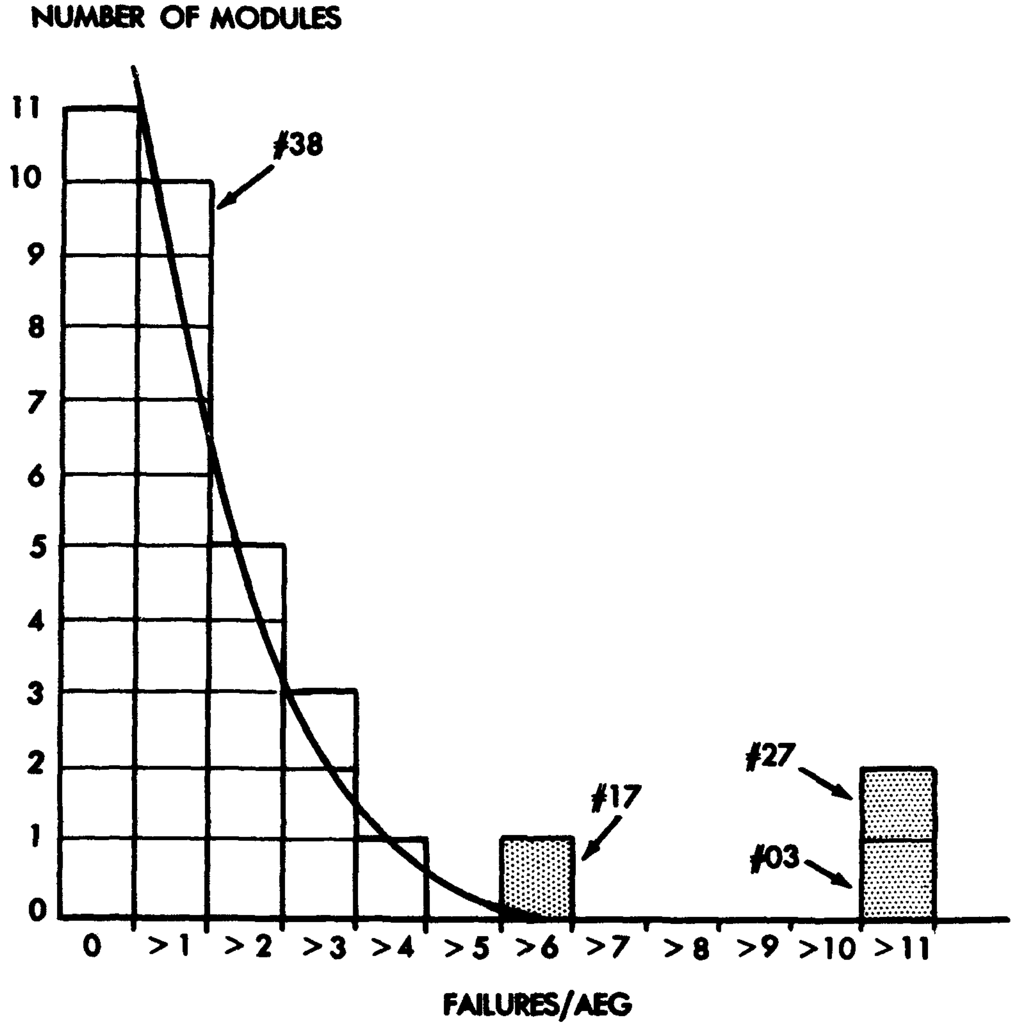


Figure 8-3. Failure Distribution per Active Element/Module

Figure 8-3 shows the number of modules exhibiting 0, 1, 2, 3, or more failures per active elements. The "expected" shape of this homogeneous set of data is shown by the smooth curve of the figure (a Poisson probability function as described in Appendix 2). In most instances, this curve can be sketched "by eye". Modules outside or beyond the curve are "statistically" different from the majority. They are the mavericks which, if corrected, should yield the most reliability improvement per dollar. Failure rate improvement in the homogeneous group should also be considered (Step 5 of Paragraph 8-3) as a longer-range objective, following a clean-up of the mavericks.

Normalized module failures are obtained by dividing the number of observed module failures by the number of active element groups in the module. The number of AEG's determined from the design data on the example equipment are given in Figure 8-4.

For illustration, consider Modules #38 and #17. The observed failures were 34 and 5, respectively, Normalization to number of failures per AEG gives the following results:

Module #38 34/29 = 1.2 failures per AEG

Module #17 5/1 = 5 failures per AEG

Module #17 is classified as a maverick with 5 failures, whereas Module #38 is within the range of "expected" AEG failure rates even though it produced 34 failures during the same period.

Module	AEG's	Module	AEG's
1	1 (Antenna)	20	Omitted
2	3	21	4
3	2	22	18
4	3	23	26
5	1	24	6
6	3	25	8
7	1	26	9
8	2	27	1 (Hydraulic dr.)
9	2	28	3
10	1	29	1
11	6	30	7
12	4	31	6
13	6	32	4
14	1	33	0
15	4	34	Omitted
16	2	35	Omitted
17	1	36	2
18	Omitted	37	0
19	Omitted	38	29
		Total	165

Figure 8-4. Number of Active Elements in Each Module of the Example Equipment

STEP 4 – Evaluate Problems Within Maverick Modules.

Steps 2 and 3 are now repeated within each of the modules designated as maverick problems, using *part reference designators* in lieu of modules as the common denominator. Where relatively few parts are involved, a review of the part data will usually permit determination of those parts and related applications which are largely responsible for the high failure rate of maverick modules, as illustrated in the following examples:

Module #03 Most removals were of Transistor Q 302. Two other transistors and several diodes, resistors, and capacitors were not removed. The 21st removal was a faulty terminal strip.

Module #27 All ten removals were contributed by one connector.

Module #17 Four out of the five removals were of one relay.

Module #38 No single part produced more than two failures.

Removals or repair actions that have been itemized by module and part reference designator normally contain information which may lead directly to a definition of the problem, as illustrated in the following examples:

Module #03 Transistor removals from Q 302 were primarily the result of inability to adjust the module. However, removed transistors tested "OK". This indicates a circuit/part incompatibility problem requiring an engineering analysis for solution.

Module #27 Connectors were removed during routine maintenance because of corrosion. Review of equipment is in order to determine how condensate gets into the connector.

Module #17 Relays were removed after equipment became inoperative and the part failure code indicated "Contacts DO NOT open/close". The relay application should be reviewed relative to the relay specification.

STEP 5 – Define Maintainability Problem Area.

The preceding steps, as applicable, should be repeated in order to extract all available maintainability data from the reporting forms – fault location time, repair time, waiting time, post-repair checkout time, maintenance problems, instrumentation difficulties.

STEP 6 – Follow Up.

The analysis illustrated in the preceding steps can prove useful and effective only if follow-on detailed engineering changes are conceived, tested, and introduced into existing equipments. The summarized problems and solutions should therefore be fed back to design and engineering groups for the development of field modifications for existing systems, as well as to guide the design of future systems.

8-3 RELIABILITY EVALUATION

The failure-reporting procedures in use today are also useful for the estimation of equipment reliability under Fleet "use" conditions. The accuracy of the estimate is, of course, dependent upon the accuracy of data reporting and the availability of adequate supporting information. Steps which may be followed in estimating equipment reliability are given below.

STEP 1 – Determine the Number of Equipment Failures in a Selected Time Period.

Failure reports provide an estimate of the total number of equipment removals, and recent forms also cover adjustment and alignment failures where no parts were removed. The data should be ordered by date/time sequence and equipment time-meter readings, as well as by report number, in order to aid in grouping those part removals which occurred as a "cluster" during each repair action following an equipment failure.[1] Preventive maintenance removals should not be considered in determining the number of equipments to be used in the reliability computation.

Continuing with the example, a total of 108 removals was reported during service evaluation of the four equipments. Of these, 23 were removed during preventive maintenance, with no indication that the equipment was in a failed state. This left 85 removals associated with equipment failures. Analysis of date/time and time-meter readings produced 62 independent equipment operational failures (44 during flight, and 18 during ground operation).

[1] Experience has shown that between 1.2 and 2 parts are removed per repair action.

STEP 2 – Obtain Total Number of Equipment Operating Hours.

The number of equipment operating hours accumulated during the same time period by those equipments from which data were obtained (include equipments which accumulated time but did not fail) must be secured from equipment operating logs and major system logs (flight logs, ships logs, etc.). Where large numbers of equipments and several months are involved, the estimated number of hours of operation per month per equipment may be sufficiently accurate for estimating purposes.

Time records for the example equipments, based on aircraft log data during the evaluation period, are summarized as follows:

Equipment Number	Ground Time	Flight Time
1	368 hours	163 hours
2	980 "	420 "
3	530 "	213 "
4	276 "	210 "
Totals	2154 hours	1008 hours

STEP 3 – Estimate Reliability or MTBF.

Flight MTBF

$$= \frac{\text{Total Flight Operate Hours}}{\text{Total Number of In-Flight Failures}}$$

$$= \frac{1008}{44} = 22.4 \text{ Hours}$$

Ground MTBF

$$= \frac{2154}{18} \approx 120 \text{ Hours}$$

(Note: If the data do not include adjustment and alignment failures, this estimate is likely to be optimistic.)

Statistical confidence intervals can be placed upon these estimates, if desired (see Appendix 3).

STEP 4 – Assess Possible Improvement by Reduction of Problem Unit Failures.

A comparison between the observed reliability computed in Step 3 and that predicted by elimination of the problems in maverick modules provides a measure of potential gain which may be used as justification for corrective action.

Assume that failures from maverick modules of the example equipment can be reduced to the average number of failures of the remaining modules (.54 failures/AEG/module). This represents a reduction of 24 failures for the time accumulated during service evaluation, as shown in Figure 8-5. It can be assumed that this reduction would be proportionately divided between ground and in-flight failures.

Predicted Potential Flight MTBF

$$= \frac{\text{Total Operate Hours}}{\text{Observed Failures - Potential Reduction}}$$

$$= \frac{1008}{44 - 17} \approx 37 \text{ Hours}$$

There is, therefore, a potential increase from 22 to 37 hours in flight reliability (MTBF) by treating maverick problems alone. This is illustrated in Figure 8-6.

Module	Removals		Expected Reduction
	Observed	Avg/AEG/Module	
#03	21	.54 × 2 = 1.1	19.9
#17	5	.54 × 1 = .5	4.5
#27	10 (Preventive Maintenance)	.54 × 1 = .5	– *

Total Reduction = 24.4

In-Flight Reduction $= \frac{44}{62} \times 24.4 = 17$

* Preventive maintenance removals do not indicate a failed system. Thus, reduction of preventive maintenance removals will not influence system reliability directly.

Figure 8-5. Potential Reduction in Maverick Problems

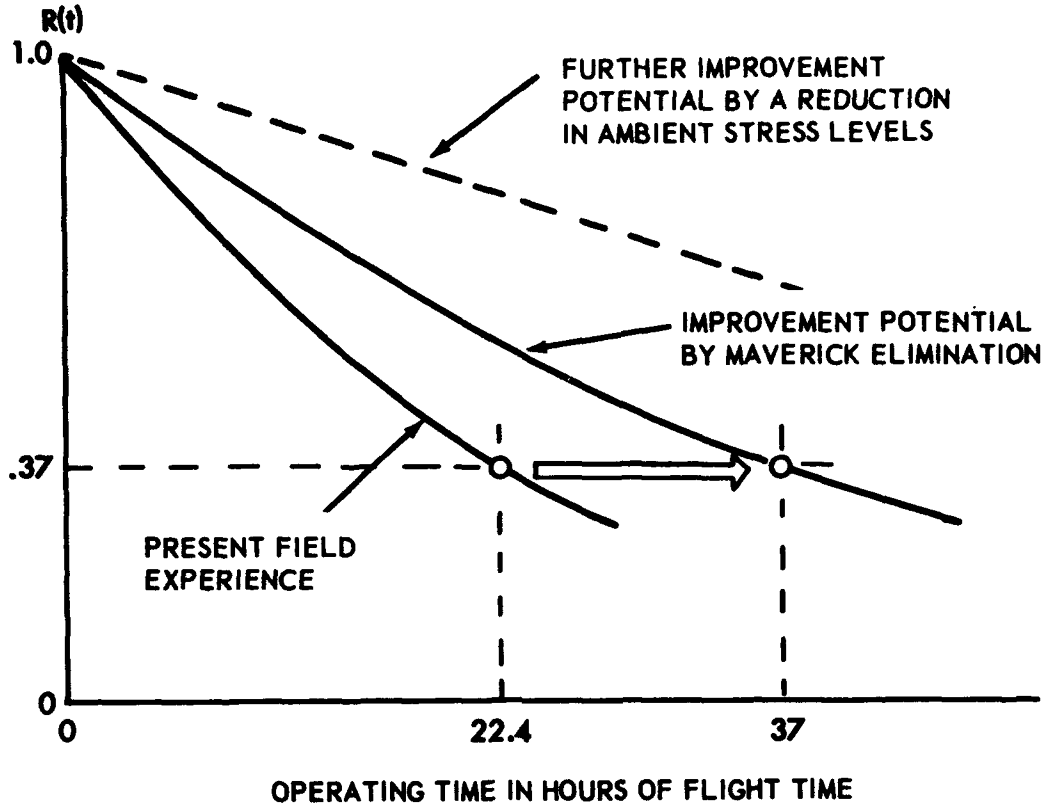


Figure 8-6. Estimated Equipment Flight Reliability, Present and Potential, Based on an Analysis of Failure Data

STEP 5 – Evaluate the "Ambient" Problem.

The preceding steps have dealt with the more outstanding "maverick" problems that jeopardize equipment reliability – problems readily apparent to the data analyst. The other general classification of problems hidden in the failure pattern of Figure 8-3 is called the "ambient" problem – not so readily discernible, and generally not so easily corrected. This ambient problem, accounting for the high average failure rate of the homogeneous group of the distribution, is largely attributable to environmental factors (thermal, shock,. and vibration) and application stresses (use conditions versus rated conditions) which are peculiar to a particular installation requirement. These are the factors which explain the seven-to-one ratio in MTBF experience between shipboard systems and airborne systems of comparable complexity, as discussed in Chapter 1.

Significant reliability improvements can be achieved through effective treatment of stringent ambient conditions, although the cost in space, weight, repackaging, cooling, voltage control, and overall parts derating may be prohibitive as a redesign "retrofit" measure. Effective treatment of the problem requires supplemental information not generally available through the failure reporting system. In the airborne systems just discussed, for example, it would be necessary to perform in-flight measurements of ambient and "hot-spot" temperatures, vibration levels and frequencies, voltage levels, and transients. With these measurements, operating stress-levels can be precisely defined at the part level, to indicate the need for, and the nature of, required design improvements.

EXAMPLE: An in-flight thermal survey conducted on the airborne equipment used in the foregoing example discloses a steady state ambient temperature within modules. ranging from 60°C to 120°C. Control of this ambient range to a 60°C upper limit would reduce the average failure rate of the homogeneous portion of the failure distribution by a factor of three-to-one. This, combined with the reduction of maver previously discussed, could yie five-to-one improvement in equipm TBF. The predicted reliability function for this case is also shown in Figure 8-6.

8-4 MAINTAINABILITY EVALUATION

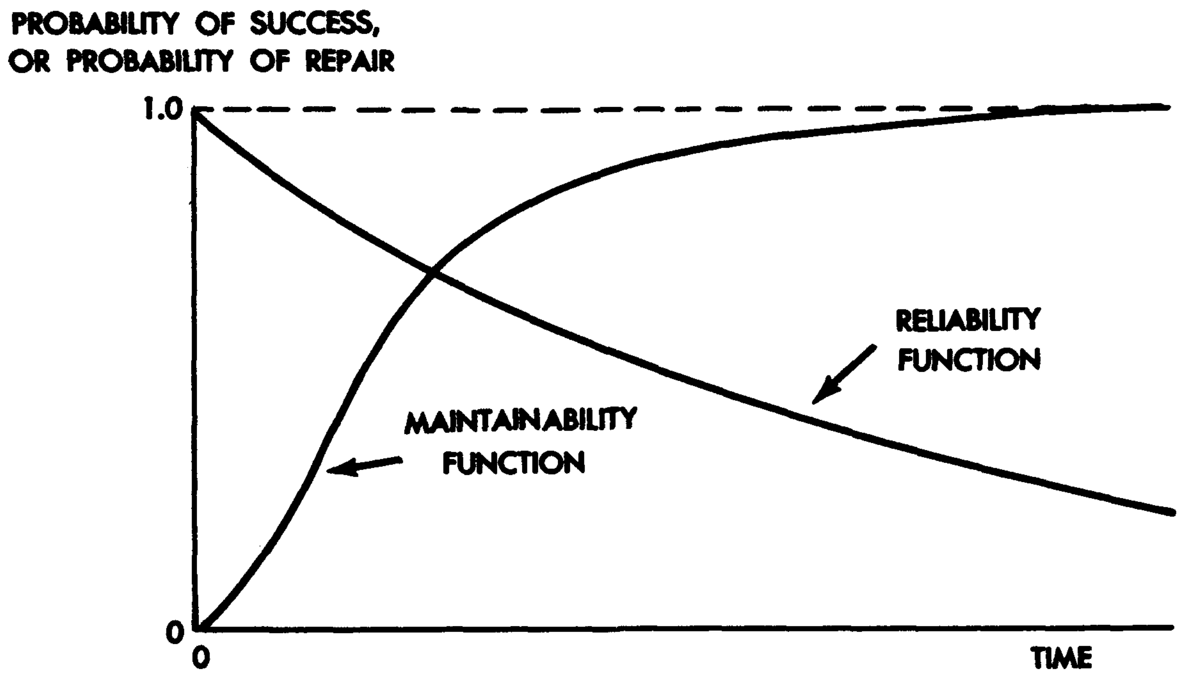


Figure 8-7. Reliability and Maintainability Time Functions

Maintainability is expressed as a probability of repair in a given time. The maintainability function, as contrasted with the reliability function, is generally of the log-normal shape illustrated in Figure 8-7.

The step-by-step procedure for estimating the maintainability of an equipment follows.

STEP 1 – Tabulate Reported Maintenance Times in Ascending Order.

Repair times were reported on 72 of the 108 repairs performed on the example equipments. These 72 times were arranged in ascending order, as illustrated:

15 minutes	1 hours	8 hours
15 "	1 "	9 "
15 "	1 "	19 "
15 "	1 "	Total 216 hours
20 "	1.5 "	
20 "	1.5 "	
1 hours	2 "	
1 "	2 "	
"		

STEP 2 – Estimate Mean-Time-To-Restore

Divide the total repair or maintenance hours by the number of repair actions (maintenance hours are the calendar hours the equipment is being worked on and should not be confused with maintenance man-hours per repair action).

The mean-time-to-restore of the example equipment is calculated as:

$$\frac{216}{72} = 3 \text{ hours}$$

STEP 3 – Plot the Repair Times to Determine the Frequency Distribution.

The repair times grouped into equal intervals are plotted as shown in Figure 8-8. This plot is called a frequency distribution, which represents the number of instances in which various repair times were required to correct a failure.

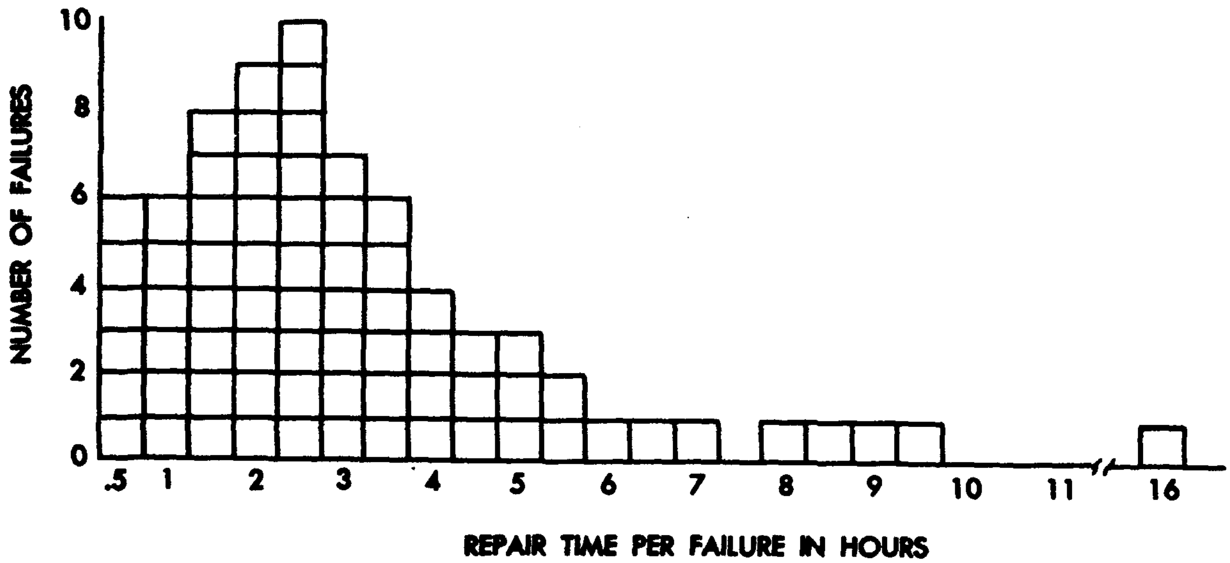


Figure 8-8. Plot of Repair Times per Repair Action

STEP 4 – Estimate the Maintainability Function.

The maintainability function can be estimated by computing the following probability of repair for each time interval in Figure 8-8:

$$\text{Probability of Repair} = \frac{\text{Total Repair Actions Completed in Time t or Less}}{\text{Total Repair Actions}}$$

A plot of these values versus the time t provides the desired maintainability function (Figure 8-9). From this plot it can be stated that 50% of the repair actions will be completed in 2.5 hours or less and 90% will be completed in 6 hours or less.

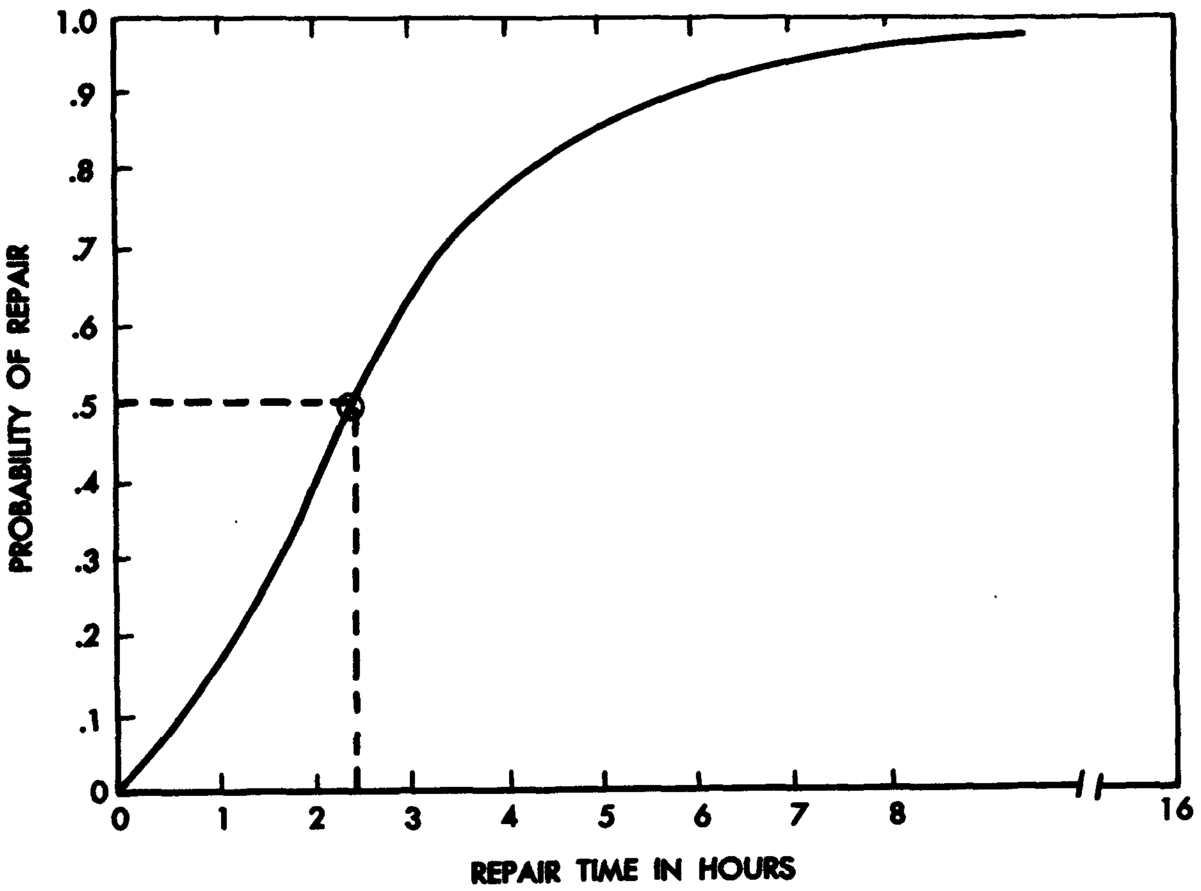


Figure 8-9. Maintainability Function

STEP 5 – Assess Potential Maintainability Improvement.

Upon determining that the long time to repair items can be reduced, it is possible to plot a predicted maintainability function for comparison with the observed function. Either the median values (i.e., 50th percentile) or the mean values of the two distributions can be used to assess the degree of improvement as shown in Step 2 of Paragraph 8-3.

8-5 CORRECTIVE ACTION

8-5-1. Failure Patterns

The preceding failure isolation and evaluation analyses describe the system status and indicate whether corrective action will produce significant improvements. These analyses do not indicate the specific corrective actions which may be employed in system improvement. Additional analysis of failure reports will often suggest the appropriate corrective action. Typical questions which may be answered by this analysis are:

- Does the recorded cause of failure or reason for removal indicate a repetitive application, environmental, or design problem?
- Is there a difference in unit, module, or part removals due to physical location or application?
- Would a scheduled replacement time reduce the number of in-service failures of short-lived components?
- Are long maintenance times consistently related to given repair actions?

Each of the above questions, plus others of a more specific nature on a given equipment or part type, may be answered fully or in part by failure data *when combined* with supporting information on the equipment and its usage.

> *EXAMPLE:* Connectors were too frequently removed from Module #27 of the example equipment. The reason for the removal was reported as "corrosion". Occasional remarks indicated that water dripped into the compartment. Recommended approach: (1) Determine if the compartment was designed to be or could easily be made watertight; (2) if not, initiate a field change to install waterproof connectors.
>
> Actual Solution: The compartment was essentially waterproofed by installing a sheet metal drip guard over the compartment opening to prevent condensation from dripping onto the connectors.

8-5-2. Scheduled Replacement

The question of replacement time for components which appear to operate satisfactorily for some length of time and then begin to fail rapidly (or, conversely, for components which never seem to last more than a few hundred hours) can be partially answered through failure data that give the reference designation and time-meter readings on the equipment. Gyros, magnetrons, other high-power tubes, rotating and high-wear devices, sealed modules, and so on, can exhibit wearout phenomena prior to the end of equipment service life.

The shape of the reliability function can be easily estimated by a plot of the probability of survival versus various time intervals.

EXAMPLE: A gyro installed in 12 equipments repeatedly failed after a few hundred hours of operation. For analysis, field data on this gyro from the 12 equipments were ordered by failure report number (or date/time sequence) within a fixed time interval, as illustrated:

Equipment #	Failure Report #	Time Meter (Equipment)	Elapsed Time
1	62	108	? (Estimate 108)
1	138	432	324
1	171	1042	610
1	200	1643	601
1	216	2267	624
2	76	1061	? (Do not use this observation)
2	129	1665	604

The total elapsed time was arranged in ascending order for the 30 observations:

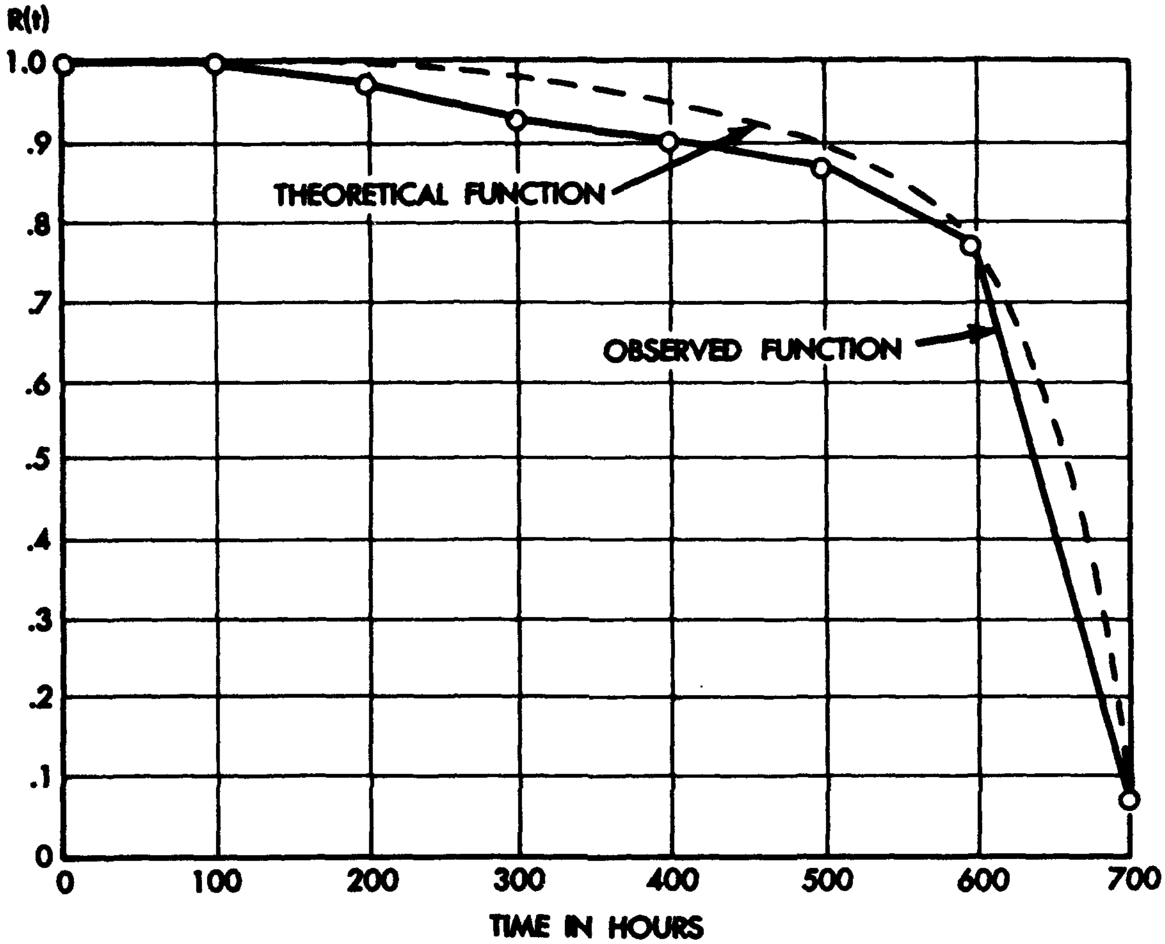
108	617	668
289	617	670
324	624	673
446	640	673
516	641	679
538	652	680
580	657	688
601	658	698
604	661	730
610	662	809

These data were then employed to obtain the reliability function given in Figure 8-10 (the computational procedures are shown on the figure). From Figure 8-10, it can be estimated that a replacement schedule of 600 hours would decrease in-flight failures by as much as 50%.

8-5-3. Laboratory Analysis

Other questions can be answered by similar methods of organizing, classifying, and combining the basic data. It must be borne in mind that field failure data, in their present state, do not always provide irrefutable results, but they do provide an indication of problem areas and estimates which are useful to design engineers, project engineers, and management personnel in improving existing designs and avoiding repetition of errors in the next generation of equipments.

More often, however, additional detailed laboratory studies are required in order to fully establish the causes of failure and to recommend a "fix". Regression analysis (step-by-step procedures are given in Chapter 6) is the most widely used approach to these laboratory studies. A brief dis-



PROPORTION SURVIVING	30/30	30/30	29/30	28/30	27/30	26/30	23/30	2/30
PROBABILITY OF SURVIVAL	1.0	1.0	.97	.93	.90	.87	.77	.07

Figure 8-10. Observed Flight Reliability for a Gyro

cussion of the application of regression techniques to a typical circuit problem will illustrate the laboratory approach to closing the feedback loop.

> *EXAMPLE:* A multivibrator circuit in an airborne application was classified as a maverick because of an excessive number of Q_1 transistor removals. Upon testing the transistors, however, it was found that they checked "OK" relative to specification limits.
>
> To determine the effect of transistors on circuit performance (Figure 8-11), 4 production circuits were randomly selected and each was operated with 60 different transistors. The results are shown in Figure 8-12.
>
> Figure 8-12 indicates that the average or mean frequency was very close to the specified minimum for the circuit. A large percentage of the outputs fell below the lower limit. It is apparent that the output distribution of the circuit must be shifted so that the average output will fall on the design center value. Simple regression, defined as the relationship between a dependent variable and one independent variable, was used to determine what part or parts values are related most directly to circuit output.
>
> In this circuit, the coupling capacitor (.022μf) was the suspected culprit. Verification of this was obtained by taking one model of the circuit and measuring the frequency as a function of three different values of the coupling capacitor. Three sets of data were obtained by running a sample of 47 transistors through the circuit for each value of the capacitor. All circuit components except transistors were held at fixed values. This made it possible to obtain distributions in which output frequency variability was due to transistors alone at each of

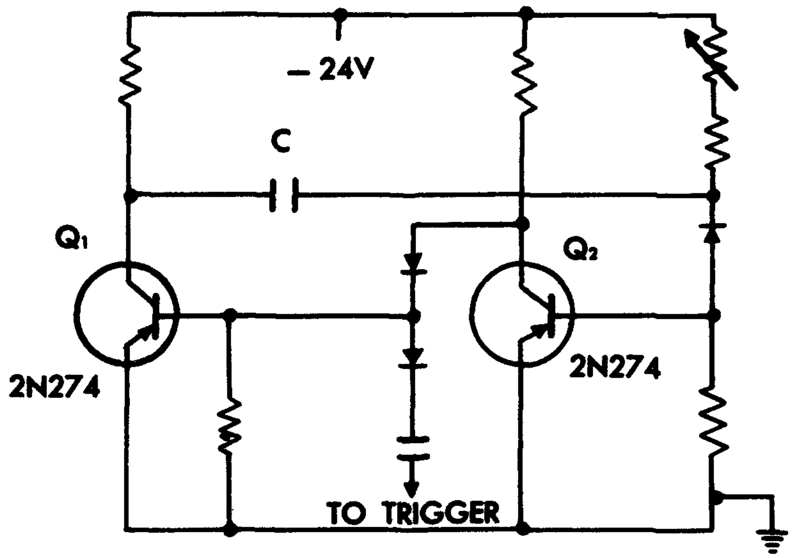


Figure 8-11. Circuit Schematic of Troublesome Multivibrator

three capacitor values. Figure 8-13 shows, for each value of the coupling capacitor, the frequencies obtained. The means of the distributions are connected by a *regression line* from which can be read the expected frequency for any value of cathode-coupling capacitance.

Figure 8-13 indicates that the design frequency of 115 cycles per second would most often be obtained if the capacitance were .016 microfarads. The statistical 95% tolerance limits on multivibrator outputs were calculated and drawn on the figure. These limits show that frequencies in the range of 104 to 126 cycles will be achieved 95% of the time if the coupling capacitor is changed to .016 microfarads.

This example illustrates the use of simple regression, not only to determine that an erroneous capacitor value was employed, but also to give a solution to the field problem – which was uncovered through detection of an excessive number of transistor removals.

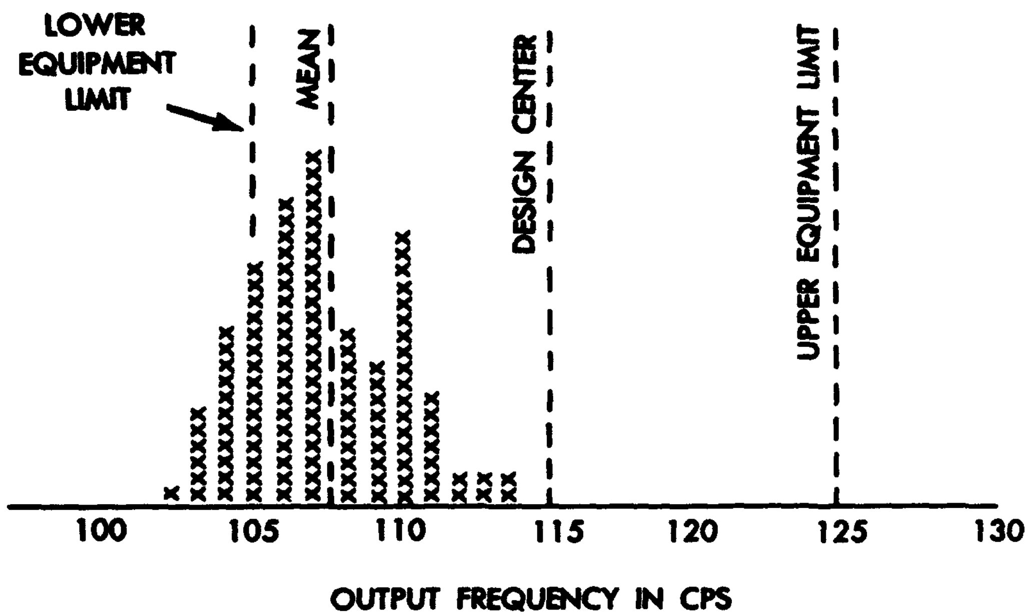


Figure 8-12. Distribution of Output Frequency of Four Multivibrator Modules

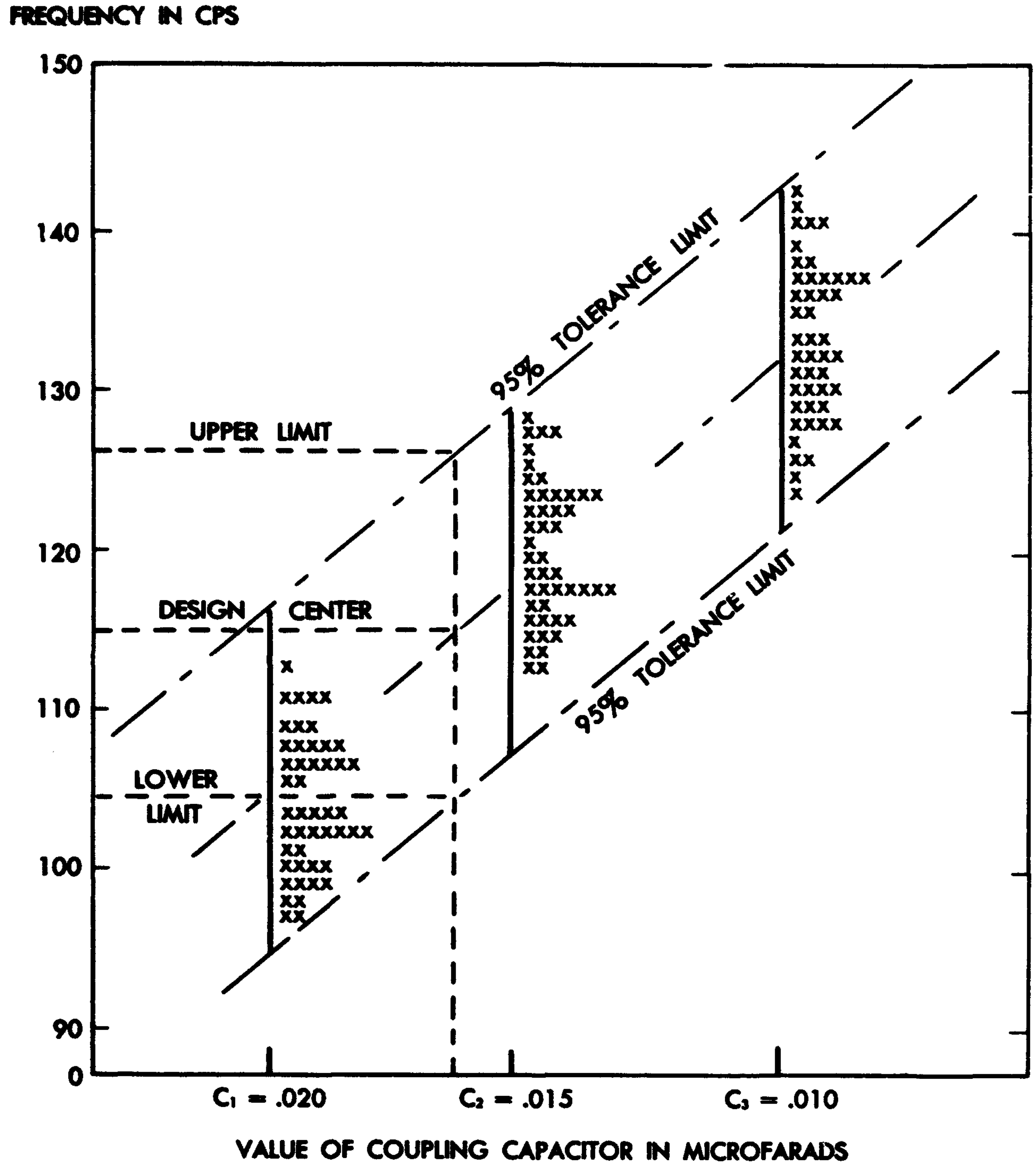


Figure 8-13. Regression of Frequency Versus Capacitance in Multivibrator.

8-6 RELIABILITY DATA SOURCES

The design engineer is dependent upon the feedback of part performance and failure data from a wide range of applications and use environments if he is to optimize design reliability and avoid the pitfalls which befell his predecessors. This type of data is made available to the designer of new Navy systems through the following:

- MIL-HDBK-217, "Reliability Stress Analysis for Electronic Equipment." This handbook, referenced in Chapter 1 and Chapter 5, provides a source of parts failure rate data for standard electronic and electromechanical parts. Catastrophic part failure rates observed over wide ranges of electrical and thermal stresses have been analyzed and presented in a form which permits determination of the most likely failure rate for a given set of stresses. This handbook is available from the Government Printing Office.

- Bureau of Naval Weapons Failure Rate Data (FARADA) Program. FARADA is a Navy-sponsored effort to provide reliability design data to contractors engaged in the design, development, and production of systems for the Navy. Part failure rates are obtained from the various contractors and service organizations. They are summarized, analyzed, and published in the form of a part "Failure Rate Data Handbook." MIL-HDBK-217, described above, is one of the prime data sources for FARADA. The principal difference between MIL-HDBK-217 and FARADA is that the latter integrates many individual sets of established failure rates while MIL-HDBK-217 converts failure data into failure rates versus stress levels.

- Inter Service Data Exchange Program. IDEP is a tri-service program for the exchange of part test reports to assist system designers in the selection and application of reliable part types. The test data exchanged includes, but is not limited to, that obtained from:

  a. Qualification or Certification Tests
  b. Production Acceptance Tests
  c. Diagnostic or Design and Development Tests
  d. General or Comparative Evaluation Tests
  e. Reliability, Exaggerated Stress, and Life Tests

  The IDEP exchange program does not summarize or edit test reports; instead the three distribution centers (one for each service) act as clearing houses. Contractor test reports are forwarded to their appropriate service distribution center (e.g., Navy IDEP Office, NOL, Corona) where they are reproduced and forwarded to other participants in the program.

- Guided Missile Data Exchange Program. GMDEP is similar in purpose and intent to the IDEP program, except that it is devoted primarily to the exchange of data generated by Navy contractors who are engaged in the research, development, and production of guided missiles. In addition to test reports, specification and part application data sheets are also exchanged.

Contractors who are developing Navy systems and who are not presently participating in FARADA., IDEP., and GMDEP should be encouraged to inquire at the Naval Ordnance Laboratory, Corona, California, for information on how to join.

CHAPTER 9
DEVELOPMENT TIME AND COST ESTIMATION

9-1 INTRODUCTION

9-1-1. Purpose

The project engineer is ultimately confronted with the task of cost and time estimation with respect to his development program –

- To estimate the time and funds required to develop the proposed new system, as a basis for documenting Sections 5 and 6 of the Technical Development Plan (TDP);
- To evaluate the realism and validity of contractor cost and time estimates submitted in response to bid requests;
- To assess the feasibility of completing the development phase within the time span and funds finally allotted to the program; and
- To estimate the effect of cost differentials and system reliability; or, conversely, to estimate development program costs for specific levels of reliability.

