TNO Physics and Electronics Laboratory

TNO Defence Research

TD 93-2438 III

Fax +31 70 328 09 61
Phone +31 70 326 42 21

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Assessment of benefits and drawbacks of using fuzzy logic, especially in fire control systems

author(s):

N.M. de Reus

TDCK RAPPORTENCENTRALE
Frederikkazerne, gebouw 140
v/d Burchlaan 31 MPC 16A
TEL. : 070-3166394/6395
FAX. : (31) 070-3166202
Postbus 90701
2509 LS Den Haag

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MANAGEMENTUITTREKSEL

Titel	:	Beoordeling van voor- en nadelen van 'fuzzy logic', in het bijzonder in vuurleidingssystemen
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Fuzzy logic trekt momenteel de wereldwijde aandacht van onderzoekers. Het idee stamt uit 1965; toen introduceerde Professor Zadeh de "vage verzamelingen theorie" (fuzzy set theory).
Tegenwoordig zien we een steeds toenemend aantal toepassingen, de meeste afkomstig van Japanse bedrijven. In de USA en (nog sterker) in Europa heeft men daarom steeds meer het idee dat men fuzzy logic ook meer moet gaan toepassen om de boot niet te missen.
De aanleiding voor deze studie, uitgevoerd door TNO-FEL in opdracht van de KM, is geweest dat in een voorstel tot verbetering van het vuurleidingsproces van het Goalkeeper kanonsysteem het gebruik van "fuzzy reasoning" technieken voorkwam. Dit voorstel is gedaan door SIGNAAL, in een opdracht van de KM. Het TNO-FEL had een evaluerende rol in deze predictieverbeteringsstudie.
Dit onderzoek had tot doel om de mogelijkheden van fuzzy logic in het algemeen te bekijken, en de toepassing binnen vuurleidingssystemen in het bijzonder. Het is uitgevoerd in de vorm van een literatuurstudie.
De nadruk is gelegd op voor- en nadelen van het gebruik van fuzzy logic in vergelijking met 'conventionele' methoden. Geconcludeerd wordt dat er in principe goede mogelijkheden voor toepassing bestaan (met name voor rapid prototyping) maar dat de vele voorbeelden van projecten die een succes werden dankzij fuzzy logic, sceptisch moeten worden bekeken.
Aanbevolen wordt om in vervolgstudies daadwerkelijk problemen op te lossen met fuzzy logic, om zo de vergelijking met conventionele methoden mogelijk te maken. Gedacht wordt hierbij bijvoorbeeld aan IFF binnen het Nato Identification System (NIS), aan plot extractie binnen het ARTIST programma (Advanced Radar Techniques for Improving Surveillance and Tracking) en aan automatisering van de instelling van camera's met helderheidsversterker.

CONTENTS

1 INTRODUCTION

Currently, fuzzy logic has drawn the attention of researchers all over the world. The idea goes back to 1965, the year in which Professor Zadeh introduced the "fuzzy set theory". Today, an increasing number of applications of fuzzy logic is encountered, most of these coming from Japanese companies. So in the US and even more in Europe the current idea is that we may miss the connection if we will not explore and exploit fuzzy logic as soon as possible.

A very wide range of applications of *fuzzy logic* or related terms like *fuzzy sets, fuzzy reasoning* and *fuzzy systems* is covered in the literature. Examples range from train control to vacuum cleaners. Even in defence applications that may not seem fuzzy or vague at first sight, examples can be found. One of these examples is the fire control system of the Goalkeeper Close In Weapon System, which is an air defence gun system defending a ship against anti-ship-missiles like the well known Exocet. In a study about possible prediction process improvements, the use of fuzzy reasoning techniques has been proposed. This prediction process study has initiated this study for fuzzy logic applications in fire control systems. The study is performed for the Ministry of Defence by TNO-FEL.

