This final report covers the work done by our group of neural network computing at the University of Maryland for the past three years. We studied the neural network's capability of processing temporal or sequential data. Recurrent neural networks were used to perform inference on grammars. An external memory stack was constructed to work with the neural network to perform inferences on context-free languages. And finally, a spatially homogeneous locally connected recurrent neural network that could simulate any given Turing machine, including the universal Turing machine was devised. It is capable of performing universal computations and demonstrated the universal power of recurrent neural network architectures. To train these sequential neural network machine, we have investigated the forward propagating learning algorithms.
FINAL REPORT

CONNECTIONIST MODELS
FOR INTELLIGENT COMPUTATION

Covering Period
May 1, 1991 — April 30, 1994

Submitted to
Air Force Office of Scientific Research

by

Laboratory for Plasma Research
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ABSTRACT

This final report covers the work done by our group of neural network computing at University of Maryland for the past three years under the sponsorship of AFOSR. During this grant period, we studied the neural network's capability of processing temporal or sequential data. Recurrent neural networks were used to perform inference on grammars. An external memory stack was constructed to work with the neural network to perform inferences on context free languages. And finally, a spatially homogeneous, locally connected, recurrent neural network that could simulate any given Turing machine, including the universal Turing machine was devised. It is capable of performing universal computations and demonstrated the universal power of recurrent neural network architectures. In order to train these sequential neural net machines, we have investigated the forward propagating learning algorithms. A fast learning algorithm is proposed that could reduce the computation complexity from $O(N^4T)$ to $O(N^3T)$. This algorithm was tested on a continuous temporal problem that will be the next phase of our research effort.

I. INTRODUCTION

Artificial neural networks are very powerful constructs. It is supposed to simulate the brain structure of human to perform intelligent tasks. However, most of the current research in this area seem to treat neural net as only that of a functional mapper. It is used to conduct an input-output mapping. We think that most intelligent tasks involve the processing of temporal or sequential signals. Therefore, neural networks must be able to extract generation rules from sequential patterns, i.e. grammatical inferences. In the past three years, our project dealt with topics surrounding such issues. Neural networks with recursive connections were chosen to process such sequential data that are correlated in both space and time. The recurrent connections that serve as a memory of history of the sequence and empower the neural network to extract temporal orders out of the sequential patterns. A recurrent network in itself behaves like a finite state machine if we can cluster the neuronal state and quantize them. Indeed, numerical simulations showed that recurrent neural net can be easily trained to do just that. Perfect finite state machine can be extracted from only a handful amount of data from a sequence. Theoretical analysis also established that the computational power of a finite state machine would increase tremendously if it is coupled with a memory stack. The resulting pushdown automata could recognize an extended set of language called context free language which is much more expressive than the finite state machine grammar. Even more computational power could be achieved by replacing the stack memory with an infinite tape and empower the neural network finite state controller to erase and write on the tape and thus becoming a Turing machine. Once established that neural net can fully simulate universal Turing machine, we have no doubt that neural net should have the full power to simulate any given intelligent task.
II. Simulation of Finite State Machine

As in most applications of neural nets, the topology of the connection weights in the net is crucial to the success of the applications. We studied the issue regarding the order of connections to the simulation of finite state machines and pushdown automata. Specifically, we established that:

**Theorem 1**: For any given finite state machine (FSM) with \( N \) states and \( M \) input symbols, there are at least one second order connected recurrent neural net (RNN) with \( N \) state neurons and \( M \) input neurons that can directly simulate the FSM.

This theorem ensures us that RNN can be used to learn regular grammar since the existence of the solution is guaranteed. On the other hand, we can also show that

**Theorem 2**: There are some FSM structure that cannot be directly simulated by any RNN with first order connections without hidden neurons.

A notable example is the four state loop transition diagram of a dual parity finite state grammar. However, this does not imply that first order recurrent net cannot learn the dual parity grammar. Because we can also prove the following

**Theorem 3**: There are at least one first order RNN with at most \( NM \) neurons that can simulate a given FSM indirectly. That is, it can simulate an equivalent FSM to the given FSM and this equivalent machine can be obtained automatically.

**Theorem 4**: In a second order RNN with \( S \) state neurons and \( M \) input neurons, the probability of it to simulate a given finite state machine with \( N \) states and \( M \) input symbols is given by

\[
P = \left[ \frac{L(N, S)}{2^N} \right]^{SM}
\]

where \( L(N, S) \) is the number of dichotomies that can be implemented by a \( S \)-dimensional perceptron for an \( N \)-input pattern.

These theorems give us good guidance in our choice of different RNN connection topologies for various grammatical inference tasks. Similar results for neural net simulation of PDA were also obtained.

**Theorem 5**: For any given deterministic pushdown automata with \( N \) states, \( M \) symbols and \( M \) stack symbols, there exists a third order RNN coupled with an external stack memory that can simulate it completely.
Likewise we also have an estimate of the capacity of a K-th ordered RNN to simulate finite state machines.

Theorem 6: The capacity of a K-th order RNN with N recurrent neurons, $C(N,K)$, can be inferred from the recursion formula:

$$C(N,K) = C(N-1,K) + C(N-1,K-1)$$

III. Simulation of Pushdown Automata with enhanced RNN:

In previous work we have developed a RNN pushdown that uses a third order connected recurrent neural net controller to operate an analog stack memory. Many context free grammars were learned by this construct from a limited set of positive and negative samples. A typical example is parenthesis balance checker.