In any case, the project engineer must draw on past experience with other programs of similar intent and complexity, to formulate an estimate of development time and funding requirements for the new program. The estimating problem is made difficult by many factors, the most significant of which is the degree to which the new design concept will depend upon state-of-art advances or "breakthroughs" in component development and design techniques. If the new design is relatively free of these dependencies, a fairly accurate prediction of *minimum* time and cost can be made. On the other hand, if the system concept employs several unique or untried design approaches or depends upon achieving a state-of-art breakthrough in the development of a critical component or part, an estimate of *average* cost expectancy derived from past experience can prevent overoptimism on the part of the project engineer in planning and budgeting the development program.

9-1-2. Basis

The procedures outlined in this section represent a first attempt to translate and quantify the collective experience[1/] of twenty-one weapon system development programs conducted under the cognizance of the Bureau of Naval Weapons during the past ten years. While the procedures set forth below are straightforward, the input data available at this time are understandably limited to experience on predominantly electronic systems. The *time*-estimating procedures embrace the period between initial contract award and final acceptance of the prototype model. The *cost*-estimating procedures apply to the costs incurred by the contractor and his subcontractors during this time period. They do not include costs of functions performed by the Bureau and its centers in project management and technical direction.

[1/] "Cost and Time Factors Relating to Reliability in Development Planning", Final Report dated 1 October 1963 submitted by Bird Engineering–Research Associates, Inc., under BuWeps Contract NOW-62-0990-c.

9-2 FUNDAMENTAL COST-TIME RELATIONSHIPS

9-2-1. General

There are many factors which influence time and cost of a development program – the prior experience of the contractor on similar systems; the degree of finality and realism with which mission characteristics and performance requirements are specified at the outset of the development program; the continuity and stability of scheduling and funding; the complexity and conventionality of the design concept; and the relative level of reliability to be achieved in development. Among these, the following are the principal factors which determine the time and cost of a weapon system development program:

(1) Functional complexity of the system concept;

(2) Conventionality of the proposed design approach – i.e., "within state-of-art";

(3) Relative level of reliability to be achieved with respect to the average value observed in conventional designs of this complexity.

9-2-2. Cost Determining Factors

Figure 9-1 presents a family of *minimum* cost curves relating the overall cost of a system development program to the complexity of the system to be developed, the degree of freedom from state-of-art problems, and the level of achievable reliability actually sought. Assume for example, a development program for an avionics system of 100 AEG's[2] estimated complexity, employing a conventional design approach. Past experience indicates that a minimum of $460,000 (1963 dollars) was required to produce a prototype capable of demonstrating approximately 21 hours MTBF, the lowest level of reliability observed on other avionics systems of this complexity in the Fleet today.[3] Where a 150-hour MTBF goal (upper boundary of the MTBF curve of Figure 2-19) was achieved, the *minimum* development cost increased to $1,150,000 – nearly a 3-to-1 increase in minimum funding requirements for a 7-to-1 gain in system MTBF. Note that this example has assumed a "conventional" design with no outstanding state-of-art problems to overcome. When such problems have existed, however, development costs have greatly exceeded the minimums shown in the chart – ranging in the 100 AEG example up to $6,400,000.

Clearly, then, if the project engineer is to estimate the costs of a new development program on the basis of past experience on predecessor systems, he must evaluate the design for possible state-of-art problems and adjust his minimum estimates to reflect any departure from convention. A cost curve for designs dependent on state-of-art advancements is also shown in Figure 9-1.

The value of the dollar continues to change from year to year. It is necessary for long-term estimating purposes to predict its valuation for each fiscal year's budget period. For an approximation of overall costs, it is permissible to take the predicted dollar value at the midpoint in the program in order to derive a correction

[2] See Chapter 2 for a discussion of the AEG method of system complexity measurement.

[3] See Figure 2-19.

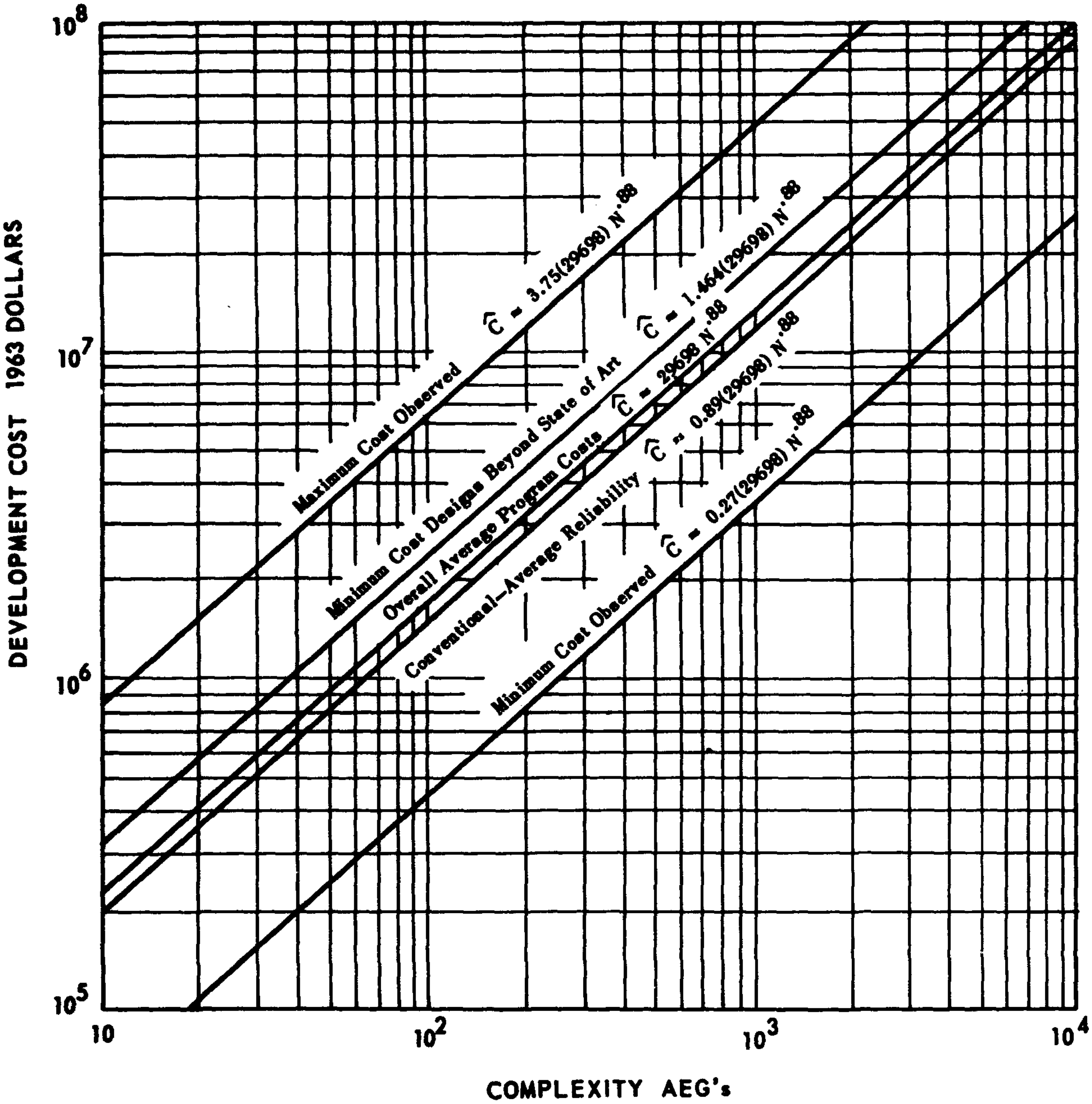


Figure 9-1. Development Program Costs Related to System Complexity, Reliability, and Design Concept

factor for current value. Figure 9-2 is a plot projecting the dollar valuation for the period 1960 to 1970.4/

9-2-3. Time Determining Factors

Figure 9-3 presents a preliminary regression model derived from the development experience of the twenty-one system programs in the study. Development time appears to increase approximately with the tenth root of system complexity, for a given degree of design conventionality. The lower line, representing the straightforward conventional design approach, should be used for estimating *minimum* development time required for a new program. The center line is a more conservative *average time* estimator, to be used when state-of-art problems are expected. The upper boundary is realistic when it is known that the system concept is dependent upon components or design techniques that are themselves still in the development stage – and consequently are easily identifiable as potential state-of-art problems.

4/ From Proposed Cost-of-Research Index Report by E. A. Johnson and H. S. Milton of Operations Research Office, The Johns Hopkins University, Bethesda, Maryland, September 1960. Published in December 1961 by IRE Transactions on Engineering Management.

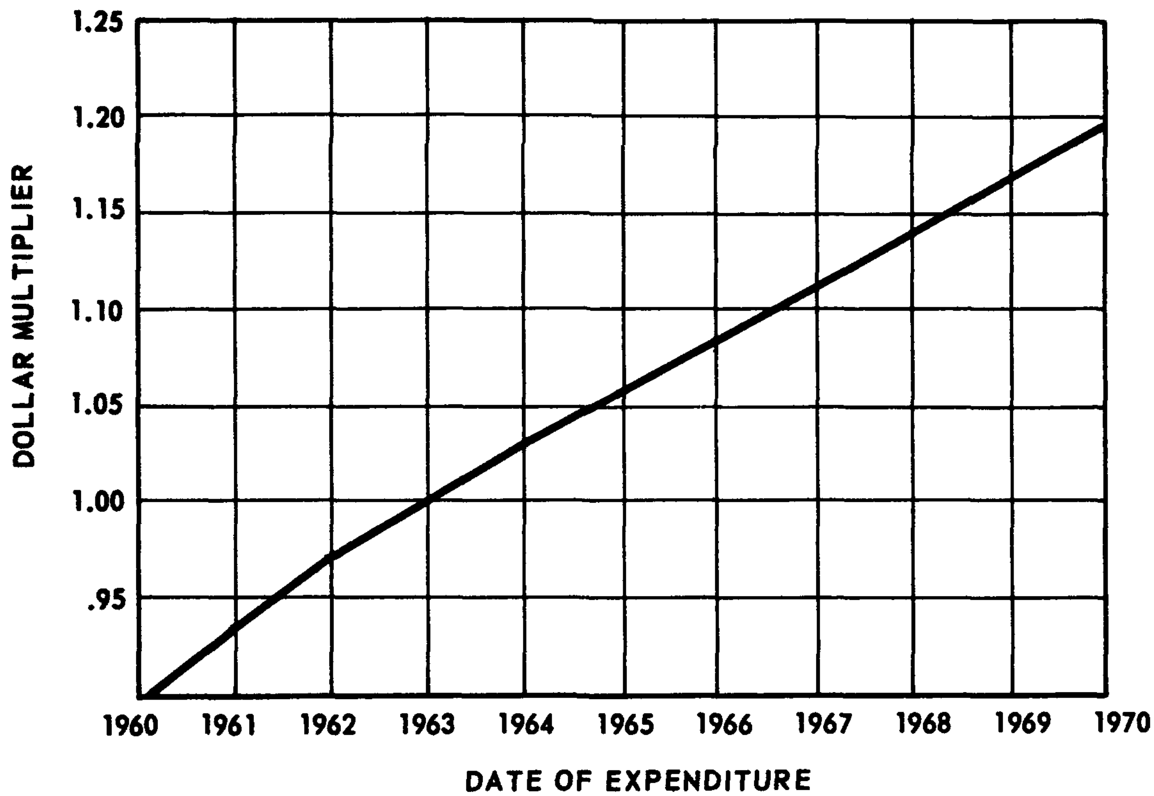


Figure 9-2. Cost Correction Multiplier for Estimating Future Program Costs

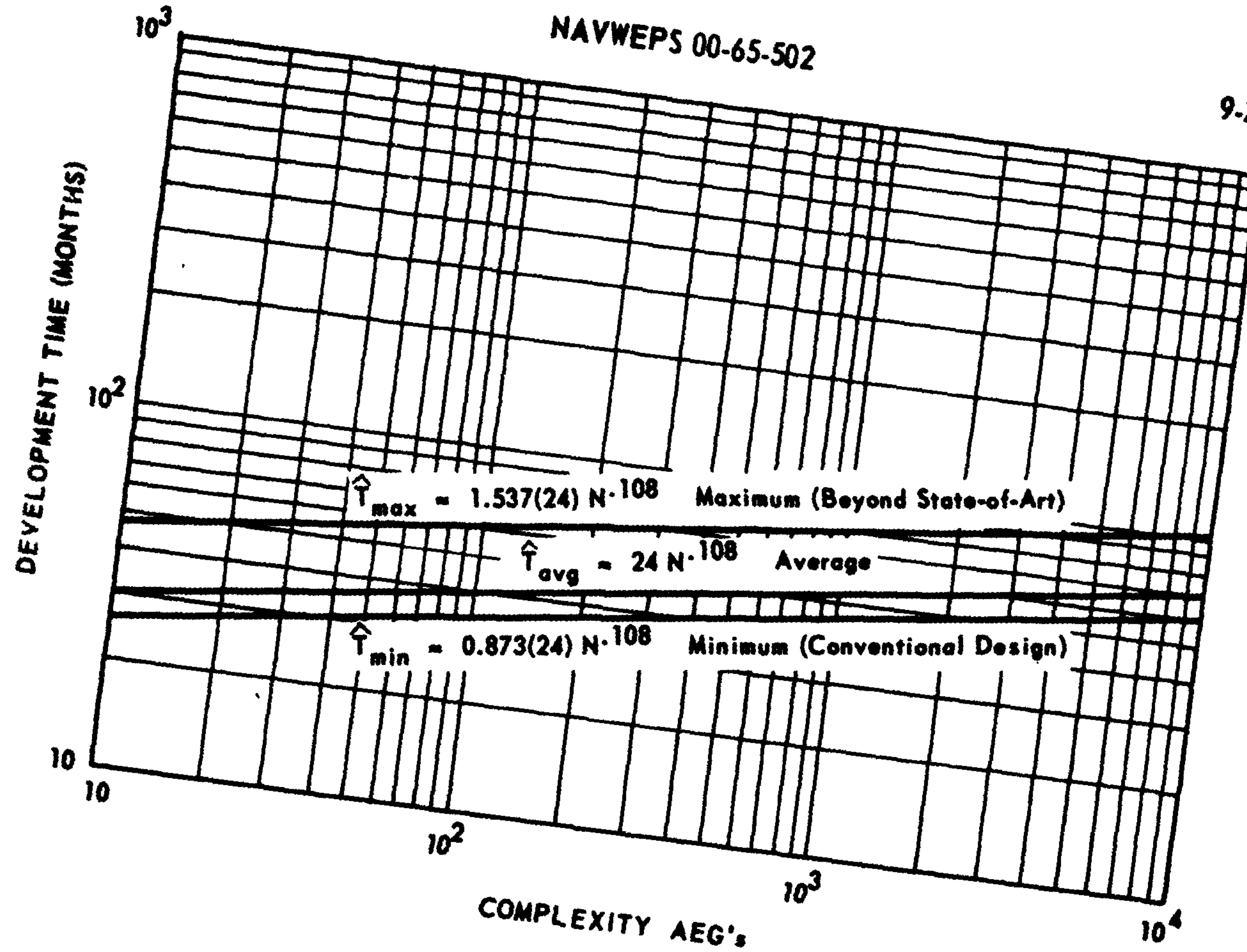


Figure 9-3. Development Time, Complexity, and State-of-Art

Assume for example that the 100-AEG avionics system of the preceding example must be scheduled for delivery in 24 months to meet an airframe (major system) prototype delivery schedule. Entering the figure at 100 AEG's, it appears that 24 months is realistic if no state-of-art problems are encountered.

The relationship between development time and reliability is not yet separately definable, but is hidden in the time/state-of-art relationship – i.e., as long as the reliability requirement is feasible by conventional design (for the particular level of complexity involved) the T_{min} curve should be generally applicable. When the reliability requirement exceeds this value, then reliability itself becomes one of the state-of-art problems, and the other curves become applicable.

9-3 ESTIMATION PROCEDURE

9-3-1. Conventionality Assessment

Procedures for assessing the conventionality of a proposed design concept are outlined in Chapter 2. The feasibility of achieving specified performance and reliability objectives is evaluated according to these same procedures. Key steps are summarized here:

STEP 1 – Obtain Reliability Requirements

In the requirements analysis (Chapter 2-2, Step 6), the reliability requirements were defined and should have been expanded to the subsystem level. The subsystem level is selected as best for time and cost feasibility estimation and evaluation because it generally consists of a major piece of equipment, is designed to perform a complete function, and will certainly be accomplished by one prime contractor.

STEP 2 – Develop the Reliability Block Diagram to Subsystem Level

This step, illustrated in Figure 2-20 of Chapter 2, will have been taken in the reliability feasibility estimation (Chapter 2-3, Step 1). The same "model" is applicable for cost estimation purposes.

STEP 3 – Estimate the Complexity and Range of Feasible Failure Rates at the Subsystem Level

This is the same as Step 3 of Chapter 2-3, and the same information should be brought forward. Complexity is again expressed in AEG's.

As an example, assume that Block 4 of Figure 2-20 is the fire-control radar of a weapon control system to be developed for airborne use. A range of probable complexity is estimated at between 250 to 500 AEG's. From the complexity, the range of feasible MTBF can be estimated, as illustrated in Figure 2-21. If the stated requirement falls within this range, it can be considered feasible by conventional design. Assume a complexity of 300 AEG's for this example.

The feasible failure rates for this example, at the subsystem level, are shown in Figure 9-4.

STEP 4 – Compare Feasible Reliability Estimates With the Requirements at Subsystem Level

A comparison is now made between the expected failure rates determined in Step 3, above, and the allocated failure rate as determined in Step 6 of Chapter 2-3. Figure 9-5 illustrates the comparison within System 4.

STEP 5 – Classify Each Subsystem as to State-of-Art

The project engineer must next classify each subsystem as to state-of-art. Figure 9-5 is an example of the manner in which this comparison might be made.

In classifying subsystems as to state-of-art, consideration is given to the following:

- Are performance characteristics, as determined in Step 5 of Chapter 2-2, achievable by conventional design?
- Is the stated reliability requirement achievable by conventional design?

Subsystem	AEG Complexity	Failure Rate × 10^{-6}	
		Per AEG	Per Subsystem
a	40 Power [6/]	340	13,600
b	500 Digital [5/]	180	9,000
c	120 Analog	240	28,800
d	40 Power [6/]	340	13,600
e	50 Analog	180	9,000
Total Block 4 Failure Rate			74,000

Figure 9-4. Calculation of Expected Subsystem and System Failure Rates

Subsystem	Expected Failure Rate	Percent of Total	Allocated Failure Rate	Reliability Allocation	State-of-Art
a	13,600	18.38	6,850	.98	Conventional
b	9,000	12.17	4,200	.99	Beyond
c	28,800	38.90	13,200	.96	Conventional
d	13,600	18.38	6,850	.98	Conventional
e	9,000	12.17	4,200	.99	Beyond

Figure 9-5. Allocation and Comparison of Failure Rates

Requirements which fall outside the shaded region of Figure 2-22 pose problems which cannot be solved by conventional non-redundant methods.

If the answer to each of these questions is not a firm "Yes", accept the fact that costs will greatly exceed those which would be expected for development of conventional equipment of equal complexity, together with the risk that performance and/or reliability will fall short of requirements.

5/In a system employing both analog and digital AEG's, divide the number of digital AEG's by ten and treat as analog.

6/The number of power AEG's is a system are multiplied by two and treated as analog.

9-3-2. Budget Requirements Estimation

STEP 1 – Calculate the Cost of Each Subsystem

The estimate of cost is calculated using either the formulae or the graph of Figure 9-1. The AEG count is used in the same manner as in estimating reliability. The AEG count is in terms of power and analog AEG's. Ten digital AEG's are the equivalent of one analog AEG. Estimated costs should be corrected to the middle of the time period when money will actually be spent, using the graph of Figure 9-2.

STEP 2 – Compare the Cost of Each Subsystem with Budget Allocations

Continuing with the example used earlier, Figure 9-6 compares estimated costs with the preliminary budget allocation, as a basis for developing Section 6 of the TDP.

STEP 3 – Evaluate the Feasibility of Developing Each Subsystem and the Complete System Within Budget Limitations

In the example, the project engineer would conclude that Subsystems a, c, and d can be developed within cost limitations. Subsystem e will probably overrun, but this may be compensated for by slight underruns in a, c, and d. Since these represent minimum cost estimates, careful management and fiscal control will be required to hold them within the bounds of the financial plan. Subsystem b cannot be successfully developed without providing additional funding. A total of $716,000 additional must be requested and included in Section 6 of the TDP if a subsequent overrun is to be avoided.

9-3-3. Schedule Requirements Estimation

STEP 1 – Calculate the Development Time for Each Subsystem

The estimate of time is calculated using either the formulae or the graph of Figure 9-3. The estimates are picked off of the curve using the state-of-art classification and the complexity of each subsystem. If there is some doubt of state-of-art aspects, the center (average) time curve is satisfactory for a preliminary rough estimate.

In development programs whose designs are beyond the state-of-art, feasibility studies or supporting research and development (component development) are required. If such is the case, additional time is required. The amount of additional time depends upon how long it takes to produce the information, design, or material that will permit the start of design of the subsystem.

Continuing with the example, analysis of System 4, development times are tabulated as shown in Figure 9-7. In addition to the subsystem development times, a feasibility

Subsystem	Complexity	Analog Equiv.	State-of-Art	Estimated Minimum Cost, 1963	Corrected Min. Cost, Mid-1967	Financial Plan
a	40 power	40	Conventional	550,000	633,000	700,000
b	500 digital	50	Beyond	1,300,000	1,500,000	800,000
c	120 analog	120	Conventional	1,700,000	1,950,000	2,000,000
d	40 power	40	Conventional	550,000	633,000	650,000
e	50 analog	50	Beyond	1,300,000	1,500,000	1,350,000
				5,400,000	6,216,000	5,500,000

Figure 9-6. Comparison of Estimated and Allocated Costs

study of Subsystem b must be performed before its development can be started, and special research is needed to develop a component for Subsystem e. In each case 12 months are estimated as required prior to the start of design.

STEP 2 – Plot a Network of the Events Showing Relationships and Dependencies; Find the Critical Path

If all the subsystems could be developed independently, it would be a simple matter to take the longest total development time in Figure 9-7 as being that of the system. Frequently a subsystem or component is dependent upon information which will be available only after the design of another is complete. The integration of a system takes time after all of its components are complete.

A simple PERT/time network at system level showing the dependencies among the five subsystems which might be encountered in the development program is the best way to estimate the total system development time. Such a network may already have been constructed at an earlier planning stage. If so, a check of development times is facilitated.

Figure 9-8 is a simple network showing the development of System 4 using the estimates found and tabulated in Figure 9-7. The critical path is the sequence of events and activities which adds up to the longest time, in this case 12, 48, 30, and 4 months, or a total of 94 months. This estimate of minimum development time is that expected for a normal development, without overtime or special priority.

Subsystem	Complex-ity	State-of-Art	Estimated Development Time	Studies Special R & D	Total Development Time	Action
a	40	Conventional	30 months	No	30 months	*
b	50	Beyond	56 months (Minimum)	Feas. Study 12 months	68 months (Minimum)	**
c	120	Conventional	35 months	No	35 months	**
d	40	Conventional	30 months	No	30 months	*
e	50	Beyond	56 months (Minimum)	Component Develop. 12 months	68 months (Minimum)	***
	300					

*Defer development
**Start feasibility & development
***Start component development and special R&D

Figure 9-7. Estimated Subsystem Development Time

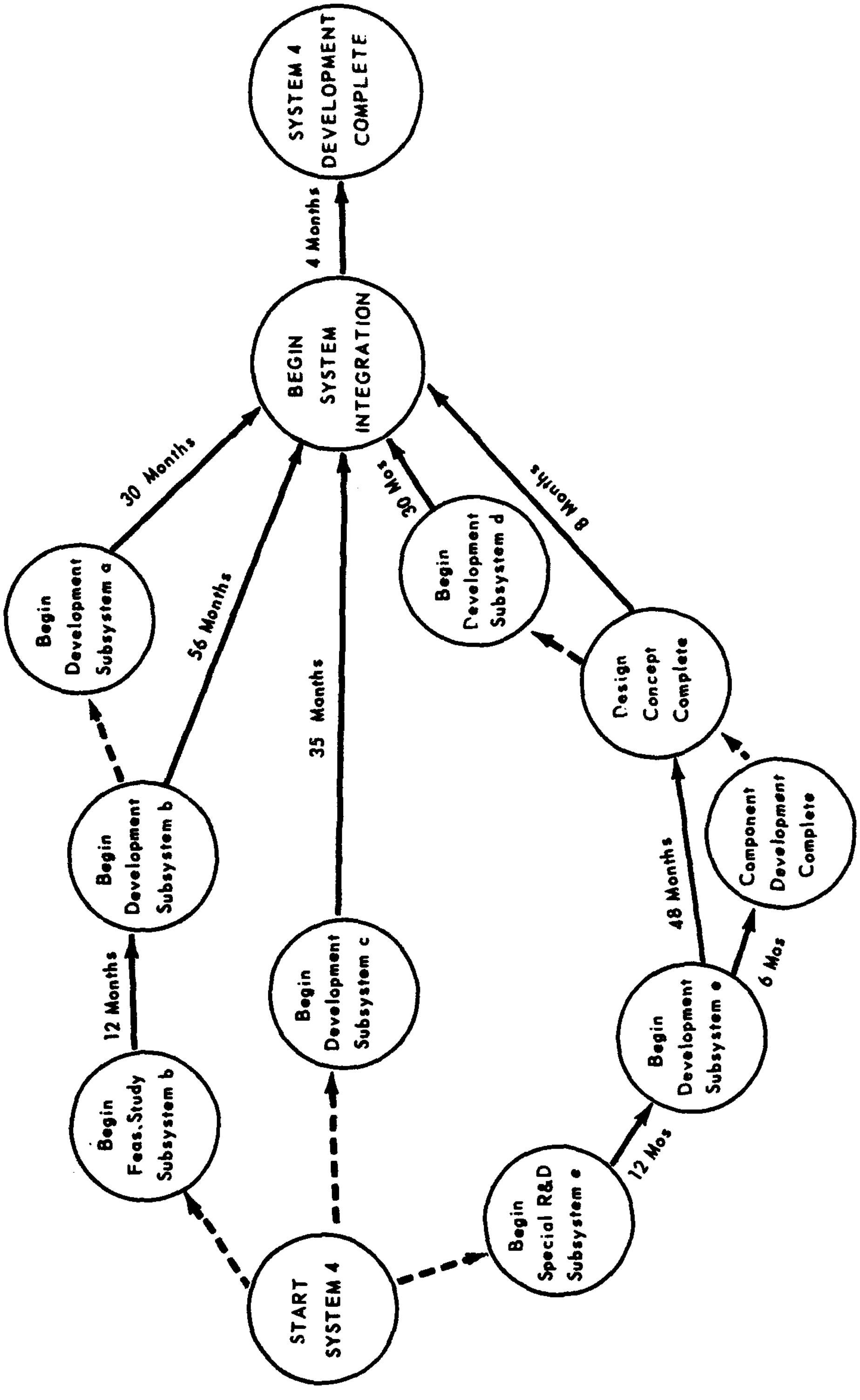


Figure 9-8. PERT/Time Network for System 4

STEP 3 – Evaluate the Feasibility of Meeting the Allocated Time Schedule

A comparison of the estimated time required for development with the lead time allocated by the operational force provides a measure of the feasibility of completing the development within the scheduled time limit. When allocated time is less than estimated required time, the probability of meeting the schedule is lessened; the greater the differential, the lower the probability. The PERT/time estimating and analysis procedure gives an actual measure of this probability.

Such an evaluation is made under the assumption that the development program will be normal and can be completed at the most economical rate. Although development progress can be accelerated to a minor extent, costs will be considerably increased thereby. A crash program with sudden acceleration will not improve progress, even at several times the normal cost.

In the example, assume that System 4 is approved for development and that definitive planning has been started. Thus, the TDP is under preparation for initial submission. If System 4 is to become operational with the Fleet on schedule, production must begin in March 1968. The earliest date a contract can be let is March 1964, permitting an elapsed time for development of 48 months. The schedule is infeasible, and a more realistic date to commence production would be January 1970.

Since the time allocated for development is grossly short of that estimated, the infeasible schedule must be reconsidered prior to completion of Section 5 of the TDP.

CHAPTER 10
NEW CONSIDERATIONS

10-1 INTRODUCTION

The familiar chart of MIL-STD-756A was presented in Figures 1-12 and 1-13, with several of today's Naval weapon systems superimposed for a graphical portrayal of "where we are today, on the basis of yesterday's designs". On the average, we have been buying equipment that ultimately demonstrated about one-seventh the MTBF that had been specified as a "requirement", because the quality assurance provisions of specifications have lacked the necessary acceptance-test "teeth" to *assure* product compliance with specified requirements. The most serious result of this absence of a statistically meaningful reliability acceptance test is that it has led development contractors to bid on the relatively simple task of assembling conventional building blocks into a particular configuration to satisfy *performance* requirements, with negligible engineering emphasis on the *reliability* aspects of design.

Absence of a firm requirement reduces the prospect of generating enough development test data to adequately determine the nature of the tolerance problems that are being designed into the system. There is no motive for effectively analyzing the data that are generated. Frequently, those analyses that are made do not get back to the designer in time. And when they do, there is seldom any follow-on evaluation of resulting design changes to assess the net gain or loss in reliability at the equipment level. Thus, the *extremely vital* feedback loop is not the dynamic guiding force that it must be if reliability is to be a "designed-in" system parameter.

Further, the absence of a firm reliability requirement leads to "passive" monitoring. It is not enough to passively monitor a development program – to maintain the "finger-on-pulse" awareness of its progress and problems. The project engineer must make "controlling" decisions that can enhance or degrade the prospects for reliability in his product. This active control function must depend upon close and informed monitoring based on the review of progress reports, failure analyses, prediction studies, and contractor reliability program activities. He cannot rely on PERT alone to force his decision – to do so can result in an innocent trade of reliability for "slack" for the sake of a target date that may be of secondary importance. Instead, he must require that PERT events specifically include reliability as one of the essential system characteristics.

In short, we seem to be up against a "reliability barrier" that defies penetration by the conventional standards of design. We can now forecast, within reasonably accurate bounds, the levels of reliability that can be achieved by a *conventional* design approach. These levels are not good enough.

What can the project engineer do to break the longstanding reliability barrier, to assure success in future system development programs? This section discusses some of the management and technical considerations which, if applied early enough in the planning stage of system development, can help break the so-called reliability barrier of conventional design. Cautionary notes are also sounded about the dangers of relying solely on the "wonders" of some of the newer concepts of system design and management.

10-2 DESIGN CONSIDERATIONS

10-2-1. General

At the outset of development program planning, the project engineer should have the practical feasibility of the stated reliability requirement evaluated, to determine where the requirement falls on the MTBF/complexity chart. Depending upon where the requirement falls, he should contemplate soliciting several alternate design proposals, to verify that his design requirement is fully understood by prospective designers in order to assure at the outset that his requirement will be satisfactorily demonstrated at the conclusion of the development program. If the requirement falls in the shaded area of the chart, he need have little worry, because conventional design has usually achieved this level without a formal reliability assurance and test program. If the requirement falls above the shaded area, there are several possible design choices to be made by the project engineer or to be suggested by him to prospective bidders as acceptable approaches.

The following alternate design approaches are keyed to the degree of reliability improvement required over conventional levels. The tradeoffs that will likely be required are discussed, as are specific applications in which one approach is more applicable than another.

10-2-2. Conventional Design

Considerations:

When the reliability requirement falls within the shaded area of the chart (Figures 1-12 and 1-13), it should be feasible to achieve and demonstrate the reliability requirement using conventional non-redundant design with conventional parts, if the following factors are given primary consideration:

- The initial choice of parts for the design should be based on *certified* life test data, to assure high inherent part reliability without excessive (and duplicative) parts testing. When such data are not available for a particular part type, a parts test program may be justified to verify vendor's "claimed" failure rates in the particular application.

- The choice of circuits for the design should be made from those of *proven* stability and reliability, supported by documented contractor experience.

- Parts should be derated for minimum failure rate consistent with circuit performance requirements. Derating should in general be accomplished in accordance with the procedures and factors presented in MIL-HDBK-217.

- Circuits involving semiconductors and diodes should be protected against transient voltage spikes, decoupled from common power sources, isolated from adjacent module interactions, and "buffered" in series configurations, to minimize failures due to interactions.

- Analog circuits should be stabilized against the variability of critical parts characteristics through temperature compensation and feedback stabilization.

Possible Tradeoffs:

The application of derating to conventional design can be expected to increase either the number or the physical size of active elements required for load-sharing in power circuits. Feedback stabilization of analog circuits can be expected to increase the number of series active elements to achieve the required level of performance. Thus, consideration should be given to both parallel and series redundancy at critical points in the design. The project engineer must be prepared to negotiate a trade of weight and space, and in some instances power, for the improved reliability promised by the proposed design.

10-2-3. Micro-Electronics

Considerations:

The micro-electronics program is off to a good start and is being properly evaluated on a continuing basis as development progresses, making it possible to draw on current test data for a prediction of reliability feasibility of equipment designs using the micro-module as the basic building block. A plot of micro-AEG failure rate as a function of temperature is shown in Figure 10-1, based on two sources of test data.[1] Each of the AEG's plotted consists of one transistor, one resistor, and one capacitor in a digital (switching) function. Test data from another source[2] indicate perhaps a 3-to-1 ratio between analog and digital AEG failure rates. On this basis, the figure can be adapted to analog AEG derating by multiplying the ordinate scale by three.

1/ Texas Instruments Report, First Quarter 1962; and Litton Interim Report dated 20 November 1962, corroborating the T.I. test results.

2/ RCA 17th Quarterly Report, July 1962.

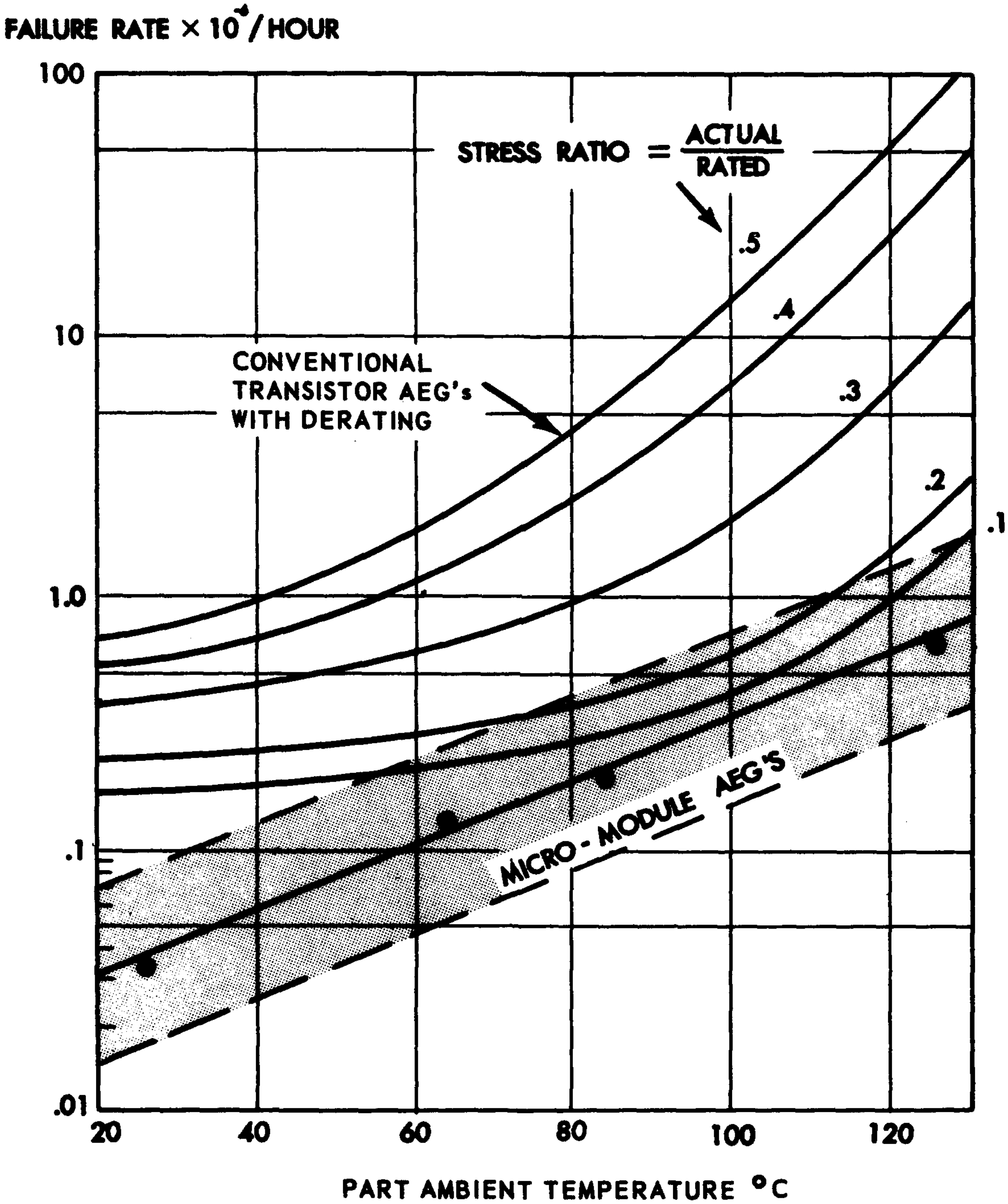


Figure 10-1. Derating Curves for Micro-Module and Conventional AEG's (Digital)

Although Figure 10-1 can be considered only as an "interim" plot, on the basis of very limited data early in the program there is evidence that the micro-AEG can achieve an order of magnitude (10-to-1) improvement in catastrophic failure rates over conventional transistor AEG's. As the micro-AEG program advances, it is expected that temperature/stress derating curves will be developed comparable to those that can now be developed from MIL-HDBK-217 for transistor AEG's. Further, as the life test program broadens in scope, various analog AEG reliability characteristics should become available for use in equipment planning and design. It will then be possible for the project engineer to precisely specify those applications within his equipment in which micro-electronics (or equivalent) will be required.

The fact that the micro-AEG may exhibit a great reduction of catastrophic failure rate below that of its transistor-AEG counterpart is only one of the attractive features of this new concept in electronic building blocks. Other equally important features include a 10-to-1 reduction in space and weight per circuit function and a comparable reduction in power requirements. These latter features make the micro-AEG a natural candidate for multiple redundancy in future designs.

Possible Tradeoffs:

Problems in micro-AEG's can and must be anticipated, however. They are constructed of the same basic materials used in present semiconductor devices. The project engineer should expect the same characteristic variability, instability, and deterioration problems in micro-AEG's as now are experienced in transistor AEG's. He must therefore contemplate the need for isolation, decoupling, transient protection, stabilization, and derating, in order to fully exploit the inherent reliability characteristics of the new AEG. The extent to which these considerations will apply has not yet been full assessed. For planning purposes, it must be anticipated that from 15 to 30 of present-day circuit functions will *not* be replaceable by micro-AEG's – these are the power generation and conversion functions and other special functions that must still depend on the use of conventional parts.

10-2-4. Redundancy

Considerations:

Whether the basic building block is the conventional transistor AEG or the prospective micro-AEG, there will arise instances in which the only solution to a circuit reliability problem is the application of redundancy. Some of these instances were pointed out above – when power devices are derated and it becomes necessary to provide load-sharing (parallel) redundancy or stabilization feedback (series) redundancy. In other cases, certain critical circuit functions whose failure rates still remain relatively too high (even with derating) must be protected with redundancy. Thus, to achieve a 50-to-1 improvement in reliability above conventional design, it is practically certain that redundancy in one form or another will be required.

Redundancy can be divided into two general classes:[3/] operative and standby. Operative redundancy is required when manual switching of standby units is out of the question. Automatic switching requires the added complexity of sense/switch devices to detect failure of one element and switch to the standby. On the

3/ Refer to Appendix 4 for a detailed discussion of redundancy in design.

other hand, in the operative case, all redundant elements continually draw power, thereby increasing load demands on the power supply which instead should be further derated.

The design engineer must assess the proposed design for the most advantageous application of redundancy, in order first to determine the points in the series chain at which redundancy is required and then to evaluate the level of complexity at which an optimum yield in reliability is assured.

EXAMPLE: A critical module is the "weak link" in a proposed equipment design. Predicted reliability of the module is .9 for the period of time of interest; yet a reliability of .99 is required. The designer considers two alternatives:

(1) **Redundant Modules:**
Figure 10-2 shows two modules in parallel, each with reliability, R = .9. Reliability of the pair is R = .99.

(2) **Redundant Parts:**
The module is exploded into its parts configuration, as shown in Figure 10-3. All blocks except No. 5 have a reliability of .9999. Block No. 5 has a reliability of slightly higher than .9 (actually .901). Block No. 5 can be made redundant to produce the desired module reliability of .99.

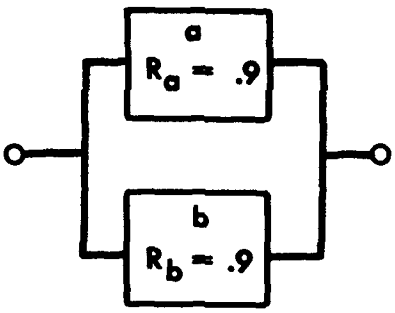


Figure 10-2. Redundancy at Module Level

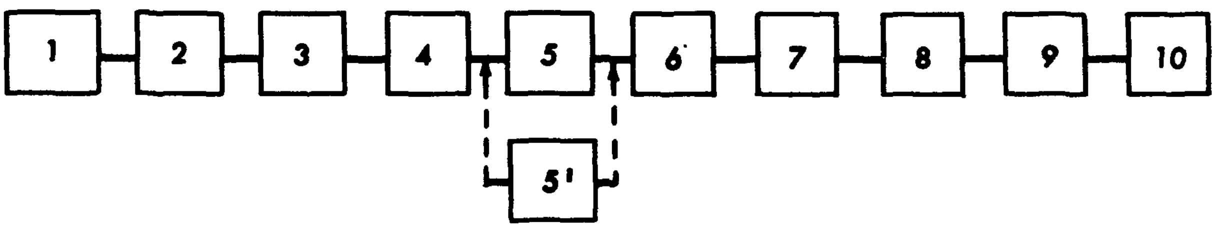


Figure 10-3. Reliability at Part Level Within Module

The differences in the two alternatives in this example are:

(1) This design requires twice the weight, space, and power consumption, but is easier to test at preflight checkout to verify its operational status; cost is approximately twice the cost of a single module.

(2) This design requires only a 5% increase in weight, space, and power consumption, but requires additional test-point provisions for preflight determination of operational status.

Possible Tradeoffs:

Space, weight, and power must be traded for reliability with redundancy. In addition, availability must be traded, to the extent that additional preflight test time is required to verify that all elements in a redundant configuration are operational at the start of a mission – an essential condition if the reliability advantages of redundancy are to be realized.

10-2-5. Digital/Analog Hybrids

Considerations:

Many analog functions can be performed with digital circuits by appropriate analog-to-digital conversion at the equipment input and a corresponding digital-to-analog conversion at the equipment output. Although a reliability gain of about 10-to-1 is reasonable (digital AEG over analog AEG), it generally requires several times more digital AEG's to perform the analog function. The net gain to be expected is further reduced by the difficulty of achieving reliable D/A and A/D conversion.

Some of the principal advantages of digital switching logic over analog servo loops are the relative simplicity of design, permitting a higher degree of standardization among modules and circuit cards; the relative freedom of digital circuits from drift and tolerance problems; and the ease with which redundancy can be applied.

10-2-6. Redundancy-With-Repair

Considerations:

If the proposed new system is to be accessible to maintenance *during* the mission cycle (shipboard and shore-based systems), it may be more desirable to accept conventional design *reliability* and consider an improved "availability" design concept. Consideration should then be given to redundancy-*with-repair* techniques, which provide means for the detection, isolation, and replacement of redundant element failures without producing system failure. These techniques are discussed in more detail in Appendix 4.

Possible Tradeoffs:

The redundancy-with-repair concept depends upon an *effective* monitoring system to detect and localize the loss of redundant elements. Standby elements must then be switched in (either automatically or manually); or, in the operative case, the failed unit must be switched out and a replacement made before the remaining element of the redundant pair fails, producing a system failure. These monitoring systems can become very complex – to the extent that the practical advantages of the with-repair concept is lost. To avoid this, [illegible]s necessary to specify reliability requ[illegible]ments for the monitoring function itself.

10-3 PROGRAM PLANNING CONSIDERATIONS

10-3-1. General

Following the procedures of Chapter 2, the project engineer must establish a reliability requirement for the product he is to develop, define the requirement in clear terms in the design specification, and reference the specification in both the RFP and the contract statement of work.

Often the requirement has not been defined in the implementing TDP or it cannot be derived from the system or "airframe" project office because certain conditions will not become known until later. It is then necessary to establish requirements on the basis of past conditions and requirements. In this case, the equipment specification becomes a design guidance document for the larger system.