In the past few years an enormous amount of popular literature dealing with introduction to fuzzy logic has been published, some of which are referred to in the next section. So it seems a waste of effort to make this report just another introduction to fuzzy logic. This is the reason why a somewhat different approach has been taken. After a short introduction to the concept 'fuzzy' in section 2 and fuzzy systems in section 3, fuzzy logic is introduced as a function approximation and realisation technique. Section 4 discusses several examples of fuzzy systems and section 5 deals with the application of fuzzy systems in the air defense fire control problem.
The discussion in this report was based on a literature study.

2 THE FUZZY CONCEPT

This section deals with the concept of fuzziness. It will not go into very much detail; lots of articles are available for this purpose. The aim of this section is to give a concise introduction, detailed enough to enable the reader to grasp the ideas of subsequent sections. In ref. [1-6] a simple practical introduction can be found, ref. [7] gives an overview of approximate reasoning techniques, also discussing fuzzy logic and ref. [8-9] give a more profound, theoretically more sound discussion. Ref. [10] introduces fuzzy sets from a geometrical view as points in a (hyper)space, and elaborates on the relation with neural networks. For those interested in the latest developments in fuzzy logic, the "International Journal on Fuzzy Sets and Systems" is recommended. It handles profound theoretical aspects as well as applications of fuzzy logic.

Contrary to the idea presented in many articles on fuzziness, it is my opinion that nature is not fuzzy. Only people handle it in an imprecise, ambiguous or fuzzy way. In other words, knowledge that people have about natural phenomena tends to be vague or imprecise to a certain degree. In [11] a clear survey of fuzziness, ambiguity and imprecision is given. The author distinguishes the *fuzzy concept*, the *ambiguous* or *intentionally fuzzy concept* and *imprecision*.

The *fuzzy concept* is a concept which is well defined, but where objects exist which fulfil this concept only to a certain degree. Consider for instance the concept 'unemployed'. What to think of a person who only works for 10 hours a week compared to the full 40 hours. Is that person unemployed ? A reasonable answer is to call such a person partially unemployed, or unemployed to a certain degree (say 0.75 on a scale from 0 to 1).

An *ambiguous or intentionally fuzzy concept* is a concept where there is disagreement over the properties which constitute this concept. This kind of fuzziness appears when (slightly) different concepts are designated with the same name.

A concept is called *imprecise* if it does not totally match with the described phenomenon. For example if it takes you 6 hours and five minutes to travel from Amsterdam to Paris but you say 6 hours then you are imprecise. Even in more exact, technical situations imprecision is a normal phenomenon. A bolt, for instance, will have a length which is given with a certain tolerance.

In fact, when people (using a natural language) deal with continuous quantities, they almost always encounter inexactness; we can consider this fuzziness. Only when people deal with abstract things, for instance continuous mathematics, absolute precision is possible. Other areas where this is possible, at least in principle, are where discrete numbers are concerned. This appears in counting problems. However, the result of counting may also be imprecise. Consider

for example the problem of determining the number of sand grains in a heap of sand. This will be practically impossible without errors.
Categorisation or classification is a process that inherently uses fuzziness. The problem of determining to which category a particular object belongs typically gives rise to inexactness. Consider for instance the following examples: (1) determining if a bottle is empty or full, given its contents, (2) determining if a person is old or young, given his age, (3) determining if a person is short or tall, given his length, (4) determining if an object is expensive or cheap, given its price.
These are all classification problems; for a given value of the describing quantity (the contents, age, length and price in the above examples) it is to be determined to which category or class the 'objects' belong. Even in the example of the bolt length, where the imprecision stems from inaccuracy rather than inherent vagueness, one can speak of a classification problem.

