This report summarizes our work on four general problems during the three-year period of the contract: (1) estimation of frequency-wavenumber, (2) matched subspace filters, (3) maximum likelihood estimation of modes from space-time data, and (4) statistical inference within the wavelet representation. We have generalized the theory of multiboard estimators of the power spectrum to multiboard estimators of the frequency wavenumber spectrum and of the related correlation sequence. We are now applying these ideas to the derivation of adaptive filters. We have developed a theory of matched subspace detectors for detecting signals which lie in low-dimensional model subspaces. The theory bridges the gap between generalized likelihood ratio theories and invariance theories. We have generalized the theory of maximum likelihood for identifying time domain modes and space domain directions of arrival. We have characterized subband decompositions for perfect reconstruction, developed filter design algorithms for constructing near perfect reconstruction filterbanks from nonorthogonal analysis filters, and derived algorithms for predicting and filtering in periodically correlated time series.
SIGNAL PROCESSING IN THE LINEAR STATISTICAL MODEL

PRINCIPAL INVESTIGATOR: Louis L. Scharf, 303/492-8283
DEPARTMENT OF ELECTRICAL & COMPUTER ENGINEERING
UNIVERSITY OF COLORADO
BOULDER, CO 80309-0425

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R&T Number:

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Principal Investigators: Louis L. Scharf and C. T. Mullis

Mailing Address: Electrical & Computer Engineering, Box 425
University of Colorado
Boulder, CO 80309-0425

Phone Number: (303) 492-8283  E-Mail Address: scharf@prony.colorado.edu, mullis@prony.colorado.edu

a. Number of Papers Submitted to Refereed Journals but not yet published: 10  (list attached)
b. Number of Papers Published in Refereed Journals: 11  (list attached)
c. Number of Books/Chapters Submitted but not yet Published: 0  (list attached)
d. Number of Books or Chapters Published: 2
e. Number of Technical Reports & Non-Refereed Papers: 3
f. Number of Patents Filed: 0
g. Number of Patents Granted: 0
h. Number of Invited Presentations at Workshops or Professional Society Meetings: 3  (list attached)
i. Number of Contributed Presentations at Workshops or Professional Society Meetings: 16  (list attached)
j. Honors/Awards/Prizes for Contract/Grant Employees: 2  (list attached)
k. Total number of Graduate Students and Post-Docs Supported at Least 25% Under This Grant: Graduate Students: 6  Post-Docs: 0

Of these:
# Female Graduate Students: 0  # Female Post-Docs: 0
# Minority Graduate Students: 0  # Minority Post-Docs: 0

Note: Minorities includes African Americans, Hispanics, American Indians, and Aleutians only.

l. (added by PI) Graduate Theses submitted: 4

m. (added by PI) Work in Progress: 6
Overview of Research

In 1991 we submitted a proposal entitled “Matched Subspace Filtering for Detection, Estimation, and Time Series Analysis.” In it we proposed to study four problems:

1. Estimation of the Frequency-Wavenumber Spectrum;
2. Matched Subspace Filters for Matched Field Processing;
3. Maximum Likelihood Identification of Modes from Space-Time Data; and
4. Statistical Inference Within the Wavelet Representation.

With ONR support under contract N00014-89-J-1070, we studied these problems. Let’s review our findings.

1. In references [CSM91] and [CIS92], we generalized the results of [MuS91] to characterize the class of quadratic estimators of the frequency-wavenumber spectrum that are required to be non-negative and invariant to temporal and spatial modulation. These estimators take the form

   \[ \hat{S}(\kappa, \theta; y) = y^* D^*(\kappa, -\theta) V^* V D(\kappa, -\theta) y, \]

   where \( \kappa \) is wavenumber, \( \theta \) is frequency, the vector \( y \) is a lexicographically-ordered version of space-time data, \( D \) is a Kronecker product of modulator matrices for time and space, and \( V \) is a matrix of space-time windows [CSM91], [CIS92]. The mean-squared error for this quadratic estimator is bounded by

   \[ \text{mse}(\kappa, \theta) \geq \frac{1}{M + 1} (S(\kappa, \theta))^2, \]

   where \( M \) is the number of windows used to construct the estimator. This bound for mean-squared error actually obscures the selectivity and the variance of the estimator. Generally, we wish to resolve a cell of frequency-wavenumber space in order to concentrate energy, but the more we concentrate it the higher is the variance of the concentration. So, the problem is to choose a number of windows equal to the time-bandwidth product for the data and the cell to be resolved. The selection of separable space-time windows is discussed at length in [CIS92] and [Cla92]. More generally, the designer of an estimator that resolves energy or power into frequency-wavenumber cells can design a multiplicity of orthogonal windows which have quite general spectral shapes in frequency and wavenumber.