Once the level of tactical reliability is established, there are certain factors to consider in the definition of reliability program plans which will assure with confidence that the stated requirement *will be met*, or, conversely, will reveal far in advance of prototype acceptance testing that the requirement *cannot be met* with the proposed design approach.

The following new considerations are outlined, for planning the development of Naval weapon systems in the future.

10-3-2. Application of a "Margin of Reliability Safety"

Design margins are applied to other system design parameters as generally accepted good engineering practices – where the "strength" distribution of the design is kept at a safe distance from the distribution of anticipated stresses. Consideration should be given to incorporating a margin of reliability safety in the specified requirement, to account for errors in prediction, measurement, and test conditions.

It is important at the outset to insure that specified reliability requirements remain consistent from one phase to the next in the equipment life cycle – to make clear what is meant by "required MTBF" or "required reliability". A *good guide to follow* is based on a complete definition of the tactical requirement, which then sets the basis for all subsequent requirements. Figure 10-4 illustrates how the tactical requirement should be expressed and interpreted.

The interpretations made in the example used in the figure are always on the conservative side – to provide a *margin of reliability safety*, just as we provide safety margins in all other design procedures. By this procedure, the project engineer will occasionally observe *more* equipment reliability than he actually asked for – a predicament that in no way reflects discredit on his engineering management capability!

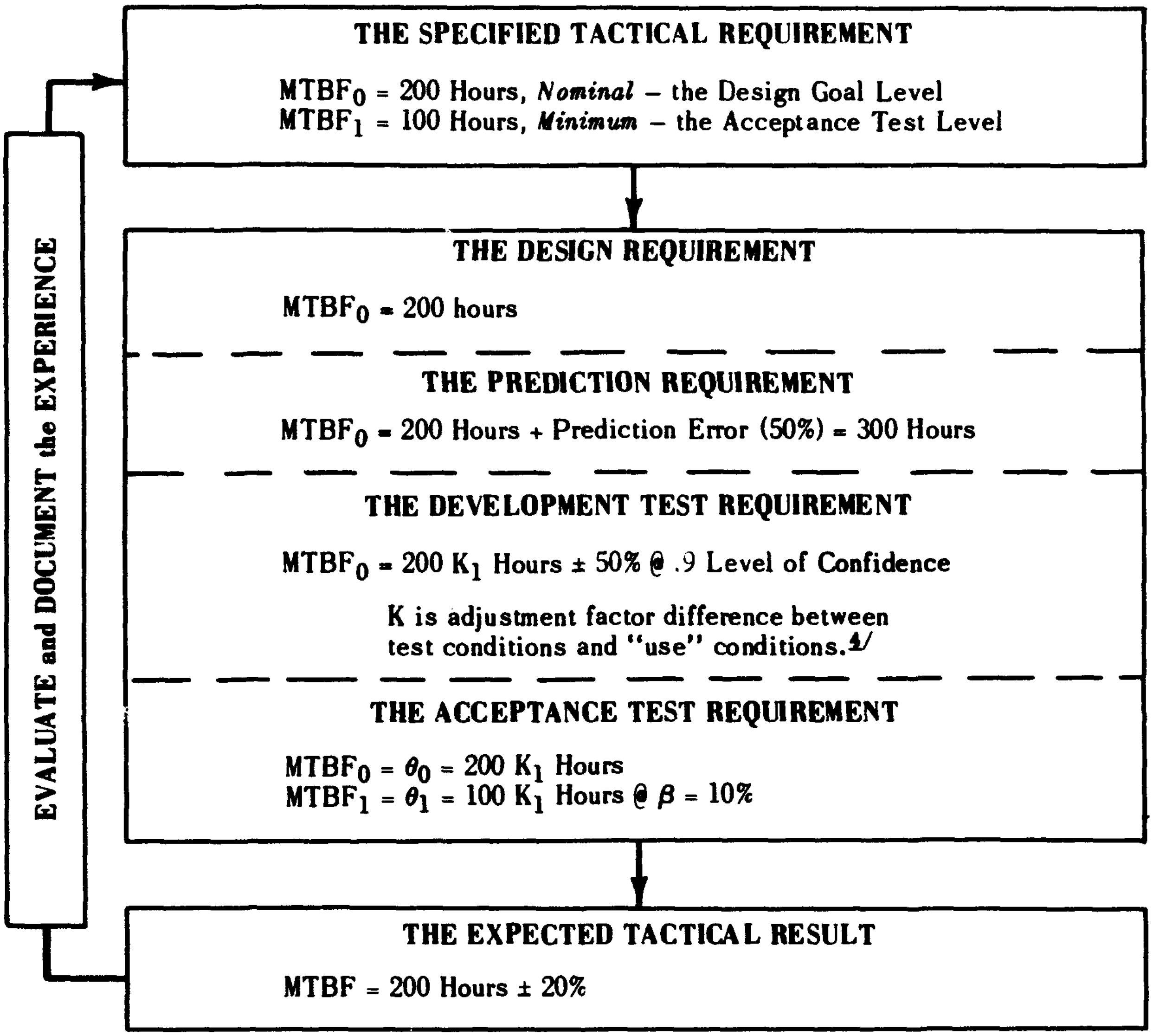


Figure 10-4. Safety Margins in Reliability Definition

4/ Values of K_1 depend upon the realism of the test environment. $K_1 = 1$ when test conditions are approximately equivalent to anticipated use conditions.

10-3-3. Reliability Monitoring by PERT

Management monitoring of system reliability status and reliability program operations can and should be accomplished coincidentally with PERT time and cost monitoring, for obviously there is little value in monitoring the production of an "unknown". Erratic management decisions are frequently forced by the pressure of a PERT schedule slippage, in complete ignorance of the reliability consequences of the decision. The insensitivity of most PERT monitoring systems to the reliability aspects of development is attributable to the absence of adequate reliability *requirements* documentation in the PERT network.

Project management should specify requirements for PERT reliability monitoring in the TDP, the RFP, and subsequent contract documentation pertaining to the proposed new development program. The contractor's proposed schedule of PERT activities and events (milestones) should be reviewed to determine that the following monitoring requirements are satisfied:

- Reliability activities are either integrated into the PERT/time network, or are shown in a "reliability program network" as an overlay to the basic PERT/time network.
- Reliability milestones are quantitatively documented as PERT event requirements, to clearly define event success/failure criteria.
- PERT reporting format and procedures have provisions for reporting reliability growth status with respect to goals, and problem status with respect to schedule.
- PERT input data requirements include contractor estimates of reliability/time and reliability/cost tradeoffs to provide management with the necessary "consequence" information for decision making.

10-3-4. Reliability as a Contract Incentive

As outlined in Chapter 7, two values of reliability should be specified: one for design guidance; the other for acceptance testing. Consideration should be given to the award of incentive type contracts which provide that the government pays *no fee* for anything less than the "minimum acceptable", but pays fee according to a sliding scale for demonstration of reliability in excess of minimum requirements. This furnishes incentive for the contractor not only to keep his reliability techniques "sharp" but to develop and apply new improved techniques as a matter of "good business" policy. Two major requirements must be satisfied, however, to insure that an incentive contract is both equitable and workable:

- Minimum acceptable requirements must be realistically compatible with levels achievable by good (current) state-of-art techniques;
- Decision criteria for determination of incentive fee eligibility must be clearly defined by test parameters that include α and β errors that are mutually understood and agreed upon.

APPENDIX 1. DEFINITIONS OF TERMS AND SYMBOLS USED IN THE HANDBOOK

The more important terms and symbols used in the handbook are presented alphabetically in this section of the appendix. Wherever possible the definitions are drawn directly from MIL Standard 721, IRE and ASQC Standards, MIL Handbook 217, and other sources. Some liberties have been taken, however, to simplify and clarify certain of these definitions for the benefit of the handbook user. A more comprehensive glossary of reliability and quality control definitions can be found in the IRE-ASQC Reliability Training Text.[1]

1.1 LEGEND OF REFERENCED SYMBOLS

α	(Alpha) Producer's Risk
A	Availability
A_c	Acceptance Number
AEG	Active Element Group
AQL	Acceptable Quality Level
ASN	Average Sample Number
β	(Beta) Consumer's Risk
B	Base Failure Rate
CEP	Circular Error Probability
D	Dependability
E	Effectiveness
f	Failure
f.r.	Failure Rate (see also λ)
G	Acceleration Level
H_0	Null Hypothesis
H_1	Alternate Hypothesis
k_t	Tolerance Factor
k_u	Use Factor
λ	(Lambda) Failure Rate
L	Longevity
Ln	Natural Logarithm = Log_e
LTPD	Lot Tolerance Percent Defective
M	Maintainability
M_c	Corrective Maintenance
MC	Military Characteristic
MCF	Mean Cycles to Failure
M_p	Preventive Maintenance
MTBF	Mean-Time-Between-Failures
MTF	Mean-Time-To-Failure
MTR	Mean-Time-To-Repair or Restore
μ	(Mu) Repair Rate; Mean of Normal Distribution; Average Life
N	Number of AEG's
n	Sample Size
OC	Operating Characteristic (Curve)
π	(Pi) Product of a Series
P	Performance Capability
P	Probability
p	Probability of success of an element
p	Actual or true proportion defective of a quantity of items constituting a "lot" or "test group".
p_0	Nominal desired or specified value of proportion defective associated with the producer's risk (α) of a test plan.
p_1	Maximum acceptable value of proportion defective accepted by a test plan with consumer's risk (β).
P_a	Probability of Acceptance
P_s	Probability of Survival or Success
P_r	Probability of Repair (Repairability)
P_k	Kill Probability

[1] Bibliography item 13.

Q	Failure Probability (Q = 1 - P)	R(t)	Reliability as a function of time
q	Probability of Element Failure	r	Number of failures
QA	Quality Assurance	r_0	The number of failures at which an acceptance test is truncated
QC	Quality Control	σ	(Sigma) Standard Deviation
QPL	Qualified Products List	Σ	(Sigma) Sum of a Series
$\underline{R}$	Operational Reliability ($R = R_i \times R_u$)	S	Stress
$\bar{R}$	Unreliability ($\bar{R}$ = 1 - R)	SOR	Specific Operational Requirement
R_i	Inherent Reliability	θ	(Theta) Mean Life, MTBF
R_0	Nominal or desired level of reliability stated as a requirement. (Either θ_0 or p_0, as applicable, is calculated from R_0 for the design of an acceptance test.)	$\hat{\theta}$	(Theta Caret) Estimated Mean Life
		TDP	Technical Development Plan
		t	Time
		t_a	Administrative Downtime
R_1	Minimum acceptable reliability stated as a requirement. (Either θ_1 or p_1, as applicable, is calculated from R_1 for the design of an acceptance test.)	t_m	Mission Time
		t_r	Repair Time
		U	Unreliability
		$\bar{x}$	Mean or Average Value

1.2 DEFINITIONS OF TERMS

Accelerated Test Conditions – Test conditions that are made more severe than recommended use conditions, in order to "accelerate" the occurrence of failures and thus shorten the test time required for evaluation of reliability.

Acceptable Quality Level – The value of percent defective associated in a sampling plan with the producer's risk.

Acceptance Number – The largest number of defectives that can occur in a sample from an inspection lot and still permit the lot to be accepted.

Acceptance Sampling Plan – A procedure which specifies the number of units of product which are to be inspected (sample size or series of sample sizes) and the criteria for determining acceptability (acceptance and rejection numbers).

Acceptance Sampling – A procedure in which decisions to accept or reject are based on the examination of samples.

Acceptance Tests – Tests to determine conformance to design or specifications as a basis for acceptance. They may apply to parts, equipments, or systems.

Active Element – A part that converts or controls energy; e.g., transistor, diode, electron tube, relay, valve, motor, hydraulic pump.

Active Element Group – An active element and its associated supporting (passive) parts; e.g., an amplifier circuit, a relay circuit, a pump and its plumbing and fittings.

Active Repair Time – That portion of downtime during which one or more technicians are working on the system to effect a repair. This time includes preparation time, fault-location time, fault-correction time, and final checkout time for the system, and perhaps other subdivisions as required in special cases.

Administrative Downtime – That portion of system or equipment downtime not included under active repair time and logistic time.

Arithmetic Mean – The sum of a set of values divided by the number in the set.

Assembly – A number of parts or subassemblies joined together to perform a specific function.

Assurance – The relative confidence or certainty that specific program objectives will be achieved.

Attribute – A characteristic or property that a product either does or does not have; e.g., shorts and opens in electronic parts, leaks in hydraulic lines, "stiction" in bearings.

Attributes Testing – "Go/no-go" testing to evaluate whether a property does or does not fall within specification limits. The product is accepted if the property falls within these limits but is rejected if the product does not fall within them; the specific value of the property in either case is not tested.

Availability (Operational Readiness) – The probability that at any point in time the system is either operating satisfactorily or ready to be placed in operation on demand when used under stated conditions.

Average – The arithmetic mean; the average of a set of n numbers, $x_1, x_2, \ldots x_n$, is the sum of the numbers divided by n;

$$\bar{x} = \frac{x_1 + x_2 + \ldots + x_n}{n}$$

Average Life – The mean value for a normal distribution of lives. The term is generally applied to mechanical failures resulting from "wearout".

Average Sample Number – The average number of sample units inspected per lot in reaching a decision to accept or to reject.

Basic Failure Rate – The basic failure rate of a product derived from the catastrophic failure rate of its parts, before the application of use and tolerance factors. The failure rates contained in MIL-HDBK-217 are "base" failure rates.

Breadboard Model – An assembly of preliminary circuits and parts to prove the feasibility of a device, a circuit, an equipment, a system, or a principle in rough or breadboard form, without regard to the eventual overall design or form of the parts.

Catastrophic Failure – A sudden change in the operating characteristics of an item resulting in a complete loss of useful performance of the item.

Censored Data – Data from sample items when the actual values pertaining to such data are unknown; e.g., when it is known merely that the data either exceed or are less than some value.

Chance Failure – That failure which occurs at random within the operational time of an equipment, after all efforts have been made to eliminate design defects and unsound components and before wearout becomes predominant.

Characteristic – A trait, quality, or property distinguishing an individual, group, or type.

Checkout Time – The time required to determine that a system or equipment is in satisfactory operating condition.

Circular Error Probable – The radius of the circle within which 50% of the shots are designated to land.

Complexity Level – A measure of the number of active elements required to perform a specific system function.

Confidence Level – The probability that a given statement is correct; the chance that a given value lies between two confidence limits (the confidence interval).

Confidence Limits – Extremes of a confidence interval within which the true value has a designated chance (confidence level) of being included.

Consumer's Reliability Risk (β) – The risk, or probability, that a product will be accepted by a reliability test when it should properly be rejected.

Controlled Process – A process tested or verified by counter or parallel evidence of experiment.

Controlled Test – A test designed to control or balance out the effects of environmental differences and to minimize the chance of bias in the selection, treatment, and analysis of test samples.

Critical Defect – A defect that judgment and experience indicate could result in hazardous or unsafe conditions for individuals using or maintaining the product; or–for major end item units of product such as ships, aircraft, or tanks–a defect that could prevent performance of their tactical function.

Debugging – A process of "shakedown operation" of a finished equipment performed prior to placing it in use. During this period, defective parts and workmanship errors are cleaned up under test conditions that closely simulate field operational stresses. The debugging process is not, however, intended to detect inherent weaknesses in system design. These should have been eliminated in the preproduction stages by appropriate techniques.

Degradation Failure – A failure which occurs as a result of a gradual or partial change in the characteristics of some part or parameter; e.g., drift in electronic part characteristics, changes in lubricant with age, corrosion of metal.

Derating – The technique of using a part, component, or equipment under stress conditions considerably below rated values, to achieve a "reliability margin" in design.

Design Adequacy – The probability that the system will satisfy effectiveness requirements, given that the system design satisfies the design specification.

Discrimination Ratio – A measure of steepness of the OC curve for an acceptance test between the AQL and the LTPD; i.e., the capability of the test to discriminate between "good" and "bad" product. Numerically, $k = LTPD/AQL$.

Downtime – The total time during which the system is not in condition to perform its intended function. (Downtime can in turn be subdivided in the following categories: corrective maintenance time, preventive maintenance time, logistic time, and administrative time.

Early Failure Period – That period of life, after final assembly, in which failures occur at an initially high rate because of the presence of defective parts and workmanship.

Effectiveness – The probability that the product will accomplish an assigned mission successfully whenever required.

Element – One of the constituent parts of anything. An element, in fact may be a part, a subassembly, an assembly, a unit, a set, etc.

Environment – The aggregate of all the external conditions and influences affecting the life and development of the product.

Equipment – A product consisting of one or more units and capable of performing at least one specified function.

Exponential Case – The reliability characteristics of those products known to exhibit a *constant* failure rate. Reliability in the exponential case is given by $R = e^{-\lambda t}$, where λ is the failure rate and t is the period over which reliability is measured.

Exponential Reliability Function – A frequency distribution that is completely defined by its mean (MTBF) which occurs when x = 1, ($R = e^{-1} = .368$).

Failure – The inability of a product to perform its required function.

Failure Mode Analysis – A study of the physics of failure to determine exactly how a product fails and what causes the failure.

Failure Rate – The expected number of failures in a given time interval. (For an exponential distribution of times to failure, the failure rate is approximately equal to the reciprocal of the mean life.)

Failure Probability – The probability of failure in a specified period of time.

Free Time – The time during which operational use of the product is not required. This time may or may not be downtime, depending on whether or not the system is in operable condition.

Gaussian Distribution – A density function which is bell-shaped and symmetrical. It is completely defined by two independent parameters, the mean and standard deviation.

Geometric Mean – The arithmetic mean of the sum of the logarithms of a series of numbers, or, algebraically,

$$\bar{X}_G = \sqrt[n]{A_1 A_2 A_3 . \ldots A_n}$$

Goal – A long-term requirement implied by specification or contract and used primarily for guidance. Goals are usually not legally binding because no acceptance test requirements are imposed.

Heterogeneity – A state or condition of dissimilarity of nature, kind, or degree.

Homogeneity – A state or condition of similarity of nature, kind, or degree; e.g., two tube types found to have the same probability of removal are said to be homogeneous.

Human Error Reliability Criteria – Criteria used in the design of a complex system to adapt its physical features to the response characteristics of the man who is ultimately to be charged with its operation, in order to minimize reliability degradation due to operator (and maintenance technician) error. Typical criteria include size, shape, and location of critical controls; illumination and configuration of visual displays; use of automatic error detection and warning devices; modularization and physical arrangement for maintenance ease.

Human Factor Engineering – A branch of engineering that treats a complex equipment as a unified man-machine system, to assure quantitative consideration of operator and maintenance influence on system performance, reliability, and maintainability.

Hypothesis – An unproven proposition which remains subject to doubt until proven true.

Importance Factor – The relative importance of a particular equipment to total mission effectiveness, expressed as the permissable ratio of the number of mission failures due to the equipments failing to the total number of failures of the equipment.

Independent Failures – Those failures which occur or can occur without being related to the malfunctioning of associated items. In the development of the exponential failure law, it is essential to assure that each source of potential independent failure which results in the complete malfunction of the equipment under consideration be included. In electronic systems, signals are usually cascaded and power sources are non-redundant so that nearly all component parts introduce independent sources of catastrophic failure. Such independent failures are, therefore, the normal occurrence rather than the exception.

Induced Environment – The conditions of shock, vibration, temperature, acceleration, pressure, and so forth, that are imposed upon the system by its particular application.

Infant Mortality – Premature catastrophic-type failures occurring at a rate substantially greater than that observed during subsequent life prior to wearout. Infant mortality is usually reduced by stringent quality control.

Inherent Reliability – The reliability potential in a given design configuration.

Inspection by Attributes – Inspection wherein the unit of product is classified simply as defective or nondefective with respect to a given requirement or set of requirements. If desired, the degree of nonconformance may be further categorized through the use of such classifications as critical, major, and minor.

Inspection by Variables – Inspection wherein a specified quality characteristic of a unit of product is measured on a continuous scale, such as pounds, inches, or feet per second, and a measurement is recorded; or inspection wherein certain characteristics of the sample units are evaluated with respect to a numerical scale and are expressed as precise points along this scale. The distribution of these points, as established by measures of their central tendency and dispersion, are mathematically related to specified requirements to determine the degree of conformance of the characteristics.

Inspection Level – A term used to indicate the number of sample units required for inspection of a given amount of product. All other things being equal, a higher inspection level entails a lower risk of acceptance by the government of a lot of inferior quality, and a lower inspection level entails a higher risk.

Inspection Lot – A collection of units of product manufactured or processed under substantially the same conditions and offered for inspection at one time, or during a fixed period of time.

Interaction – The influence of one subsystem or subassembly on the performance and reliability behavior of another. Although gross effects are qualitatively predictable, specific interaction effects must usually be determined by breadboard and development testing.

Interchangeability – The ability to interchange, without restriction, like equipments or portions thereof in manufacture, maintenance, or operation.

Interfaces – Boundary conditions and requirements existing between two or more "mating" subsystems or components - - e.g., impedance matching, structural fitting, thermal and vibration levels.

Intrinsic Availability – The probability that the system is operating satisfactorily at any point in time when used under stated conditions, where the time considered is operating time and active repair time.

Kill Probability – The probability that a target will be destroyed. (See also "Effectiveness".)

Level of Significance – The probability that a decision to reject a null hypothesis will be made.

Logistic Downtime – That portion of downtime during which repair is delayed solely because of the necessity for waiting for a replacement part or other subdivision of the system.

Longevity – Length of useful life of a product, to its ultimate wearout requiring complete rehabilitation. This is a term generally applied in the definition of a safe, useful life for an equipment or system under the conditions of storage and use to which it will be exposed during its lifetime.

Lot Size – A specific quantity of similar material or collection of similar units from a common source; in inspection work, the quantity offered for inspection and acceptance at any one time. It may be a collection of raw material, parts, or subassemblies inspected during production, or a consignment of finished product to be sent out for service.

Lot Tolerance Percent Defective – That value of percent defective associated in a sampling plan with the consumer's risk; i.e., the value of lot percent defective on an OC curve corresponding to the value of β.

Maintainability – The probability (when maintenance action is initiated under stated conditions) that a system will be restored to its specified operational condition within a specified period of downtime.

Maintainability Function – A plot of the probability of repair within time t, versus maintenance time.

Maintenance Capabilities – The facilities, tools, test equipment, drawings, technical publications, trained maintenance personnel, engineering support, and spare parts required to restore a system to serviceable condition.

Maintenance Ratio – The number of maintenance man-hours of downtime (t_m) required to support each hour of operation (t_o); i.e., $M = t_m/t_o$. This figure reflects the frequency of failure of the system, the amount of time required to locate and replace the faulty part, and to some extent the overall efficiency of the maintenance organization. This method of measurement is valuable primarily to operating agencies since, under a given set of operating conditions, it provides a figure of merit for use in estimating maintenance manpower requirements. The numerical value for maintenance ratio may vary from a very poor rating of 5 or 10 down to a very good rating of 0.25 or less.

Marginal Testing – A procedure for system checking which indicates when some portion of the system has deteriorated to the point where there is a high probability of a system failure during the next operating period.

Mean Life – The arithmetic average of population life.

Mean Cycles to Failure – The average number of cycles to failure of nonrepairable items; i.e., the total number of cycles under specified conditions divided by the number of failures (the mean cycles to failure is the reciprocal of the failure rate per cycle).

Mean Cycles Between Failure – The average number of operating cycles between failures (applicable to repairable items).

Mean-Time-To-Failure – The average length of time to failure of nonrepairable items, i.e., the total operating time under specified conditions divided by the number of failures during this time (in the exponential case, the mean-time-to-failure is the reciprocal of the failure rate per unit time).

Mean-Time-Between-Failures – The mean operate time between failures (applicable to repairable items).

Mean-Time-To-Repair – A measure of repairability, expressed as the total repair time over a specified period divided by the total repairs made during that period.

Micro-Electronics – A name that has been adopted to indicate the use of miniaturization techniques in the fabrication of replaceable modules, e.g., micromodules, solid-state circuits.

Military Characteristic – Those essential qualities which a system must possess to fulfill a specific military requirement. (See also "Specific Operational Requirement".)

Mission Profile – A description of system environmental and use duty cycles throughout the mission period for which reliability is to be specified.

Mission Reliability – The probability that, under stated conditions, the system will operate in the mode for which it was designed (i.e., with no malfunctions) for the duration of a mission, given that it was operating in this mode at the beginning of the mission.

Mission Time – See "Operating Time".

Module – An assembly, subassembly, or unit packaged for ease of maintenance of the next higher level of assembly, usually in "plug-in" form.

Module – An assembly, subassembly, or component packaged for ease of maintenance, usually in "plug-in" form.

Multiple Sampling – Sampling inspection in which, after each sample is inspected, the decision is made to accept, to reject, or to take another sample; but in which there is a prescribed maximum number of samples, after which decision to accept or to reject must be reached.
NOTE: Multiple sampling as defined here sometimes has been called "sequential sampling" or "truncated sequential sampling".

Natural Environment – External conditions, such as temperature, humidity, pressure, solar radiation, rain, snow, hail, or wind, under which the system is to operate when tactically deployed.

Natural Logarithm – Log to the base 2.71828.

Normal Distribution–See "Gaussian Distribution".

Null Hypothesis – An assumed proposition used for the purpose of statistical test.

Objectives – See "Goals".

On-Line Maintenance – Maintenance performed on a system or equipment without interrupting its operation. (See also "Reliability-With-Repair".)

Operating Characteristics (OC) Curve – The quality curve which shows for a particular sampling plan the relation between (1) the fraction defective in a lot and (2) the probability that the sampling plan will accept the lot.

Operating Time – The time during which a system or equipment is actually operating (in an "up" status). Operating time is usually divisible among several operating periods or conditions, e.g., "standby time", filament "on-time", preflight "checkout" time, flight time.

Operating Mode – A specific function or level of performance by which system performance is described.

Operational Equipment–An equipment which when given the opportunity to perform its intended function does so within its design limits.

Operational Maintenance – Maintenance that is performed without interrupting the satisfactory operation of the system. (See also "On-Line Maintenance".)

Operational Readiness – See "Availability".

Operational Reliability – The probability that the system will give specified performance for a given period of time when used in the manner and for the purpose intended. It consists of the inherent equipment reliability as degraded by various application factors peculiar to each particular field condition (use reliability). The operational reliability is thus peculiar to individual situations and is not a measure of inherent equipment reliability. As the conditions of use approach those under which the inherent equipment reliability was measured, and as the operation and maintenance approach the quality of that provided during the factory evaluation, then the operational reliability will approach the inherent equipment reliability.

Part – An element of a subassembly, or an assembly, of such construction that it is not practical to disassemble the element for maintenance purposes.

Part Failure – A breakdown or a partial change in some parameter or characteristic necessitating replacement of the part to restore satisfactory operation of a higher assembly; e.g., drift in resistor value, shorted motor winding.

Percent Defective – That proportion of a lot which is defective.

Performance Capability – The probability that the system or equipment will perform its intended function when operating within specified design limits.

Pilot Production – Production of a limited quantity of an item using as nearly the same tooling, methods, and inspection techniques as will be used in the full production.

Population – In statistical terminology, any set of individuals, objects, or measurements–real or hypothetical, finite or infinite in number–having some common characteristic.

Precision of Estimate – The size of the interval within which the population parameter can be expected to lie for a fixed proportion of the times it is estimated, when the parameter is being estimated by means of a sample statistic. Precision of estimating varies with the square root of the number of observations on which it is based.

Prediction Techniques – Methods for estimating future behavior of a system on the basis of knowledge of its parts, functions, and operating environments, and of their interrelationships.

Preventive Maintenance – A procedure in which the system is periodically checked and/or reconditioned in order to prevent or reduce the probability of failure or deterioration in subsequent service.

Probability – The likelihood of occurrence of a particular event, measured by the ratio of the number of ways an event actually occurs to the total number of possibilities.

Probability of Acceptance – Probability that a lot or process will be accepted.

Probability of Survival – The likelihood of an item's performing its intended function for a given period of time or number of duty cycles, measured by the ratio of the number of survivors at time, t, to the population at the beginning of the period.

Producer's Reliability Risk (α) – The risk that a batch of goods of acceptable reliability will be rejected by a reliability test.

Prototype – A model suitable for complete evaluation of mechanical and electrical form, design, and performance. It is of final mechanical and electrical form, employs approved parts, and is completely representative of final equipment.

Qualification Test – Such testing of a product as may be necessary to determine whether or not the product conforms to qualification requirements in the applicable specification. Qualification testing is normally conducted independently of a procurement action and at the request of a supplier seeking inclusion of his product in a Qualified Products List.

Qualified Products List (QPL) – A list of items that have been tested and approved for use in military systems, as authorized by Armed Service Procurement Regulation (ASPR).

Quality – An attribute or characteristic of a product. In the broadest sense, "quality" embraces "reliability"; i.e., "reliability" is a characteristic of the product.

Quality Assurance – A broad term used to include both quality control and quality engineering. (See MIL-Q-9858.)

Quality Characteristics – Those properties of an item or process which can be measured, reviewed, or observed, and which are identified in the drawings, specifications, or contractual requirements. Reliability becomes a quality characteristic when so defined.

Quality Control – A production-oriented operation for causing a process to manufacture a uniform product within specified limits of percent defective in accordance with design requirements.

Quality Engineering – A production-oriented operation for establishing quality tests and quality acceptance criteria and for interpreting quality data. Quality engineering begins in the early design phase, however, to assure the required level of inherent quality in the design ultimately to be produced.

Random Failure – A failure which occurs at an unpredictable point in time.

Random Sample – A sample in which each item in the lot has an equal chance of being selected in the sample.

Redundancy – The existence of more than one means for accomplishing a given task.

Rejection – An action by the customer indicating nonacceptance of material. In most cases material is rejected as being nonacceptable with regard to certain features, with the understanding that upon correction the material may be resubmitted for inspection and acceptance.

Reliability – The probability of performing without failure a specified function under given conditions for a specified period of time.

Reliability Assurance – The exercise of positive and deliberate measures to provide confidence that a specified reliability will be obtained.

Reliability Control – The coordination and direction of technical reliability activities through scientific planning from a system point of view. There is no sharp distinction between modern reliability control and the usual engineering management and production methods of improving reliability. Nevertheless, it is important to recognize that reliability control differs in degree from conventional methods in three respects: first, overall system planning is emphasized; second, statistical analysis of failure data and reliability accomplishment is used as a control; and third, constant surveillance of the feedback of pertinent data is required in all phases of development, design, production, and use.

Reliability Goal – The level of reliability desired of a design, often expressed as the reliability design "objective" for development *guidance*, as contrasted with the minimum acceptable reliability which is expressed as a development *requirement*.

Reliability Life Test – Testing of a sample under specified conditions for predetermined periods of time or until a predetermined number of failures has occurred, for the purpose of estimating the mean-time-to-failure or mean-time-between-failures at a specified confidence level.

Reliability Operating Characteristic Curve – The operating characteristic of a reliability acceptance test.

Reliability Requirement – A level of reliability expressed in an equipment specification as a design requirement and supported with a reliability acceptance test.

Reliability-With-Repair – Reliability achieved through the use of redundancy to permit "on-line" repairs or replacement of redundant units without interruption of system operation. (See also "On-Line Maintenance".)

Reliability Index – A figure of merit, such as a ratio or factor, that is used to denote relative reliability. For example: (a) the number of failures per 100 or 1000 operations; (b) the number of failures per 1, 10, 100, 1000, or 10,000 equipment operating hours as may be appropriate to the equipment application; (c) the mean-time-between-failures in equipment operating hours.

Regression Analysis – An analytical method for determining the correlation between several variables.

Repair Rate – A measure of repair capability; i.e., number of repair actions completed per hour (reciprocal of mean-time-to-repair in the exponential case).

Repairability – The probability that a failed system will be restored to operable condition within a specified active repair time.

Risk – The probability of making an incorrect decision. (See also, Producer's Reliability Risk; Consumer's Reliability Risk.)

Safety – The quality of being devoid of whatever exposes one to danger or harm.

Sampling Plan – A specific plan which states (a) the sample size and (b) the criteria for accepting, rejecting, or taking another sample.

Sequential Test – A test of a sequence of samples in which it is decided at each step in the sequence whether to accept or reject the hypothesis, or to take an additional sample and continue the test.

Specific Operational Requirement (SOR) – A document prepared by OPNAV which states a need for a capability, outlines a system or equipment to satisfy the need, and states the reasons for the requirement. The SOR constitutes a directive to the appropriate Bureau for the preparation of a Technical Development Plan (TDP) that will accomplish the objectives stated in the SOR.

Specification – A detailed description of the characteristics of a product and of the criteria which must be used to determine whether the product is in conformity with the description.

Standard Deviation – The square root of the variance of a random variable (and of its distribution). The standard deviation of a set of n numbers, $x_1, x_2, \ldots x_n$, is the root-mean-square (r.m.s.) deviation of the numbers (x_i) from their average ($\bar{x}$):

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}}$$

Stress Analysis – The evaluation of stress conditions (electrical, thermal, vibration, shock, humidity, etc.) under which parts are applied in the design of a system or equipment. On the basis of a stress analysis, failure rates are appropriately adjusted to reflect the deleterious effects of the stresses on the reliability of the parts involved.

Subassembly – Two or more parts which form a portion of an assembly, or form a unit replaceable as a whole, but having a part or parts which are replaceable as individuals.

Subsystem – A major subdivision of a system that performs a specified function in the overall operation of a system.

Support Equipment – Items that are necessary for the operation and/or maintenance of the system but are not physically part of the system.

System – A combination of complete operating equipments, assemblies, components, parts, or accessories interconnected to perform a specific operational function.

System Compatibility – The ability of the equipments within a system to work together to perform the intended mission of the system. In a broader sense, system compatibility is the suitability of a system to provide the levels of field performance, reliability, and maintainability required by the military services.

Systems Engineering – The process of applying science and technology to the study and planning of an overall system, whereby the various parts of the system and the utilization of various subsystems are fully planned and comprehended prior to the time that hardware designs are committed.

Tactical Capability – See "Performance Capability".

Technical Development Plan (TDP) – A plan for the fulfillment of an Advanced Development Objective or Specific Operational Requirement, serving as a basic decision-making document at Bureau management level. When funded, the TDP becomes the primary management control and reporting document for the life of the development program. It is essential that it be kept up to date on a continuing basis.

Test to Failure – The process of subjecting an item to stress levels until failure occurs.

Thermal Survey – The prediction or actual measurement of part ambient temperatures in order to detect the existence of "hot spots" and to determine the need for cooling.

Tolerance Factor – A factor by which the base failure rate of a series system is multiplied to account for failures due to drift characteristics of active elements in the system. The tolerance factor in analog systems of conventional design is proportional to complexity and is given by

$$k_t \approx \sqrt[3]{N}$$

where N is the number of active elements (the measure of complexity) used in the performance of the particular system function.

Tolerance Failure – A system or equipment failure resulting from multiple drift and instability problems, even though part failures may not have occurred.

Truncation – Deletion of portions of a distribution greater than or less than a certain value. Truncation of a sequential test means termination of the test prior to reaching a decision under the sequential plan.

Unit – An assembly or any combination of parts, subassemblies, and assemblies mounted together, and normally capable of independent operation in a variety of situations.

Uptime – The time in which a system is in condition to perform its intended function.

Useful Life – The total operating time between debugging and wearout.

Use Factor (k_u) – A factor for adjusting base failure rate, as determined from MIL-HDBK-217, to specific use environments and packaging configurations other than those applicable to ground based systems; e.g., for Avionics equipment, the use factor (k_u) is 6.5 on the basis of conventional design (current experience reflected in MIL-STD-756).

Use Reliability – The probability of performing a specified function without failure under actual use conditions.

Variables Testing – A test procedure wherein the items under test are classified according to quantitative rather than qualitative characteristics. Variables testing yields more information than attributes testing.

Wearout Failure Period – That period of time after the normal failure period during which the equipment failure rate increases above the normal rate.

APPENDIX 2. RELIABILITY FORMULAE

2.1 RELIABILITY AND PROBABILITY

Reliability, by definition, is a probability concept –

> ***"Reliability is the probability of success...under specified conditions....."***

A knowledge of basic probability theory is therefore necessary for a full understanding of the prediction and evaluation methods used in the study of reliability. This section reviews some of the probability concepts, shown in Chart 2-I.

2.1.1 Probability Definitions and Concepts Applicable to Reliability

a. *Definition:* The probability of an event is the ***proportion*** of times it is expected to occur in a large number of trials. Specifically, if event A occurs in s out of n trials, the probability of its occurrence–P(A), or A–is the ratio s/n as n goes to infinity. This is often referred to as the "relative frequency" definition of probability.

> *EXAMPLE:* Ten missiles are launched. Eight successfully intercept the target drone. The estimate of missile reliability on future flights is thus 0.8, or 80%, under the same set of "use" conditions which prevailed during the test.

As a special case, the probability of an event is the ratio of the m ways it can occur to the (m+n) ways it can occur and fail to occur, respectively, provided the ways are equally likely and mutually exclusive.

> *EXAMPLE:* A die is rolled. There is one way for a six to appear. There are five ways for a six not to appear. If the die is not "loaded", each of the six ways the die can come to rest is equally likely. The six ways are also mutually exclusive; that is, only one way can occur at a time. Probability of a six on one roll of a single die is then $m = 1$, $m + n = 5 + 1$, $P(6) = 1/(5 + 1) = 1/6$.

b. *Symbolic Representation:* Reliability is usually symbolized mathematically as R( ), where the time period or element of interest is indicated parenthetically. Probability is usually represented by p, P, or P( ), with the time period or element of interest indicated parenthetically. R( ) and P( ) are interchangeable.

c. *Ranges of Probability and Reliability Values:* The probability scale ranges from zero (denoting impossibility) to 1.0 (denoting certainty). If the probability of event A occurring is p, the probability of A not occurring is 1-p. Similarly, if the reliability of A is P_A, then its unreliability, U_A, is $1-P_A$. For simplicity in mathematical computation, P(A) and R(A) can be denoted by A; while 1-P(A) and 1-R(A) can be denoted by $\bar{A}$ (not A), as shown in Figure 2-1.

2.1.2 Two Basic Principles

In evaluating probabilities, all possible outcomes of a chance event must be enumerated. Two basic principles apply:

(1) If event A can occur "a" ways and event B can occur "b" ways, then event A *or* B (usually written A+B) can occur in a+b ways, provided that A and B cannot occur simultaneously.

(2) If event A can occur "a" ways and event B can occur "b" ways,

CHART 2-1. FUNDAMENTAL PROBABILITY FORMULAE

Probability Definitions and Notation:

The probability that event A will occur is the relative frequency with which it occurs (s) in a large number of trials (n); or	$P(A) = s/n$
The number of ways event (A) can occur (m) or can fail to occur (n) in m+n mutually exclusive ways.	$P(A) = m/m+n$ $P(\text{not } A) = P(\bar{A}) = n/m+n$ $P(A) + P(\bar{A}) = 1$

Addition Theorem:

The probability that either A or B mutually exclusive events will occur.	$P(A+B) = P(A) + P(B)$
The probability that either A or B (but not both) will occur when the events are not mutually exclusive.	$P(A+B) = P(A) + P(B) - P(AB)$

Multiplication Theorem:

The joint probability that A and B will occur, given that B has occurred.	$P(A\|B) = P(A)P(B\|A) = P(B)P(A\|B)$
The probability that both A and B will occur, when A and B are independent events.	$P(AB) = P(A)P(B)$

Permutation Theorem:

The number of possible ways to arrange (permute) n events, k at a time.	$P(n,k) = P_k^n = \frac{n!}{(n-k)!}$ $n! = n(n-1)(n-2) \ldots 3 \cdot 2 \cdot 1$ $o! = 1$

Combination Theorem:

The number of possible combinations of n events, k at a time.	$C_k^n = \binom{n}{k} = \frac{n!}{k!(n-k)!}$

Binomial Law:

Probability of an event occurring k times in n independent trials with probability p per trial.	$P_{(k,n\|p)} = \binom{n}{k} p^k (1-p)^{n-k}$

then events A *and* B (written AB) can occur a·b ways.

Principle (1) is illustrated in Figure 2-2a. The system is successful if A or B is operating. A has two redundant elements and B has three redundant elements. Since A has two paths for successful performance and B has three paths, the total number of ways for A or B to occur is 2 + 3 = 5.

Figure 2-2b illustrates Principle (2). Since A can occur two ways and B can occur three ways, A and B can occur 2·3 = 6 ways.

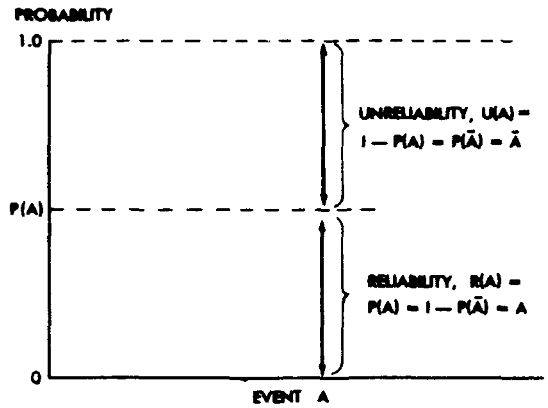
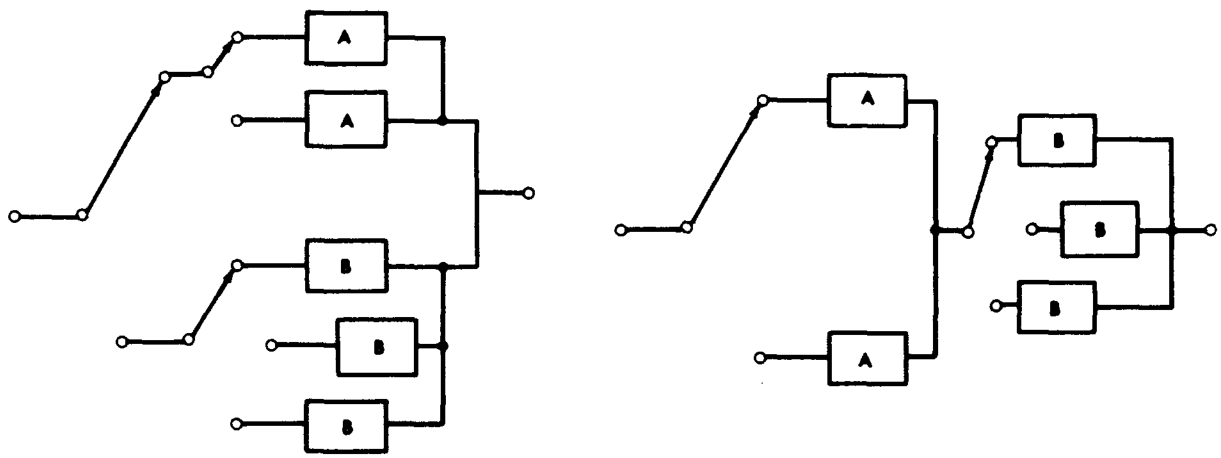


Figure 2-1. Probability Relationships for an Event, A



(a) A OR B CAN OCCUR IN 5 WAYS: A + B = 2 + 3 = 5

(b) A AND B CAN OCCUR IN 6 WAYS: A × B = 2 × 3 = 6

Figure 2-2. Basic Probability Combinations (a) the Addition Theorem (b) the Multiplication Theorem

These two basic principles can be extended to more than two events. For example, if three mutually exclusive events, A, B, and C, can occur in a, b, and c ways, respectively, then events A *or* B *or* C can occur in a+b+c ways, and events A and B and C can occur in a·b·c ways.

2.1.3 Permutations

If the order in which events occur is important, we are concerned with permutations. A permutation is defined as a collection of events arranged in a specific order. The total number of permutations of three events, A, B, and C, can be found in the following manner: There are three choices for the first event; either of the two remaining events may fill the second position; and the one remaining event must fill the last position. By Principle (2), the total number of permutations possible is $3 \cdot 2 \cdot 1 = 6$. In general, the total number of permutations possible in n distinct events or objects is equal to $n(n-1)(n-2)\ldots 3 \cdot 2 \cdot 1$ or $n!$ (n factorial).