In conventional thinking, classification of an object is done using sharp boundaries. An example is a bottle which may be considered empty when it is for less than 50% filled and full when it filled for 50% or more. When for a certain problem this separation into two classes is not sufficient, usually more classes are defined.
Classifying the degree to which the bottle is filled in this manner does not seem very realistic. When one bottle is filled 49.9% and another 50.1% their degree to which they are filled are pretty much the same and classifying the one in the category empty and the other in the category full seems nonsense.
The idea of Lotfi Zadeh now was to have more levels of truth compared with binary logic which uses two levels, ref.[8]. In binary logic a statement can be false or true with corresponding values 0 and 1. Zadeh proposed to use infinitely many values between 0 and 1 for truth values. In this way the degree to which a statement can be true can for instance be 0.9, 0.5 or 0.2.

Consider a variable like 'age', which can have values in the set [0,120], also called *domain* or *universe of discourse*. Categorizing ages from this set into 'young' and 'old' in a conventional way for this variable amounts to defining subsets of [0,120] such that for each element of [0,120] it is clear whether it is in the subset 'old' or 'young'.
These subsets can be represented by so-called *indicator functions*, i.e. functions with function value 1 for elements that are in the subset and 0 for elements that are not, see also appendix A. For the variable 'age', the subsets 'young' and 'old' could for instance be defined by the following functions:

'young': I(x)=1 for x in [0,40)
I(x)=0 for x in [40,120]

'old': I(x)=0 for x in [0,40)
I(x)=1 for x in [40,120]

Such indicator functions have sharp boundaries, or in other words *crisp* boundaries. The subsets obtained this way are therefore often called '*crisp sets*'. Also values from the domain ([0,120] in the age example) are therefore called '*crisp values*'. From now on, in the text, the words 'real' and 'crisp' will be used interchangeably.

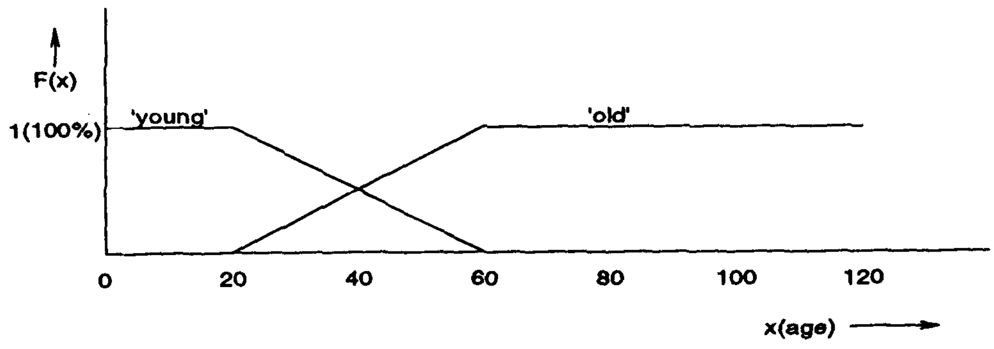
Now that crisp sets are defined, it is not very difficult to make the step to a fuzzy approach. Let's again take the age example. Someone who is 39 years old would more realistically be called a little younger than someone of age 40. So when the degree to which a statement like 'John is old' is true is taken into account, for every real (crisp) value of 'age' it can at least in principle be defined to which degree this value is 'old' or 'young'. A possible example of the functions 'young' and 'old' may for instance be as follows:

'young': F(x)=1 for x in [0,20)
F(x)=(60-x)/40 for x in [20,60)
F(x)=0 for x in [60,120]

'old': F(x)=0 for x in [0,20)
F(x)=(x-20)/40 for x in [20,60)
F(x)=1 for x in [60,120]

These functions are called *fuzzy sets,* they represent the sets 'old' and 'young'. They are also called *membership functions*. For every crisp element from the domain, the function value represents the degree to which the crisp element belongs to, or is a member of, the fuzzy set.

In the figure below these functions are displayed.