It came to our attention later that several hard grammar such as the Palindrome is very difficult to learn by the above system. To solve this problem, we devised an enhanced recurrent neural net with “full order” connections. It turns out that this enhanced neural net can learn grammars such as the Palindrome very easily. It automatically figured out some very tricky transition rules associated with such grammar. After quantization, the learned rules are again exact and the generalization to test other samples of this grammar is again infinite.

IV. Neural net Turing Machine

Turing machine is the most powerful sequential machine that is capable of universal computations. The ability of the neural net to simulate Turing machines is therefore an important issue in neural computing. In the past year, we have succeeded in constructing just such a neural net Turing machine. The finite state controller and the tape symbols are represented by neurons arranged into a row of columns. Each neuron is locally connected to other neurons in the same and neighboring columns. The detailed values of these weights and the specific construction of the network is beyond this report. We shall summarize our results in the following two theorems.

Theorem 1: Given an arbitrary deterministic Turing machine with $M$ symbols and $N$ states, there exists a neural network with $M+2N+1$ rows of neurons and a set of second order locally connected weights that can simulate it in 2-to-1 time.

Theorem 2: Given an arbitrary deterministic Turing machine with $M$ symbols and $N$ states, there exists a neural net with $M+N+2$ rows of neurons and two sets of second order locally connected weights that can simulate it in real time.

Currently, we are studying the training of the neural net Turing machine to recognize some
simple grammars, for example, the parenthesis checker.

V. Green Function Method for Fast On-line Training of Recurrent Neural Networks

In processing temporal or sequential signals, recurrent neural network is found to be able to capture most of the complex temporal orders and correlations. However, a particular pressing issue concerning recurrent net is the lack of an efficient on-line training algorithm especially when we are dealing with applications that would require large number of internal states or neurons.

Among the currently popular training algorithms, the error back propagation method is not an on-line algorithms since we have to wait until the signals propagate all the way to the output layer to obtain the error message and then propagate back the error to each layer for weight corrections. The Williams and Zipser error prediction forward propagation algorithm is indeed on-line. However, it is very expensive since it needs $O(N^4 T)$ number of calculations for each updating of the weights. Recently, Toomerian and Barhen modified their adjoint operator approach into an on-line algorithm and claimed that it only needs $O(N^3 T)$ number of calculations. However, a careful examination of their scheme revealed some flaw in their derivation and therefore invalidated their claim.

In the past year, we developed an alternative approach in which we tried to avoid the redundant calculations presented in the forward propagation method and use a common Green function to integrate the error sensitivity matrix. We also exploit the special form of the driving term in the equation to reduce the number of calculations. The combined effect is an algorithm that is truly $O(N^3 T)$.

VI. Controlling Chaos with Neural Networks

Many of the everyday signal processing problems are temporal in nature. They are the continuous or analog counterpart of the symbolic sequential patterns. The extraction of temporal orders from such continuous temporal signals would found many real world applications in signal processing. As a preliminary study of the neural net capability in this respect, we studied the interesting chaos control problem.

The control of chaos means that to stabilize a chaotic system settling around an unstable fixed point or periodic orbit. The system we chose is the two dimensional Henon map:

$$X_{t+1} = A - X_t^2 + BY_t$$

$$Y_{t+1} = X_t$$

where the parameters are chosen as $A=1.29$ and $B=0.3$ and is in a typical chaotic regime.
One of the unstable fixed point for this attractor can be found as

$$X_F = Y_F = \frac{1}{2} \left[ B - 1 + \sqrt{(B - 1)^2 + 4A} \right] = 0.838486$$

Our objective is to construct a neural net controller that can be trained to locate the unstable fixed point automatically and to guide the chaotic system to this point and settled there indefinitely. Our study seems successful. The neural net is easily trained to accomplish this goal with an objective function given by:

$$E_t = \frac{1}{2} \left[ (X_t - \bar{X}_t)^2 + (Y_t - \bar{Y}_t)^2 \right]$$

It measures the deviation of the orbit from an averaged orbit. In the vicinity of the fixed point, the orbit is in general sticky and therefore contribute heavily to the averaging of orbits. the minimization of the objective function therefore requires the system to stay around the fixed point.

We also add noises to the system either in the mapping itself or in the emulating neural net weights. The system turns out to be rather robust against noises. It learned to control the system very smoothly without the appearance of uncontrollable outbursts seen in the original OGY model. It also located the fixed point by itself. Since chaotic system has the characteristics of moving around the whole chaotic attractor in an ergotic fashion, the system is bound to travel by the fixed point location frequently. This makes the controllability of a chaotic system even better than a regular system.

VII. Future Directions:

In the previous research, we have developed knowledge of extracting temporal orders mainly from discrete symbolic sequences. Sequential machines that generate these sequences can be constructed from a few hundred of positive and negative samples each about ten symbols or less. The constructed machine are usually complete and exact capable to generalize to the infinite number of member sequences that belong to the same grammar. The rich phenomena associated with these discrete symbolic sequences should be a subset of what could be described in an analog or continuous sequences. In one sense, a continuous sequence is a discrete sequence with an infinite number of different discrete symbols. It is therefore much more intricate to deal with. However, our preliminary study of such problems indicated that neural net could be easily adapted to the analog situation. Instead of a discrete recurrent neural net that simulate a finite state machine, we should use an infinitesimally incremented constructed so that it simulated a differential or integral system.

An especially interesting realm of research could be directed toward the chaotic system. It is known that very simple system could exhibit very complicated orbit. In a study of automata, it was
pointed out by Wolfram that it can generate sequences with arbitrary complexity. An understanding of controlling these subset of dynamical systems would be the most fruitful endeavor in the neural net research.

Publications