In considering the number of permutations of k objects out of n, there are n ways for the first position, (n-1) ways for the second, and so on. When we come to the k^{th} position, (k-1) of the objects will have been used, so that the k^{th} position can be filled in n-(k-1) ways. The symbol P(n,k) is used to denote the number of permutations of k out of n objects:

$$P(n,k) = n(n-1)(n-2)\ldots(n-k+1)$$

$$= \frac{n!}{(n-k)!} \qquad (2\text{-}1)$$

EXAMPLE: Find the probability of losing output E_o, of Figure 2-3, after three diodes have shorted. From Equation (2-1), the total number of permutations of three out of five diodes is $(5!)/(2!) = 60$. Output E_o will be absent through short circuiting only after diodes A, B, and C short. This can occur in $3! = 6$ ways. If all possible permutations are equally likely, the probability of loss of E_o after three diodes have shorted then is $6/60 = 0.10$

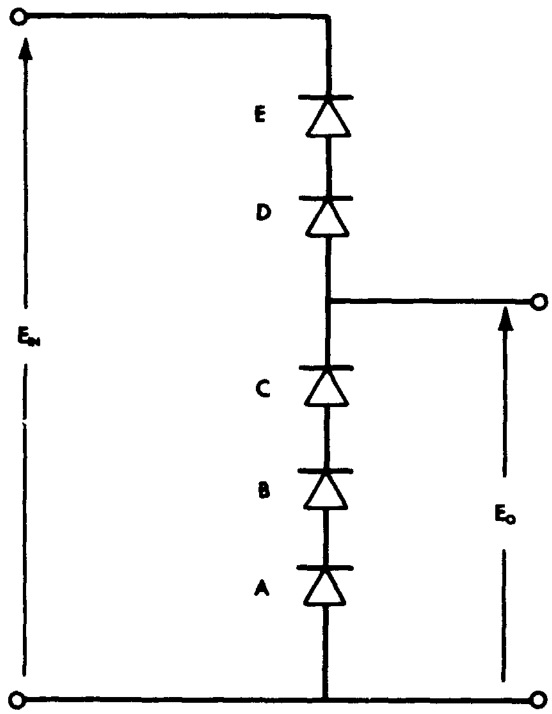


Figure 2-3. Failure by Shorting of A, B and C Result in Loss of E_0

2.1.4 Combinations

A combination is the number of different ways k out of n objects can be selected without regard to the order of arrangement. This is denoted by:

$$\binom{n}{k} = \frac{P(n,k)}{k!} = \frac{n!}{k!(n-k)!} \qquad (2\text{-}2)$$

Equation (2-2) can be used to solve the example given in 2.1.3. From the circuit in Figure 2-3 there is only one combination of three diodes shorting which will result in the loss of E_o, namely ABC. The total number of combinations of five diodes taken three at a time is

$$\binom{5}{3} = \frac{5!}{3!2!} = \frac{1\cdot2\cdot3\cdot4\cdot5}{(1\cdot2\cdot3)(1\cdot2)} = \frac{120}{(6)(2)} = 10,$$

of which only one (ABC) will produce a short. Therefore, the probability of ABC shorting $= 1/10 = 0.10$, as before.

2.1.5 Fundamental Rules for Probability Computations

(1) ***The Addition Rule:*** If A and B are two ***mutually exclusive*** events, i.e., occurrence of either event excludes the other, the probability of either of them happening is the sum of their respective probabilities:

$$P(A \text{ or } B) = P(A+B) = P(A) + P(B) \quad (2\text{-}3)$$

This rule follows directly from principle (1) of 2.1.2 and can apply to any number of mutually exclusive events.

$$P(A+B\ldots+N) = P(A) + P(B)\ldots+P(N) \quad (2\text{-}4)$$

(2) ***The Addition Rule (non-exclusive case):*** If A and B are two events not ***mutually exclusive***, i.e., either or both can occur, the probability of at least one of them occurring is

$$P(A \text{ or } B) = P(A+B) = P(A) + P(B) - P(AB) \quad (2\text{-}5)$$

The equation for three events becomes:

$$P(A+B+C) = P(A) + P(B) + P(C) - P(AB) - P(AC) - P(BC) + P(ABC) \quad (2\text{-}6)$$

Rule (2) can be extended to any number of events.

> *EXAMPLE:* If event A is a face card and event B is a spade, they are not mutually exclusive, i.e., the occurrence of one does not preclude the occurrence of the other. There are 12 ways to draw a face card; there are 13 ways to draw a spade. There are 3 ways to draw a face card in the spade suit. The probability of at least a face card or a spade on the first draw is:
>
> $$P(A+B) = P(A) + P(B) - P(AB)$$
>
> $$= \frac{12}{52} + \frac{13}{52} - \frac{12}{52} \cdot \frac{13}{52}$$
>
> $$= \frac{12}{52} + \frac{13}{52} - \frac{3}{52} = \frac{22}{52}$$

(3) ***The Multiplication Rule:*** If events A and B are ***independent***, i.e., the occurrence of one does not affect the probability of occurrence of the other, the probability that both will occur is equal to the product of their respective probabilities.

$$P(A \text{ and } B) = P(AB) = P(A)P(B) \quad (2\text{-}7)$$

Equation (2-7) may be extended to any number of independent events:

$$P(AB\ldots N) = P(A)P(B)\ldots P(N)$$

This is known as the product or multiplication law for independent events used in reliability prediction techniques.

> *EXAMPLE:* A weapon system is made up of a radar set, computer, launcher, and a missile. Each has an independent probability of successful operation over a particular time period of 0.87, 0.85, 0.988, and 0.80, respectively. The probability of successful system operation for the same time interval is the product of the individual subsystem probabilities, or (0.87)(0.85)(0.988)(0.80) = 0.585.

(4) ***Conditional Probabilities:*** If events A and B are ***not independent***, i.e., the occurrence of one affects the probability of occurrence of the other, a ***conditional*** probability exists. The probability of *A given that B has occurred* is denoted by $P(A|B)$, and similarly *B given A* is denoted by $P(B|A)$. Thus if A and B are not inde-

pendent, then the probability of both occurring is

$$P(AB) = P(A)P(B|A) \qquad (2\text{-}8)$$
$$= P(B)P(A|B)$$

If A and B are independent, $P(A|B) = P(A)$ and $P(B|A) = P(B)$ and Equation (2-8) reduces to Equation (2-7).

For three events A, B, and C

$$P(ABC) = P(A)P(B)P(C|AB) \qquad (2\text{-}9)$$

EXAMPLE: The probability of drawing two hearts in sequence from a deck of cards in two draws is conditional on the first draw. Since there are 13 hearts in a deck of 52 cards, the probability of a heart on the first draw, P(A), equals 13/52, or 0.25. If the first draw was a heart, there are 12 hearts left in a reduced deck of 51 cards. Thus, the probability of drawing a heart on the second draw if the first draw was a heart, P(B|A), is 12/51, or 0.235. The probability of drawing two hearts in sequence, P(AB), is then

$$P(AB) = P(A)P(A|B)$$
$$= (0.25)(0.235)$$
$$= 0.058$$

2.1.6 The Binomial Law

If the probability of an event occurring in a single trial is p, the probability of it occurring exactly k times out of n *independent* trials is given by the binomial law:

$$P(k,n|p) = \binom{n}{k} p^k(1-p)^{n-k}$$

$$\text{where } \binom{n}{k} = \frac{n!}{k!(n-k)!} \qquad (2\text{-}10)$$

EXAMPLE: A redundant circuit has five components. The circuit will operate successfully if *at least two of the* five components are operating. p is the probability of each component failing. The failure of one component has no effect on the performance of the other components. The probability of system success is equal to 1 - [(probability of exactly four components failing) + (probability of exactly five components failing)]. Using Equation (2-10) and letting k equal the number of failures,

$$R = 1 - \left[\binom{5}{4} p^4(1-p)^1 + \binom{5}{5} p^5(1-p)^0\right]$$
$$= 1 - \left[5p^4(1-p) + p^5\right]$$
$$= 1 - \left[5p^4 - 4p^5\right] \qquad (2\text{-}11)$$

The binomial law is treated as a discrete distribution in more detail in 2.2.

2.1.7 Application of Basic Rules of Probability

The probability of the simultaneous occurrence of A and B is the product of the unconditional probability of event A and the conditional probability of B, given that A has occurred:

$$(AB) = (A)(B|A) \qquad (2\text{-}8)$$

This more general version of the rules takes account of instances in which the events are not independent nor mutually exclusive. These instances do not give rise to different rules; however, care must be taken to separate the events into independent groups before adding or multiplying probabilities, as the case may be. For example, consider the failure of a particular electronic equipment. There are failures arising from transistors and failures arising from capacitors—to mention only two possibilities. These failures are not mutually exclusive since a failure from a transistor does not exclude failure from capacitors. If these events are separated into mutually exclusive subclasses, the following events and probabilities are apparent (let A be failure from transistors and B failure from capacitors):

(1) Failure from transistors alone, assuming no simultaneous failures from capacitors, with probability

$$(A) - (AB)$$

(2) Failure from capacitors alone (no simultaneous failures from transistors), with probability

$$(B) - (AB)$$

(3) Failure from both transistors and capacitors simultaneously (assuming that they are independent), with probability

$$(AB)$$

The probability of failure from either transistors or capacitors (A or B) is obtained by applying the additive rule—a valid procedure since, as written, the three events are mutually exclusive. This gives

$$[(A) - (AB)] + [(B) - (AB)] + (AB)$$

$$= (A) + (B) - (AB)$$

The same procedure is extendible to more than two cases.

The probability of occurrence of either A or B can also be obtained as the probability of A plus the probability of "not A" and B. This is

$$(A) + (\bar{A})(B) = (A) + [1 - (A)]\,(B)$$

$$= (A) + (B) - (A)(B)$$

$$= (A) + (B) - (AB),$$

as before.

Similarly, the probability of occurrence of either A, B, or C is obtained as the sum of two probabilities:

(1) The probability of A, and
(2) The probability of "not A" but either B or C

These are

$$(A)$$

and

$$[1 - (A)]\,[(B) + (C) - (BC)],$$

respectively. The sum is

$$(A) + (B) + (C) - (A)(B) - (A)(C) - (BC) + (A)(BC)$$

$$= (A) + (B) + (C) - (AB) - (AC) - (BC) + (ABC)$$

$$= A + \bar{A}B + \bar{A}\bar{B}C$$

These events can be seen graphically in Figure 2-4. All events appear as overlapping circles, i.e., points in the circles represent ways A, B, or C can occur. The first event in the series is the A circle. The second event is the part of the B circle that is not inside the A circle. The sum of the first two events is the sum of these two areas. The third event is that part of the area in the C circle which is not in the A and B circles, or, stated another way, it is the portion of the C circle not included in the area represented by the first two events in the series, and so on.

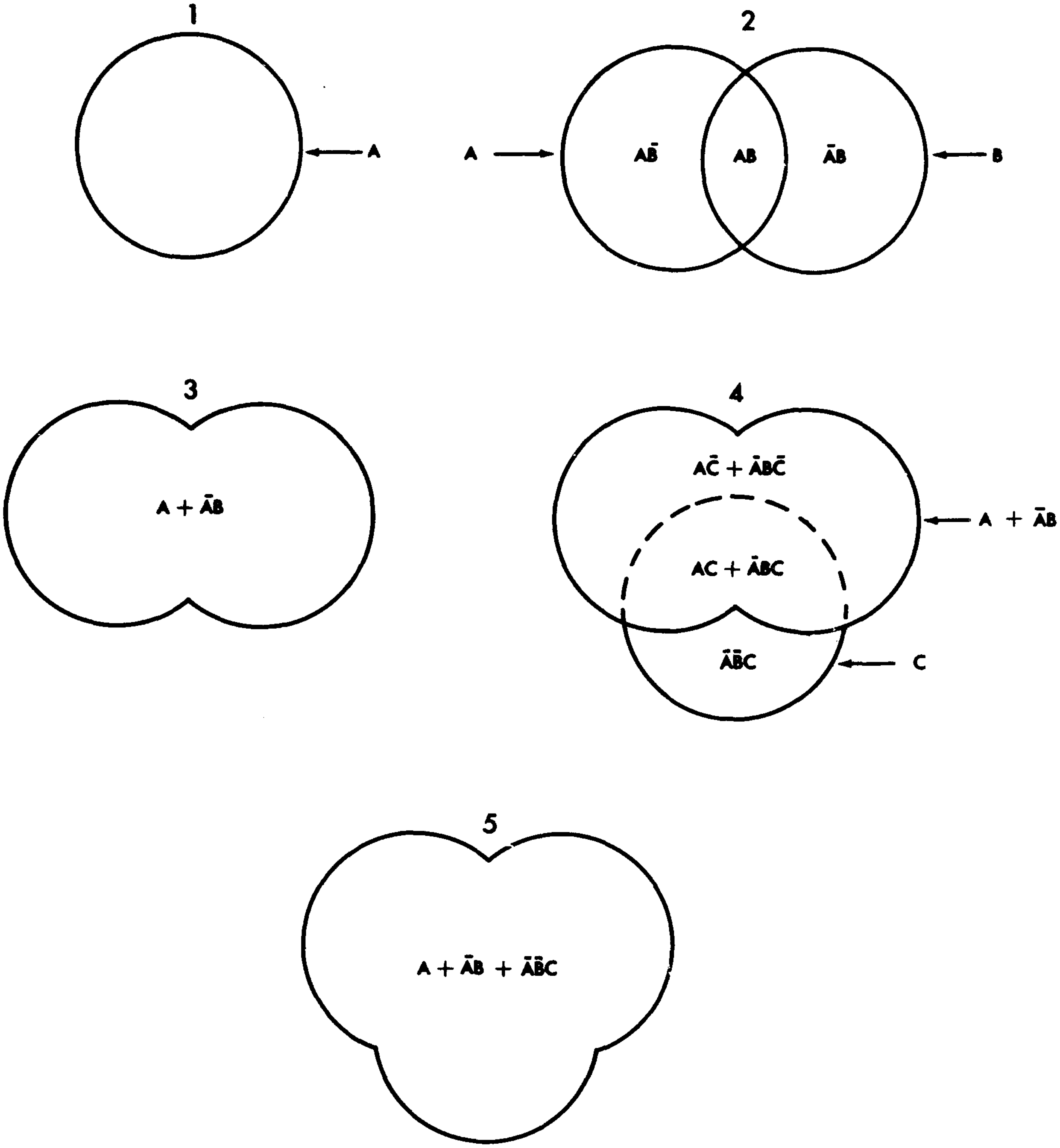


Figure 2-4. Probabilities of Mutually Exclusive Events: Five Examples

2.1.8 A General Method for Solving Probability Problems—An Example

The relative-frequency definition of probability (2.1.1) is by itself a very useful tool for solving probability problems. It does two things:

(1) It permits the use of observed data on the proportion of successes as an estimate of the probability of success, and

(2) It permits the use of the estimate of probability in predicting the proportion of future successes.

The first item is the input data needed in solving the problem; the second item is the inference or prediction.

As an example, consider an equipment consisting of three black boxes denoted by A, B, and C. In a particular use of this equipment, the failure of one of the boxes does not influence the failure of either of the others. Denote by a, b, and c the successful operation of boxes A, B, and C, respectively; and denote by $\bar{a}$, $\bar{b}$, and $\bar{c}$ the failure of A, B, and C, respectively. Assume that success and failure have occurred in the following proportions of trials:

	Success	Failure
A	$a = 1/2$	$\bar{a} = 1/2$
B	$b = 2/3$	$\bar{b} = 1/3$
C	$c = 4/5$	$\bar{c} = 1/5$

A "trial" is defined as a mission involving a time period of fixed duration. The table expresses the equality of the observed relative frequencies of the corresponding probabilities.

From the above probabilities, the failures expected in a number of future missions can be computed. Each of the three boxes will or will not fail in all possible combinations. Probabilities for the separate combinations of A, B, and C are estimated in sequence as follows.

Consider 60 future missions. In $60a = 30$ of these, A will operate properly. Of the 30 in which A operates properly, B will operate in $30b = 20$. Similarly, of the 20 missions in which B will operate properly, C will operate satisfactorily in $20c = 16$ cases. Thus, for the next 60 missions, 16 of them will have A, B, and C working satisfactorily.

The process is easier to follow when the computations are expressed in a systematic form, as shown in the table.

60	a(30)	b(20)	c	16	abc
			$\bar{c}$	4	$ab\bar{c}$
		$\bar{b}$(10)	c	8	$a\bar{b}c$
			$\bar{c}$	2	$a\bar{b}\bar{c}$
	$\bar{a}$(30)	b(20)	c	16	$\bar{a}bc$
			$\bar{c}$	4	$\bar{a}b\bar{c}$
		$\bar{b}$(10)	c	8	$\bar{a}\bar{b}c$
			$\bar{c}$	2	$\bar{a}\bar{b}\bar{c}$
			TOTAL	60	

Probabilities for any combination can now be computed as the ratio of the number of missions with the particular failure combination to the total number of missions attempted (60). Thus, the probability that A and B fail while C does not is $8/60 = 2/15$, the 8 being indicated above by $\bar{a}\bar{b}c$.

There is a special significance in the method of identification: the indicated product is the probability. For example, the $\bar{a}\bar{b}c$ product is (1/2)(1/3)(4/5) = 2/15 as computed above. Furthermore, the 8 missions noted for this case can be computed by the formula 60 $\bar{a}\bar{b}c$, if $\bar{a}$, $\bar{b}$, and c are used to denote probabilities. The product $\bar{a}\bar{b}c$ illustrates the "both-and" theorem in probability. The multiplication by 60 reflects the definition of probability as the number of occurrences of an event in a given number of trials.

It is also possible to illustrate the "either-or" theorem in which probabilities are added. Thus, the probability that A and B both fail, regardless of C, is shown as 10/60 = 1/6, the numerator 10 being obtained by the computations listed above. It could be denoted by $\bar{a}\bar{b}$, the formula for this probability. It can also be obtained as $\bar{a}\bar{b}c + \bar{a}\bar{b}\bar{c} = \bar{a}\bar{b}(c+\bar{c}) = \bar{a}\bar{b}$. In terms of the number of missions, this is 8 for $\bar{a}\bar{b}c$ and 2 for $\bar{a}\bar{b}\bar{c}$.

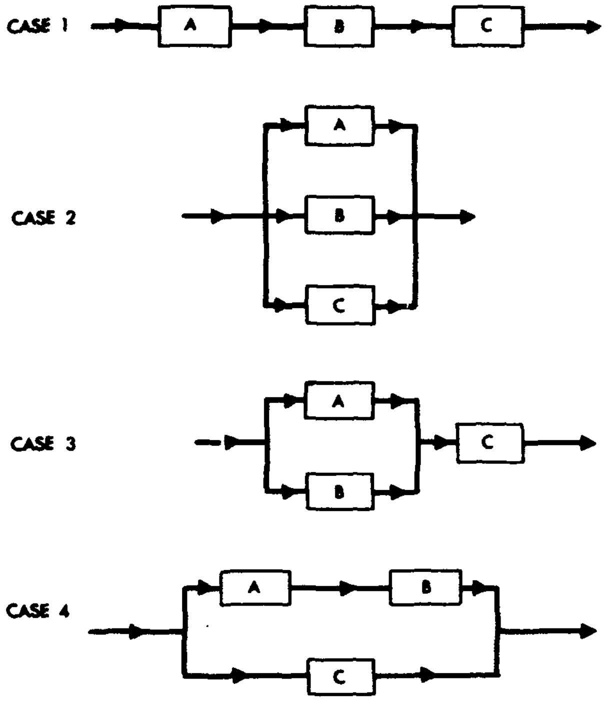


Figure 2-5. Four Possible Combinations of Units A, B and C

The failure of a component or "black box" may or may not mean system failure, depending on the series or parallel arrangement of the boxes in a particular operating mode. Hence, it is necessary to look at a number of possible arrangements of boxes A, B, and C in the present example. Four possible cases are diagramed in Figure 2-5.

Failure-free operation of the system in each of the four cases requires the following conditions:

Case 1. None of the three boxes can fail.

Case 2. Not more than two of the boxes can fail.

Case 3. Box C must not fail and at least one of the other two boxes must operate without failure.

Case 4. Boxes A and B must operate without failure if box C fails, but the system will operate if box C does not fail regardless of what happens to boxes A and B.

Using these conditions, it is now possible to tabulate the number of successful missions for each of the four cases. The failure combinations are identified by the associated probabilities in the following table.

SUCCESSFUL MISSIONS

Failure Combination	Case 1	Case 2	Case 3	Case 4
abc	16	16	16	16
$ab\bar{c}$		4		4
$a\bar{b}c$		8	8	8
$a\bar{b}\bar{c}$		2		
$\bar{a}bc$		16	16	16
$\bar{a}b\bar{c}$		4		
$\bar{a}\bar{b}c$		8		8
$\bar{a}\bar{b}\bar{c}$				
Total	16	58	40	52
Probability	16/60	58/60	40/60	52/60

The identification of the failure combination is the formula for the probability of the combination. Therefore, the formula for the probability of successful equipment operation in each of the four cases can be written as the sum of the identifications for the recorded entities. These are as follows:

Case	*Probability of Success*
1	abc
2	$abc + ab\bar{c} + a\bar{b}c + a\bar{b}\bar{c} + \bar{a}bc + \bar{a}b\bar{c} + \bar{a}\bar{b}c$
3	$abc + a\bar{b}c + \bar{a}bc$
4	$abc + ab\bar{c} + a\bar{b}c + \bar{a}bc + \bar{a}\bar{b}c$

These formulae can be simplified for cases 2, 3, and 4. Thus, for case 2 the formula is

$$1 - \bar{a}\bar{b}\bar{c} \text{ or } a + b + c - ab - ac - bc + abc$$

For case 3, the formula is

$$ac + \bar{a}bc \text{ or } c(a + \bar{a}b) \text{ or } c(a + b - ab)$$

For case 4, the formula is

$$c + ab\bar{c} \text{ or } c + ab - abc$$

2.1.9 Probability and Time

The probability formulae and examples given thus far have related to missions of fixed time duration, where time was considered a constant factor. In most applications of probability theory to reliability engineering, the events being studied are expressed as continuous functions of time. Hence the probabilities are not constants but are functions of the time variable, denoted by t. The probability formulae given above hold equally well when interpreted as functions of time. For example, the reliability at time t is equivalent to a probability of no failure before t. If we have a system composed of two equipments, a and b, each having an independent probability of successful operation, then by the multiplication law the probability of successful system operation at time t is

$$R_{ab}(t) = R_a(t) \cdot R_a(t) = R_S(t)$$

If we have two equipments a and b and if the system is successful at time t if either or both a, b are operable, then by the addition law

$$R_S(t) = R_a(t) + R_b(t) - R_{ab}(t)$$

In the example of 2.1.8, the system success was defined as failure-free performance for the duration of a mission of a fixed length. If the equipment were used for a longer time, one would expect lower probabilities of success. This can be represented in general by replacing the numerical probabilities by functions of time. Let $R_a(t)$, $R_b(t)$, and $R_c(t)$ represent the probabilities of failure-free operation for a mission of length t for boxes A, B, and C, respectively. Failure probabilities will be $1-R_a(t)$, $1-R_b(t)$, and $1-R_c(t)$. If individual boxes follow the exponential law (usually the case in non-redundant designs), then:

$$R_a(t) = e^{-xt}$$

$$R_b(t) = e^{-yt}$$

$$R_c(t) = e^{-zt}$$

The probability of equipment success, using these reliability expressions, is:

Case	Probability of Success
1	$e^{-(x+y+z)}$
2	$e^{-xt} + e^{-yt} + e^{-zt} - e^{-(x+y)t} - e^{-(x+z)t} - e^{-(y+z)t} + e^{-(x+y+z)t}$
3	$e^{-zt}(e^{-xt} + e^{-yt} - e^{-(x+y)t})$
4	$e^{-zt} + e^{-(x+y)t} - e^{-(x+y+z)t}$

The function for case 1 is of the same form as the functions for the black boxes—namely, an exponential. In all other cases, the equipment probability of success is not exponential, but instead is quite complex. This point is discussed further in the section on redundancy.

2.2 PROBABILITY DENSITY AND DISTRIBUTION FUNCTIONS

A probability distribution is the relative frequency with which certain events occur. In reliability work, the term "event" is usually related to failure time by considering the time at which failures occur or by considering the number of failures that occur in a fixed time interval. In order to predict system reliability, it is necessary to know the part or component failure probability distributions. The more common failure probability distributions used in reliability engineering are discussed in this section.

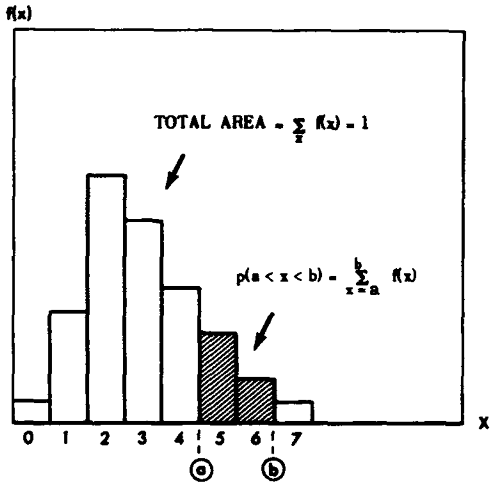


Figure 2-6. A Discrete Probability Distribution

Discrete and Continuous Probability Distributions

Probability distributions are classified in two general categories, ***discrete*** and ***continuous***. In a discrete distribution the random variable can take on only isolated values and can change only by set increments, as shown in Figure 2-6.

If a random variable can take on any value within an interval, then the associated probability distribution is continuous, as illustrated in Figure 2-7.

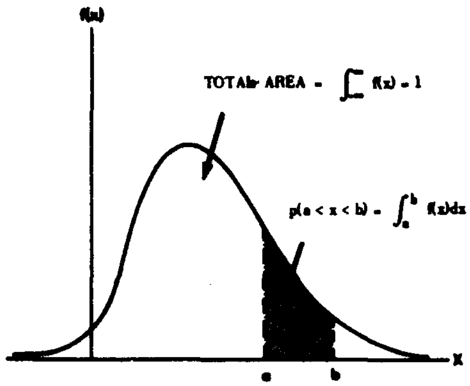


Figure 2-7. A Continuous Probability Distribution

In both cases, x represents the random variable, and f(x) the probability distribution or ***probability density function*** (pdf) of x. f(x) has the following properties:

(1) f(x) is always positive, with unity area:

$$\sum_x f(x) = 1 \text{ if x is discrete}$$

or

$$\int_{-\infty}^{\infty} f(x)\,dx = 1 \text{ if x is continuous}$$

(2) The probability that x will take on a value in the interval [a,b] is equal to the area between the two points:

$$\sum_{x=a}^{b} f(x) \text{ if x is discrete}$$

or

$$\int_a^b f(x)dx \text{ if x is continuous}$$

$$= \int_{-\infty}^{b} f(x)dx - \int_{-\infty}^{a} f(x)dx$$

Cumulative Density Functions

Associated with every pdf is a ***cumulative density function*** (cdf) of x, denoted by F(x). The cdf is defined as the probability that the value of the random variable x will be less than or equal to some specific value of x, such as "b" for example, shown in Figures 2-8 and 2-9.

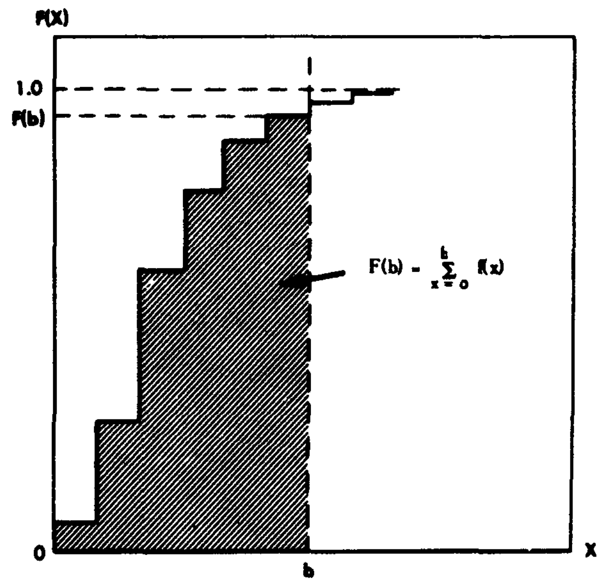


Figure 2-8. A Discrete Cumulative Distribution Function

For a discrete random variable, at x = b,

$$F(b) = \sum_{x=0}^{b} f(x)$$

where 0 is the lower limit of the range of x.

For a continuous random variable, at x = b,

$$F(b) = \int_{-\infty}^{b} f(x)dx$$

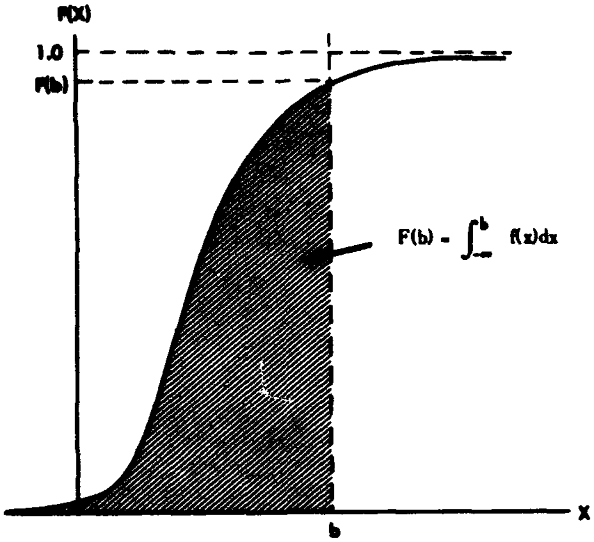


Figure 2-9. A Continuous Cumulative Distribution Function

Parameters and Moments of Distribution

Most probability distribution or density functions contain certain constants called parameters. These parameters completely specify the function.

Probability distributions are described by their ***moments***. Moments can be thought of as descriptive properties of a probability distribution and are usually related to the parameters in the probability distribution function.

The first two moments are of major importance. The first is the mean (μ) of the distribution. This is the x coordinate of the center of gravity of the area under the probability density curve. Essentially, the mean is the arithmetic average of the values of all the members in a population.

The population mean is defined as

$$\mu = \int_{-\infty}^{\infty} xf(x)dx \quad \text{for continuous variables}$$

$$\mu = \sum_{x} x_i f(x) \quad \text{for discrete variables}$$

The population mean is estimated from sample readings as the average value of x in n readings:

$$\hat{\mu} = \bar{x} = \frac{1}{n} \sum_{x=1}^{n} x_i \quad \text{for n samples}$$

The second moment of a distribution, the variance (σ^2), is a measure of dispersion from the mean. It is the average value of the square of the deviation of individual items from the mean. It corresponds to the moment of inertia of the distribution about the mean. Variance is defined by the equations

$$\sigma^2 = \sum_{x} (x_i - \mu)^2 f(x) \quad \text{for discrete variables}$$

$$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x)dx \quad \text{for continuous variables}$$

The variance is usually estimated by

$$\sigma^2 = S^2 = \frac{1}{n-1} \sum_{x=1}^{n} (x_i - \bar{x})^2 \quad \text{for n samples}$$

Two of the most commonly encountered discrete distributions are the *Binomial* and the *Poisson*. Both are described in detail in 2.2.1 and 2.2.2, respectively. In general, discrete data do not furnish as much information as continuous measurements, but in many cases only discrete data are available because of time or economic limitations, or because of the inherent characteristics of the phenomenon being examined. Continuous probability distributions are presented in more detail in 2.2.3 through 2.2.6.

2.2.1 The Binomial Distribution

If a variable can be classified into two mutually exclusive and exhaustive categories (i.e., every value will lie in one of the categories and no value will lie in both) –for example, head or tail, black or white– and if the probability of observing each of the categories on any trial is constant, then the variable is distributed by the binomial law.

The usual procedure is to term one of the two categories a success and the other a failure. If p is the constant probability of success and $q = 1-p$ is the constant probability of failure, then the distribution of the number of successes x in n total trials is given by the binomial pdf

$$f(x) = \binom{n}{x} p^x q^{n-x}, \quad x = 0, 1, 2 \ldots n$$

$$\text{and } \binom{n}{x} = \frac{n!}{x!(n-x)!} \qquad (2\text{-}12)$$

The probability that $x \leq X$ is given by the binomial cumulative distribution function (cdf)

$$F(X) = \sum_{x=0}^{X} \binom{n}{x} p^x q^{n-x}, \quad X \leq n \qquad (2\text{-}13)$$

The mean of the binomial variable x is equal to np, and the variance is equal to npq.

EXAMPLE: Assume a very large lot of identical parts. Past experience has shown that the probability of a defective part is equal to 0.05. The acceptance sampling plan for lots of these parts is to randomly select 30 parts for inspection and accept the lot if 2 or less defectives are found. We wish to find the probability of accepting the lot.

A part will be in one of two categories –defective or non-defective. The probability of a defective part, p, is 0.05 and the probability of a non-defective part, q, is 0.95. Sample size, n, is 30, and the specific value of the random variable "number of defectives," s, is less than or equal to 2. Using the *cumulative binomial density function*, Equation (2-13), the probability of accepting the lot, P(a), is equal to the probability of zero, one, or two defectives in a sample of 30:

$$P(a) = \sum_{x=0}^{2} \binom{30}{x} (.05)^x (.95)^{30-x}$$

$$= \frac{30!}{0!30!} (.05)^0 (.95)^{30}$$

$$+ \frac{30!}{1!29!} (.05)^1 (.95)^{29}$$

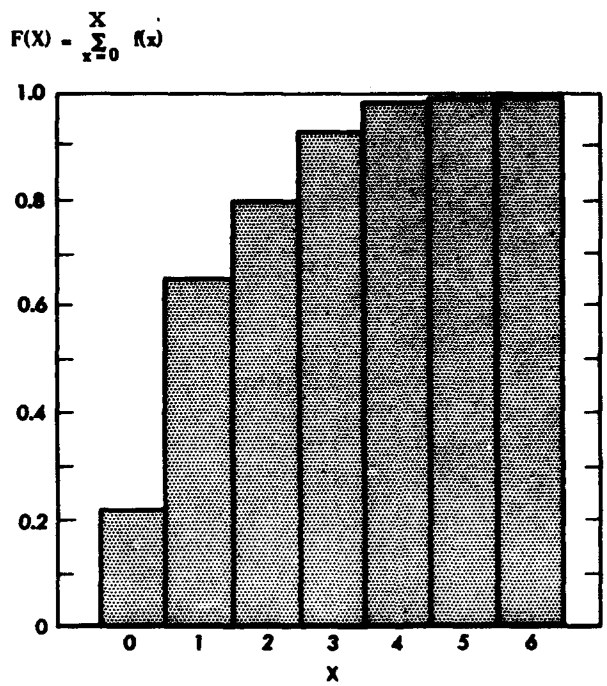


Figure 2-10. Cumulative Binomial Probability Density Function for n=30, p=.05

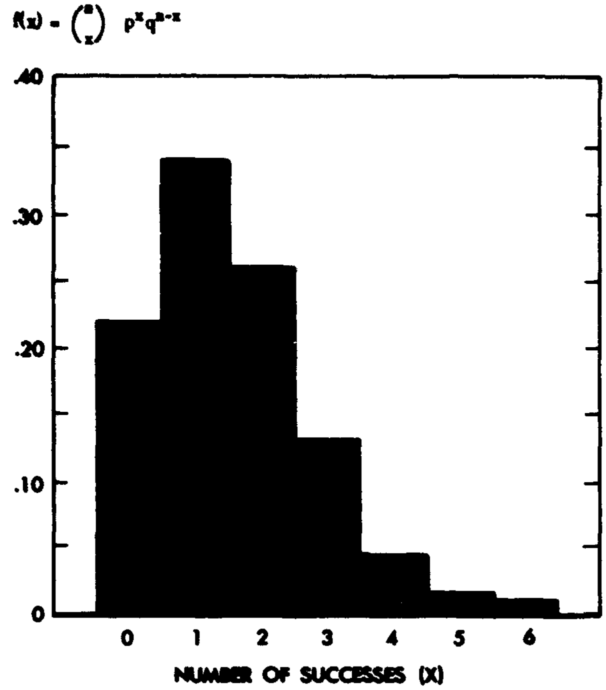


Figure 2-11. Binomial Probability Distribution Function for n=30, p=.05

$$+ \frac{30!}{2!28!} (.05)^2 (.95)^{28}$$

$$= 0.812$$

Figure 2-10 shows this cumulative density function for the above parameters. Figure 2-11 shows the binomial probability distribution function for this same problem. From it, the probability of no defectives in the lot is 0.22, of exactly one defective, is 0.33, etc.

EXAMPLE: The binomial is useful for computing the probability of system success when the system employs partial redundancy. Assume a five-channel VHF receiver as shown in Figure 2-12.

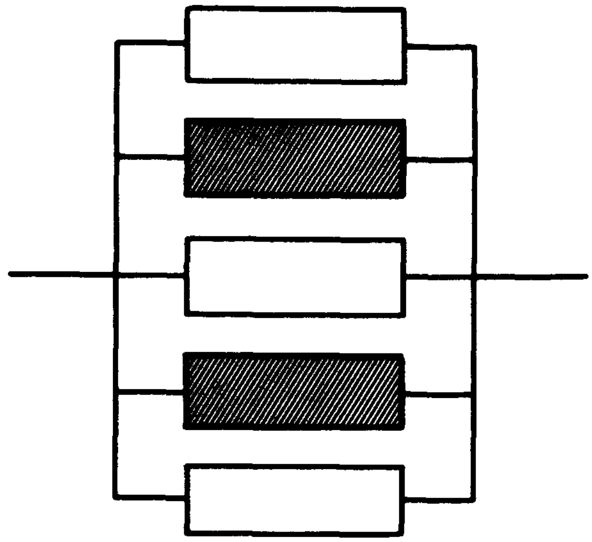


Figure 2-12. Five Channel Receiver with Two Failures Allowed

As long as three channels are operational, the system is classified as satisfactory. Each channel has a probability of .9 of surviving a 24-hour operation period without failure. Thus two channel failures are allowed. What is the probability that the receiver will survive a 24-hour mission without loss of more than two channels?

Let $n = 5$ = number of channels

$r = 2$ = number of allowable channel failures

$p = .9$ = probability of individual channel success

$q = .1$ = probability of individual channel failure

x = number of successful channels

and $P(S)$ = probability of system success

Then

$$P(S) = \sum_{x=3}^{n} \frac{n!}{x!(n-x)!} p^x q^{n-x}$$

$$= \frac{5!}{3!2!}(.9)^3(.1)^2 + \frac{5!}{4!1!}(.9)^4(.1)^1$$

$$+ \frac{5!}{4!0!}(.9)^5(.1)^0$$

$$= .99144$$

This is the probability that three or more of the five channels will survive the 24-hour operating period.

The problem can be solved another way, by subtracting the probability of three or more failures from one, e.g.:

$$P(S) = 1 - P(F)$$

$$= 1 - \sum_{x=(r+1)}^{n} \frac{n!}{x!(n-x)!} q^x p^{n-x}$$

$$= 1 - \left[\frac{5!}{3!2!}(.1)^3(.9)^2 + \frac{5!}{4!1!}(.1)^4(.9)^1 + \frac{5!}{5!0!}(.1)^5(.9)^0\right]$$

$$= 1 - [.00856] = .99144 \text{ as before}$$

Note the change in notation (only) that x now represents the number of failures and q^x is the probability of x failures whereas before x represented the number of successes and p^x was the probability of x successes.

Computations involving the binomial distribution become rather unwieldy for even small sample sizes; however, complete tables of the binomial pdf and cdf are available in many statistics texts.

Values of the binomial coefficient,

$$C_x^n = \binom{n}{x} = \frac{n!}{x!(n-x)!}$$

are shown in Table 2-I to values of n and x up to 20. For values beyond the table, it is often practical to resort to simple arithmetic as in the case of the third coefficient in the first example above, where n = 30, x = 2:

$$\binom{n}{x} = \frac{30!}{2!28!} = \frac{28!(29)(30)}{2!28!} = \frac{29 \cdot 30}{2}$$

$$= 435$$

2.2.2 The Poisson Distribution

The probability distribution function of the Poisson is

$$f(x) = \frac{e^{-m}m^x}{x!}, \quad x \geq 0 \qquad (2\text{-}14)$$

where m = np

x = the number of failures (or successes, according to the problem statement)

The parameter m is the expected or average number of failures (or successes) in n trials. The variance is also equal to m. The cumulative Poisson distribution function is

$$F(X) = \sum_{x=0}^{X} \frac{e^{-m}m^x}{x!} \qquad (2\text{-}15)$$

When n, the sample size or number of observations, becomes large, and p, the probability of failure, is very small, the binomial distribution can be closely approximated by Poisson's limit.

EXAMPLE: The first example given for the binomial distribution can be

TABLE 2-1. BINOMIAL COEFFICIENTS, $C_x^n = \binom{n}{x} = \frac{n!}{x!(n-x)!}$, to n = x = 20*

x \ n	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2			1	3	6	10	15	21	28	36	45	55	66	78	91	105	120	136	153	171	190
3				1	4	10	20	35	56	84	120	165	220	286	364	455	560	680	816	969	1140
4					1	5	15	35	70	126	210	330	495	715	1001	1365	1820	2380	3060	3876	4845
5						1	6	21	56	126	252	462	792	1287	2002	3003	4368	6188	8568	11628	15504
6							1	7	28	84	210	462	924	1716	3003	5005	8008	12376	18564	27132	38760
7								1	8	36	120	330	792	1716	3432	6435	11440	19448	31824	50388	77520
8									1	9	45	165	495	1287	3003	6435	12870	24310	43758	75582	125970
9										1	10	55	220	715	2002	5005	11440	24310	48620	92387	167960
10											1	11	66	286	1001	3003	8008	19448	43758	92378	184756
11												1	12	78	364	1365	4368	12376	31824	75582	167960
12													1	13	91	455	1820	6188	18564	50388	125970
13														1	14	105	560	2380	8568	27132	77520
14															1	15	120	680	3060	11628	38760
15																1	16	136	816	3876	15504
16																	1	17	153	969	4854
17																		1	18	171	1140
18																			1	19	190
19																				1	20
20																					1

*For complete tables, see National Bureau of Standards, Tables of the Binomial Probability Distribution, GPO 1949 (Applied Mathematics Series 6) solved using the Poisson approximation. Here $m = np = 30(.05) = 1.5$.

$$P(a) = \sum_{i=0}^{2} \frac{e^{-m} m^i}{i!}$$

$$= e^{-1.5} \left[\frac{(1.5)^0}{0!} + \frac{(1.5)^1}{1!} + \frac{(1.5)^2}{2!} \right]$$

$$= 0.809$$

The true probability given by the binomial is 0.812, hence the Poisson approximation is reasonably close. Thus for large n and small p, the Poisson can be used to approximate binomial probabilities.

For the binomial we had a sample of a definite size and could count the number of times an event occurred and *also the number* of times it did not occur. There are many situations in which the number of times an event did not occur are meaningless. For example, the number of defects in a sheet of steel can be counted, but we cannot count the number of non-defectives. Similarly, for a fixed time period we can count the number of telephone calls made, but the number of telephone calls not made has no meaning.

If m, the expected number of events in a given interval of time, is constant, and if the number of events produced in any subinterval is independent of the number of events produced in any other time interval, then the probability of x events for the interval is a Poisson distribution given by Equation (2-14). The Poisson frequency distribution then predicts the number of failures in a given time interval, if time effect is negligible.

EXAMPLE: Assume a partially redundant system of ten elements. An average of λ failures per hour can be expected if each failure is instantly repaired or replaced. Find the probability that x failures will occur if the system is put in operation for t hours and each failure is repaired as it occurs.

If λ is the average number of failures per element for one hour, then $m = \lambda t$ is the average number of element failures for t hours. Hence,

$$f(x) = \frac{e^{-\lambda t}(\lambda t)^x}{x!} \qquad x \geq 0$$

With n of these elements in the system, the average number of failures in t hours would be $n\lambda t$, and

$$f(x) = \frac{e^{-n\lambda t}(n\lambda t)^x}{x!}$$

If $\lambda = 0.001$ per hour, $t = 50$ hours, for $n = 10$, then

$$m = n\lambda t = 10(.001)50 = 0.5$$

$$f(x) = \frac{e^{-0.5}(.5)^x}{x!}$$

$f(x=0) = .607$

$f(x=1) = .303$

$f(x=2) = .076$

etc., as shown in Figure 2-13.