It must be stated that the actual functions are a matter of expert opinion and depend on the application. The influence of expert opinion is clarified by the following example. Children in elementary school were once asked to define 'young' and 'old' in terms of fuzzy sets. The answers that were given are summarized in the functions below:

'young':	$F(x)=1$	for x in [0,8)
	$F(x)=(12-x)/4$	for x in [8,12)
	$F(x)=0$	for x in [12,120]
'old':	$F(x)=0$	for x in [0,8)
	$F(x)=(x-8)/4$	for x in [8,12)
	$F(x)=1$	for x in [12,120]

Also it must be stressed that, although in the example, 'young' is the negation of 'old', this is not a necessary condition. The functions could easily be defined in a different way. For instance the term middle-aged could be introduced; this fuzzy set could for instance have nonzero values in the region near the age of 40 while 'young' could then be zero for ages larger than 30 and 'old' for instance for ages smaller than 50.

Summarizing:

- Variables like: age, contents, price, length can have crisp (real) values but also can have linguistic values like 'young', 'old', 'tall', 'cheap'. When linguistic values are assigned to the variables, these are called *linguistic variables*.
- The linguistic variables can be linked with their real counterparts by the use of fuzzy sets. So for every crisp value it can be determined to which degree the linguistic variable is associated with a linguistic value or in other words, fuzzy set.
- Fuzzy sets are functions on a domain (universe of discourse) of crisp variables.

3 FUZZY SYSTEMS

Fuzzy systems are systems that transform (or map) fuzzy sets to fuzzy sets. Actually, the basic feature of fuzzy systems is the fact that fuzzy reasoning techniques are used. In this section, first the logic behind fuzzy reasoning is discussed, then a categorisation of fuzzy sets is given in terms of the kind of input/output signals. This may sound strange because, strictly speaking, fuzzy sets are input as well as output of fuzzy systems. In practice one often encounters systems with crisp input and/or output that still are called a fuzzy systems.

3.1 Fuzzy Logic

Logic deals with truth of statements and how truth values of statements can be derived from the truth values of other statements. The truth values that statements may have, depend on the particular kind of logic used. In this section, *fuzzy logic* is derived from the so-called *multivalued logic*, which in its turn is derived from *binary logic*.

In *binary (two-valued) logic*, propositions (statements) can be either true or false. Truth tables of the operators AND, OR, NEGATION and IMPLICATION are given in appendix A. The result of these operators is called the *consequent* and the values on which the operators are applied are called the *antecedents*. Now the contents of the tables can be given as a function of the values of the antecedents. This is done in the appendix as well.

In *multivalued logic*, operators must be defined for the single-input / single-output operator NEGATION and for the multiple-input / single-output operators like AND, OR and IMPLICATION. This is done by extending the domain of the binary logic functions. The binary functions are defined on the domain {0,1}*{0,1} for the multiple input case and {0,1} in the single input case (the * denotes Cartesian product, the definition of which can be found in appendix B). In multivalued logic these domains are extended to [0,1]*[0,1] and [0,1] respectively while the function descriptions remain unchanged. So in the limit case when the inputs are zeros and ones, the same results as with binary logic are obtained. Multivalued logic derived in this way from binary logic is called a generalisation of binary logic.

Fuzzy logic can be viewed as an extension of multivalued logic. In fuzzy logic, statements can not only have multivalued truth values, so values from the domain [0,1] but the values are allowed to range over all possible fuzzy subsets of the universe of discourse. In other words, in fuzzy logic, functions from the universe of discourse to [0,1] form the possible values of the antecedents and the consequents.

