ADVANCED DETECTOR DEVELOPMENT, LABORATORY SIMULATIONS, DIAGNOSTIC DEVELOPMENT, AND DATA ANALYSIS ON WAKE PHYSICS

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**we have developed a theoretical analysis of the current scaling issues of a negatively biased probe in a plasma wake. the theoretical assumptions have been analyzed qualitatively and quantitatively by the use of the POLAR computer code. for a plasma with a constant beam energy, the analysis shows that the I-V collection characteristics are unaffected by Mach number variations for Mach numbers above the value of two. for the same case, the current was found to scale with the $3/7$ power of the density.**
1. INTRODUCTION
The investigation of both theoretical effects and computer simulation models of the collection of ion current by a Langmuir probe in the plasma wake of a shadowing body is the focus of this report. Results of such experimentation have been used in flight predictions [Cooke et al., 1994] of the WSF-CHAWS STS-60 spaceflight mission. It has been found that the collection of ion-current in a plasma wake is a space-charge-limited process with a heavy dependence upon an angular-momentum mechanism. Previous attempts [Enloe et al., 1993; Cooke et al., 1994] have been made to model effects of scaling plasma parameters and pose theoretical models to assess the variations of current collection with the variations of plasma parameters. Although expected by orbit-limited models of ion collection in the plasma wake, the magnitude of the collected current is not a linear function of plasma temperature (T), plasma density (N), nor ion Mach number (M).

2. THEORETICAL MODELS
Given that the angular momentum of the particles is a dominant characteristic of particle collection in the wake [Enloe et al., 1993], we must speculate on the impact of the scaling of particular plasma parameters for both orbit-limited and space-charge-limited analytical approaches. For the scope of our study, we look at the first order impact of the most dominant plasma parameters—plasma density, temperature, Mach number, and atomic mass number. In any plasma environment, we know that the ion thermal velocity is proportional to the square root of the plasma temperature to mass (AMU) ratio, more conveniently written

\[ v_{th} = (kT / m_i)^{1/2} \times (T / AMU)^{1/2} \quad (2.1) \]

For a moving object in a stationary plasma or a stationary object in streaming plasma, we know that the relative plasma stream velocity \( \nu_0 \), ram current density \( J_{ram} \), and the plasma beam energy \( E_0 \) are related as

\[ \nu_0 = M \nu_{th} \times M (T / AMU)^{1/2} \quad (2.2) \]
\[ J_{ram} = e N M \nu_{th} \times N M (T / AMU)^{1/2} \quad (2.3) \]
\[ E_0 = (1/2) m_i \nu_0^2 \times M^2 T \quad (2.4) \]

where \( M \) is the mach number of the streaming plasma relative to the object immersed in the plasma. From orbit-limited theory, we would expect that an increase in the ram current density by a factor of two would be followed by an increase in current collection by a factor of two. From space-charge-limited theory for a body, such as the WSF-CHAWS experiment, where the...
dimensionless characteristic radius \((r = R/\lambda_D)\) is much greater than unity, the current collection of a
probe in a plasma is limited by the Debye shielding of the plasma sheath set up by the probe
potential [Langmuir and Blodgett, 1924]. Otherwise, we can assume that the current collection in
the wake of the WSF-CHAWS experiment is orbited limited when the dimensionless characteristic
radius is less than unity. Such a case can occur when the density is at a critical minimum, thus
making the influence of the Debye length more appreciable to the characteristic radius \((R)\) of the
experiment. In the case of the WSF-CHAWS experiment, a ram density on the order of \(10^4\)
becomes highly unlikely for a body in LEO. Therefore, we can assume that a simple scaling of
the ram flux density and the associated collected wake current, as in accordance with the previous
orbit-limited assumption, once again is inaccurate.