By cumulatively adding the probabilities of consecutive values of x, e.g., 0, 0+1, 0+1+2, the cumulative probability function can be generated. This function is shown in Figure 2-14 for x = 0, 1, 2, 3. Mathematically it is represented by Equation

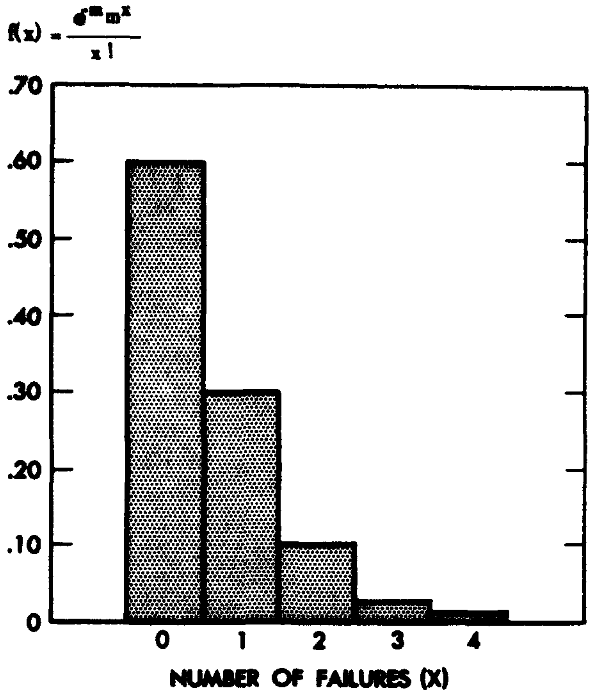


Figure 2-13. Poisson Probability Function for m=0.5

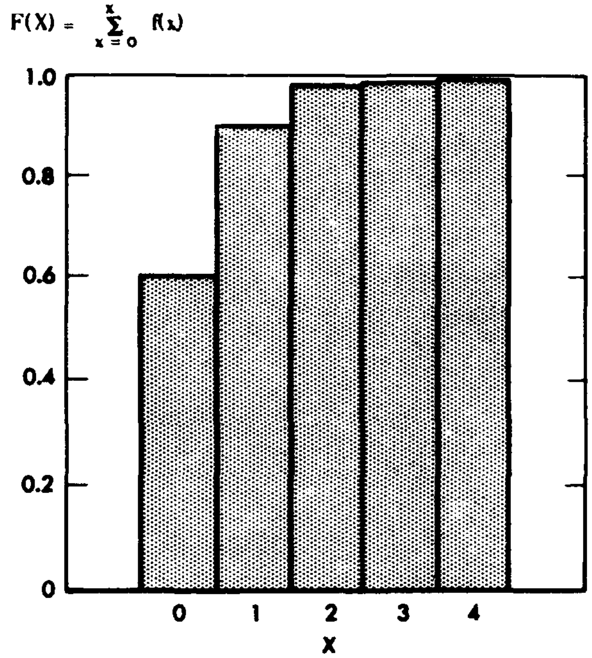


Figure 2-14. Cumulative Poisson Probability Distribution Function for m=0.5

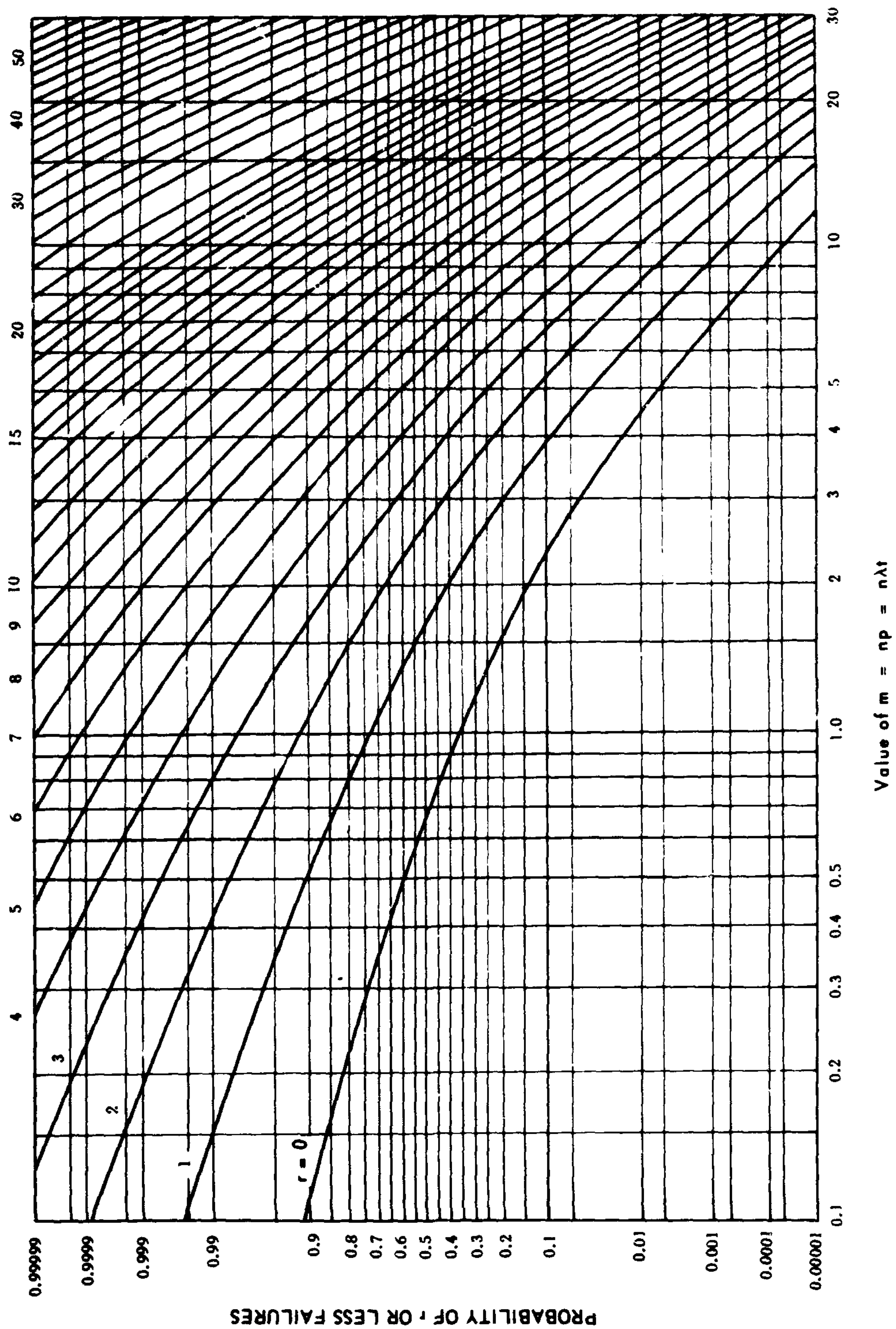
(2-15), where m is the Poisson parameter which, for the example given, is equal to $n\lambda t$ and represents both the mean and the variance.

The system then has a probability of .607 of surviving the 50-hour mission with no element failures; a probability of .91 (the sum of P(0) and P(1)) of surviving with no more than one element failure. There is a 9% chance that two or more failures will occur during the mission period. If the system will perform satisfactorily with nine elements, and if further we are permitted one on-line repair action during the mission (to repair a second failure) then system reliability *with one repair* during the mission is .986 (assuming instantaneous repair or replacement capability). This illustrates the advantage of on-line repairs, to permit failure occurrence without sacrificing reliability.

Chart 2-II is a Poisson cumulative probability graph for values of m ranging from 0.1 to 30, useful for graphical solution of the Poisson equation. In the above case, for example, enter the chart at m = .5 and go vertically to the curve r = 0. The ordinate corresponding to this point is approximately 0.6—the probability of zero failures in the 50-hour mission. Proceeding to the r = 1 curve, again at m = .5, the probability of surviving a 50-hour mission with one or less (no more than one) failure is approximately 0.91 as was derived before. To find the probability of two or more failures, merely subtract the probability of one or less from unity, e.g.:

$$P(r \geq 2) = 1 - P(r \leq 1)$$
$$= 1 - 0.91 = 0.09 = 9\%$$

CHART 2-II. POISSON CUMULATIVE PROBABILITIES



EXAMPLE: What is the probability of finding three or more defective transistors in a sample of n = 100 whose percent defective p = 5% = .05?

Enter chart at $m = np = (100)(.05) = 5$.

Go vertically to r = 2.

Read 0.125 as the probability of 2 or less.

Then the probability of 3 or more = 1 - 0.125 = 0.875

EXAMPLE: What is the probability of a 10-hour mission without failure in a system whose mean life is known to be 50 hours?

n = 1 system

λ = 1 failure per 50 hours
= .02 failures per hour

t = 10 hours

$m = n\lambda t = (1)(.02)(10) = .2$

r = 0 = allowable failures

Enter chart at m = .2.

Go vertically to r = 0.

Read 0.82 on left-hand scale.

This is the reliability of the system for a 10-hour mission.

EXAMPLE: If 10 aircraft take off for ASW service, each with the system described above, what is the probability that at least 8 will complete the 10-hour mission without failure?

$m = n\lambda t = 2$ failures expected

r = 2 or less

Probability of at least 8 operational systems for the full 10-hour mission is then 0.67. This can be interpreted as a level of confidence on the estimated reliability, i.e., 67% confidence that at least 80% operational reliability will be realized.

As another application of the Poisson Chart, determine the number of aircraft that should be dispatched to assure with 90% confidence that at least 10 will remain on patrol for the 10-hour period. From the previous example, n is unknown and r is unknown. But $n = m/\lambda t = 5m$, for $\lambda t = .2$ as before. Then n - r = 5m - r = 10 at 90% confidence will satisfy the requirement. From the Chart, m = 3 and c = 5 is the combination that satisfies the 90% probability ordinate. Thus, 15 aircraft should be dispatched to be 90% confident that 10 will remain on patrol throughout a 10-hour mission.

Confidence limits and levels are discussed in more detail in 2.3.

2.2.3 The Normal Distribution

The normal distribution has the probability density function shown by the following equation:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

for values of x between $-\infty$ and $+\infty$
$(-\infty < x < \infty)$ *(2-16)*

The formula shows that the two parameters of the normal distribution are the mean, μ, and the standard deviation, σ. Figure 2-15 shows the normal curve. The ordinate of the probability density function (pdf) indicates the relative probabilities of various values occurring.

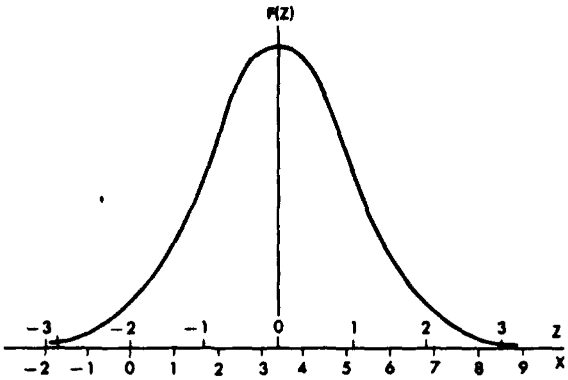


Figure 2-15. Probability Density Function of the Normal Distribution

The area under a density curve between two points, a and b, is equal to the probability that a value will occur between a and b. To find this probability it is necessary to integrate the pdf between a and b. That, for the normal distribution, is:

$$P[a < x < b] = \int_a^b \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$$

Tables of the cumulative normal distribution, shown in Figure 2-16, have been tabulated in Table 2-II for a distribution with mean 0 and variance 1. This is called the standard or normalized form and is obtained by transforming the original values of x into a new variate Z where

$$Z = \frac{X - \mu}{\sigma}.$$

The density function of Z is

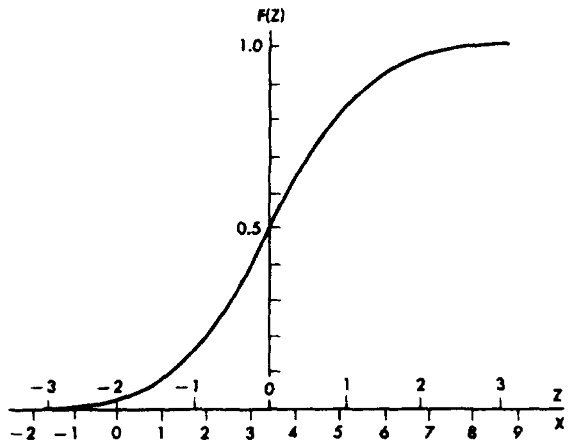


Figure 2-16. Cumulative Distribution Function of the Normal Distribution

$$f(Z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{Z^2}{2}}$$

The table therefore gives

$$F(Z) = \int_{-\infty}^{Z} f(Z) dZ^{1/}$$

For a known mean and variance of the variable x, various probabilities can be found by computing $Z = (x - \mu)/\sigma$ and referring to the table of areas.

EXAMPLE: Assume $\mu = 100$ and $\sigma = 5$. Find the probability that a value will occur between 95 and 110 as shown in Figure 2-17.

$$\text{Let } Z_1 = \frac{95 - 100}{5} = -1$$

1/ Other limits often given are $\int_0^Z$, $\int_{-Z}^Z$, and $\int_Z^\infty$. Because of the symmetry of the normal distribution, it is easy to use any set of tables to find particular probabilities.

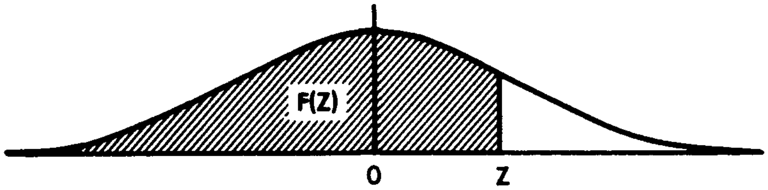


TABLE 2-II CUMULATIVE NORMAL DISTRIBUTION

z	.000	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5336	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9206	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

$$F(Z) = \int_{-\infty}^{Z} \frac{1}{\sqrt{2\pi}} e^{-Z^2/2} \, dz$$

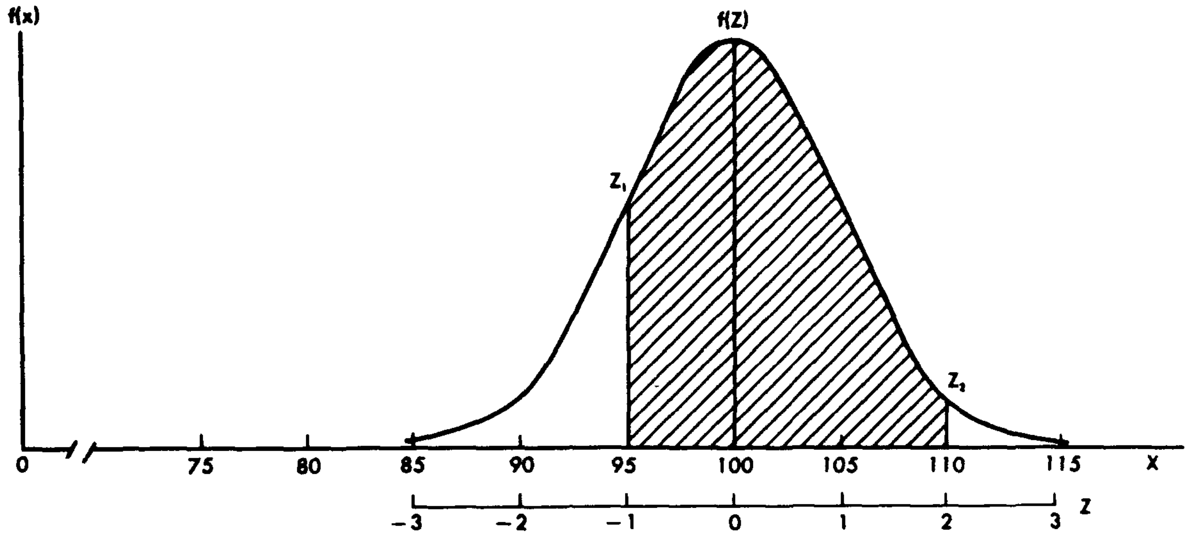


Figure 2-17. Probability of a Value between Two Points under the Normal

$$Z_2 = \frac{110 - 100}{5} = 2$$

Then

$$P[95 < x < 100] = P[-1 < Z < 2] = F(2) - F(-1)$$

From Table 2-II,

$F(2) = 0.977$, and

$F(-1) = 0.159$,

hence $P[-1 < Z < 2] = 0.977 - 0.159 = 0.818$

In reliability engineering, the normal distribution is frequently found to adequately represent the failure time distribution of items whose failure modes are a result of wearout. The failure rate of a normally distributed time-to-failure variable is an increasing function of the age of the product, which is consistent with a wearout process.

It must be pointed out that the normal distribution implies that the variable can theoretically take on values anywhere between plus and minus infinity. In reliability work where the variable is time-to-failure, values less than zero cannot occur. The use of the normal distribution rests upon its ability to describe the observed phenomena. Normal assumptions appear valid for those cases described below.

(a) *Probability of failure, before time zero, is small.*

The table of the cumulative standard normal shows that the probability of a value less than three standard deviations below the mean is negligibly small—approximately .001 compared to the total area under the curve, which is equal to one. If μ is greater than 3σ, then the theoretical probability of a value falling below zero is small enough to ignore.

(b) *Truncated normal.*

The truncated normal distribution may be appropriate. By truncation of a distribution is meant that a portion of the curve is deleted and its area is distributed over the remaining portion. In this particular case, it is assumed that population values less

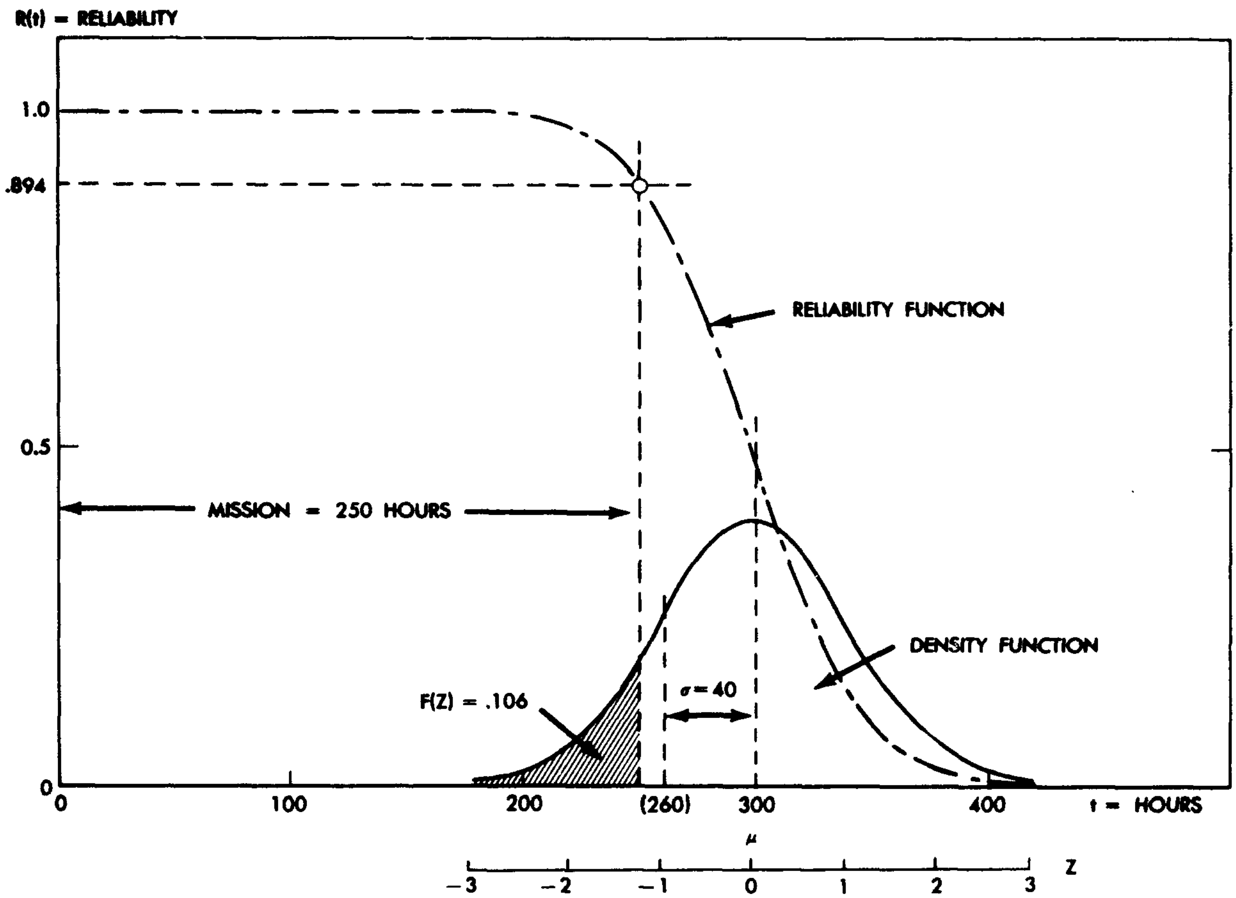


Figure 2-18. Reliability Function = One Minus F(Z), the Failure Density Function

than zero are impossible and the probability area represented by these values is to be distributed over the range 0 to ∞.

> *EXAMPLE:* Assume an item whose mean life and standard deviation is estimated to be 300 hours and 40 hours, respectively. If its mission length (or time before maintenance or replacement) is 250 hours, the probability that the item will complete its mission is
>
> $R(250) = 1 - F(250)$

$$= \int_{-\infty}^{250} \frac{1}{40\sqrt{2\pi}} e^{-\frac{(x-300)^2}{2(40)^2}} dx$$

By letting

$$Z = \frac{x-300}{40},$$

the upper limit of x = 250 becomes

$$Z = \frac{250-300}{40} = -1.25$$

and

$$R(Z) = 1 - \int_{-\infty}^{-1.25} \frac{1}{\sqrt{2\pi}} e^{-\frac{Z^2}{2}} dz$$

$$= 1 - 0.106$$

$$= 0.894$$

A probability distribution function for this example is shown in Figure 2-18.

2.3 THE EXPONENTIAL DISTRIBUTION

The exponential density is a direct consequence of the assumption that the probability of failure in a given time interval is directly proportional to the length of the interval and is *independent* of the age of the product. The exponential density derived from this basic assumption has the form

$$f(t) = \frac{1}{\theta} e^{-\frac{t}{\theta}} \qquad (2\text{-}17)$$

where θ = mean life and t is the time period of interest. The reliability at time t is

$$R(t) = \int_t^{\infty} \frac{1}{\theta} e^{-\frac{t}{\theta}} dt = e^{-\frac{t}{\theta}} \qquad (2\text{-}18)$$

Figure 2-19 shows the exponential reliability function where time is given in units of θ.

Mean life is the arithmetic average of the lifetimes of all items considered. A lifetime may consist of time-between-malfunctions, time-between-repairs, time-to-removal-of-parts, etc. Mean life for the exponential distribution is MTBF = θ.

The property of the exponential implies two significant failure characteristics. First, individual failures occur in a random or unpredictable manner. Second, the failure rate or hazard rate is constant, which implies that deterioration is not a failure cause.

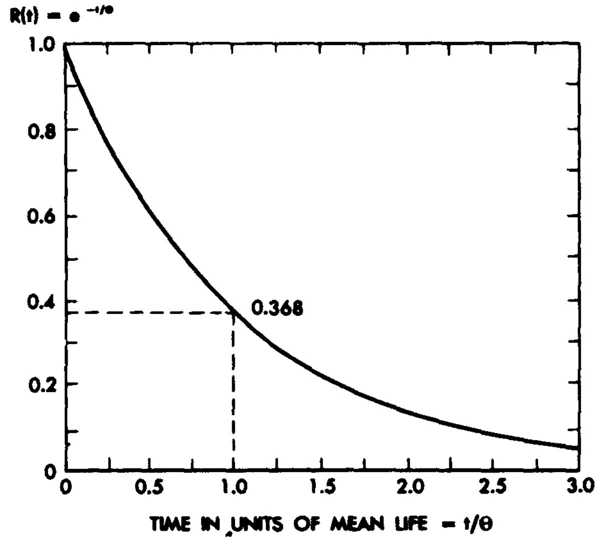


Figure 2-19. The Exponential Reliability Function

The constant failure rate per h hours can be shown to equal

$$1 - e^{-\frac{h}{\theta}}$$

and, similarly, the failure rate per hour is

$$1 - e^{-\frac{1}{\theta}}$$

When θ is large relative to h, the failure rate per h hours is usually approximated by h/θ.

The instantaneous failure rate, λ, equals $1/\theta$ and is usually used as the constant exponential failure rate. Thus

$$R(t) = e^{-\lambda t} \qquad (2\text{-}19)$$

If an item has a constant failure rate, the reliability at its mean life, θ, is 0.368.

In other words, the mean life will occur at the point where there is 36.8% probability of survival. This follows from

$$R(\theta) = e^{-\frac{\theta}{\theta}}$$

$$= e^{-1}$$

$$= 0.368$$

This is shown in Figure 2-19. Thus there is a 36.8% probability that a system will survive to its mean life, without failure.

Mean life and failure rates are related by the following equations:

$$\theta = \frac{-t}{\log_e R(t)} \qquad (2\text{-}20)$$

$$\lambda = \frac{1}{\theta} \qquad (2\text{-}21)$$

2.3.1 Relationship of the Exponential to the Poisson

The exponential and the Poisson distributions are equivalent except for the choice of the random variable. For the exponential, the random variable is the time-to-failure; for the Poisson, it is the number of failures per given time period where failure times are exponentially distributed. The exponential variable is continuous; the Poisson variable is discrete.

The Poisson density of number of failures, x, is

$$f(x) = \frac{e^{-m} m^x}{x!}$$

$$x = 0, 1, 2 \ldots$$

Letting $m = \lambda t$, the expected number of failures over the interval (0,t) in a replacement situation, the density becomes

$$f(x) = \frac{e^{-\lambda t} (\lambda t)^x}{x!}$$

The probability of zero failures in the interval (0,t) is therefore

$$f(0) = e^{-\lambda t}$$

which is the exponential reliability function.

2.3.2 The Exponential Function as a Failure Model

The mechanism underlying the exponential reliability function is that of random or chance failures which are independent of accumulated life and consequently are individually unpredictable. The use of this type of "failure law" for complex systems is usually justified by the fact that many forces can act upon the system and produce failure. Varying deterioration mechanisms, different part failure rates, varying environmental conditions, and so on, result in stress-strength combinations that produce failures randomly in time.

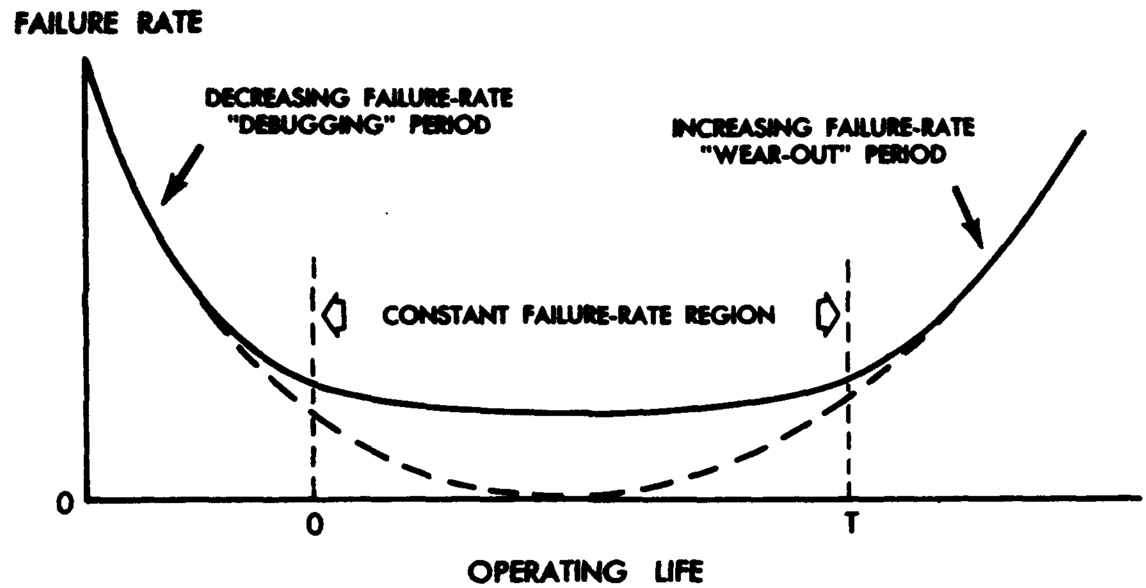


Figure 2-20. Typical Failure-Rate Curve

A typical failure-rate curve is shown in Figure 2-20. If a life characteristic can be represented by this type of curve, then for some time period—say, (0,T)—the failure rate is approximately constant; that is, failures in this period can be classified as random occurrences. After time T, wearout effects become apparent with increasing frequency, so that the probability of failure increases. Infant mortality, represented by a decreasing failure rate in early life, is usually detected during system debugging, and therefore should not remain as a continuing problem after the system gets into the Fleet.

Chart 2-III presents normalized functions for both R(x) and U(x) = 1 - R(x), with x expressed in terms of proportion of mean life, t/θ. Tables of the Exponential Function are available from the Department of Commerce.[2/]

e^{-x} may also be derived from the series expansion:

$$e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \frac{x^5}{5!} + \dots$$

EXAMPLE:

$$e^{-.3} = 1 - .3 + \frac{.09}{2} - \frac{.027}{6} + \frac{.0081}{24}$$

$$\approx 1 - .3 + .045 - .0045 + .0003$$

$$\approx .7408$$

[2/] *Tables of Exponential Function*, National Bureau of Standards Applied Mathematics Series No. 14.

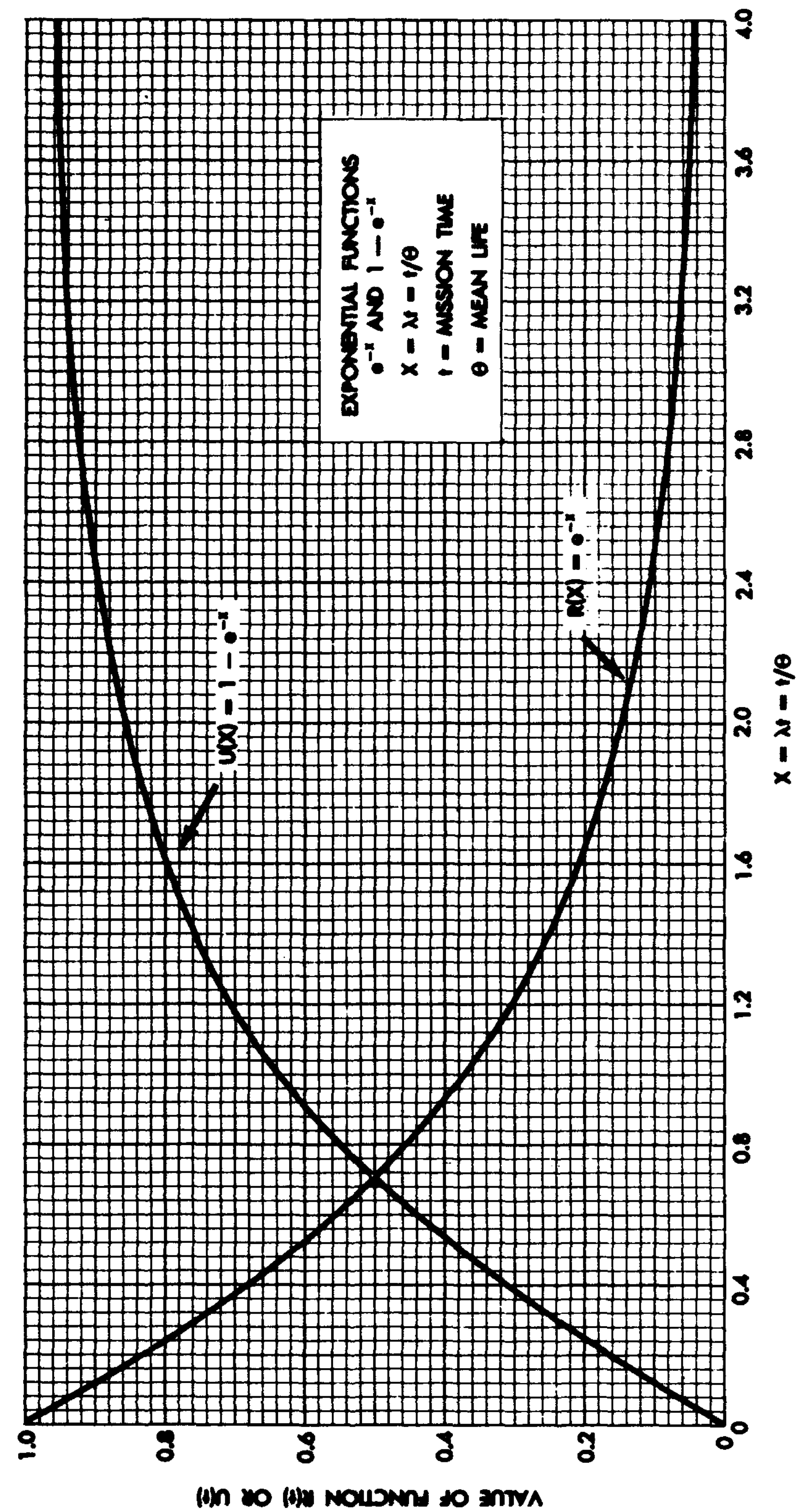


CHART 2-III. EXPONENTIAL FUNCTIONS

APPENDIX 3. RELIABILITY ESTIMATION

During the course of system development, evaluation, and Fleet use, many opportunities—both planned and unplanned—become available for the estimation of reliability on the basis of data generated in system testing and operational use. Other sections of the appendix describe procedures for test design and data reporting. This appendix describes the most commonly used procedures for analyzing and presenting such data for practical application by management and engineering in system improvement.

3.1 ESTIMATION OF MEAN LIFE AND FAILURE RATE IN THE EXPONENTIAL CASE

The mean life of an equipment whose failure times have the exponential distribution is approximately

$$\hat{\theta} = \frac{\text{Total Operating Time, } T(t)}{\text{Number of Observed Failures}}$$

$$= \frac{T(t)}{r} \qquad (3\text{-}1)$$

where $\hat{\theta}$ denotes estimated mean life.

Since the hazard or instantaneous failure rate is equal to the reciprocal of mean life, all estimates of θ can be used to estimate λ.

Total operating time is defined to be the total number of operating hours accumulated before the test is terminated, or the total test time in a test to failure of several items. For example, if a test of n items was run for T hours and failed items were not replaced,

$$T(t) = \sum_{i=1}^{r} t_i + (n-r)T \qquad (3\text{-}2)$$

where t_i is the time of the i^{th} failure and T is length of test in hours.

If c items were censored before T (removed before failure, accidentally broken, etc.) and were not replaced in the test, then

$$T(t) = \sum_{i=1}^{r} t_i + \sum_{j=1}^{c} t_j + (n-r-c)T \qquad (3\text{-}3)$$

where t_j is the time the j^{th} censorship took place.

EXAMPLE: Ten traveling wave tubes were placed on reliability demonstration test. The test was terminated with the fifth failure. One tube had been removed from the test because of accidental damage after 100 hours of operation. Total accrued test time was then

Time to 1st failure	10 hours
Time to 2nd failure	70 hours
Time to 3rd failure	120 hours
Time to 4th failure	210 hours
Time to 5th failure	300 hours
Time to censorship (damaged tube)	100 hours
Time to censorship (remaining 4 tubes)	1,200 hours
Total Operating Time	2,010 hours

Mean Life $= \hat{\theta}$

$$= \frac{\text{Total Operating Time}}{\text{Number of Failures}}$$

$$= \frac{2{,}010 \text{ hours}}{5}$$

$$= 402 \text{ hours}$$

Failure Rate $= 1/\hat{\theta} = 1/402$
$= 2{,}480 \times 10^{-6}$ failures per hour.

EXAMPLE: An airborne system has been out on 20 successive 3-hour missions. In five instances the system failed during a mission. Times to failure were recorded to the nearest half-hour as:

Failure #1	1.5 hours
Failure #2	.5 hours
Failure #3	2.5 hours
Failure #4	1.0 hours
Failure #5	2.0 hours
Total Time to Failure	7.5 hours
Total Successful Time = 3 x 15 =	45.0 hours
Total "Up" Time	52.5 hours

Mean Life,

$$\hat{\theta} = \frac{52.5 \text{ hours}}{5} = 10.5 \text{ hours}$$

System failure rate = 1/10.5 = one failure per 10.5 hours or .095 failures per hour, usually expressed as 95 failures per thousand hours or 95×10^{-3} failures per hours.

Reliability Nomograph

A reliability nomograph[1/] is shown in Chart 3-I. The nomograph relates reliability to operating time and failure rates or mean life for the exponential case.

EXAMPLE: Mean time to failure of an airborne fire control system is 10 hours. What is the probability that the system will satisfactorily perform throughout a 3-hour mission? Connect $\theta = 10$ to $t = 3$ hours with a straight edge and read $R = .75$ for an estimate of reliability for the 3-hour mission. This is the graphical solution to the equation

$$R(3 \text{ hours}) = e^{-t/\hat{\theta}} = e^{-.3} = .7408$$

3.2 VERIFICATION OF VALIDITY OF THE EXPONENTIAL ASSUMPTION

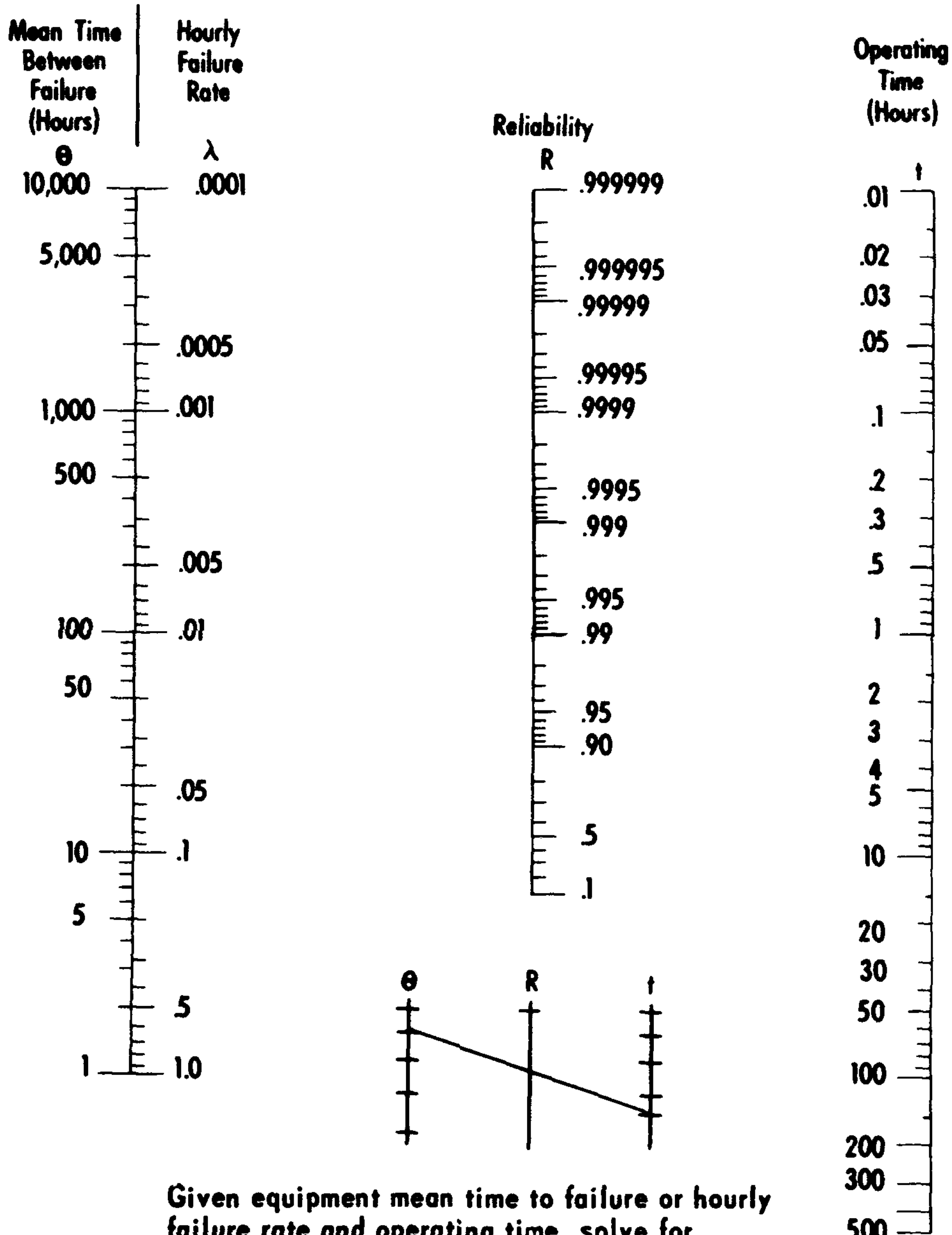
The exponential function is generally valid for complex systems and most parts. However, if failure rate data do not support the exponential assumption, or if failures do not occur randomly in time and wear out becomes an important factor, then the exponential assumption is no longer valid.

A graphical procedure is useful for a quick indication of the validity of the exponential assumption provided that the number of observed failures is relatively large. The procedure is to plot the cumulative test or operating time against the cumulative number of failures r, as shown in Figure 3-1.

1/ Reprinted with permission from Reliability Data Sheet #1, Curtiss Division of Curtiss-Wright Corporation, Caldwell, New Jersey.

CHART 3–1. RELIABILITY NOMOGRAPH FOR THE EXPONENTIAL FAILURE DISTRIBUTION

(Multiply "R" values by 100 for % survival)



Given equipment mean time to failure or hourly failure rate and operating time, solve for reliability. Connect "θ" and "t" values with straight line. Read "R".

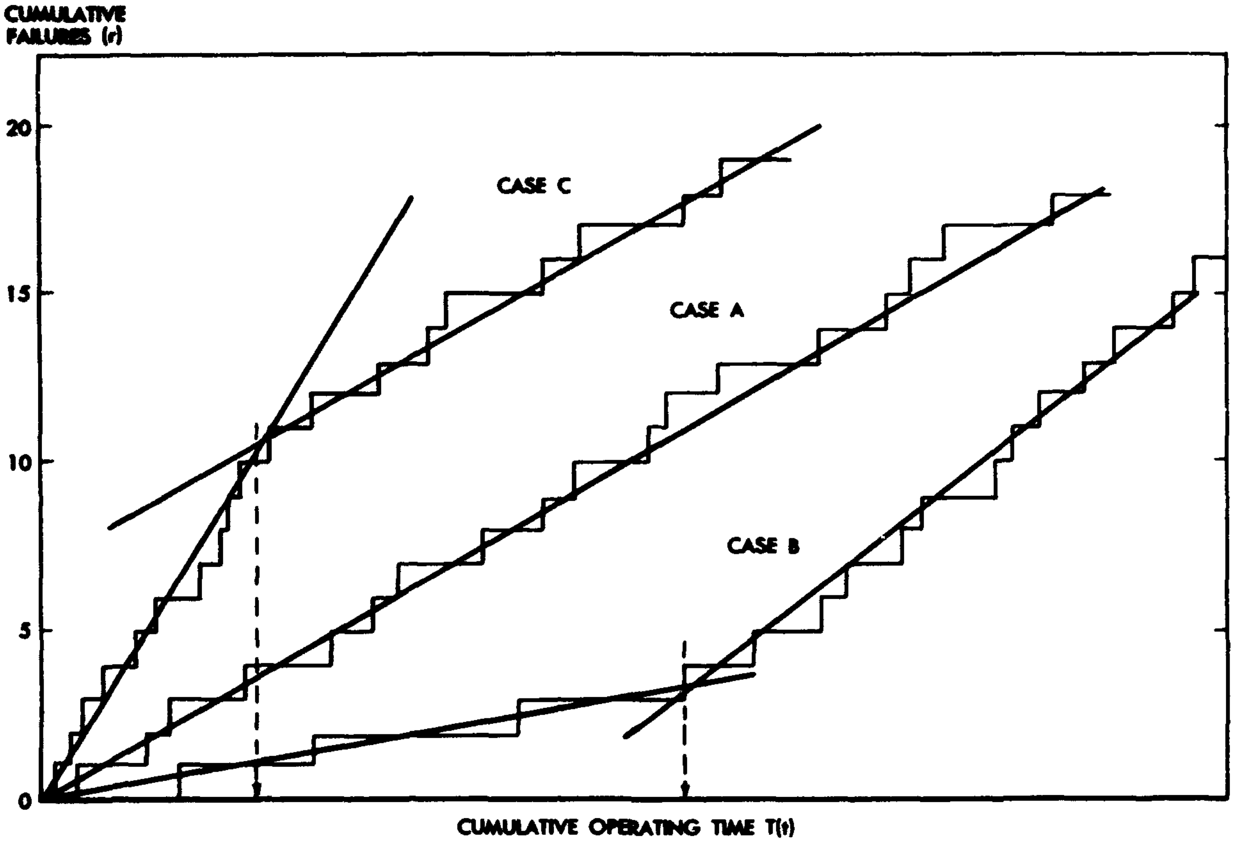


Figure 3-1. Plot of Time versus Failures

Figure 3-2 shows a further refinement on the graphical procedure. Here cumulative time at each failure is plotted against the quantity.

$$Y_i = \text{Ln}\left(\frac{n+1}{n-i+1}\right), \tag{3-4}$$

where n is the number of items at the start of a non-replacement test, or the number of original items plus replacements in a replacement test, and times are recorded as time between failure. The method is applicable to system studies, where n now becomes the number of operating periods between failures.

The figures show three cases:

CASE A–Exponential assumption is valid.

CASE B–Exponential valid over prescribed time period. Change in slope may be due to change in use conditions or maintenance procedures.

CASE C–Exponential valid over prescribed time period. Change in slope indicates debugging or user training period in early life.