Of special meaning are the so-called Fuzzy Assosiative Memories or FAMs (ref. [10]). These are continuous fuzzy systems that map fuzzy sets to fuzzy sets. Continuity must here be regarded in the strict mathematical sense. So a continuous fuzzy system maps balls of fuzzy sets in the domain space to balls of fuzzy sets in the image space. In other words they map close inputs to close outputs. The simplest FAM encodes the FAM rule or association: [*if* (x is A) *then* (y is B)] which is denoted (A,B) where A and B are fuzzy sets. A FAM system consists of a parallel bank of FAM rules : $(A_1,B_1),\ldots,(A_n,B_n)$. Each input fuzzy set A to such a FAM system activates each stored FAM rule to a different degree. The rule (A_i,B_i) results in B_i' when activated by A. The more A resembles A_i, the more B_i' resembles B_i. The consequent fuzzy sets B_i' $(i=1,\ldots,n)$ are combined (usually using a weighted average) to form B, the consequent fuzzy set of the bank of rules. Determining the consequent from antecedents is called logical inference.

In fuzzy literature, several inference mechanisms are encountered. The problem with all these different possibilities is that newcomers to the fuzzy field are easily confused when reading several articles on fuzzy systems because most articles present a particular solution without mentioning different methods.

Therefore in appendix A, three inference methods that one can encounter are discussed. I have called them *minmax*, *minsum* and *prodsum*.

One view of this situation is that one may conclude that multivalued logic is such a 'rich' theory in which many solutions exist for one particular problem. A more realistic view is to say that the designer of a fuzzy system is not supported enough by theory in making design decisions.

Fuzzy systems are mappings from the family of fuzzy sets to the family of fuzzy sets. In other words, they are operators transforming fuzzy sets to fuzzy sets. The fuzzy sets may be one-dimensional as well as multidimensional.

In practice, often the input of the fuzzy system as well as the output are crisp numbers. But a fuzzy system requires a fuzzy set as input and generates a fuzzy set as output. Therefore schemes like the one displayed are normally used.

Viewing a crisp number as a fuzzy system actually is *fuzzification*. In this way, even crisp inputs can be viewed as fuzzy sets with a membership value 1 for the crisp value and all zeros for the other values in the universe of discourse. Compare this concept to the Dirac function in function theory. Fuzzy systems with crisp input as well as crisp output are called Binary Input Output FAM (or BIOFAM) systems, (ref. [10]).

Transforming fuzzy sets to crisp numbers is called *defuzzification*. Several methods exist of which two main ones are the centroidal defuzzification and the maximum membership defuzzification scheme. Both are a kind of averaging method, described in appendix A.

3.2 Fuzzy reasoning as a function approximation and realisation technique

Approximation and realisation of functions are the key items of this section. Therefore the ideas are defined here. For the formal definition of a function, one is referred to appendix B.

Approximation:

Function approximation is the problem of constructing a function which resembles a given function.

A function $f: N \rightarrow M$ is said to approximate a given function g if both f and g have the same domain and if $|f(x)-g(x)|<\varepsilon$ for some given (small) value of ε and every x from N where $|.|$ is a norm defined on M. More loosely defined, f approximates g if $|f(x)-g(x)|$ is small for every element x in the domain.

Realisation:

Function or mapping realisation is the problem of constructing an image given a domain element of that mapping or function. Possibilities are: formulas, lookup tables or algorithms.

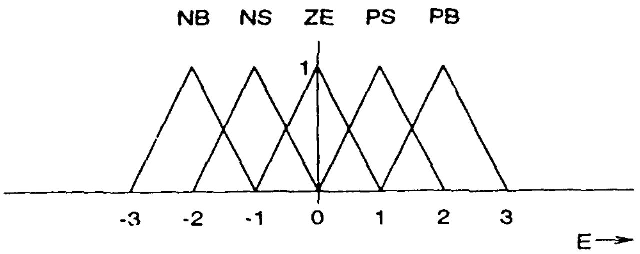
In ref. [12] it is proven that any real continuous function can be approximated with linear combinations of fuzzy basis functions.