2. In references [ScF94] and [BeS94], we have generalized what was previously known about detection and estimation of rank-one signals to the detection and estimation of rank-\( r \) signals obscured by rank-\( t \) interferences. This work produces linear and quadratic forms in oblique projections. These projections project data onto one subspace along the direction of another. The singular values of the oblique projections determine the performance of the linear and quadratic forms, and these singular values depend on the geometrical angles between the subspaces. With these results, we have been able to gain new insights into the resolution of closely spaced subspaces \( \langle H \rangle \) and \( \langle S \rangle \) and into the detection of signals that lie in a subspace \( \langle H \rangle \) that is near to an interfering subspace \( \langle S \rangle \). As an example of what our results have produced, we list here the signal-to-noise ratio (SNR) in an optimum subspace detector and the gain against noise (G) when resolving two closely spaced subspaces:

   \[ \text{SNR} = \frac{\mu^2}{\sigma^2} \theta^* H^* P_{\theta}^{\dagger} H \theta, \]

   \[ G = \text{tr} [H^* P_{\theta}^{\dagger} H]^{-1}, \]

   \[ = \sum_{i=1}^{M} \frac{1}{\sin^2 \theta_i}, \]
\[ P_S^\perp : \text{projection onto } (S)\perp. \]

These formulas illustrate that the angles \( \theta_i \) between the signal subspace \( \langle H \rangle \) and interference subspace \( (S) \) determine performance. In our work on the geometry of the Cramer-Rao bound [ScM93], we have shown that the variance for resolving the subspace components of a signal is bounded as

\[
V \geq \text{tr}[H^* P_S^\perp H]^{-1} = \sum_{i=1}^{M} \frac{1}{\sin^2 \theta_i},
\]

where now \( H \) and \( S \) are sensitivity matrices that describe how parameter variations influence the measured data. In summary, performance bounds, noise gains, and signal-to-noise ratio gains all depend on

\[
\text{tr}[H^* P_S^\perp H]^{-1} = \sum_{i=1}^{M} \frac{1}{\sin^2 \theta_i},
\]

where \( \theta_i \) are principal angles between subspaces. When these angles are small, parameter estimation variance is high, noise gain is high, and signal-to-noise ratio is low. These findings clarify, for example, the difficulty of solving matched field processing problems when medium parameters produce normal mode subspaces which are nearly collinear. We consider our results on these problems to be essentially complete, except for their extensions to Hilbert spaces and to special subspaces such as subbands of wavelet decompositions.

3. In references [Cla92] and [CIS94], we have derived maximum likelihood estimators for the parameters of two-dimensional damped harmonic signals that are sensed by a two-dimensional array. The results apply to the processing of data in linear arrays that extend over space and take data over time. The likelihood maximization for this problem was actually achieved by minimizing residual error in the orthogonal subspace of the 2-D signals, using a new characterization of this subspace and a computing algorithm that uses a prediction polynomial in one dimension and a Lagrange interpolating polynomial for interpolating between the two dimensions. The algorithm minimizes the quadratic form

\[
\text{tr}[(Y - FS)^* R^{-1} (Y - FS)],
\]

where \( Y \) is the array data, \( \langle F \rangle \) is a candidate 2-D subspace, \( S \) is a candidate set of mode parameters, and \( R^{-1} \) is the broadband covariance matrix. In [CIS94], the authors derive Cramer-Rao bounds for 2-D deterministic modal analysis. We consider this work to be complete.

4. In reference [Spu94], Spurbeck has established the connection between algebraic resolutions of identity and subband decompositions of \( l_2 \). He has constructed oblique projections onto arbitrary subsampled subspaces along nearby subsampled subspaces. These projections generalize the usual projections associated with QMF filters and suggest that one can build oblique projections for resolving nonorthogonal subsampled time series that might arise in multiuser communication. He has also shown what happens to the second-order properties of wide-sense stationary sequences that propagate through subband decompositions which do not have the perfect reconstruction property: they become periodically correlated. In references [SpS92] and [SpS94], the authors have imbedded scalar periodically correlated time series in vector WSS time series and constructed least-squares filters for equalizing and estimating periodically correlated time series. These results illustrate that filterbanks or subband decompositions are very natural structures for filtering periodically correlated time series. We are continuing to develop a toolkit for filtering periodically correlated time series by filtering the corresponding (internal) vector WSS time series.
References


a. Papers submitted to refereed journals (and not yet published):


b. Papers published in refereed journals:


c. Books (and sections thereof) submitted for publication:

None.

d. Books (and sections thereof) published:


e. Technical reports and nonrefereed papers:


f. Patents filed:

None (one in preparation, disclosure filed 12/93).

g. Patents granted:

None.

h. Invited presentations at topical or scientific/technical society conferences:


i. Contributed presentations at topical or scientific/technical society conferences:


j. Honors/awards/prizes:

C. T. Mullis promoted to full Professor effective January 1, 1993.


k. Graduate students and post-doctorals supported under the contract for the three years ending 1 October 1993: (no female/minority)

Knut Aas
Richard T. Behrens
Michael P. Clark
Mark Spurbeck
John K. Thomas
Fred Ziel
1. Graduate Theses Submitted:


m. Work in Progress:


M. Spurbeck. "Wavelet Decompositions of \(L^2\)," to be submitted.


C. T. Mullis, Text on "Advanced Linear Systems," to be submitted.
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