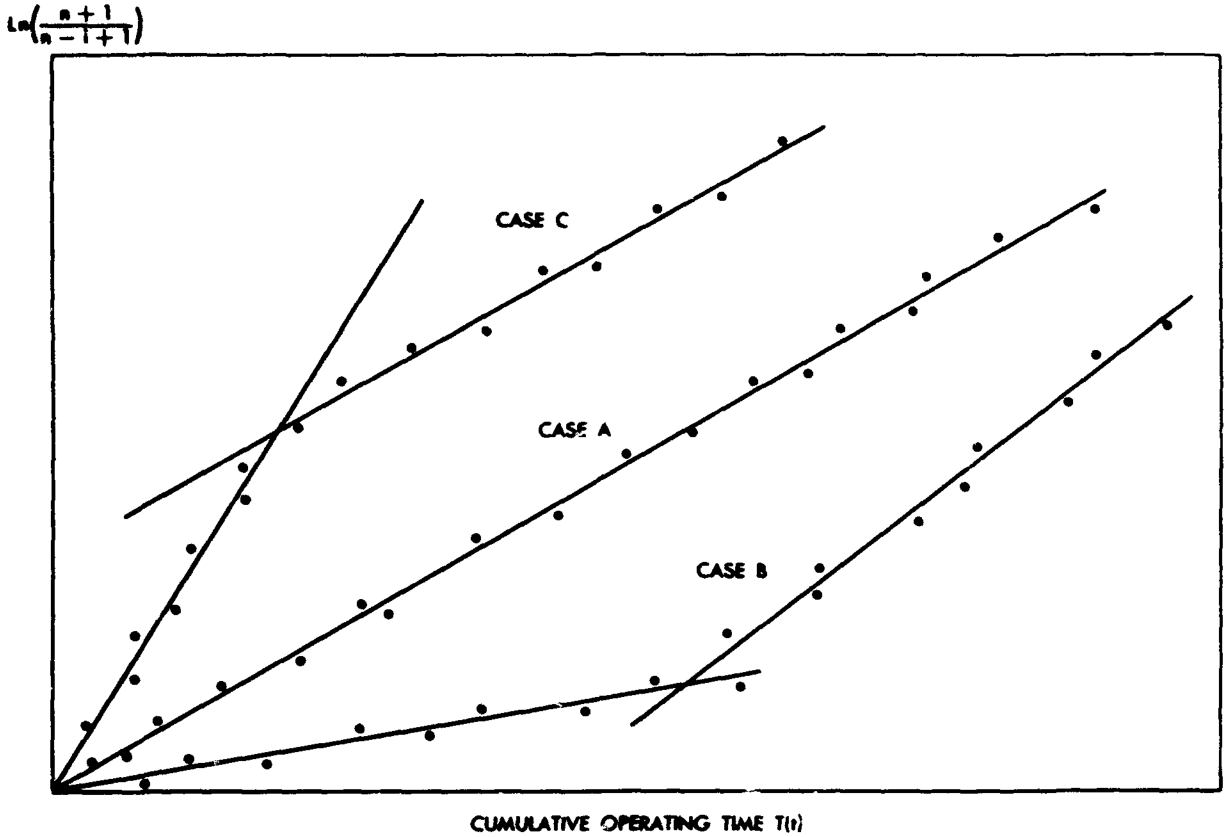


Figure 3-2. Plot of Time versus $\ln\left(\frac{n+1}{n-i+1}\right)$

An Analytical Method

This is the best test to use for testing the hypothesis of an exponential distribution versus the alternative that the failure rate is not constant. Compute the value of k

$$k = -2 \sum_{i=1}^{r} \log_e \left[\frac{T(ti)}{T(t)}\right] \qquad (3\text{-}5)$$

where T(ti) = total operating time at the i^{th} failure, T(t) = total operating time at conclusion of test. r = total number of failures.

k is a χ^2 variate with 2r degrees of freedom. The two critical values of χ^2 are found in tables of the Chi-square distribution.[2/] Alpha (α) represents the risk of rejecting a true hypothesis (Type I error) which, for this test, is equivalent to concluding that the failure rate is not constant when in fact it is. For a fixed sample size, the lower the specification on α, the greater the chance of accepting a false hypothesis (Type II error) by concluding that the failure rate is constant when in fact it is not. The usual range of α is from 0.01 to 0.10 depending on the consequences of making a Type I error for the particular situation. 2r is the number of degrees of freedom (r is the number of

[2/] See Appendix 2.

failures) and is used for entering the χ^2 table. If k lies between the two values of Chi-square, i.e.,

$$(\chi^2_{\alpha/2,2r} < k < \chi^2_{1-\alpha/2,2r})$$

then the hypothesis of the exponential pdf is accepted.

EXAMPLE: Assume 20 items were life tested (non-replacement) for 100 hours. A total of 9 failures occurred at 10, 17, 17, 25, 31, 46, 52, 65, and 79 hours. To test whether the exponential assumption is valid, compute

$$k = -2 \left[\log_e \frac{20 \times 10}{1442} + \log_e \frac{10 + 19 \times 17 +}{1442} \ldots + \log_e \frac{263 + 12 \times 79}{1442} \right]$$

$$= -2 \left[-8.552 \right]$$

$$= 17.104$$

For $\alpha = 0.05$ (95% confidence level),

$$\chi^2_{0.025,\,18} = 8.23;$$

$$\chi^2_{0.975,\,18} = 31.5$$

Since k falls between the two critical limits, the assumption of an exponential distribution is valid.

If it is desired to test against the alternative hypothesis that the failure rate is increasing, then only the lower critical limit,

$$\chi^2_{\alpha,2r'}$$

is used. Similarly, only the upper critical limit,

$$\chi^2_{1-\alpha,\,2r'}$$

is used if the alternative hypothesis is that the failure rate is decreasing.

3.3 ESTIMATION OF CONFIDENCE LIMITS ON EXPONENTIAL MEAN LIFE, FAILURE RATE, AND RELIABILITY

The mean life of an item is estimated from sample operating periods and failure data. Therefore allowances must be made for sampling fluctuations. Since it is quite unlikely that any two "samples" drawn from the same population will produce the same results, an interval is computed for which there is a high degree of confidence that it will contain the true population value. If we compute a 95% confidence interval, it means there is a probability of 0.95 that the interval will contain the true parameter value.

The limits associated with the confidence interval are called confidence limits (C.L.), and the measure of confidence is the confidence level denoted by $(1 - \alpha)$ where α is the probability that the interval will *not* contain the true value. A one-sided confidence interval is used when we wish to determine only a maximum or a minimum value of a parameter, such as the lower limit on mean life or the upper limit on failure rate.

The χ^2 (Chi-square) distribution can be used to derive the confidence limits on the exponential mean life. Table 3-I gives upper and lower factors (UF and LF) which provide the two confidence limits when multiplied by the point estimate of θ given above. Hence, the probability that the true mean life lies above some lower limit (at

TABLE 3–1. UPPER AND LOWER FACTORS FOR DETERMINING CONFIDENCE LIMITS FOR THE EXPONENTIAL MEAN LIFE (TWO-SIDED LIMITS)

Number of Failures Observed (r)	90% Confidence Level		95% Confidence Level	
	Lower 95% Factor	Upper 95% Factor	Lower 97.5% Factor	Upper 97.5% Factor
1	.334	19.417	.271	39.526
2	.422	5.624	.359	8.264
3	.476	3.670	.415	4.850
4	.516	2.027	.456	3.670
5	.546	2.538	.488	3.080
6	.571	2.296	.514	2.725
7	.591	2.130	.536	2.487
8	.608	2.010	.555	2.316
9	.624	1.917	.571	2.187
10	.637	1.843	.585	2.085
11	.649	1.783	.598	2.003
12	.659	1.733	.610	1.935
13	.669	1.691	.620	1.878
14	.677	1.654	.630	1.829
15	.685	1.622	.639	1.787
16	.693	1.594	.647	1.749
17	.700	1.569	.654	1.717
18	.706	1.547	.661	1.687
19	.712	1.527	.668	1.661
20	.717	1.509	.674	1.637
21	.723	1.492	.680	1.616
22	.727	1.477	.685	1.596
23	.732	1.463	.690	1.578
24	.737	1.450	.695	1.561
25	.741	1.438	.700	1.545
26	.745	1.427	.704	1.531
27	.748	1.417	.709	1.517
28	.752	1.407	.713	1.505
29	.755	1.398	.717	1.493
30	.759	1.389	.720	1.482

Confidence limits are determined by multiplying the estimated mean life $\hat{\theta}$ by factors which correspond to the desired confidence level and the observed number of failures in the life test, i.e.:

$$P\left[(LF)_{\alpha/2,r}\,\hat{\theta} \leq \theta \leq (UF)_{\alpha/2,r}\,\hat{\theta}\right] = 1 - \alpha$$

$\alpha/2$) and below some upper limit (at $1 - \alpha/2$) is equal to $1 - \alpha$, the area between the two limits, i.e.

$$P\left[(LF)_{\alpha/2,r}\hat{\theta} \leq \theta \leq (UF)_{\alpha/2,r}\hat{\theta}\right] = 1-\alpha$$

where $\hat{\theta}$ = point estimate of mean life

θ = true mean life *(3-6)*

$(LF)_{\alpha/2,r}$ = lower factor for $(1-\alpha)$% C.L. based on r failures

$(UF)_{\alpha/2,r}$ = upper factor for $(1-\alpha)$% C.L. based on r failures

The table gives LF and UF for 90% and 95% two-sided confidence intervals for values of r from 1 to 30.

> *EXAMPLE:* Assume 15 failures occurred on a life test, giving an estimated mean life, $\hat{\theta}$, of 2,000 hours. From Table 3-I the lower and upper 97.5% factors are 0.639 and 1.787, respectively. Therefore the 95% confidence interval is
>
> $$\left[(.639)(2000) \leq \theta \leq (1.787)(2000)\right]$$
>
> or
>
> $$\left[1278 \leq \theta \leq 3754\right]$$
>
> Thus from this test we can be 95% confident that the true mean life is between 1,278 and 3,754 hours.

Note that if the data is based on a life test where there is a pre-assigned truncation time, enter the table with (r + 1) failures rather than r.

For r greater than 30, the approximate values for LF and UF are:

(1) For 90% confidence level (r>30)

$$(LF)_{.05,r} = \frac{4r}{(\sqrt{4r-1} + 1.645)^2} \quad (3\text{-}7)$$

$$(UF)_{.05,r} = \frac{4r}{(\sqrt{4r-1} - 1.645)^2} \quad (3\text{-}8)$$

(2) For 95% confidence level (r>30), replace 1.645 in the above equations by 1.96.

Table 3-II gives the lower factor $(LF)_{\alpha,r}$ for one-sided lower confidence limits on the exponential mean life. Multiplying the point estimate of θ by (LF) gives the one-sided $(1 - \alpha)$% confidence limit. For r greater than 30,

$$(LF)_{\alpha,r} = \frac{4r}{(\sqrt{4r-1} + \chi^2_\alpha)^2} \quad (3\text{-}9)$$

where

$\chi^2_\alpha = 0.84$ if $\alpha = 0.20$ (80% confidence limit)

$\chi^2_\alpha = 1.28$ if $\alpha = 0.10$ (90% confidence limit)

$\chi^2_\alpha = 1.645$ if $\alpha = 0.05$ (95% confidence limit)

Chart 3-II is a plot of Table 3-II.

Reliability Estimates from Test Data

Since the reliability function for the exponential distribution is $R(t) = e^{-t/\theta}$, the estimate $\hat{\theta}$ can be used to estimate R(t). That is

$$\hat{R}(t) = e^{-t/\hat{\theta}}$$

The confidence interval for R(t) is then approximately

$$(e^{-t/\hat{\theta}_L} < R(t) < e^{-t/\hat{\theta}_U})$$

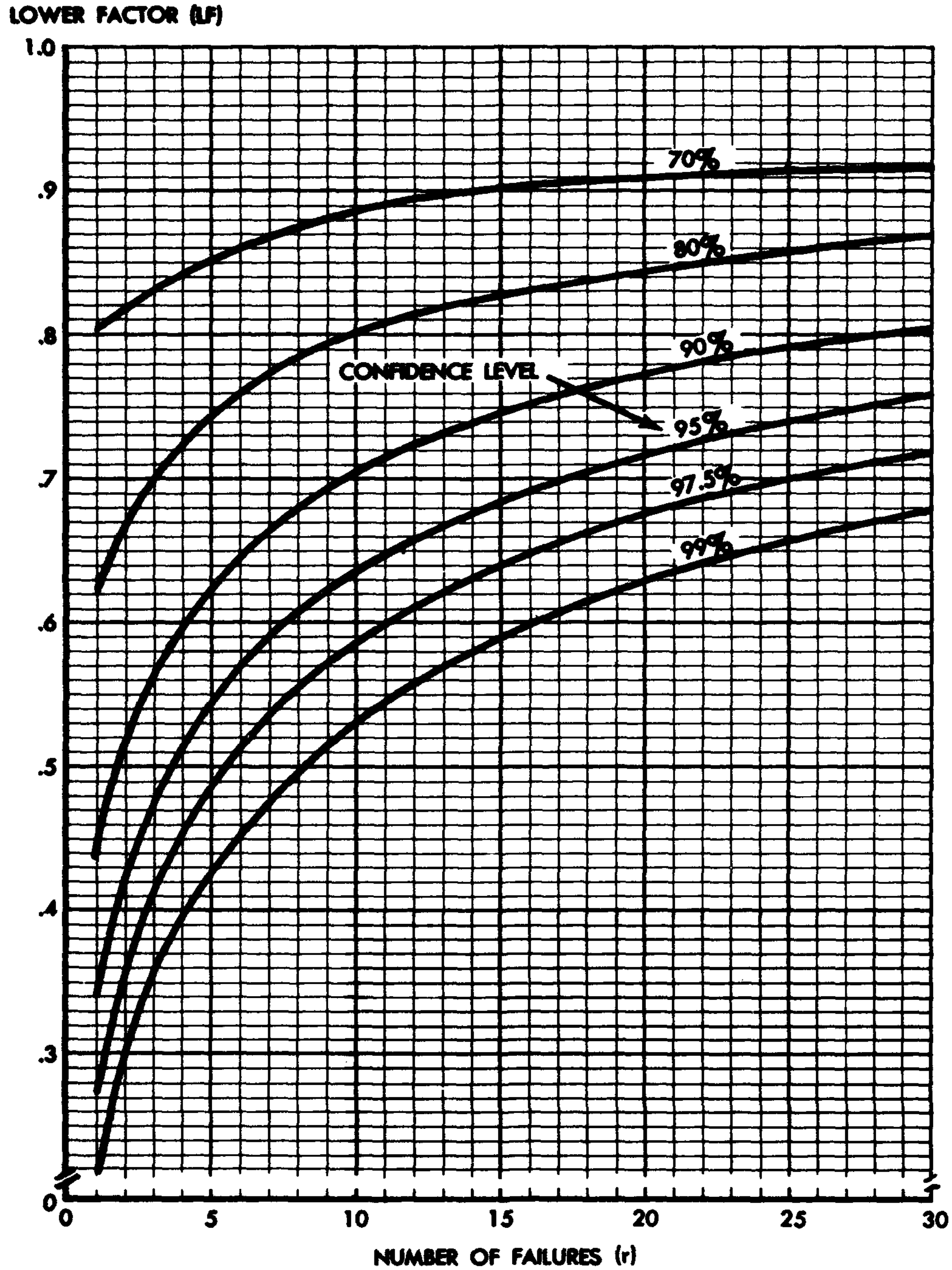
TABLE 3–II. FACTORS FOR DETERMINING LOWER CONFIDENCE LIMIT FOR THE EXPONENTIAL MEAN LIFE

Number of Failures Observed (r)	Lower Confidence Limit				
	80%	90%	95%	97.5%	99%
1	.621	.434	.334	.272	.217
2	.668	.514	.422	.360	.300
3	.701	.566	.476	.416	.358
4	.727	.597	.516	.457	.398
5	.746	.625	.546	.486	.432
6	.759	.649	.571	.515	.458
7	.769	.664	.591	.537	.480
8	.780	.681	.608	.555	.500
9	.789	.692	.624	.571	.516
10	.800	.704	.637	.583	.531
11	.806	.714	.649	.596	.546
12	.811	.723	.659	.608	.558
13	.818	.730	.669	.620	.570
14	.824	.739	.677	.630	.580
15	.826	.744	.685	.639	.590
16	.831	.751	.693	.645	.598
17	.835	.757	.700	.654	.606
18	.839	.763	.706	.662	.614
19	.842	.768	.712	.669	.620
20	.846	.772	.717	.765	.628
21	.848	.776	.723	.680	.635
22	.853	.780	.727	.685	.640
23	.855	.785	.732	.690	.645
24	.857	.788	.737	.695	.650
25	.859	.791	.741	.700	.656
26	.862	.795	.745	.705	.660
27	.864	.798	.748	.709	.666
28	.866	.801	.752	.713	.670
29	.868	.803	.755	.718	.675
30	.870	.806	.759	.720	.678

The lower confidence limit is determined by multiplying the estimated mean life by the factor (LF) which corresponds to the desired confidence level and the observed number of failures, i.e.:

$$P\left[(LF_{\alpha,r})\,\theta \leq \theta_{\infty}\right] = 1 - \alpha$$

CHART 3-11. RATIO OF LOWER LIMIT ON MTF, TO OBSERVED MTF AT SEVERAL LEVELS OF CONFIDENCE, AS A FUNCTION OF THE NUMBER OF FAILURES USED IN DETERMINING THE OBSERVED MTF



where $\hat{\theta}_L$ = lower confidence limit on θ

$\hat{\theta}_U$ = upper confidence limit on θ

In the same manner, the one-sided (lower) limit of R(t) is found by using the one-sided limit of θ found in Chart 3-II. As an example, if the total accumulated test time was 5,300 hours after 10 failures had occurred, then

$\hat{\theta} = \frac{5300}{10} = 530$ hours is the observed mean life

From Chart 3-II, the lower one-sided 90% confidence limit on θ is (.704)(530) = 373 hours. Hence, we can be 90% confident that the reliability at t hours is at least

$$R(t) = e^{-t/370}$$

Figure 3-3 illustrates the application of this lower 90% confidence limit to the exponential reliability function. In Figure 3-4 a 90% confidence interval has been derived using Table 3-I for values of $\hat{\theta}_L$ and $\hat{\theta}_U$ at the 90% level. In this instance the upper and lower bounds imply 95% confidence that θ is at least 338 hours but no greater than 977 hours, or 90% confidence that θ lies between these two bounds.

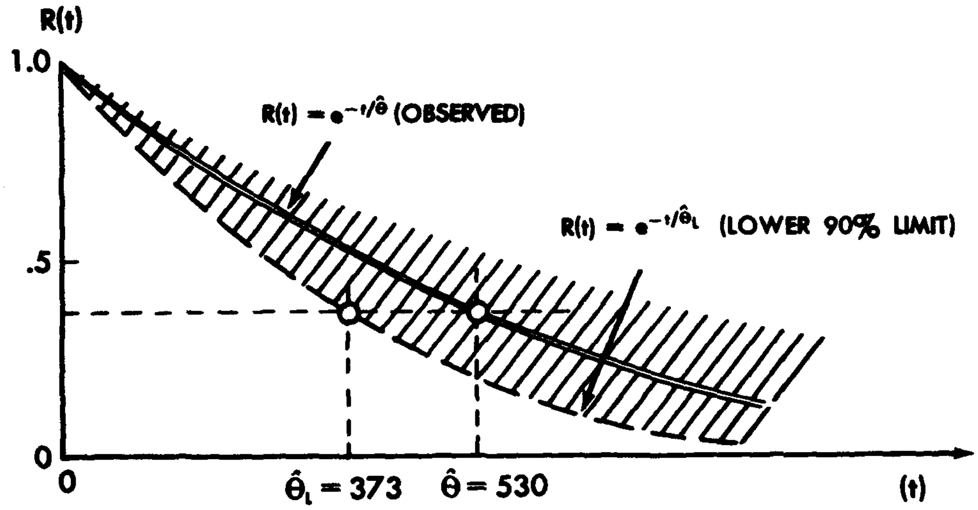


Figure 3-3. One-Sided (Lower) 90% Confidence Limit of θ Applied to the Exponential Reliability Function

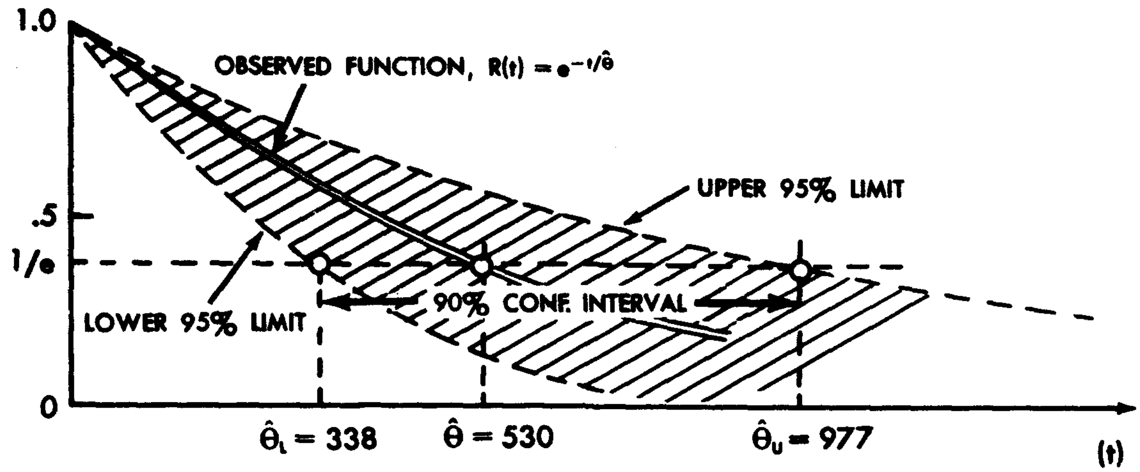


Figure 3-4. Two-Sided 90% Confidence Limits of θ Applied to the Exponential Function, Where $\hat{\theta}_L$ and $\hat{\theta}_U$ are 95% Limits for the 90% Confidence Interval

3.4 ESTIMATION OF CONFIDENCE LIMITS ON MISSION RELIABILITY AND FAILURE PROBABILITY

Confidence limits for a proportion of an attribute based on a sample are the limits that contain the true proportion of that attribute in the population from which the sample was drawn. The "attribute" may be the percent defective in a production lot of parts; the probability of system failure in a given number of operating cycles; or the probability of mission success in a given number of trials–mission reliability. Table 3-III[3] shows the upper confidence limits for sample sizes ranging from 2 to 30. Charts 3-III, IV and V extend the table from sample size 30 to sample size 5000.

[3] *Statistics Manual*, E. L. Crow, Frances A. Davis, and Margaret W. Maxfield, Dover Publications, Inc., p. 262.

EXAMPLE: Ten missiles are fired at target drones during a system exercise. All ten successfully intercept their respective targets. The observed reliability in this sample of ten is therefore 1.0 (the proportion failing is zero). From Table 3-III it can be stated with 90% confidence that the probability of missile failure in future tests of this system under the same set of test conditions will not exceed .206 on the basis of this sample of data. Estimated reliability of future missile firings should be at least .794 or approximately 80% at the 90% level of confidence.

EXAMPLE: In a sample of 50 transistors, 20% are observed to be defective.

Chart 3-III may be used to determine the limits for the true percentage of defectives in the population from which the sample was drawn. The chart shows that, for a proportion of r/N = .2 (20% in a sample of 50), the upper 95% confidence limit is .32. It may be stated with 95% confidence that the true percent defective of the population from which the sample was drawn is less than 32%.

EXAMPLE: A complex weapon control system is subjected to operational evaluation in the Fleet. In 100 randomly scheduled system exercises (mission trials) the system failed to respond to command on five occasions (r/N = .05) for an estimated *availability* of .95. It can be stated with 90% confidence that the availability of the weapon control system for any future demand is at least .9; that is, it will be available for tactical use approximately 9 times in 10.

When the sample size of interest does not appear on the chart, the following approximate formula may be used to compute confidence intervals on the true proportion in the population from which the sample is drawn:

$$p = \frac{(r+Z^2/2) \pm \sqrt{(r+Z^2/2)^2 - r^2/N(N+Z^2)}}{N + Z^2} \qquad (3\text{-}10)$$

where r = number of observed failures

N = Sample size, where N > 30

p = true proportion in the population

and Z has the following values for the indicated single limit confidence limits:

Upper or Lower Confidence Limit	Confidence "Band"	Z
80.0%	60%	0.840
90.0%	80%	1.282
95.0%	90%	1.645
97.5%	95%	1.960
99.5%	99%	2.576

EXAMPLE: In the 95 trials in which the weapon control system in the above example was "up" when needed, it successfully completed 80 of 95 attempted 3-hour missions (r/N = 15/95) for an observed reliability of .842. To solve for the lower 95% confidence limit on the observed reliability, substitute values of r, N, and Z into Equation (3-10) as follows, remembering that the lower limit on the reliability estimate is equal to one minus the upper confidence limit on r/N or p:

$$p = \frac{[15+(1.645)^2/2]}{95+(1.645)^2}$$

$$+ \frac{\sqrt{[15+(1.645)^2/2]^2 - 225/95[95+(1.645)^2]}}{95+(1.645)^2}$$

$$= \frac{[16.35] + \sqrt{[16.35]^2 - 231}}{97.7}$$

$$= \frac{23.05}{97.7} = .236$$

and $R_L = 1 - p_U = 1 - .236 = .764$

We can say, with 95% confidence, that reliability of the weapon control system is at least .764 under the conditions that prevailed during the test.

TABLE 3-III. ONE-SIDED CONFIDENCE LIMITS FOR A PROPORTION

If the observed proportion is r/n, enter the table with n and r for an upper one-sided limit. For a lower one-sided limit, enter the table with n and n − r and subtract the table entry from 1.

r	90%	95%	99%	r	90%	95%	99%	r	90%	95%	99%
n = 2				n = 3				n = 4			
0	.684	.776	.900	0	.536	.632	.785-	0	.438	.527	.684
1	.949	.975-	.995-	1	.804	.365-	.941	1	.680	.751	.859
				2	.965+	.983	.997	2	.857	.902	.958
								3	.974	.987	.997
n = 5				n = 6				n = 7			
0	.369	.451	.602	0	.319	.393	.536	0	.280	.348	.482
1	.584	.657	.778	1	.510	.582	.706	1	.453	.521	.643
2	.753	.811	.894	2	.667	.729	.827	2	.596	.659	.764
3	.888	.924	.967	3	.799	.847	.915+	3	.721	.775-	.858
4	.979	.990	.998	4	.907	.937	.973	4	.830	.871	.929
				5	.983	.991	.998	5	.921	.947	.977
								6	.985+	.993	.999
n = 8				n = 9				n = 10			
0	.250	.312	.438	0	.226	.283	.401	0	.206	.259	.369
1	.406	.471	.590	1	.368	.429	.544	1	.337	.394	.504
2	.538	.600	.707	2	.490	.550	.656	2	.450	.507	.612
3	.655+	.711	.802	3	.599	.655+	.750	3	.552	.607	.703
4	.760	.807	.879	4	.699	.749	.829	4	.646	.696	.782
5	.853	.889	.939	5	.790	.831	.895-	5	.733	.778	.850
6	.931	.954	.980	6	.871	.902	.947	6	.812	.850	.907
7	.987	.994	.999	7	.939	.959	.983	7	.884	.913	.952
				8	.988	.994	.999	8	.945+	.963	.984
								9	.990	.995-	.999
n = 11				n = 12				n = 13			
0	.189	.238	.342	0	.175-	.221	.319	0	.162	.206	.298
1	.310	.364	.470	1	.287	.339	.440	1	.268	.316	.413
2	.415+	.470	.572	2	.386	.438	.537	2	.360	.410	.506
3	.511	.564	.660	3	.475+	.527	.622	3	.444	.495-	.588
4	.599	.650	.738	4	.559	.609	.698	4	.523	.573	.661
5	.682	.729	.806	5	.638	.685-	.765+	5	.598	.645+	.727
6	.759	.800	.866	6	.712	.755-	.825+	6	.669	.713	.787
7	.831	.865-	.916	7	.781	.819	.879	7	.736	.776	.841
8	.895+	.921	.957	8	.846	.877	.924	8	.799	.834	.889
9	.951	.967	.986	9	.904	.928	.961	9	.858	.887	.931
10	.990	.995+	.999	10	.955-	.970	.987	10	.912	.934	.964
				11	.991	.996	.999	11	.958	.972	.988
								12	.992	.996	.999
n = 14				n = 15				n = 16			
0	.152	.193	.280	0	.142	.181	.264	0	.134	.171	.250
1	.251	.297	.389	1	.236	.279	.368	1	.222	.264	.349
2	.337	.385+	.478	2	.317	.363	.453	2	.300	.344	.430
3	.417	.466	.557	3	.393	.440	.529	3	.371	.417	.503
4	.492	.540	.627	4	.464	.511	.597	4	.439	.484	.569
5	.563	.610	.692	5	.532	.577	.660	5	.504	.548	.630

TABLE 3-III – ONE-SIDED LIMITS (Contd.)

r	90%	95%	99%
n = 14			
6	.631	.675-	.751
7	.695+	.736	.805+
8	.757	.794	.854
9	.815-	.847	.898
10	.869	.896	.936
11	.919	.939	.967
12	.961	.974	.989
13	.993	.996	.999
n = 17			
0	.127	.162	.237
1	.210	.250	.332
2	.284	.326	.410
3	.352	.396	.480
4	.416	.461	.543
5	.478	.522	.603
6	.537	.580	.658
7	.594	.636	.709
8	.650	.689	.758
9	.703	.740	.803
10	.754	.788	.845-
11	.803	.834	.883
12	.849	.876	.918
13	.893	.915+	.948
14	.933	.950	.973
15	.968	.979	.991
16	.994	.997	.999
n = 20			
0	.109	.139	.206
1	.181	.216	.289
2	.245-	.283	.358
3	.304	.344	.421
4	.361	.401	.478
5	.415-	.456	.532
6	.467	.508	.583
7	.518	.558	.631
8	.567	.606	.677
9	.615+	.653	.720
10	.662	.698	.761
11	.707	.741	.800
12	.751	.783	.837
13	.793	.823	.871
14	.834	.860	.902
15	.873	.896	.931
16	.910	.929	.956
17	.944	.958	.977
18	.973	.982	.992
19	.995-	.997	.999

r	90%	95%	99%
n = 15			
6	.596	.640	.718
7	.658	.700	.771
8	.718	.756	.821
9	.774	.809	.865+
10	.828	.858	.906
11	.878	.903	.941
12	.924	.943	.969
13	.964	.976	.990
14	.993	.997	.999
n = 18			
0	.120	.153	.226
1	.199	.238	.316
2	.269	.310	.391
3	.334	.377	.458
4	.396	.439	.520
5	.455+	.498	.577
6	.512	.554	.631
7	.567	.608	.681
8	.620	.659	.729
9	.671	.709	.774
10	.721	.756	.816
11	.769	.801	.855-
12	.815-	.844	.890
13	.858	.884	.923
14	.899	.920	.951
15	.937	.953	.975-
16	.970	.980	.992
17	.994	.997	.999
n = 21			
0	.104	.133	.197
1	.173	.207	.277
2	.234	.271	.344
3	.291	.329	.404
4	.345+	.384	.460
5	.397	.437	.512
6	.448	.487	.561
7	.497	.536	.608
8	.544	.583	.653
9	.590	.628	.695+
10	.636	.672	.736
11	.679	.714	.774
12	.722	.755+	.811
13	.764	.794	.845+
14	.804	.832	.878
15	.842	.868	.908
16	.879	.901	.935-
17	.914	.932	.959
18	.946	.960	.978
19	.974	.983	.993
20	.995-	.998	1.000

r	90%	95%	99%
n = 16			
6	.565+	.609	.687
7	.625-	.667	.739
8	.682	.721	.788
9	.737	.773	.834
10	.790	.822	.875-
11	.839	.868	.912
12	.886	.910	.945-
13	.929	.947	.971
14	.966	.977	.990
15	.993	.997	.999
n = 19			
0	.114	.146	.215+
1	.190	.226	.302
2	.257	.296	.374
3	.319	.359	.439
4	.378	.419	.498
5	.434	.476	.554
6	.489	.530	.606
7	.541	.582	.655+
8	.592	.632	.702
9	.642	.680	.746
10	.690	.726	.788
11	.737	.770	.827
12	.782	.812	.863
13	.825-	.853	.897
14	.866	.890	.927
15	.905-	.925-	.954
16	.941	.956	.976
17	.972	.981	.992
18	.994	.997	.999
n = 22			
0	.099	.127	.189
1	.166	.198	.266
2	.224	.259	.330
3	.279	.316	.389
4	.331	.369	.443
5	.381	.420	.493
6	.430	.468	.541
7	.477	.515+	.587
8	.523	.561	.630
9	.568	.605-	.672
10	.611	.647	.712
11	.654	.689	.750
12	.695+	.729	.786
13	.736	.767	.821
14	.775+	.804	.853
15	.813	.840	.884
16	.850	.874	.912
17	.885+	.906	.938
18	.918	.935+	.961
19	.949	.962	.979
20	.976	.984	.993
21	.995+	.998	1.000

TABLE 3-III – ONE-SIDED LIMITS (Contd.)

r	90%	95%	99%	r	90%	95%	99%	r	90%	95%	99%
n = 23				n = 24				n = 25			
0	.095+	.122	.181	0	.091	.117	.175-	0	.088	.133	.168
1	.159	.190	.256	1	.153	.183	.246	1	.147	.176	.237
2	.215+	.249	.318	2	.207	.240	.307	2	.199	.231	.296
3	.268	.304	.374	3	.258	.292	.361	3	.248	.282	.349
4	.318	.355-	.427	4	.306	.342	.412	4	.295-	.330	.398
5	.366	.404	.476	5	.352	.389	.460	5	.340	.375+	.444
6	.413	.451	.522	6	.398	.435-	.505-	6	.383	.420	.488
7	.459	.496	.567	7	.442	.479	.548	7	.426	.462	.531
8	.503	.540	.609	8	.484	.521	.590	8	.467	.504	.571
9	.546	.583	.650	9	.526	.563	.630	9	.508	.544	.610
10	.589	.625-	.689	10	.567	.603	.668	10	.548	.583	.648
11	.630	.665-	.727	11	.608	.642	.705-	11	.587	.621	.684
12	.670	.704	.763	12	.647	.681	.740	12	.625-	.659	.719
13	.710	.742	.797	13	.685+	.718	.774	13	.662	.695-	.752
14	.748	.778	.829	14	.723	.754	.806	14	.699	.730	.784
15	.786	.814	.860	15	.759	.788	.837	15	.735-	.764	.815+
16	.822	.848	.889	16	.795+	.822	.867	16	.770	.798	.845+
17	.857	.880	.916	17	.830	.854	.894	17	.804	.830	.873
18	.890	.910	.941	18	.863	.885+	.920	18	.837	.861	.899
19	.922	.938	.962	19	.895+	.914	.943	19	.869	.890	.923
20	.951	.963	.980	20	.925+	.941	.964	20	.899	.918	.946
21	.977	.984	.993	21	.953	.965+	.981	21	.928	.943	.966
22	.995+	.998	1.000	22	.978	.985-	.994	22	.955+	.966	.982
				23	.996	.998	1.000	23	.979	.986	.994
								24	.996	.998	1.000
n = 26				n = 27				n = 28			
0	.085-	.109	.162	0	.082	.105+	.157	0	.079	.101	.152
1	.142	.170	.229	1	.137	.164	.222	1	.132	.159	.215
2	.192	.223	.286	2	.185+	.215+	.277	2	.179	.208	.268
3	.239	.272	.337	3	.231	.263	.326	3	.223	.254	.316
4	.284	.318	.385-	4	.275-	.308	.373	4	.265+	.298	.361
5	.328	.363	.430	5	.317	.351	.417	5	.306	.339	.404
6	.370	.405+	.473	6	.358	.392	.458	6	.346	.380	.445-
7	.411	.447	.514	7	.397	.482	.498	7	.385-	.419	.484
8	.451	.487	.554	8	.436	.471	.537	8	.422	.457	.521
9	.491	.526	.592	9	.475-	.509	.574	9	.459	.494	.558
10	.529	.564	.628	10	.512	.547	.610	10	.496	.530	.593
11	.567	.602	.664	11	.549	.583	.645+	11	.532	.565+	.627
12	.604	.638	.698	12	.585-	.618	.679	12	.567	.600	.660
13	.641	.673	.731	13	.620	.653	.711	13	.601	.634	.692
14	.676	.708	.763	14	.655+	.687	.743	14	.635+	.667	.723
15	.711	.742	.794	15	.689	.720	.773	15	.669	.699	.753
16	.746	.774	.823	16	.723	.752	.802	16	.701	.731	.782
17	.779	.806	.851	17	.756	.783	.831	17	.733	.762	.810
18	.812	.837	.878	18	.788	.814	.857	18	.765-	.792	.837
19	.843	.866	.903	19	.819	.843	.883	19	.796	.821	.863
20	.874	.894	.927	20	.849	.871	.907	20	.826	.849	.888
21	.903	.921	.948	21	.879	.899	.930	21	.855+	.876	.911
22	.931	.946	.967	22	.907	.924	.950	22	.883	.902	.932
23	.957	.968	.983	23	.934	.948	.968	23	.911	.927	.952
24	.979	.986	.994	24	.958	.969	.983	24	.936	.950	.969
25	.996	.998	1.000	25	.980	.987	.994	25	.960	.970	.984
				26	.996	.998	1.000	26	.981	.987	.995-
								27	.996	.998	1.000

TABLE 3-III – ONE-SIDED LIMITS (Contd.)

r	90%	95%	99%	r	90%	95%	99%	r	90%	95%	99%
n = 29				n = 30							
0	.076	.098	.147	0	.074	.095+	.142				
1	.128	.153	.208	1	.124	.149	.202				
2	.173	.202	.260	2	.168	.195+	.252				
3	.216	.246	.307	3	.209	.239	.298				
4	.257	.288	.350	4	.249	.280	.340				
5	.297	.329	.392	5	.287	.319	.381				
6	.335-	.368	.432	6	.325-	.357	.420				
7	.372	.406	.470	7	.361	.394	.457				
8	.409	.443	.507	8	.397	.430	.493				
9	.445+	.479	.542	9	.432	.465+	.527				
10	.481	.514	.577	10	.466	.499	.561				
11	.515+	.549	.610	11	.500	.533	.594				
12	.550	.583	.643	12	.533	.566	.626				
13	.583	.616	.674	13	.566	.598	.657				
14	.616	.648	.705-	14	.599	.630	.687				
15	.649	.680	.734	15	.630	.661	.716				
16	.681	.711	.763	16	.662	.692	.744				
17	.712	.741	.791	17	.692	.721	.772				
18	.743	.771	.818	18	.723	.750	.799				
19	.774	.800	.843	19	.752	.779	.824				
20	.803	.828	.868	20	.782	.807	.849				
21	.832	.855-	.892	21	.810	.834	.873				
22	.860	.881	.914	22	.838	.860	.896				
23	.888	.906	.935-	23	.865+	.885+	.917				
24	.914	.930	.954	24	.891	.909	.937				
25	.938	.951	.970	25	.917	.932	.955+				
26	.961	.971	.985-	26	.941	.953	.972				
27	.982	.988	.995-	27	.963	.972	.985+				
28	.996	.998	1.000	28	.982	.988	.995-				
				29	.996	.998	1.000				

CHART 3-III. ONE-SIDED 95% CONFIDENCE LIMITS FOR A PROPORTION, $0 \leq r/N \leq 0.2$

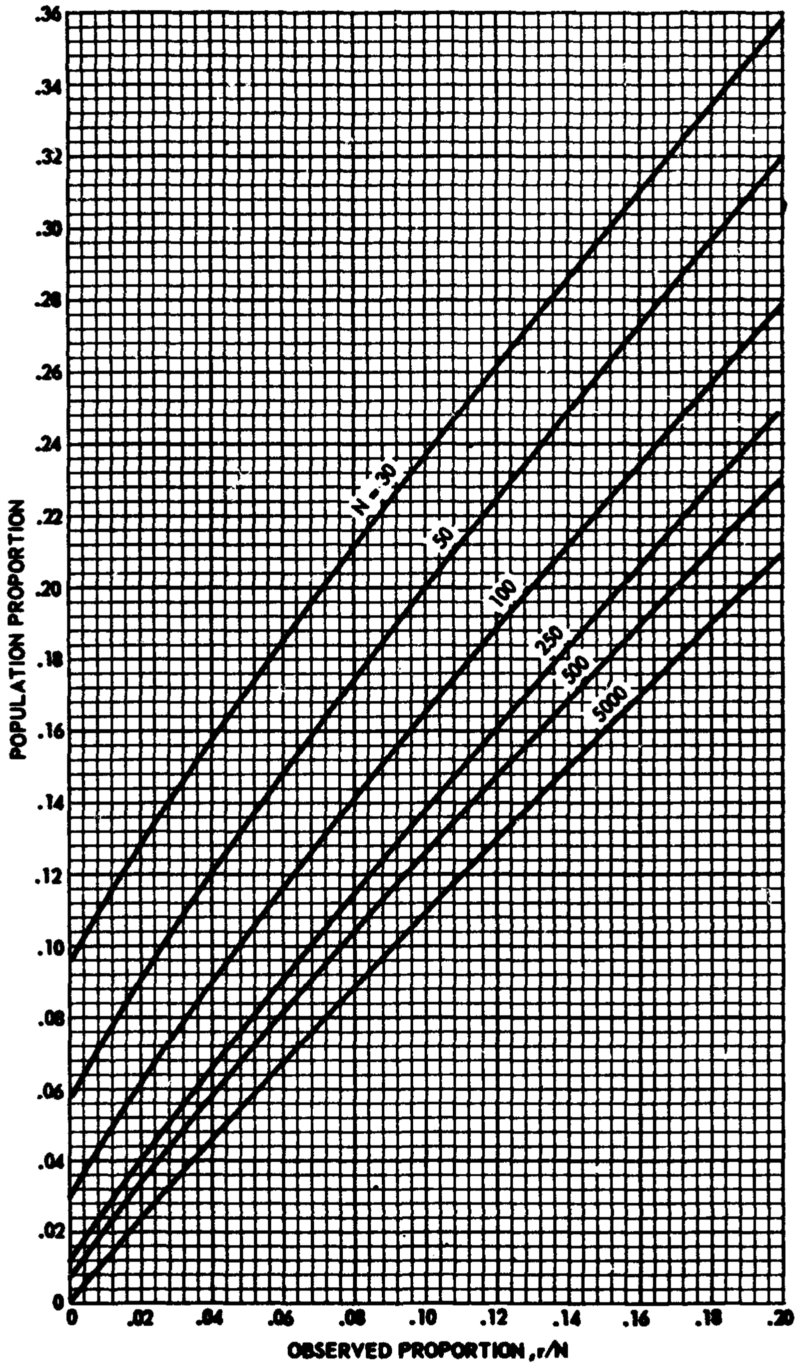


CHART 3-IV. ONE-SIDED 90% CONFIDENCE LIMITS FOR A PROPORTION, $0 \leq r/N \leq 0.2$

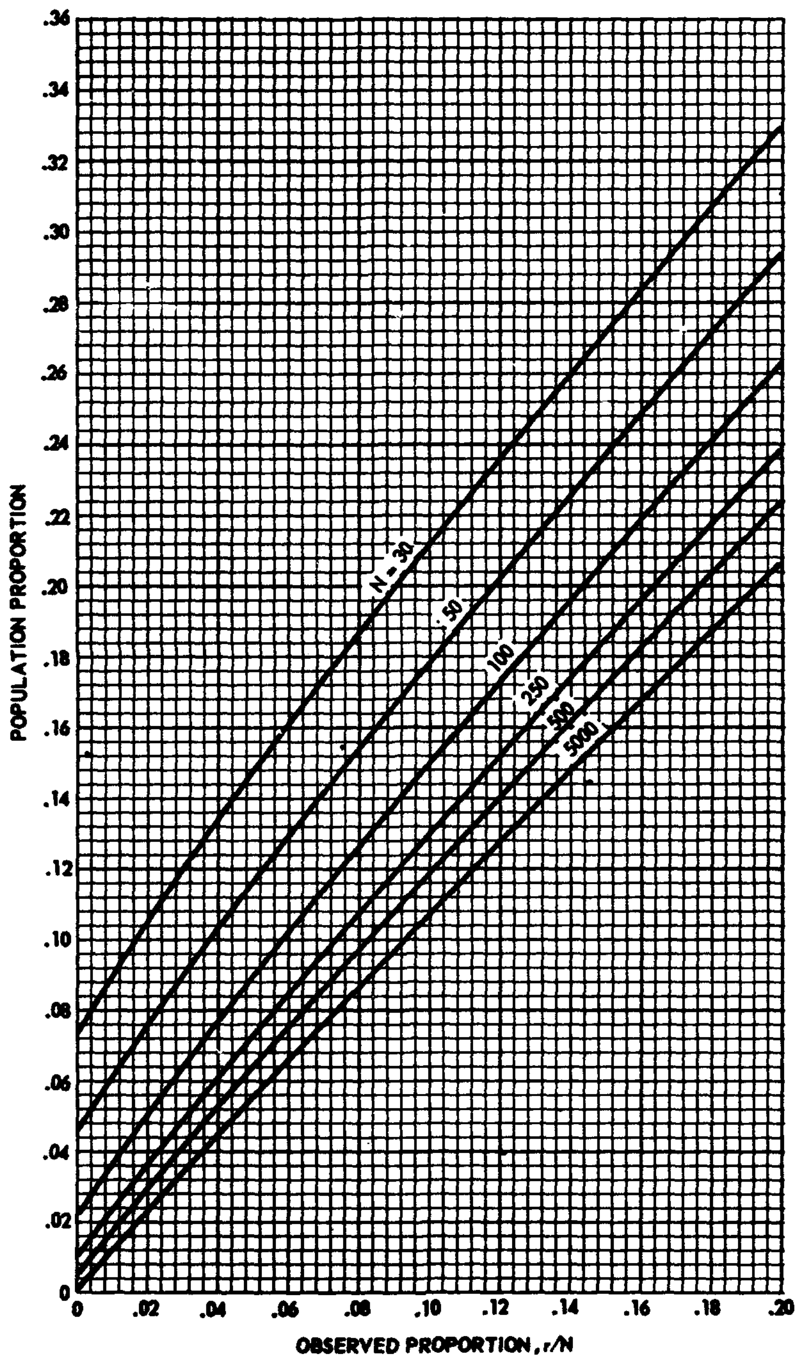
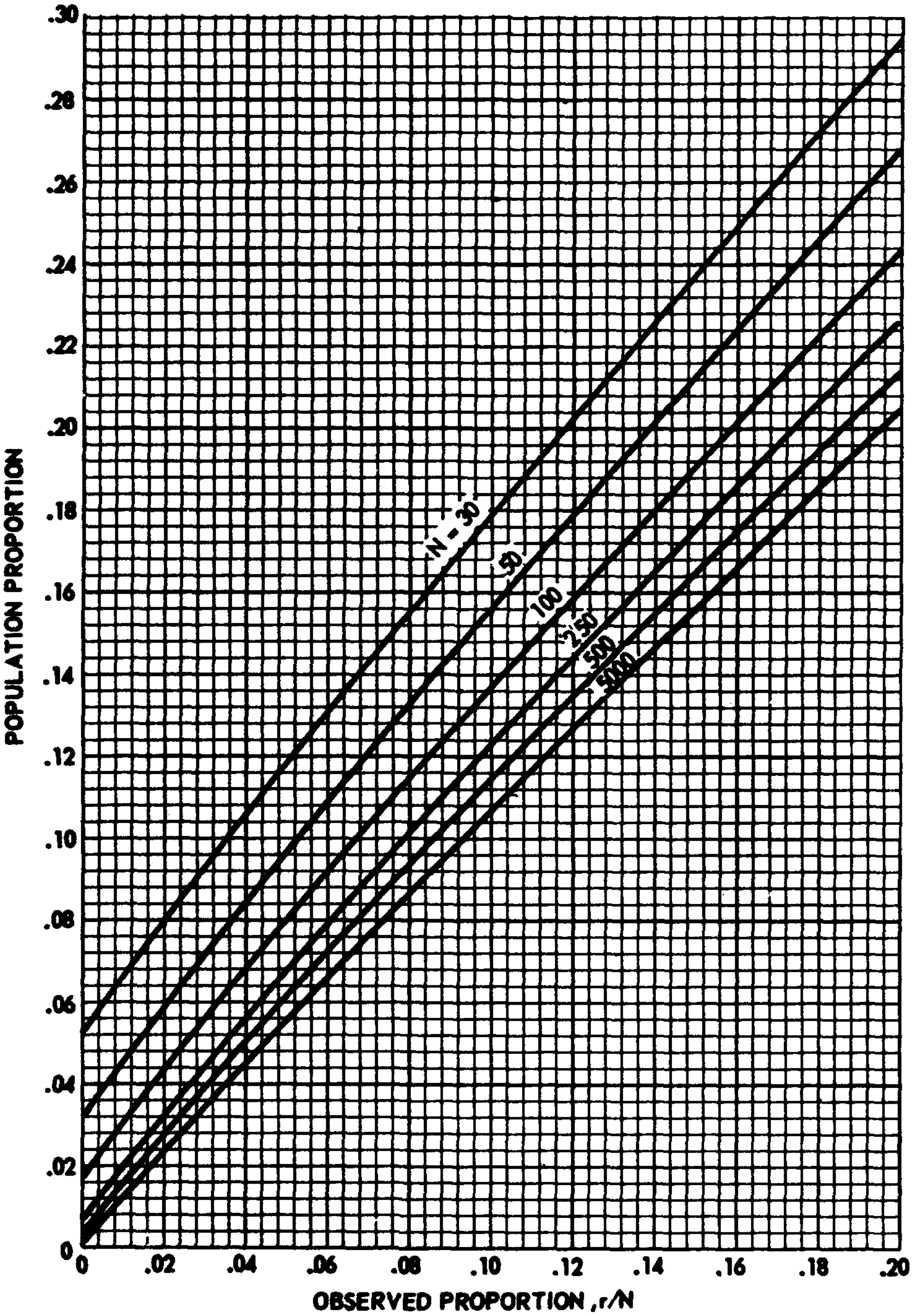


CHART 3-V. ONE-SIDED 80% CONFIDENCE LIMITS FOR A PROPORTION, $0 \le r/N \le 0.2$



APPENDIX 4. REDUNDANCY CONSIDERATIONS IN DESIGN

4.1 INTRODUCTION

Under certain circumstances during system design it may become necessary to consider the use of redundancy to reduce the probability of system failure–to enhance system reliability–by providing more than one functional path or operating element in areas that are critically important to system success. The use of redundancy is not a panacea to solve all reliability problems, nor is it a substitute for good design in the first place. By its very nature, redundancy implies increased *complexity*, increased *weight and space*, increased *power consumption*, and usually a more complicated system checkout and monitoring procedure. On the other hand, redundancy is the *only* solution to many of the problems confronting the designer of today's complex weapon systems.