The following example shows the function approximation strength of fuzzy sets. It touches on the fact that there are several methods for doing fuzzy reasoning;

Imagine that an expert composes the following rules:

if x is NB *then* y is NB
if x is NS *then* y is NS
if x is ZE *then* y is ZE
if x is PS *then* y is PS
if x is NB *then* y is PB

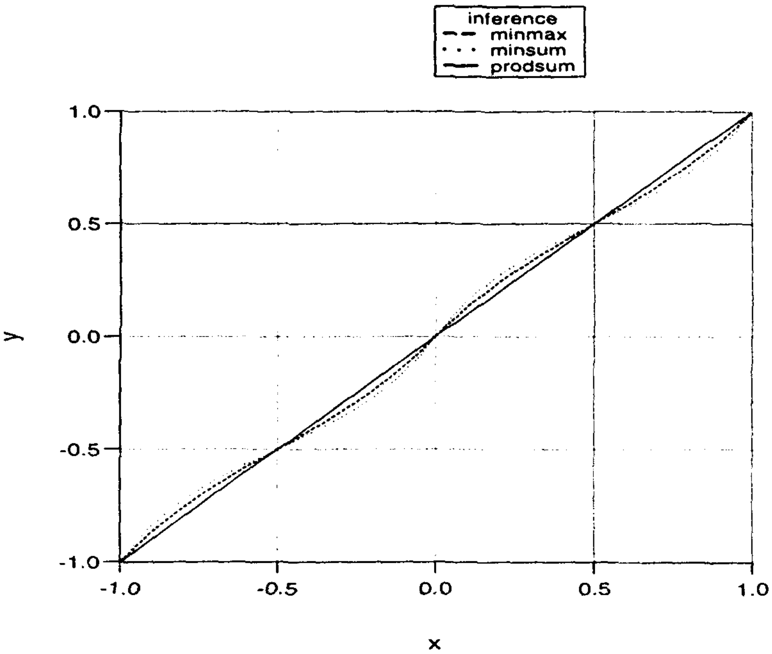
Where NB=Negative Big, NS=Negative Small, ZE=Zero, PS=Positive Small and PB=Positive Big are fuzzy sets defined as follows:



Mind the resemblance with a different description of the input/output behaviour obtained from the same expert. This description is given by the following table:

x	y
-2	-2
-1	-1
0	0
1	1
2	2

Using fuzzy inference, for every crisp value of x, a crisp value for y can be constructed from the rules given above. Three inference methods frequently encountered in literature are what I call the minmax, minsum and prodsum method. These are described in appendix A. Applying these methods results in the figure, shown below:



As one can see, the results differ for the different inference methods. The most likely function that one can think of when looking at the given rules is y=x. So a conclusion must be that in this specific case, the prodsum method outperformed the other methods when approximation quality is taken into account.

A more general conclusion is that the choice for a certain inference scheme is still an area in the theory of fuzzy sets on which little theory is available. The choice still depends on the particular situation or on the preference of the fuzzy engineer.

3.3 Characterisation of Fuzzy systems

When fuzzy systems are encountered, the application areas and typical characteristics may be quite different. The concept of fuzziness can for instance be used in a control system. In this case

the input and output of the controller will mostly be crisp, while the fuzziness is implicit in the controller. When fuzziness is for instance used in a decision model for operators of a target detection system there may be crisp as well as fuzzy inputs or outputs.
When the input and/or output of a system using fuzzy logic is crisp, then fuzzification is to be applied to the input and defuzzification to the output (see appendix). The resulting systems may then have crisp input and/or output. Although, strictly speaking, fuzzy systems have fuzzy sets as inputs and outputs, more loosely defined also crisp input or output systems will be generally called fuzzy systems in this paper.
One can come to the following main categories:

CC:	Crisp input / Crisp output
CF:	Crisp input / Fuzzy output
FC:	Fuzzy input / Crisp output
FF:	Fuzzy input / Fuzzy output

Of course, combinations of these systems are possible too, but the possibilities above are considered the main ones.