2.1. Mach Number and Plasma Temperature Variations
Reverting to a more basic approach, we must first concentrate on known relations. In congruence
with the WSF-CHAWS spaceflight operation, we know that the orbital speed of the shuttle and
WSF is nearly constant. This constant velocity, for any given species of plasma, denotes that the
streaming velocity (beam energy) of the plasma also will be a constant, relating \(T\) and \(M\)
appropriately by Eqn 2.4

\[
M^2 = \frac{1}{T}
\]  
(2.5)

To model the physical effects of the variations of these parameters, we look at the ram flux from
the origin of the plasma wake disturbance at the edge of the WSF-CHAWS shadowing body.
Since the WSF-CHAWS experiment is traveling in the \(-Z\) direction at a constant velocity \(v_{ZO}\)
and a transverse ion expansion into the wake with a mean velocity of \(v_{XO}\), we have the corresponding
distribution function

\[
f \propto N \left( \frac{\pi v_{th}^2}{2} \right)^{-1/2} e^{-\left[\left( v_{x} + v_{z} + v_{w} \right)^2 + \left( v_{x} - v_{z} \right)^2 \right]/2v_{th}^2}.
\]  
(2.6)

Integrating this function with respect to each velocity component

\[
F = N \left( \frac{\pi v_{th}^2}{2} \right)^{-1/2} \left[ \frac{1}{2\pi} \right] \left[ \frac{1}{\sqrt{2\pi}} \right] 
\nonumber
\int e^{-\left( v_{x} + v_{z} + v_{w} \right)^2/2v_{th}^2} \left[ \int e^{-\left( v_{x} - v_{z} \right)^2/2v_{th}^2} dv_{z} \right] dv_{x}.
\]  
(2.7)

yields

\[
F \propto N \frac{v_{th}}{2\sqrt{\pi}} \left[ \frac{v_{Z0}}{v_{th}} \right] \left[ 1 + \text{erf} \left( \frac{v_{Z0}}{v_{th}} \right) \right] e^{-v_{x}^2/2v_{th}^2}.
\]  
(2.8)

which agrees with Parker [1980]. Analysis of this result provides important information about the
effect of the plasma temperature (Mach number) on the ram current density. For Mach numbers
above the value of two in Eqn. 2.8, the error function saturates to unity and the negative exponential saturates to null, indicating that the flux distribution is unaffected by Mach numbers above the value of two. In comparison to oxygen ions in LEO, the WSF-CHAWS experiment operates in a Mach number range above the value of seven. Therefore, the collection of current due to oxygen ions on a probe in the WSF-CHAWS wake is invariant to perturbations in the plasma temperature. For hydrogen ions in the same environment, the WSF-CHAWS experiment operates in a Mach number range near the value of two, indicating that the rare density characteristics may be dependent upon the plasma temperature.

2.2. Plasma Density Variations

Although variations in the plasma temperature (Mach number) have little effect on the I-V characteristics of the Langmuir probe in the WSF-CHAWS wake, variations of the ambient ion density of a LEO environment exist. As Eqn. 2.8 shows, the flux density is directly proportional to the density of the plasma. As stated before by the orbit-limited theory, we can assume that the current collection in the wake will be proportional to this density, but first we must look at the effect of the density on the space-charge-limited collection of a Langmuir probe according to Langmuir and Blodgett [1924]. From the well-known Child-Langmuir law of space-charge-limited current between charged concentric spheres, we have

\[ J = \frac{4 \sqrt{2e \varepsilon_0 V^{3/2}}}{9 \sqrt{M D^2}}. \]  

(2.9)

where \( \varepsilon_0 \) is the permittivity of free space, \( V \) is the potential difference between the spheres, and \( D \) is function of the radii of the spheres. Using the three dimensional flux density to a surface (let \( J_0 = eN[kT/2\pi M]^{1/2} \)), Debye length, and dimensionless potential, Eqn. 2.9 can be written

\[ D = \left( \frac{4 \sqrt{2e \varepsilon_0 \Phi kT^{3/2}}}{9 \sqrt{M e J_0}} \right)^{1/2} = (12)\lambda_0 \Phi^{1/4}. \]  

(2.10)

From the Parker [1980] tabulation of Table II of Langmuir and Blodgett [1924].

\[ \frac{R_s}{R_s} = \left( \frac{D}{R_0} \right)^{1/4} \approx \left( \frac{D}{R_0} \right)^{1/4}. \]  

(2.11)

where \( R_0 \) is the characteristic radius of the Langmuir probe and \( R_s \) is the sheath radius. Incorporating Eqn. 2.10 into Eqn. 2.11, yields
Using Eqn. 2.12 as the radius of the spherical sheath,

\[ I = 4\pi I_0 R_s^2 = 4\pi I_0 \left[ R_o \left( \frac{\lambda_D}{R_o} \right)^{4/7} \Phi^{3/7} \right]^2 \] \quad (2.13)