It is the purpose of this appendix to present a brief description of the more common types of redundant configurations available to the designer, with the applicable block diagrams, mathematical formulae, and reliability *functions* to facilitate the computation of reliability *gain* to be expected in each case.

4.1.1 Levels of Redundancy

Redundancy may be applied at the system level (essentially two systems in parallel) or at the subsystem, component, or part level within a system. Figure 4-1 is a simplified reliability block diagram drawn to illustrate the several levels at which redundancy can be applied. System D is shown with its redundant alternate, D', at the system level. D' is in turn built up of redundant subsystems or components (C_1 and C_2) and redundant parts within components (b_1 and b_2 within Component B).

From the reliability block diagram and a definition of block or system success, the paths that will result in successful system operation can be determined, For example, the possible paths from I to O are:

(1) A, a, b_1, C_1

(2) A, a, b_1, C_2

(3) A, a, b_2, C_1

(4) A, a, b_2, C_2

(5) D

The success of each path may be computed by determining an assignable reliability value for each term and applying the multiplication theorem. The computation of system success (all paths combined) re-

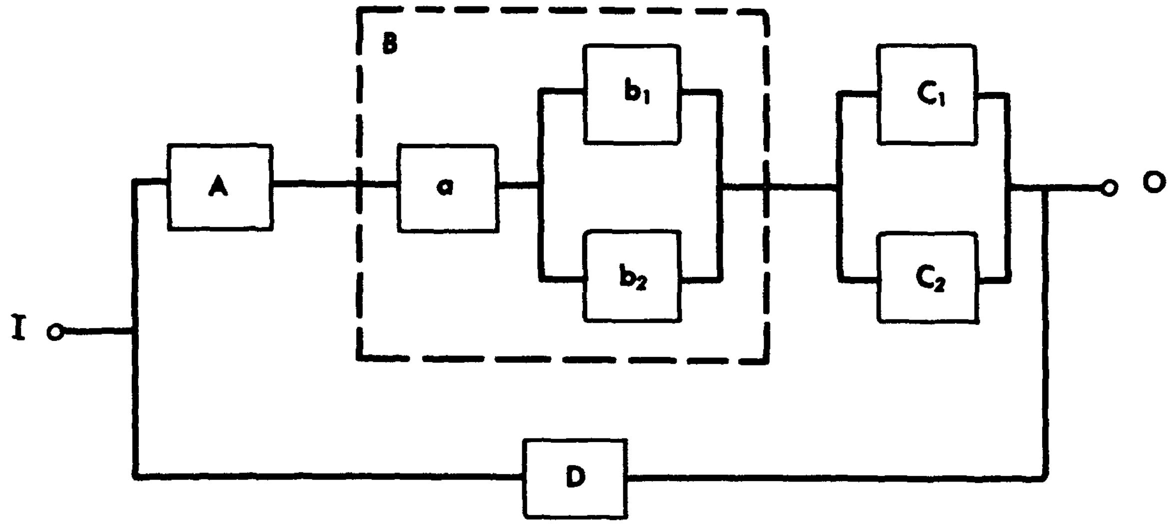


Figure 4-1. Reliability Block Diagram Depicting Redundancy at the System, Subsystem, and Component Levels

quires a knowledge of the type of redundancy to be used in each case and an estimate of individual element reliability (or unreliability).

4.1.2 Probability Notation for Redundancy Computations

Reliability of redundancy combinations is expressed in probabilistic terms of success or failure–for a given mission period, a given number of operating cycles, or a given number of time-independent "events", as appropriate. The "MTBF" measure of reliability is not readily usable because of the non-exponentiality of the reliability function produced by redundancy. Reliability of redundancy combinations which are "time-dependent" is therefore computed at a discrete point in time, as a probability of success for this descrete time period. The following notation is applicable to all cases and is used throughout this appendix:

R = probability of success or reliability of a unit or block.

$\bar{R}$ = probability of failure or unreliability of a unit or block.

p = probability of success or reliability of an element.

q = probability of failure or unreliability of an element.

For probability statements concerning an event:

$P(A)$ = probability that A occurs.

$P(\bar{A})$ = probability that A does not occur.

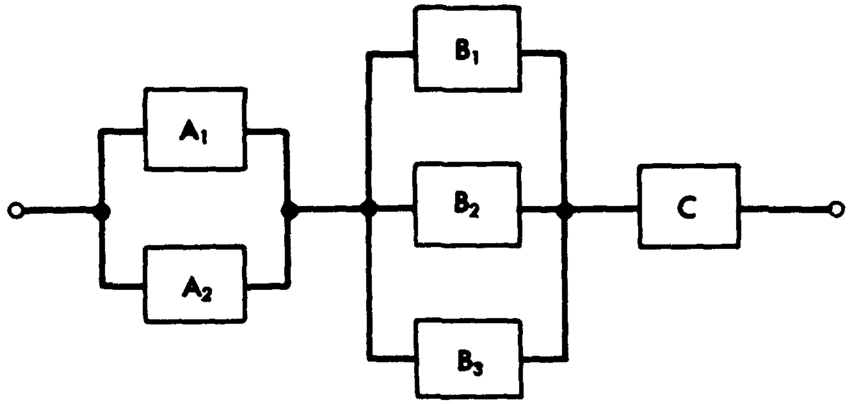


Figure 4-2. Series-Parallel Configuration

For the above probabilities:

$R + \bar{R} = 1$

$p + q = 1$

$P(A) + P(\bar{A}) = 1$

4.1.3 Redundancy Combinations

The method of handling redundancy combinations can be generalized as follows:

- *If the elements are in parallel and the units in series (Figure 4-2), first evaluate the redundant elements to get the unit reliability; then find the product of all unit reliabilities to obtain the block reliability.*
- If the elements are in series and the units or paths are in parallel (Figure 4-3), first obtain the path reliability by calculating the product of the reliabilities of all elements in each path; then consider each path as a redundant unit to obtain the block reliability.

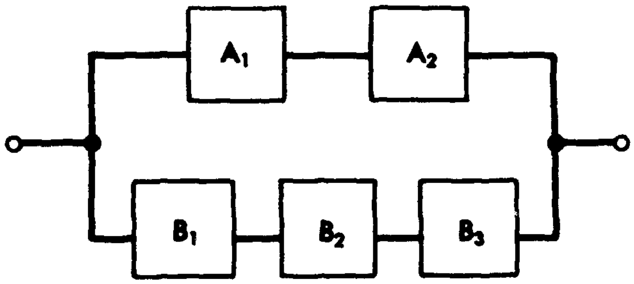
In the redundancy combination shown in Figure 4-2, Unit A has two parallel redundant elements, Unit B has three parallel redundant elements, and Unit C has only one element. Assume that all elements are independent. For Unit A to be successful, A_1 or A_2 must operate; for Unit B, B_1 or B_2 or B_3 must operate; and C must always be operating for block success. Translated into probability terms, the reliability of Figure 4-2 becomes:

$$R = [1 - P(\bar{A}_1)P(\bar{A}_2)][1 - P(\bar{B}_1)P(\bar{B}_2)P(\bar{B}_3)]P(C)$$

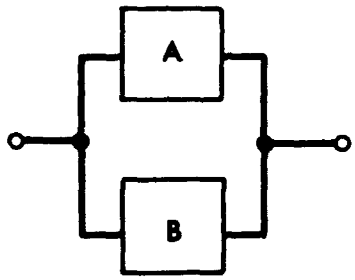
If the probability of success, p, is the same for each element in a unit,

$$R = [1 - (1 - p_A)^2][1 - (1 - p_B)^3]p_C$$

$$= (1 - q_A^{\ 2})(1 - q_B^{\ 3})p_C$$



(a)



(b)

Figure 4-3. Parallel-Series Configuration

where

$$q_i = 1 - p_i$$

Often there is a combination of series and parallel redundancy in a block as shown in Figure 4-3a. This arrangement can be converted into the simple parallel form shown in Figure 4-3b by first evaluating the series reliability of each path:

$$p_A = p_{a_1} p_{a_2}$$

$$p_B = p_{b_1} p_{b_2} p_{b_3}$$

where the terms on the right hand side represent element reliability. Then block reliability can be found from

$$R = 1 - (1 - p_A)(1 - p_B)$$
$$= 1 - \bar{p}_A \bar{p}_B$$

4.1.4 Time-Dependent Considerations

The reliability of elements used in redundant configurations is usually time-dependent. If the relation between element reliability and time is known, inclusion of the time factor does not change the basic notation and approach to redundancy computation outlined above. As an example, assume two active independent elements in parallel. System reliability is given by:

$$R = p_a + p_b - p_a p_b$$

This equation is applicable for one time interval. To express reliability over a segment of time, the reliability of each element must be expressed as a function of time. Hence,

$$R(t) = p_a(t) + p_b(t) - p_a(t)p_b(t)$$

where

$R(t)$ = system reliability at time t, $t > 0$

$p_a(t)$, $p_b(t)$ = element reliabilities at time t

The failure pattern of most components is described by the exponential distribution[1/] i.e.:

$$R = e^{-\lambda t} = e^{-t/\theta}$$

where λ is the constant failure rate; t is the time interval over which reliability, R, is measured; and θ is the mean-time-to-failure.

For two elements in series with constant failures rates λ_a and λ_b, using the product rule of reliability gives:

$$R(t) = p_a(t)p_b(t)$$

$$= e^{-(\lambda_a)t} e^{-(\lambda_b)t}$$

$$= e^{-(\lambda_a + \lambda_b)t}$$

System reliability, R(t), function is also exponential. With redundant elements present in the system, however, the system reliability function is not itself exponential, as illustrated by two operative parallel elements whose failure rates are constant. From

$$R(t) = p_a + p_b - p_a p_b$$

$$R(t) = e^{-(\lambda_a)t} + e^{-(\lambda_b)t} - e^{-(\lambda_a + \lambda_b)t}$$

which is not of the simple exponential form $e^{-\lambda t}$. Element failure rates cannot, therefore, be combined in the usual manner to obtain the system failure rate if considerable redundancy is inherent in the design.

Although a single failure rate cannot be used for redundant systems, the mean-time-to-failure of such systems can be evaluated. The mean life of a redundant "pair" whose failure rates are λ_a and λ_b, respectively, can be determined from

$$MTBF = \frac{1}{\lambda_a} + \frac{1}{\lambda_b} - \frac{1}{\lambda_a + \lambda_b}$$

If the failure rates of both elements are equal, then,

$$R(t) = 2e^{-\lambda t} - e^{-2\lambda t}$$

and

$$MTBF = \frac{3}{2\lambda} = \frac{3}{2}\theta$$

For three independent elements in parallel, the reliability function is

[1/] For a discussion of other distributions, see Appendix 2.

$$R(t) = 1 - [(1 - e^{-(\lambda_a)t})(1 - e^{-(\lambda_b)t})(1 - e^{-(\lambda_c)t})]$$

$$MTBF = \frac{1}{\lambda_a} + \frac{1}{\lambda_b} + \frac{1}{\lambda_c} - \frac{1}{\lambda_a + \lambda_b} - \frac{1}{\lambda_a + \lambda_c}$$

$$- \frac{1}{\lambda_b + \lambda_c} + \frac{1}{\lambda_a + \lambda_b + \lambda_c}$$

If $\lambda_a = \lambda_b = \lambda_c = \lambda$

$$R(t) = 3e^{-\lambda t} - 3e^{-2\lambda t} + e^{-3\lambda t}$$

$$MTBF = \frac{1}{\lambda} + \frac{1}{2\lambda} + \frac{1}{3\lambda} = \frac{11}{6\lambda} = \frac{11}{6}\theta$$

In general, for n active parallel elements, each element having the same constant failure rate, λ,

$$R(t) = 1 - [1 - e^{-\lambda t}]^n$$

and

$$MTBF = \sum_{i=1}^{n} \frac{1}{i\lambda} = \sum_{i=1}^{n} \frac{\theta}{i}$$

4.1.5 Types and Classifications of Redundancy

The following types of parallel redundancy most commonly used in equipment design are described in this appendix:

(1) *Operative Redundancy* – redundant units (or elements), all of which are fully energized during the system operational cycle. Operative redundancy may be further classified as follows:

(a) *Load-Sharing Redundancy:* redundant units are connected in such a manner that, upon failure of one unit, the remaining redundant unit(s) will continue to perform the system function. It is not necessary to switch out the failed unit nor to switch in the redundant unit. Failure of the one may or may not change the probability of failure of the remaining units, depending upon the nature of the "load" being shared.

(b) *Switching Redundancy:* operative redundant units are connected by a switching mechanism to disconnect a failed unit and to connect one of the remaining operative redundant units into the system.

(2) *Standby Redundancy* – redundant units (or elements) that are non-operative (i.e., have no power applied) until they are switched into the system upon failure of the primary unit. Switching is therefore always required.

(3) *Voting Redundancy* – the outputs of three or more operating redundant units are compared, and one of the outputs that agrees with the majority of outputs is selected. In most cases, units delivering outputs that fall in the minority group are classed as "unit failures".

(4) *Redundancy-With-Repair* – if a redundant element fails during a mission and can be repaired essentially "on-line" (without aborting the mission), then redundancy-with-repair can be achieved. The reliability of dual or multiple redundant elements can be substantially increased by use of this design concept.

Diagrams, formulae, charts, and reliability functions are presented in the following pages for the above types and classes of redundancy.

4.2 OPERATIVE OR ACTIVE REDUNDANT CONFIGURATIONS

Formulae and graphs presented in this sectior[illegible] not account for any change in failu[illegible]s which survivors in a redundant "lo[illegible]ing" configuration might experie[illegible]as a result of increased operating stress[illegible]. This aspect of redundancy design is discussed in Paragraph 4.5 of this appendix, under "Dependent Failure Probabilities". Also, except as discussed in 4.2.4, it is assumed in the operative case that switching devices are either not required or are relatively simple and failure-free.

4.2.1 Multiple Redundancy

Figure 4-4 shows a block diagram representing duplicate parallel components. There are two parallel paths for successful operation – A_1 or A_2. If the probability of each component operating successfully is p_i, the probability of circuit success can be found by either the addition theorem or the multiplication theorem of probability (see Chart 2-I, Appendix 2).

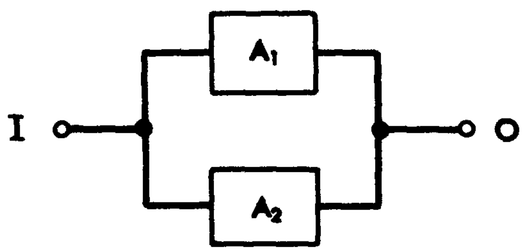


Figure 4-4. Duplicate Parallel Redundancy (Operative Case)

By the multiplication theorem, the circuit can fail only if both components fail. Since A_1 and A_2 are independent, the probability of success is equal to one minus the probability that both components fail, or

$$R = 1 - q_1 q_2$$

For example, if $p_1 = p_2 = 0.9$,

$$R = 1 - (0.1)^2 = 0.99$$

More than two redundant elements are represented by the reliability block diagram shown in Figure 4-5. There are m paths (or elements), at least one of which must be operating for system success. The probability of system success is therefore the probability that *not all* of the elements will fail during the mission period, shown as

$$R = 1 - q_1 q_2 \cdot \cdot \cdot q_m$$

where $q_1 = 1 - R_1$, etc.

If parallel elements are identical, then

$$R = 1 - q^m$$

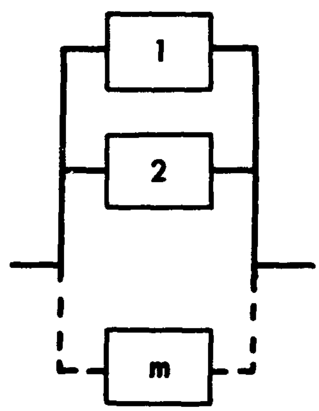


Figure 4-5. Multiple Redundant Array of m Elements, with k = 1 Required for Success

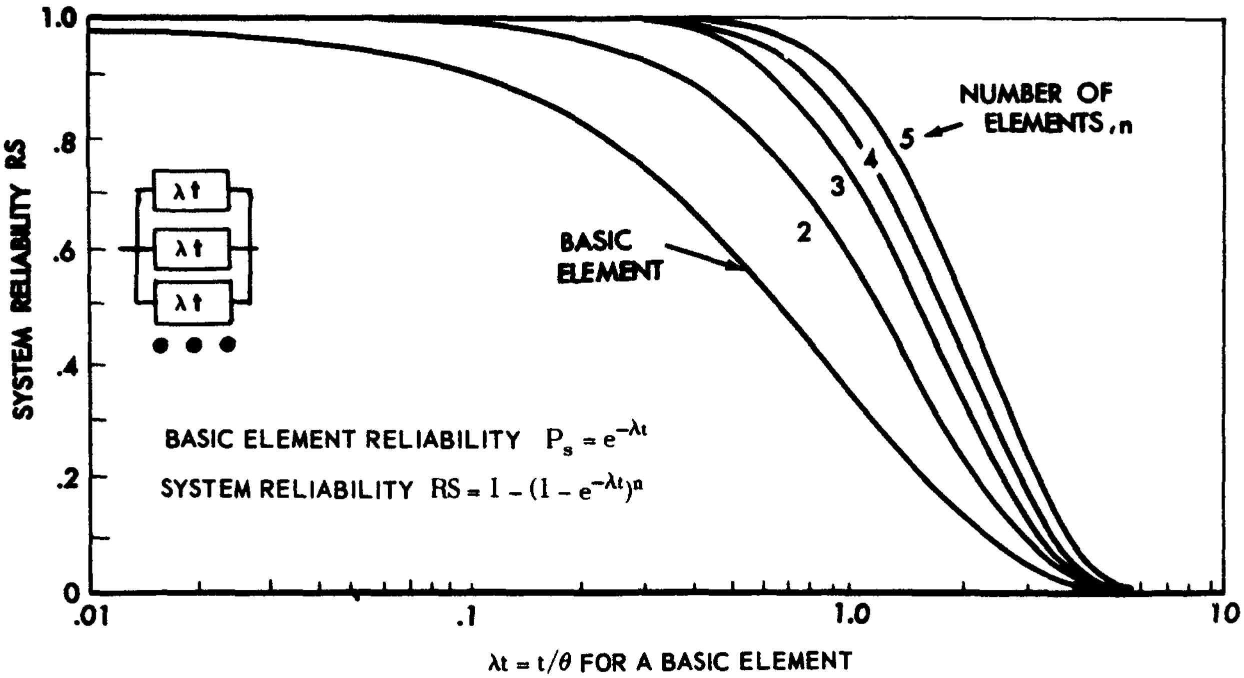


Figure 4-6. System Reliability for n Element Operative Redundant Configurations

Figure 4-6 is a chart relating system reliability to the ratio $t/\theta = \lambda t$ of individual elements making up the redundant system. Curves for n elements (from n = 1 to n = 5) are shown. One element in n must remain operative for the prescribed time interval t, to achieve the probability of system failure shown.

EXAMPLE: The inverter function for an airborne system has been allocated a reliability requirement of R(t) = .99 for a 5-hour mission. Current predictions of the MTBF feasibility by conventional design is 50 hours. Entering the chart at $t/\theta = 5/50 = 0.1$, proceed vertically to .99, the required reliability for the inverter function. n = 2 is the number of inverters that are required in active parallel, to obtain a 99% probability of survival for the inverter function.

4.2.2 Partial Redundancy

In the previous example, the system was successful if at least one of n parallel paths was successful. There may be cases where at least k out of n elements must be successful. In such cases, the reliability of the redundant group is given by a series of additive terms (binomial) in the form of

$$P_{(k,\, n|p)} = \binom{n}{k} p^k (1 - p)^{n-k}$$

EXAMPLE: Figure 4-7 illustrates three channels of a receiver. The receiver will operate if at least two channels are successful, that is, if k = 2 or k = 3. The probability of each channel being successful is equal to p; then

$$R = P_{(2,\ 3|p)} + P_{(3,\ 3|p)}$$

$$R = \binom{3}{2}p^2(1 - p) + \binom{3}{3}p^3(1 - p)^0$$

$$R = [3p^2(1 - p)] + p^3$$

$$R = 3p^2 - 2p^3$$

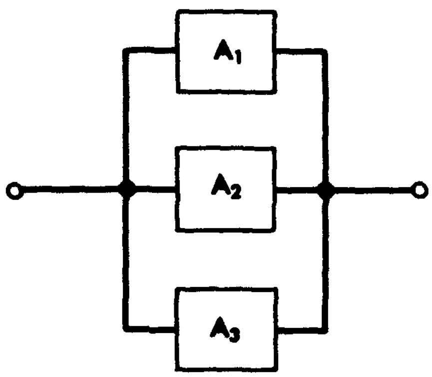


Figure 4-7. Partial Redundant Configuration of m = 3 Elements, with k = 2 Required for Success

Use of the binomial formula becomes impractical in multi-element partial redundant configurations when the values of n, k, and r become large. In these cases, the normal approximation may be used as outlined in Paragraph 2.2.3 of Appendix 2.[2/] The approach can be best illustrated by an example:

EXAMPLE: A new transmitting array is to be designed using 1000 RF elements to achieve design goal performance for power output and beam width. A design margin has been provided, however, to permit a 10% loss of RF elements before system performance becomes degraded below the acceptable minimum level. Each element is known to have a failure rate of 1000 × 10^{-6} failures per hour. The proposed design is illustrated in Figure 4-8, where the total number of elements is n = 1000; the number of elements required for system success is k = 900; and, conversely, the

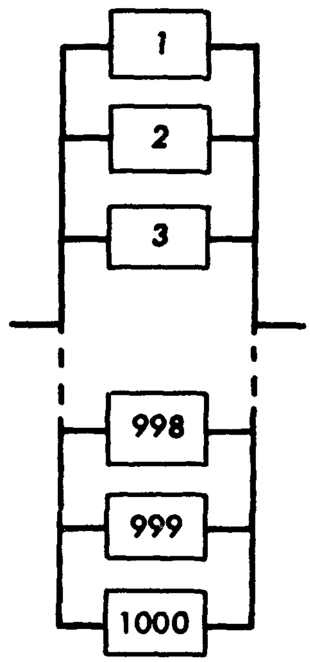


Figure 4-8. Partial Redundant Array with m = 1000 Elements, r = 0, 50, 100, 150 Permissible Element Failures

[2/] See also almost any good text book on probability and statistics.

number of element failures permitted is r = 100. It is desired to compute and plot the reliability function for the array.

Each discrete point for time (t) on the function is given by the binomial summation as:

$$R_s(t) = \sum_{x=0}^{r} \binom{n}{x} p^x q^{n-x}$$

$$= \sum_{x=0}^{100} \binom{1000}{x} (1-e^{-\lambda t})^x (e^{-\lambda t})^{n-x}$$

where

$p = 1-e^{-\lambda t}$

$q = e^{-\lambda t}$

λ = Element failure rate

This binomial summation can be approximated by the standard normal distribution function, using Table 2-II of Appendix 2 to compute reliability for the normalized statistic Z:

$$R_s(t) = F(Z)$$

and

$$Z = \frac{X-\mu}{\sigma} = \frac{X-np}{\sqrt{npq}}$$

$$= \frac{X - n(1-e^{-\lambda t})}{\sqrt{n(1-e^{-\lambda t})\,e^{-\lambda t}}}$$

By observation, it can be reasoned that system MTBF will be approximately 100 hours, since 100 element failures are permitted and one element fails each hour of system operation. A preliminary selection of discrete points at which to compute reliability might then fall in the 80- to 120-hour bracket.

At 80 hours:

$$\mu = np = 1000(1 - e^{-1000 \times 10^{-6} \times 80})$$

$$= 77$$

$$q = e^{-1000 \times 10^{-6} \times 80} = .923$$

$$\sigma = \sqrt{np} = \sqrt{71.07} = 8.4$$

$$x = 100$$

$$Z_{80} = \frac{100-77}{8.4} = 2.73$$

$$R_s(80) = F(Z_{80}) = F(+2.73)$$

$= .997$, from Table 2-II

At 100 hours:

$$\mu = np = 1000(1 - e^{-1000 \times 10^{-6} \times 100})$$

$$= 95$$

$$q = e^{-1000 \times 10^{-6} \times 100} = .905$$

$$\sigma = \sqrt{npq} = \sqrt{86} = 9.3$$

$$x = 100$$

$$Z_{100} = \frac{100-95}{9.3} = .54$$

$$R_s(100) = F(Z_{100}) = F(.54) = .705$$

Reliability at other discrete points in time, computed as above, are:

Time, t	Z	F(Z) = $R_s(t)$
90	1.54	.938
95	1.11	.867
105	0	.500
110	- .42	.337
120	-1.30	.097
130	-2.13	.017

These points are then used to plot the reliability function for the array, shown in Figure 4-9.

4.2.3 Failure Modes in the Operative Redundant Case

The previous redundant models were based on the assumption of one mode of failure, adequately protected so that failure of an individual element could not affect the operation of a surviving element. Two modes of failure are now considered – open-circuit and short-circuit – either of which can affect the surviving element unless proper design precautions are taken. In series redundant circuits, the open-circuit mode prevents surviving elements from functioning; in parallel redundant circuits, the short-circuit mode prevents the surviving elements from functioning.

The probabilities that are necessary to describe element failure can best be illustrated by an example. Assume that 100 randomly selected items are tested for a prescribed time to determine failure probabilities. The results are as follows:

80 items experienced no failure
15 items experienced an open failure
5 items experienced a short-circuit failure

Thus, the estimated probability of success is 80/100 = 0.80. The estimated probability of an open failure (q_o) is 0.15, and the

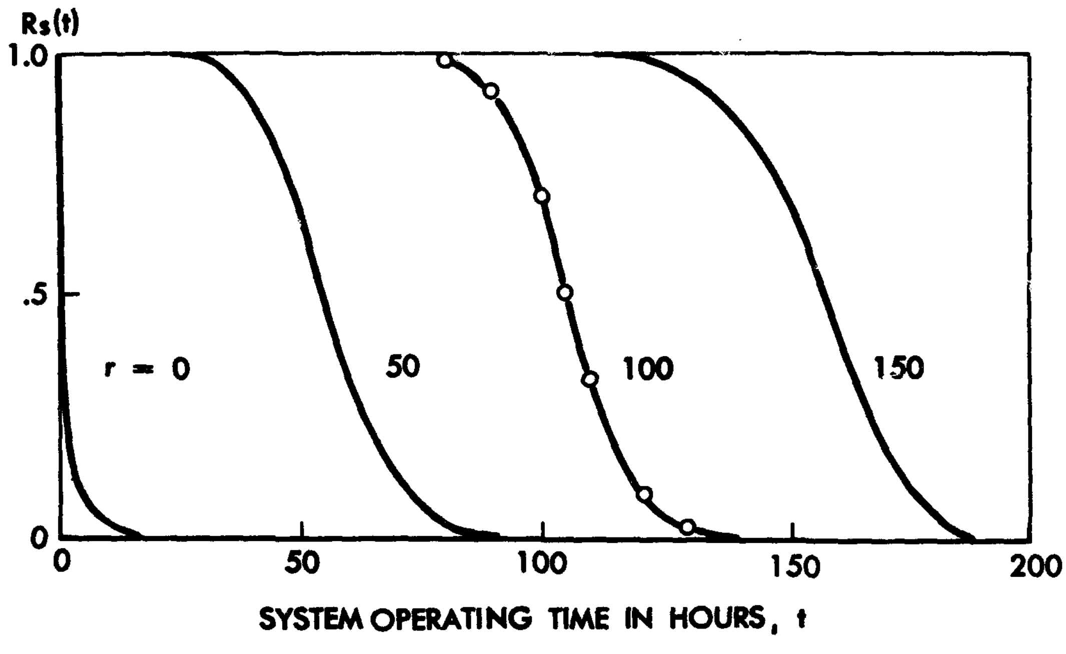


Figure 4-9. Reliability Functions for Partial Redundant Array of Figure 4-8.

estimated probability of a short-circuit failure (q_s) is 0.05. The sum of the two failure probabilities (opens and shorts are mutually exclusive events) is the probability of element failure (q), 0.20. This could have been obtained by subtracting the probability of element success (p) from one, i.e.:

$$q = 1 - p = q_o + q_s$$

The conditional probabilities of open and short failures are sometimes used to represent element failure probabilities. The data indicate that 15 of the 20 failures that occurred were due to opens. Therefore, the conditional probability of an open failure – i.e., the probability that if a failure occurs, it is an open failure – is 15/20 = 0.75. Similarly, the conditional probability of a short-circuit is 5/20 = 0.25. If

q'_o = conditional probability of an open

$= q_o/q$

q'_s = conditional probability of a short

$= q_s/q$

then the following relationship holds true:

$$q'_o + q'_s = 1$$

Parallel Elements:

For two elements, A and B in an operative-parallel redundant configuration, the unit will fail if (1) either A or B shorts, or (2) both A and B open. The probabilities of these two events are:

$$(1)\quad P_1(S) = P_a(S) + P_b(S) - P_a(S)P_b(S)$$

$$= 1 - [1 - P_a(S)]\,[1 - P_b(S)]$$

$$= 1 - [1 - q_{sa}]\,[1 - q_{sb}]$$

$$(2)\quad P_2(O) = P_a(O)P_b(O)$$

$$= q_{oa}q_{ob}$$

where $P_i(O)$ is the probability that Element i opens and $P_i(S)$ is the probability that Element i shorts. Since Events (1) and (2) are mutually exclusive, the probability of unit failure is the sum of the two event probabilities, or

$$P(F) = \bar{R} = P_1(S) + P_2(O)$$

$$= 1 - (1 - q_{sa})(1 - q_{sb}) + q_{oa}q_{ob}$$

In general, if there are m parallel elements,

$$\bar{R} = 1 - \prod_{i=1}^{m} (1 - q_{si}) + \prod_{i=1}^{m} q_{oi}$$

The reliability is equal to $1 - \bar{R}$, or

$$R = \prod_{i=1}^{m} (1 - q_{si}) - \prod_{i=1}^{m} q_{oi}$$

If all elements are equal, unit reliability is then

$$R = (1 - q_s)^m - q_o^m$$

Optimum Number of Parallel Elements:

By introducing the possibility of short-circuit failures, unit reliability may be decreased by adding parallel elements. As an example, if $q_o = 0.10$, the reliability for several values of m and q_s is as shown in Table 4-I.

TABLE 4-1. VALUES OF R FOR $q_o = 0.10$

	Case (a) $q_s = 0$	Case (b) $q_s = 0.05$	Case (c) $q_s = 0.10$	Case (d) $q_s = 0.20$
m = 1	0.900	0.85	0.80	0.70
m = 2	0.990	0.89	0.80	0.63
m = 3	0.999	0.86	0.73	0.51

For Cases (a) and (b), adding one parallel element (m = 2) increases unit reliability. For (a), the reliability increases as m increases and approaches 1 as m approaches infinity. However, for (b), increasing m from 2 to 3 decreases reliability. In fact, the reliability continues to decrease as m gets larger. Therefore, for Case (b), the optimum number of parallel elements for maximum reliability is 2. For Case (c), R is the same for m = 1 and 2, but is less for m = 3. For Case (d), the maximum reliability occurs for m = 1, the non-redundant configuration.

For any range of q_o and q_s, the optimum number of parallel elements is 1 if $q_s \geq q_o$. For most practical values of q_s and q_o, the optimum number is 2.

Figure 4-10 gives the optimum number of parallel elements for values of q_o ranging from 0.001 to 0.5 and for the ratio q_s/q_o ranging from 0.001 to 1.0 (use the left-hand and bottom axes).

Knowing the general range of element failure probabilities and possibly knowing the ratio of short to open possibilities, the figure can be used to determine the optimum number of parallel elements. For example, if it is believed that overall element reliability is somewhere between 0.8 and 0.9 and that opens are likely to occur about twice as often as shorts, then

$$0.1 \leq q \leq 0.2, \text{ and } q_s/q_o = 0.5$$

Since $q_o + q_s = q$,

$$0.1 \leq 3/2q_o \leq 0.2$$

or

$$0.7 \leq q_o \leq 0.13$$

For each of the values of q_o between 0.07 and 0.13, the optimum number is determined at $q_o/q_s = 0.5$. If this number is the same for all or nearly all possible values of q_o, then the optimum design is fairly well established. In this case, Figure 4-10 shows that 2 is the optimum number of parallel elements. If an optimum number boundary line is crossed somewhere in the interval of possible values of q_o, then it will be necessary to narrow the length of this interval by a thorough reappraisal of existing failure data or by tests specifically designed to yield more precise information.

Series Elements:

The results given above show that if $q_s \geq q_o$, the optimum number of parallel paths is 1. However, adding an element in series with another element will result in an increase in reliability if q_s is much greater than q_o. Assume we have a system made up of two series elements, A and B, in which both short-circuit and open failures are possible. The unit will fail if (1) both A and B short, or if (2) either A or B open. The probabilities of Events (1) and (2) are:

$$(1)\quad P_1(S) = P_a(S)P_b(S)$$

$$= q_{sa}q_{sb}$$

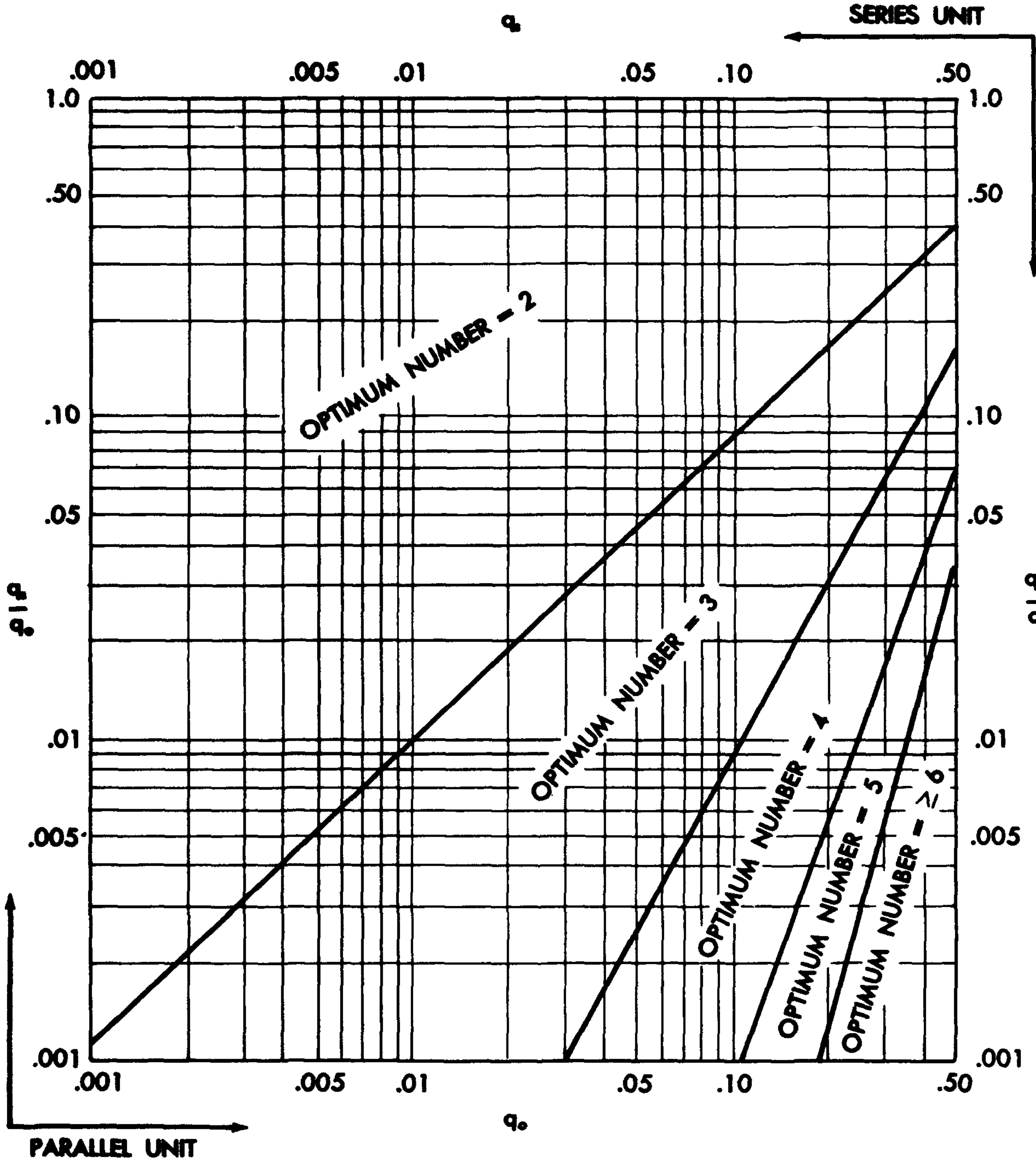


Figure 4-10. Optimum Number of Parallel Elements as a Function of Failure-Mode Probabilities

(2) $P_2(O) = P_a(O) + P_b(O) - P_a(O)P_b(O)$

$$= 1 - [1 - P_a(O)]\,[1 - P_b(O)]$$

$$= 1 - [1 - q_{oa}]\,[1 - q_{ob}]$$

Since Events (1) and (2) are mutually exclusive, the probability of unit failure is the sum of two events, or

$$P(F) = \bar{R} = P_1(S) + P_2(O)$$

$$= q_{sa}q_{sb} + 1 - (1 - q_{oa})(1 - q_{ob})$$

In general, if there are n series elements,

$$\bar{R} = 1 - \prod_{i=1}^{n} (1 - q_{oi}) + \prod_{i=1}^{n} q_{si}$$

and

$$R = \prod_{i=1}^{n} (1 - q_{oi}) - \prod_{i=1}^{n} q_{si}$$

If all elements are identical, then the reliability of an n-element series unit is

$$R = (1 - q_o)^n - q_s^n$$

Note that n replaces m in the equation for a parallel unit and the positions of q_o and q_s are reversed.

Figure 4-10 can be used to determine the optimum number of series elements by using the upper and right-hand axes. As in parallel systems, if $q_o = q_s$, the optimum number of series elements is 1.

Series-Parallel Elements:

A four-element series-parallel configuration is shown in Figure 4-11. Each element performs the same function.

Block success is defined as an output from at least one element. Therefore, the block is successful if (1) either unit has less than two opens, and (2) at least one unit has no shorts.