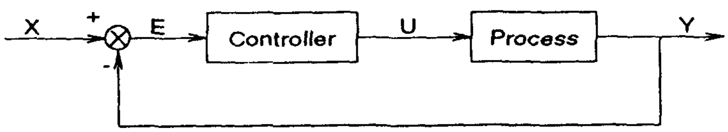
4 PRACTICAL USE OF FUZZY SYSTEMS

4.1 Typical areas of use

In this section, examples are given, clarifying typical areas of use for the categories that were given in the former section.

4.1.1 CC fuzzy systems

This input/output structure is the most widely used. It has the advantage that it behaves externally like a 'conventional' i.e. nonfuzzy system such that it can easily be implemented in technical systems without requiring an interface to an external fuzzy system, like CF or FC. In other words, in most cases, an interface to a human being is not necessary. Of course, internally such a system uses fuzzy logic. Examples can be found in control systems.
Consider for instance a single-input / single-output control system like the one displayed in the figure below.



The control system takes the (crisp) error E as input and determines the (crisp) control value U as output. The most simple continuous controller that one can think of is the so-called proportional controller, which is simply the formula: $U=K\cdot E$. The idea behind this control law is that for large errors, a large control value must be applied and for small errors, small control values.
One could imagine that the above control idea could also be stated in fuzzy rules like the ones given in the former section.

CC fuzzy systems are also frequently used in *engineering design problems*. In these cases, decision rules are used that adapt system parameters as a function of the situation. Suppose for instance, that in an engineering design process system parameters have to be chosen for optimisation. Conventionally this is done such that a compromise is reached. But in adaptive systems such a choice can be made dependent on the particular situation.

In conventional design, a designer will choose a set of parameters for a (usually relatively small) number of situations. In practice the boundaries between these situations may not be so sharp, but instead may be fuzzy. Then the parameter choice could be done based on experience (fuzzy) rules, given by the designer.
An example may be the active suspension that is used in some higher class cars. Depending on for instance road condition, the characteristics of the suspension can be chosen based on some 'comfortability' rules.
A possible rule might be:

if (measured_road_noise_power is high *and* road_trajectory is straight) *then* (suspension_damping is low)
With the fuzzy sets: measured_road_noise_power, road_trajectory and suspension_damping. These fuzzy sets can for instance be defined on the universes of discourse: Power [Watt], Acceleration [kg.m/s^2] and Damping constant (dimensionless).

It is stressed that many systems that use fuzzy logic for adaptation when compared to conventional systems are labelled new because of the fact that they use fuzzy logic. In many cases this is just an advertisement trick. Mostly, the key novelty of such systems is the fact that adaptation is used while fuzzy logic is just a way of implementing the adaptiveness! For instance when washing machines are designed to have sensors to measure for instance the dirt, temperature or type of clothing and this data is used in an adaptation scheme then the actual feedback is new compared to conventional washing machines and not (only) the fact that fuzzy is used.

4.1.2 CF and FC fuzzy systems

Examples of fuzzy systems in which only the input or output are fuzzy are:

- A loan analysis system, where a possible input is the profit of a company (crisp) and a possible output is a statement about the risk of granting a loan to that company.
- A washing machine in which a possible input is the a linguistic statement about how dirty the laundry is (very dirty, oil like stains, vaguely dirty, etc.) and a possible output is the required water temperature, sent to the heating device.

4.1.3 FF fuzzy systems

An example of an FF fuzzy system is a sports advisory software program. Possible fuzzy inputs are the degree to which one becomes tired doing ordinary day-to-day tasks and the susceptibility to heart attacks and possible output may be an fuzzy advise in terms of for instance the level of sports advised (very light, light, moderate, heavy).

Often FF systems are interfaced with CF or FC systems.

As a rule of thumb one can say that the more human involvement in a system, the more likely it is that the fuzzy concept will be used.

4.2 Examples of fuzzy systems

When a human being performs a decision task, he actually is performing mapping realisation. That is, for given elements from the domain (input), he generates images (output), or in other words for a given situation he generates decisions. He may do the generation with the help of tables, computers or even back of the envelope calculations. In those cases the problem is often well defined and structured. There are however numerous cases where the decision problem is not defined in explicit mathematical terms.