From the definition of the ion Debye length and the proportion of Eqn. 2.1, the current to the Langmuir probe is proportionally related

\[ I = 4\pi I_0 R_s^2 \propto NT^{1/2} \left[ \left( \frac{\sqrt{T}}{\sqrt{N}} \right)^{4/7} \left( \frac{1}{T} \right)^{1/7} \right]^2 \propto N^{3/7} T^{3/14}. \] \quad (2.4.14)

Therefore, we see that for a non-flowing plasma, the space-charge-limited current varies with the \(3/7\) ths power of the variation of the plasma density. The \(3/14\) ths power variation associated with the plasma temperature is considered ineffective via the derivation of Eqn. 2.8. To apply a similar reasoning to the plasma stream that evolves about the edges of the shadowing body proves to be rather difficult. We cannot assume a spherical sheath as in Eqn. 2.13, nor is the plasma density in the wake uniformly distributed about a spherical probe. What is important is the power relation of the density to the Langmuir probe current. On the other hand, if we assume that the collection potential of a Langmuir probe in a plasma wake is sufficiently high enough \((V >> kT/e)\) above some collection threshold, we can (for the moment) disregard the angular momentum properties of ion trajectories in the sheath and use the Langmuir-Blodgett relation for the current scaling with density. Such a situation is physically unrealizable because the sheath edge is not well defined and the initial velocities of the ion trajectories into the sheath are highly dependent upon the angular momentum of the ion particle. From this, we can conclude that the current collection mechanism in the plasma wake is neither orbit-limited or space-charge-limited. Rather, it is in a quasi-space-charge-orbit-limited regime where the current scaling via the density can be described mostly by the relation of Eqn. 2.14 in the space charge sense, but the particles also exhibit some behavior relevant to angular momentum concepts.
3. EXPERIMENTAL RESULTS

We have used the POLAR [Lilley et. al., 1985] computer simulation to model the chamber experimentation [Enloe et. al., 1993] and the WSF-CHAWS STS-60 mission [Cooke et. al., 1993] with good results. The POLAR code was able to model the chamber experiment as shown in Figure 1. The in situ results [Enloe et. al., 1993] and POLAR simulations are shown respectively in Figures 2 and 3. The identification of the "sweet spot" of orbiting ions and similarities in the magnitude of current collection provided a reasonable validation. A similar geometry, shown in Figure 4, was used to model the parametric variations more closely by removing the effects of the ion circulation about a Langmuir probe with quasi-radial symmetry. The small effects of Mach number and plasma temperature variations are shown in Figure 5. The effects of plasma density variations and the Langmuir-Blodgett scaling assumption is shown in Figure 6. Similar relations of the plasma density variations are also present in the Cooke et. al. [1994] data.

4. CONCLUSIONS

From the theoretical assumptions and predictions of the POLAR simulation models, we see that variations in the Mach numbers and plasma temperature have little effect on the ion current characteristics of collection when the Mach number is greater than two. It was found that the collection of ions to a Langmuir probe in the plasma wake of a shadowing body is a space charge limited process that is heavily dependent on the angular momentum of the particles. The collected ion current is expected to scale with variations in the plasma density to the 3/7 ths power.
References


Figure 1. POLAR Chamber Simulation Shadowing Body and Langmuir Probe Configurations: a.) side-view (x-z) of POLAR object, b.) three dimensional view.

Figure 2. Current collected as a function of negative potential on the Langmuir probe for varying axial positions. The change in the shape of the I-V curve for z = 3 cm is due to the effect of the ions orbiting the sphere [Enloe et. al., 1993]
Figure 3. Current collected as a function of negative potential on the Langmuir probe for varying axial positions from the POLAR simulation.

Figure 4. POLAR simulation shadowing body and Langmuir probe configurations used to limit effects of ion circulation.  a) side-view (x-z) of POLAR object, b) three dimensional view.
Figure 5. I-V collection characteristics as a function of plasma temperature and related Mach number.

Figure 6. I-V collection characteristics as a function of density. The single element Langmuir probe was replaced by an elongated probe extending from the shadowing body to prevent trajectory orbits.