(1) $P_1(O)$ = probability that at least one unit has 2 opens

= 1 - probability that both units have at least 1 "no open"

$$= 1 - [1 - P_{a_1}(O)P_{a_2}(O)]\,[1 - P_{b_1}(O)P_{b_2}(O)]$$

(2) $P_2(S)$ = probability that at least 1 element in each unit shorts

$$= \left[1 - \left(1 - P_{a_1}(S)\right)\left(1 - P_{a_2}(S)\right)\right]\left[1 - \left(1 - P_{b_1}(S)\right)\left(1 - P_{b_2}(S)\right)\right]$$

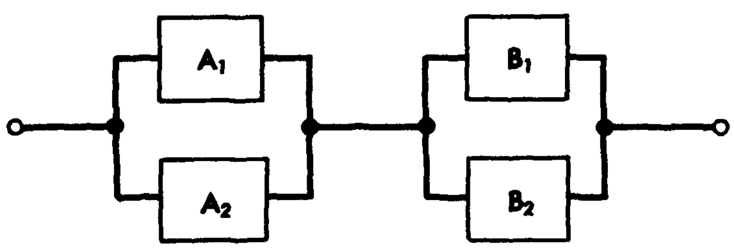


Figure 4-11. Series-Parallel Configuration

Then

$P_1(O) + P_2(S)$ = probability of block failure

$1 - [P_1(O) + P_2(S)]$ = reliability of block = R_{sp}

Since

$$P_i(O) = q_{oi}$$

and

$$P_i(S) = q_{si}$$

Then

$$R_{sp} = \left[1 - q_{oa_1}\, q_{oa_2}\right]\left[1 - q_{ob_1}\, q_{ob_2}\right]$$

$$- \left[1 - \left(1 - q_{sa_1}\right)\left(1 - q_{sa_2}\right)\right]$$

$$\left[1 - \left(1 - q_{sb_1}\right)\left(1 - q_{sb_2}\right)\right]$$

When the units are identical ($A_1 = B_1$ and $A_2 = B_2$) and all components perform the same function, then

$$R_{sp} = \left[1 - q_{oa} q_{ob}\right]^2$$

$$- \left[1 - \left(1 - q_{sa}\right)\left(1 - q_{sb}\right)\right]^2$$

For n identical units each containing m elements,

$$R_{sp} = \left[1 - \prod_{i=1}^{m} q_{oi}\right]^n - \left[1 - \prod_{i=1}^{m}\left(1 - q_{si}\right)\right]^n$$

and if all elements are identical,

$$R_{sp} = \left[1 - q_o^{\,m}\right]^n - \left[1 - \left(1 - q_s\right)^m\right]^n$$

If q_s and q_o are small, then

$$R_{sp} \approx 1 - nq_o^{\,m} - \left(mq_s\right)^n$$

Parallel-Series Elements

A 4-element parallel-series configuration is shown in Figure 4-12. Each element performs the same function. Success is defined as an output from at least one element. Therefore, the block is successful if (1) at least one path has no opens, and (2) both paths have less than two shorts.

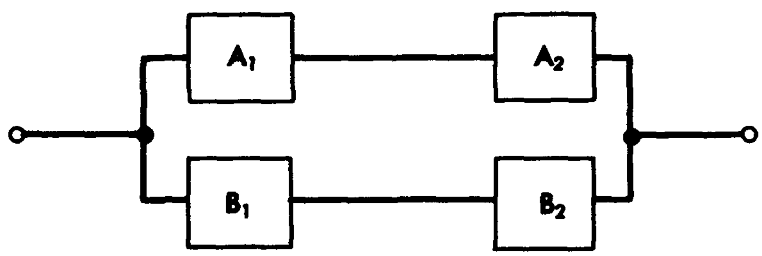


Figure 4-12. Parallel-Series Configuration

(1) $P_1(O)$ = probability that at least one element in each path has opened

$$= \left[1 - \left(1 - P_{a_1}(O)\right)\left(1 - P_{a_2}(O)\right)\right]$$

$$\left[1 - \left(1 - P_{b_1}(O)\right)\left(1 - P_{b_2}(O)\right)\right]$$

(2) $P_2(S)$ = probability that at least one path has two shorts

= one minus the probability that both paths have at least one "no short"

$$= 1 - \left[1 - P_{a_1}(S)\, P_{a_2}(S)\right]$$

$$\left[1 - P_{b_1}(S)\, P_{b_2}(S)\right]$$

Then

$P_1(O) + P_2(S)$ = probability of block failure

$1 - [P_1(O) + P_2(S)]$ = reliability of block $= R_{ps}$

Since

$$P_i(O) = q_{oi}$$

and

$$P_i(S) = q_{si}$$

then

$$R_{ps} = \left[1 - q_{sa_1} q_{sa_2}\right]\left[1 - q_{sb_1} q_{sb_2}\right]$$

$$- \left[1 - \left(1 - q_{oa_1}\right)\left(1 - q_{oa_2}\right)\right]$$

$$\left[1 - \left(1 - q_{ob_1}\right)\left(1 - q_{ob_2}\right)\right]$$

If all paths are identical ($A_1 = B_1$ and $A_2 = B_2$) and all components perform the same function,

$$R_{ps} = \left[1 - q_{sa}\, q_{sb}\right]^2$$

$$- \left[1 - \left(1 - q_{oa}\right)\left(1 - q_{ob}\right)\right]^2$$

For m identical paths each containing n elements,

$$R_{ps} = \left[1 - \prod_{i=1}^{n} q_{si}\right]^m - \left[1 - \prod_{i=1}^{n}\left(1 - q_{oi}\right)\right]^m$$

If all elements are identical,

$$R_{ps} = \left[1 - q_s^{\,n}\right]^m - \left[1 - \left(1 - q_o\right)^n\right]^m$$

If q_o and q_s are small,

$$R_{ps} \approx 1 - m q_s^{\,n} - \left(n q_o\right)^m$$

4.2.4 Operative Redundancy, Switching Required

Until now we have dealt with circuits where it was assumed that switching devices were either absent or failure free. We now deal with circuits whose redundant elements are continuously energized but do not become part of the circuit until switched in after a primary element fails. We will consider two modes of failure that can be associated with the switching mechanism:

Type (1). The switch may fail to operate when it is supposed to.

Type (2). The switch may operate without command (prematurely).

In the following discussion,

q_s = probability of a Type (1) failure

q'_s = probability of a Type (2) failure

Two Parallel Elements

Consider the system in Figure 4-13.

There are three possible element states that could lead to system failure:

1. A succeeds, B fails
2. A fails, B succeeds
3. A fails, B fails

The unreliability of the system, $\bar{R}$, is found from

$$\bar{R} = p_a q_b q'_s + q_a p_b q_s + q_a q_b$$

If we are not concerned with Type (2) failures,

$$q'_s = 0$$

and the unreliability is

$$\bar{R}_D = q_a p_b q_s + q_a q_b$$

$$= q_a q_s + q_a p_s q_b$$

As an example, assume

$$q_a = q_b = 0.2$$

and $\quad q_s = q'_s = 0.1$

Then

$$\bar{R} = p_a q_b q'_s + q_a p_b q_s + q_a q_b$$

$$= (0.8)(0.2)(0.1) + (0.2)(0.8)(0.1) + (0.2)(0.2)$$

$$= 0.072$$

$$R = 1 - \bar{R}$$

$$= 1 - 0.072$$

$$= 0.928$$

If $q'_s = 0$,

$$\bar{R}_D = q_a q_s + q_a p_s q_b$$

$$= (0.2)(0.1) + (0.2)(0.8)(0.2)$$

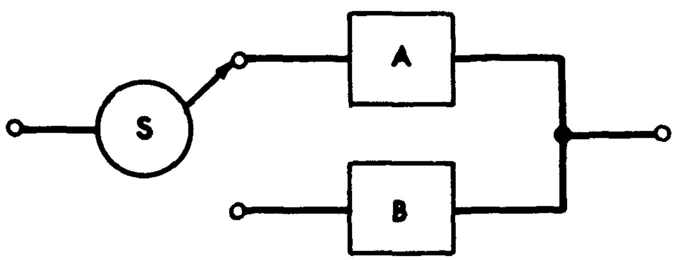


Figure 4-13. Redundancy with Switching

$= 0.052$

$R_D = 1 - 0.052$

$= 0.948$

Three Parallel Elements

Figure 4-14 illustrates this type circuit. It operates as follows: If A fails, S switches to B. If B then fails, S switches to C. Enumerating all possible switching failures shows two kinds of Type (1) failure and four kinds of Type (2) failure:

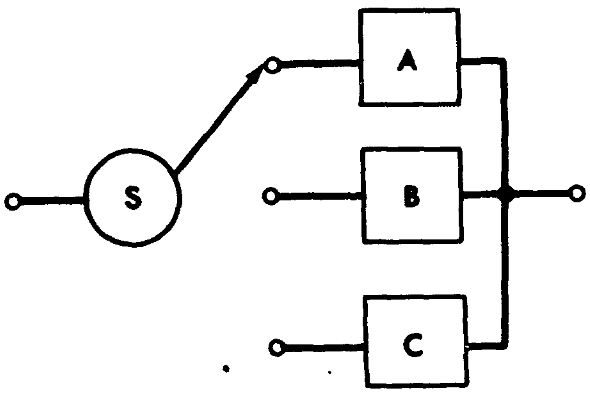


Figure 4-14. Three-Element Redundant Configurations with Switching

Type (1) Switching Failures:

1. q_{s_1} – A fails, S does not switch to B.

2. q_{s_2} – A fails, S switches to B, B fails, S fails to switch to C.

Type (2) Switching Failures:

3. q'_{s_3} – A succeeds, but S switches to B.

4. q'_{s_4} – A succeeds, S switches to B, B fails, S does not switch to C.

5. q'_{s_5} – A succeeds, S switches to B, B succeeds, S switches to C.

6. q'_{s_6} – A fails, S switches to B, B succeeds, S switches to C.

The possible states of operation of elements A, B, and C and also switching failure that will cause system failure for each state are shown in Table 4-II.

The probability of system failure can be found by summing up the probabilities of individual combinations or operating states which result in system success, each multiplied by the probability of a switching failure which would produce system failure in each state; i.e.:

$$\bar{R} = \sum_{i=1}^{8} p_i q_{s_i}$$

or, as shown in Table 4-II,

$$\bar{R} = p_a q_b q_c q'_{s_3} + p_b q_a q_c (q_{s_1} + q'_{s_6})$$

$$+ p_c q_a q_b (q_{s_1} + q_{s_2})$$

$$+ p_a p_b q_c q'_{s_5} + p_a p_c q_b q'_{s_4}$$

$$+ p_b p_c q_a q_{s_1} + q_a q_b q_c$$

(Primes denote "static" or Type (2) switch failures)

If the probability of Type (2) switching failures is very small ($q'_{si} \approx 0$), and $q_{s1} = q_{s2} = q_s$, $\bar{R}$ can be found directly from the following equation:

$$\bar{R} = q_a q_s + q_a p_s q_b q_s + q_a p_s q_b p_s q_c$$

TABLE 4-II. STATES OF OPERATION OF A THREE PARALLEL ELEMENT CIRCUIT WITH DECISION AND SWITCHING DEVICE

Operating State (i)	Operating Condition		Switching Failure Resulting in System Failure	$\bar{R} = \sum_{i=1}^{8} p_i q_s$
	Succeed	Fail		
1	A	$\bar{B}\bar{C}$	s_3	$A\bar{B}\bar{C}\bar{s}_3$
2	B	$\bar{A}\bar{C}$	s_1 or s_6	$\bar{A}B\bar{C}(\bar{s}_1+\bar{s}_6)$
3	C	$\bar{A}\bar{B}$	s_1 or s_2	$\bar{A}\bar{B}C(\bar{s}_1+\bar{s}_2)$
4	AB	$\bar{C}$	s_5	$AB\bar{C}(\bar{s}_5)$
5	AC	$\bar{B}$	s_4	$A\bar{B}C(\bar{s}_4)$
6	BC	$\bar{A}$	s_1	$\bar{A}BC(\bar{s}_1)$
7	ABC	–	Cannot fail	ABC
8	–	$\bar{A}\bar{B}\bar{C}$	Always fails	–

4.3 VOTING REDUNDANCY

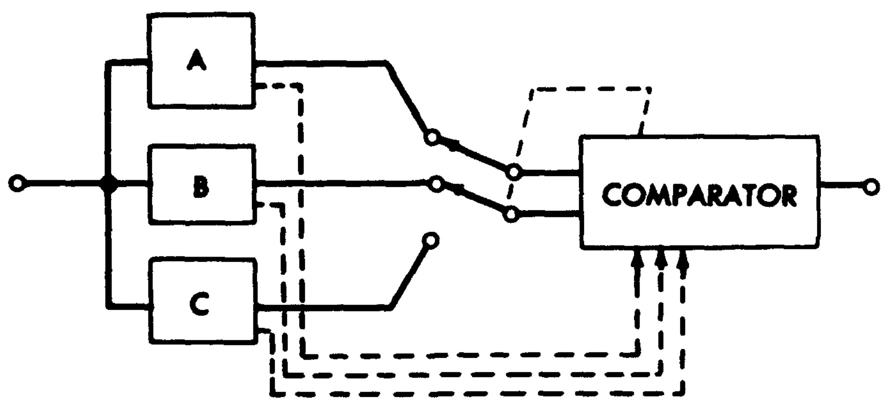


Figure 4-15. Three-Element Voting Redundancy

Figure 4-15 shows three elements, A, B, and C, and the associated switching and comparator circuit which make up a voting redundant system. The circuit function will always be performed by an element whose output agrees with the output of at least one of the other elements. At least two good elements are required for successful operation of the circuit. Two switches are provided so that a comparison of any two outputs of the three elements can be made. A comparator circuit is required that will operate the two switches so that a position is located where the outputs again agree after one element fails.

If comparison and switching are failure free, the system will be successful as long as two or three elements are successful. In this case,

$$R = p_a p_b + p_a p_c + p_b p_c - 2p_a p_b p_c$$

If failure-free switching cannot be assumed, conditional probabilities of switching operation have to be considered. To simplify the discussion, consider the probability of the comparator and switches failing in such a manner that the switches remain in their original positions. If this probability is q_s, then

$$R = p_a p_b + (p_a p_c + p_b p_c - 2p_a p_b p_c)(1 - q_s)$$

EXAMPLE: Let all three elements have the same probability of success, 0.9; i.e., $p_a = p_b = p_c = 0.9$. Assume that the comparator-switch has a probability of failing (q_s) of 0.01:

$$R = .9^2 + [(.9)^2 + (.9)^2 - 2(.9)^3]\,[1 - .01]$$

$$R = .970$$

4.4 STANDBY REDUNDANCY

In a system with redundant elements on a completely standby basis (not energized), no time is accumulated on a secondary element until a primary element fails. For a two-element system (Figure 4-16) the reliability function can be found directly as follows: The system will be successful at time t if either of the following two conditions hold (let A be the primary element):

(1) A is successful up to time t.

(2) A fails at time $t_1 < t$, and B operates from t_1 to t.

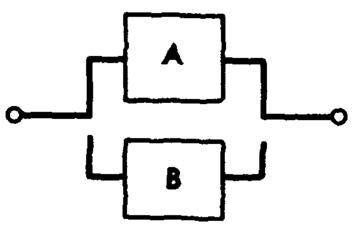


Figure 4-16. Diagram Depicting a Standby Redundant Pair

For the exponential case where the element failure rates are λ_a and λ_b, reliability of the standby pair is given by

$$R(t) = \frac{\lambda_b}{\lambda_b - \lambda_a} e^{-(\lambda_a)t} - \frac{\lambda_a}{\lambda_b - \lambda_a} e^{-(\lambda_b)t}$$

This is a form of the mixed exponential and it does not matter whether the more reliable element is used as the primary or as the standby element. If $\lambda_a = \lambda_b = \lambda$,

$$R(t) = e^{-\lambda t}(1 + \lambda t)$$

The mean-time-to-failure of the system is

$$MTBF = \frac{\lambda_a + \lambda_b}{\lambda_a \lambda_b}$$

$$= \theta_a + \theta_b \text{ when } \theta_a \neq \theta_b$$

$$= 2\theta \text{ when } \theta_a = \theta_b = \theta$$

For n elements of equal reliability,

$$R(t) = e^{-\lambda t} \sum_{r=0}^{n-1} \frac{(\lambda t)^r}{r!}$$

$$MTBF = \frac{n}{\lambda} = n\theta$$

Figure 4-17 is a chart relating system reliability to the reliability of individual standby redundant parallel elements as a function of mission time, t/θ. By entering the chart at the time period of interest and proceeding vertically to the allocated reliability requirement, the required number of standby elements can be determined.

EXAMPLE: A critical element within a system has a demonstrated MTBF, θ = 100 hours. A design requirement has been allocated to the function performed by this element of R_s = .98 at 100 hours, corresponding to a 30-to-1 reduction in unreliability below that which can be achieved by a single element. In this case, n = 4 will satisfy the design requirement at $t/\theta = 1$; i.e., a four-element standby redundant configuration would satisfy the requirement. Failure rates of switching devices must next be taken into account.

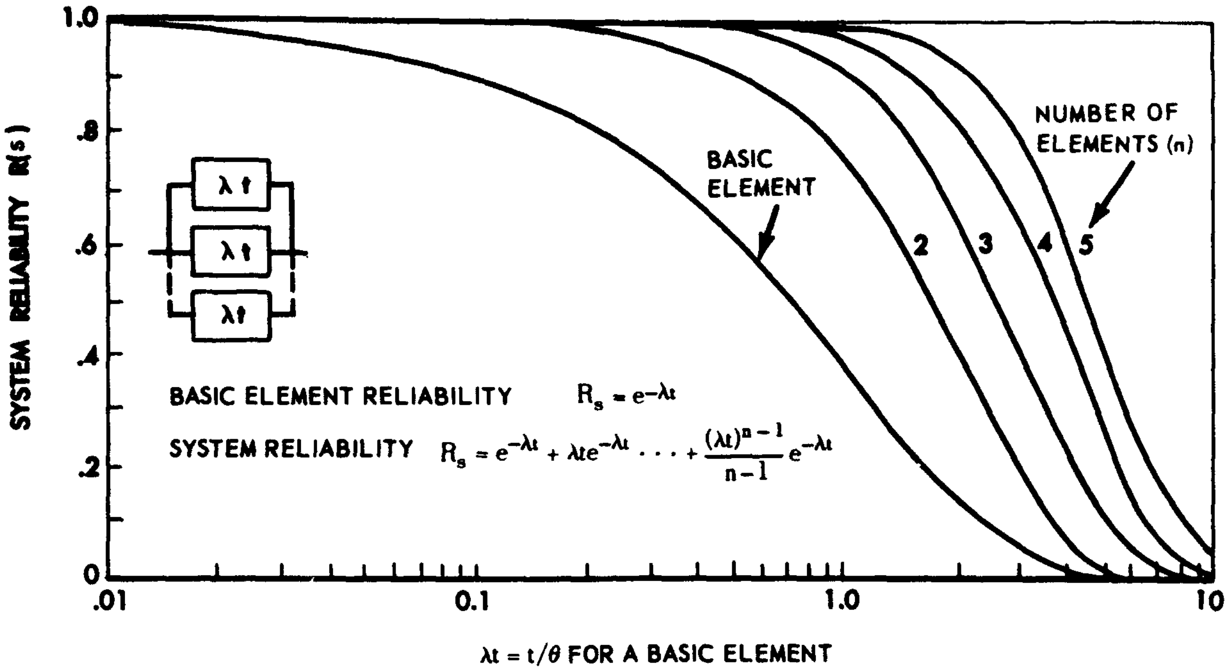


Figure 4-17. System Reliability for n Standby Redundant Elements

4.5 DEPENDENT FAILURE PROBABILITIES

Up to this point, it has been assumed that the failure of an operative redundant element has no effect on the failure rates of the remaining elements. This might occur, for example, with a system having two elements in parallel where both elements share the full load.

An example of conditional or dependent events is illustrated by Figure 4-18. A and B are both fully energized, and normally share or carry half the load, 1/2L. If either A or B fails, the survivor must carry the full

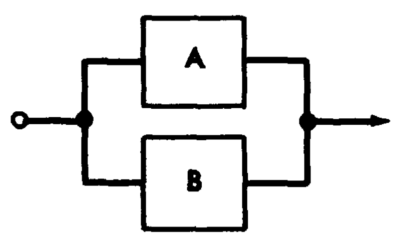


Figure 4-18. Load-Sharing Redundant Configuration

load, L. Hence, the probability that one fails is dependent on the state of the other, if failure probability is related to load or stress. The system is operating satisfactorily at time t if either A or B or both are operating successfully.

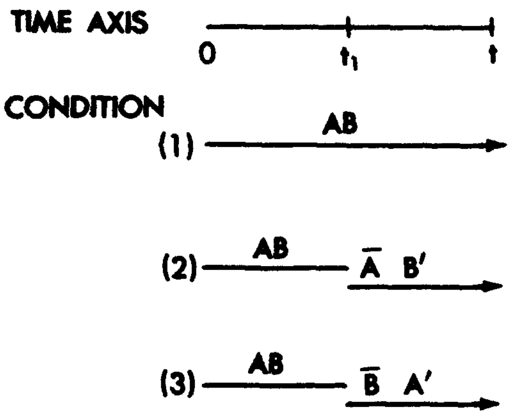


Figure 4-19. Success Combinations in Two-Element Load-Sharing Case

Figure 4-19 illustrates the three possible ways the system can be successful. The bar above a letter represents a failure of that element. A primed letter represents operation of that element under full load; absence of a prime represents operation under half load. If the elements' failure times are exponentially distributed and each has a mean life of θ under load L/2 and $\theta' = \theta/k$ under load L(k > 0), block reliability is given below without derivation:

$$R(t) = \frac{2\theta'}{2\theta' - \theta} e^{-t/\theta'} - \frac{\theta}{2\theta' - \theta} e^{-2t/\theta}$$

System mean life is equal to

$$\theta_s = \theta/k + \theta/2$$

When k = 1, the system is one in which load-sharing is not present or an increased load does not affect the element failure probability. Thus, for this case, θ_s is equal to $3\theta/2$. If there were only one element it would be operating under full load, so system mean life would be $\theta' = \theta/k$. Hence, the addition of a load-sharing element increases the system mean life by $\theta/2$. This increase in mean life is equivalent to that gained when the elements are independent, but the overall system reliability is usually less because θ' is usually less than $\theta (k > 1)$.

4.6 OPTIMUM ALLOCATION OF REDUNDANCY

Decision and switching devices may fail to switch when required or operate inadvertently. However, these devices are usually necessary for redundancy, and increasing the number of redundant elements increases the number of switching devices.

If such devices are completely reliable, redundancy is most effective at lower system levels. If switching devices are not failure-free, the problem of increasing system reliability through redundancy becomes one of choosing an optimum level at which to replicate elements.

Since cost, weight, and complexity factors are always involved, the minimum amount of redundancy that will produce the desired reliability should be used. Thus efforts should be concentrated on those parts of the system which are the major causes of system unreliability.

As an example, assume that we have two elements, A and B, with reliabilities over a certain time period of 0.95 and 0.50, respectively. If A and B are joined to form

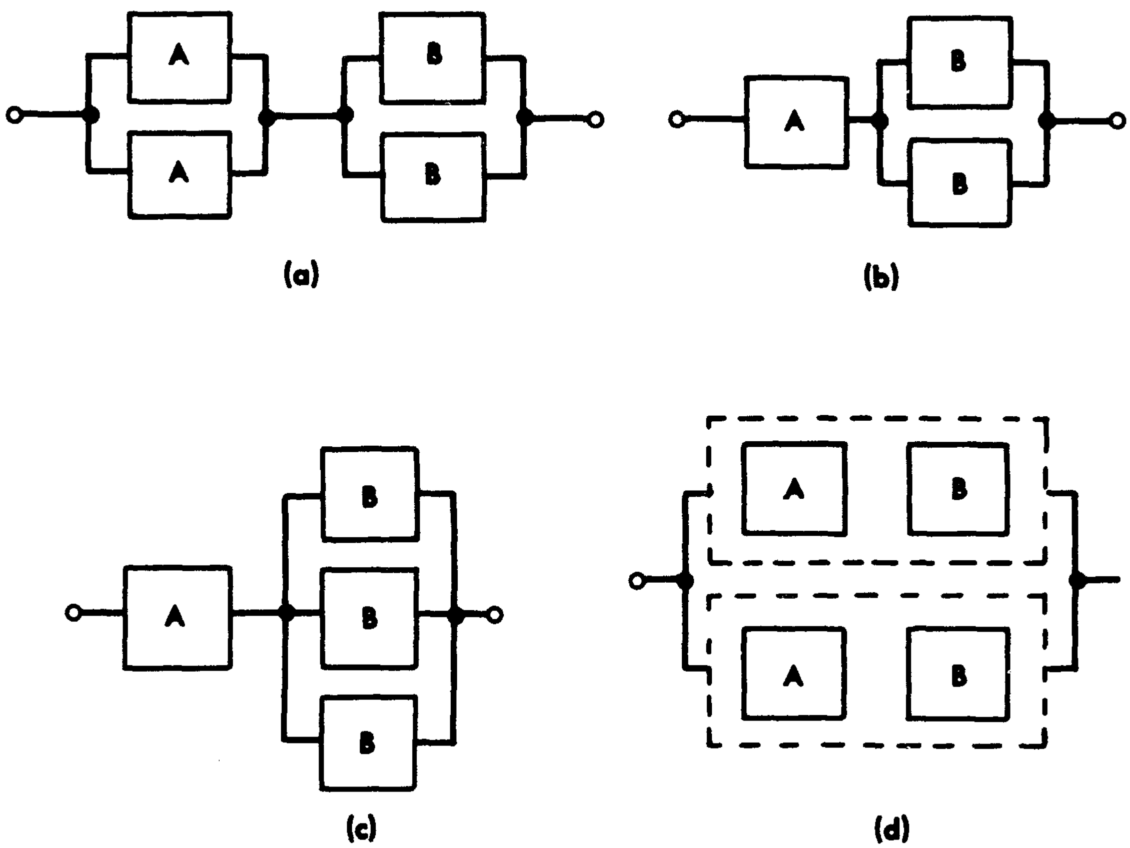


Figure 4-20. Possible Redundant Configurations Resulting from Allocation Study

a series non-redundant circuit, its reliability is

$$R = (0.95)(0.50) = 0.475$$

If we duplicate each element, as in Figure 4-20a,

$$R_1 = [1-(0.50)^2][1-(0.50)^2]$$

$$= 0.748$$

Duplicating Element B only, as in Figure 4-20b,

$$R_2 = 0.95\,[1-(0.50)^2]$$

$$= 0.712$$

Obviously, duplicating Element A contributes little to increasing reliability.

Triplication of B gives the configuration shown in Figure 4-20c, and

$$R_3 = 0.95\,[1-(0.5)^3]$$

$$= 0.831$$

R_3 gives a 75% increase in original circuit reliability as compared to the 58% increase of R_1.

If complexity is the limiting factor, duplicating systems is generally preferred to duplicating elements, especially if switching devices are necessary. If another series path is added in parallel, we have the configuration in Figure 4-20d, and

$$R_4 = 1 - (1 - .475)^2$$

$$= 0.724$$

R_4 is only slightly less than R_1. If switches are necessary for each redundant element, R_4 may be the best configuration. A careful analysis of the effect of each element and switch on system reliability is a necessary prerequisite for proper redundancy application.

4.7 REDUNDANCY-WITH-REPAIR

In certain instances it may be more practical to design a system with built-in "on-line" maintenance features to overcome a serious reliability problem than to concentrate on improving reliability of the components giving rise to the problem. Redundancy-with-repair can be made to approach the upper limit of reliability (unity), contingent on the rate with which element failures can be detected and repaired or replaced. The system thus continues on operational status while its redundant elements are being repaired or replaced, so long as these repairs are completed before their respective redundant counterparts also fail.

There are, in general, two types of monitoring that may be used for failure detection in systems employing redundant elements:

(1) *Continuous monitoring* – element failures are recognized at the instant they occur and repair or replacement action begins immediately. It is assumed that repairs can be made at the rate of μ per hour, where μ is the mean of an exponential distribution of repair times.

(2) *Interval monitoring*–the system is checked for element failures every T hours. Failed elements are replaced with operable elements. Here it is assumed that the times required to monitor the elements and make replacements are negligible.

4.7.1 Continuous Monitoring

The reliability equation for two redundant elements is:

$$R(t) = \frac{s_1 e^{s_2 t} - s_2 e^{s_1 t}}{s_1 - s_2}$$

In the case of operative redundancy,

$$s_1 = -\frac{1}{2}\left[(3\lambda + \mu) + \sqrt{\mu^2 + 6\mu\lambda + \lambda^2}\right]$$

$$s_2 = -\frac{1}{2}\left[(3\lambda + \mu) - \sqrt{\mu^2 + 6\mu\lambda + \lambda^2}\right]$$

For standby redundancy,

$$s_1 = -\frac{1}{2}\left[(2\lambda + \mu) + \sqrt{\mu^2 + 4\mu\lambda}\right]$$

$$s_2 = -\frac{1}{2}\left[(2\lambda + \mu) - \sqrt{\mu^2 + 4\mu\lambda}\right]$$

The reliability equations for these two cases are plotted in Figure 4-21 and Figure 4-22.

EXAMPLE: Two similar elements with MTBF's of 100 hours are to be used as a redundant pair. The mean-time-to-repair for each element is 10 hours. Determine the reliability of the pair for a 23-hour mission when used as (a) an operative redundant pair, and (b) a standby redundant pair.

The graphs of the reliability equations, Figures 4-21 and 4-22, are given in terms of λt and μ/λ. From the information given, $\lambda = 1/\text{MTBF} = 10^{-2}$, $t = 23$ hours, and $\mu = 1/(\text{repair time}) = 10^{-1}$. Hence, $\lambda t = .23$ and $\mu/\lambda = 10$. By means of the graphs, the reliability for the two cases is found to be:

Operative redundancy:
R(23 hours) = .9760

Standby redundancy:
R(23 hours) = .9874

When comparing the reliability of two situations that exceed .90, as above, it is more meaningful to compare the unreliabilities. In this case, a comparison of .0240 versus .0126 shows about a 2-to-1 difference in unreliability between the operative and the standby case, in favor of the latter.

4.7.2 Interval Monitoring

The reliability equations for interval monitoring require that the mission time be expressed as two components, $t = nT + d$. The number of times the elements will be monitored during the mission (t) is given by n; T is the time interval between monitoring points; and d is the time between the last monitoring point and the end of the mission. Module replacement or switching time is assumed to be zero.

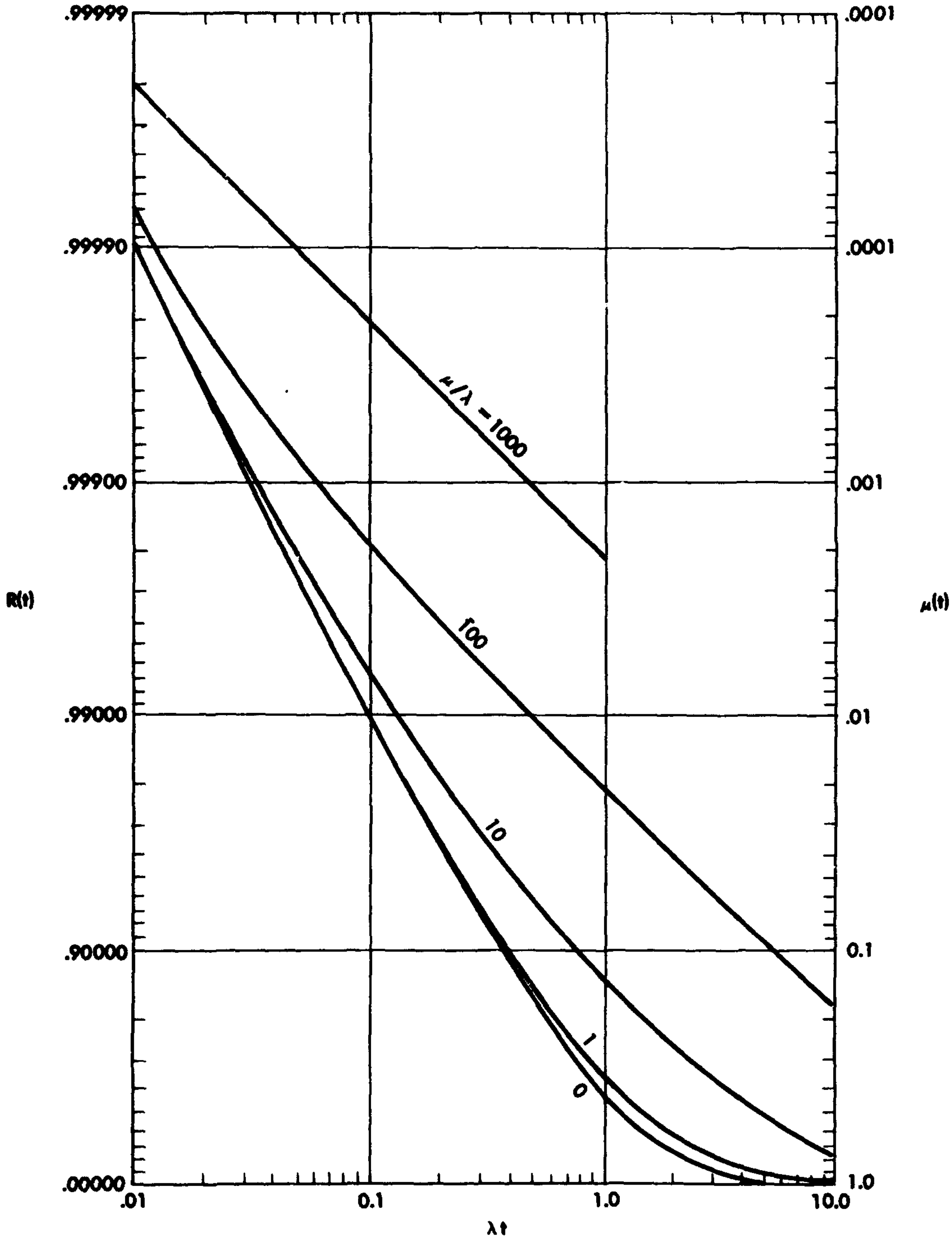


Figure 4-21. Operative Redundancy-with-Repair (Continuous Monitoring)

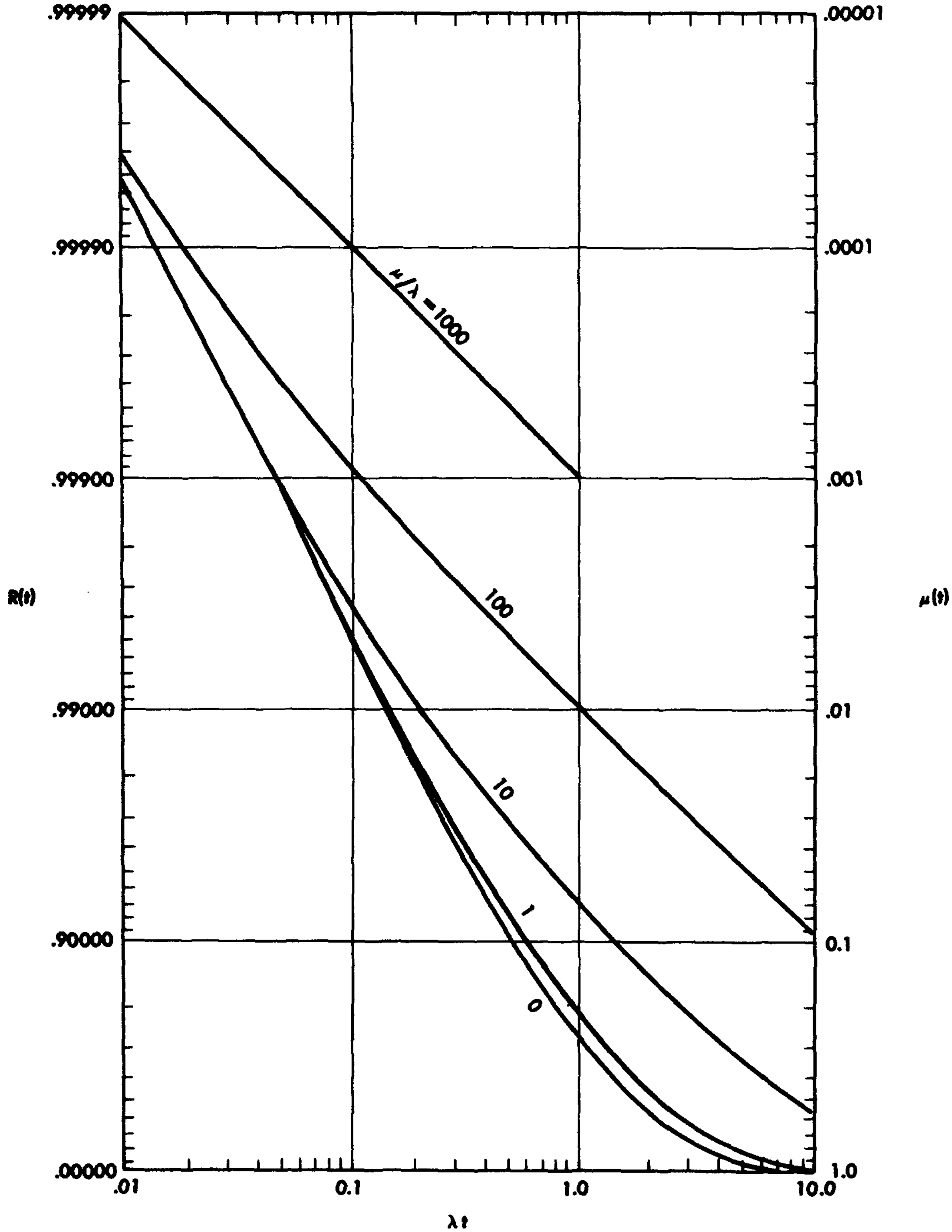


Figure 4-22. Standby Redundancy-with-Repair (Continuous Monitoring)

For operative redundancy:

$$R(t) = (2e^{-\lambda d} - e^{-2\lambda d})(2e^{-\lambda T} - e^{-2\lambda T})^n$$

For standby redundancy:

$$R(t) = (1 + \lambda T)^n (1 + \lambda d) e^{-\lambda t}$$

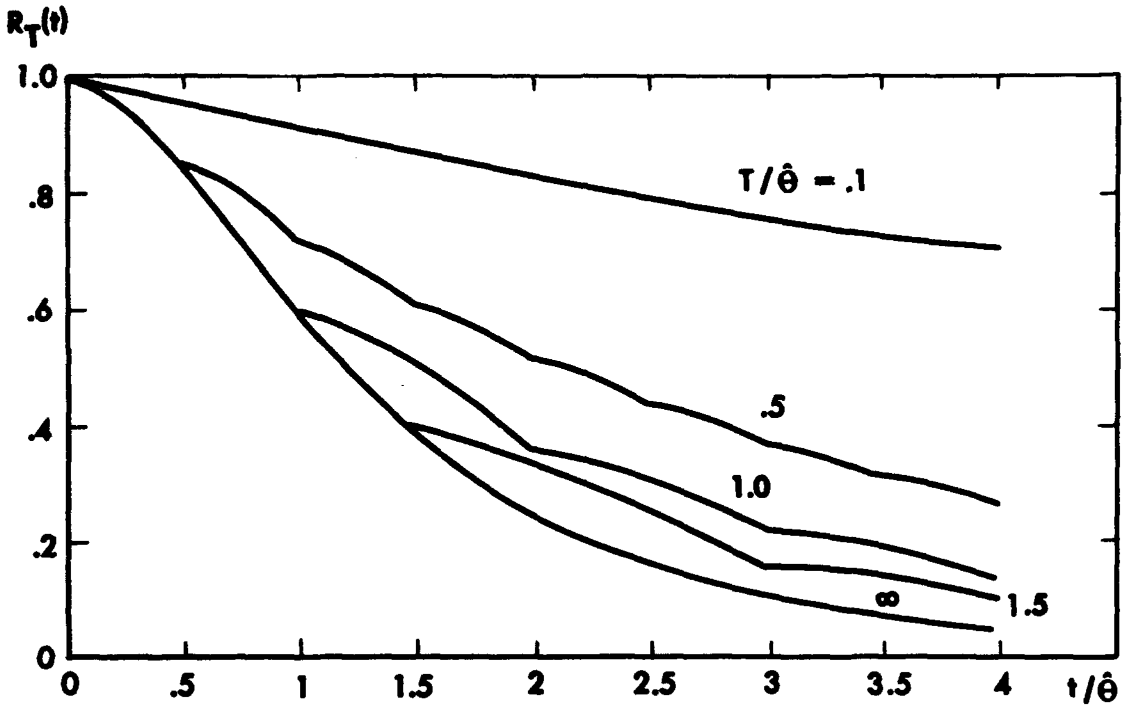


Figure 4-23. Reliability Functions for Several Cases of Interval Monitoring and Repair

EXAMPLE: Two similar elements with MTBF's of 100 hours are to be used as a redundant pair. The pair will be monitored every 3 hours. When a defective element is found, it will be replaced by an operable element immediately. We wish to determine the reliability of the pair for a 23-hour mission when used as an operative redundant pair. From the above, it is determined that t = 23 hours, n = 7, nT = 21, and d = 2 hours. As in the previous example, $\lambda = 10^{-2}$.

$$R(23 \text{ hours}) = (2e^{-.02} - e^{-.04})(2e^{-.03} - e^{-.06})^7$$

$$= .9935$$

Figure 4-23 presents reliability functions normalized with respect to operating time t/θ, for five cases of T/θ monitoring intervals, to illustrate the reliability potential of designs which provide this redundancy with interval monitoring and on-line repair capability.

BIBLIOGRAPHY ON RELIABILITY

B-1 INTRODUCTION

This extensive bibliography is included as a source of further information for those who desire more detailed coverage of specific topics of reliability engineering than has been possible in this handbook. Many of the entries carry notes which give a capsule appraisal of contents of the various papers. Certain of the documents are boxed in to suggest a few among the many that are of more general use for reference purposes.

These references are intended for information purposes only and are not necessarily consistent with the BuWeps reliability procedures contained in this handbook.

Approximately 150 books, articles, and reports are listed, dealing with various aspects of reliability. Certain periodicals, conventions, and symposia which are sources of reliability information are also listed. This selection represents only a small part of the literature within the scope of the general subject of reliability. Many excellent references have been omitted. For further reference to these, consult the other reliability bibliographies included in the list; the bibliographic material in those listed items will also be helpful.

The references have been classified by broad subject fields with subdivisions within each field, since a subject classification is often the most rapid and effective way of locating specific information. Many publications include information that falls under more than one subject heading. In such instances, the publication is listed under the subject given the major emphasis. The main subject breakdown is as follows:

- Reliability, General Coverage
- Reliability Prediction and Analysis
- Reliability Measurement and Testing
- Redundancy
- Statistics and Probability
- Reliability Bibliographies
- Periodicals
- Conventions and Symposia

Most of the references cover the period from 1956 to 1962. Reference 148 contains over 750 references concerning material published prior to June 1956. Where known, ASTIA numbers are given for those documents which can be obtained from the Armed Services Technical Information Agency (Ref. 141). These numbers are prefixed by "AD".

B-2 RELIABILITY, GENERAL COVERAGE

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Electrical Engineering (American Society of Electrical Engineers)

IBM Journal of Research and Development (International Business Machines Corporation)

Industrial Quality Control (American Society for Quality Control, Milwaukee, Wisconsin)

IRE Transactions (Transactions of Professional Group on Reliability and Quality Control)

Journal of American Statistical Association

Lubrication Engineering (American Society of Lubrication Engineers)

Operations Research (Operations Research Society of America)

Phillips Technical Review (Phillips Research Laboratories)

RCA Review (Radio Corporation of America)

SAE Journal (Society of Automotive Engineers)

Sylvania Technologist (Sylvania Electric Company)

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Technical Magazines

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B-9 CONVENTIONS AND SYMPOSIA

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Design Engineering Conference – Sponsored by A.S.M.E. (Proceedings published)

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National Convention – American Society for Quality Control. (Proceedings published. Regional meetings also held.)

National Conference on Electron Devices – Sponsored by I.R.E. Professional Group on Electron Devices.

National Convention on Military Electronics – Sponsored by I.R.E. Professional Group on Military Electronics. (Proceedings published)

National Meeting of the Operations Research Society of America. (Abstracts published)

National Symposium on Reliability and Quality Control – Sponsored by A.I.E.E., A.S.Q.C., E.I.A., I.R.E. (Proceedings published)

Navy-Industry Conference on Material Reliability – Sponsored by the BuWeps-Industry Material Reliability Advisory Board (BIMRAB). (This conference provides perhaps the best overall coverage of BuWeps reliability plans, programs, and problems. Copies of annual conference proceedings are available upon request to Secretary of BIMRAB, Attention: F. W. Snyder, Bureau of Naval Weapons, Department of the Navy, Washington 25, D.C.)

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Statistical Engineering Symposium – Sponsored by Army Chemical Center, Maryland. (Proceedings published)

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