Control systems are an area which perfectly lend themselves for the illustration of the use of fuzzy sets. Especially in control systems where human operators are involved, fuzzy sets often are applicable.

In this section, two example applications are given. These will all be given by stating (a few) rules that could be used to define a fuzzy input/output relation.

The first example is the problem of steering a ship. This is a typical naval control system application where a human operator outperforms automated control systems (autopilots) in difficult or dangerous situations. The example is completely taken from ref.[14].

The second one is the target tracking problem. This actually is also a control problem but not one where normally human operators are involved. For this problem two fuzzy approaches are given; a parameter tuning fuzzy approach and a pure fuzzy approach. The last one is compared to a Kalman filter. The last approach was again taken from literature, ref.[10].

4.2.1 Ship steering

Suppose that in a simplified situation the helmsman is performing the task of steering the ship to a required heading or keeping a required heading. He will constantly be looking at the turning rate $d\psi/dt$ and the error ε between the current heading ψ and the commanded heading ψ_c, applying a rudder angle δ in order to obtain the required heading. Actually, he constantly determines (decides) which rudder angle is required. In other words, he is performing function realisation.

Two different operating modes for the fuzzy controller are distinguished because of their different control character. These are the course-changing procedure with corresponding far-away control mode and the course-keeping procedure with the corresponding close-by control mode.

Far-away control:

For this mode, the following FAM matrix was assembled:

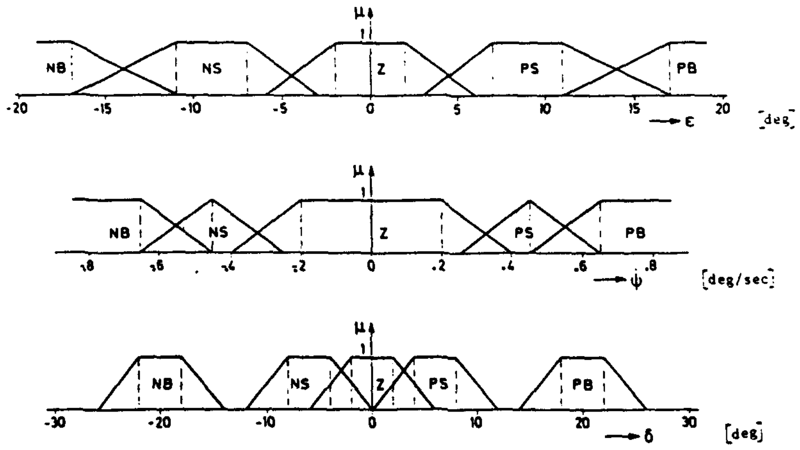
δ	ε :	NB	NS	Z	PS	PB
dψ/dt :						
PB		NB	NB	NB	Z	PB
PS		NB	NB	NS	PS	PB
Z		NB	NS	*	PS	PB
NS		NB	NS	PS	PB	PB
NB		NB	Z	PB	PB	PB

* = close by control

The following fuzzy sets are used in the table above:

NB Negative Big
NS Negative Small
Z Approximately Zero
PS Positive Small
PB Positive Big

These fuzzy sets are displayed below (taken from ref.[14]).



By choosing an inference scheme (including fuzzification) and a defuzzification scheme the value δ can be determined for every combination of values for ε and $d\psi/dt$. So actually, the function $\delta = f(\varepsilon, d\psi/dt)$ is determined from fuzzy, human operator, knowledge.

Close-by control:
This control mode is less straightforward than far-away control. The problems that one encounters are the fact that ship motion is rather noisy. High frequency components cannot be compensated for by rudder control but moving averages of the heading error must be compensated. On the other hand, rudder motion must not be too excessive because of steering machine wear.
In practice, helmsmen apply so-called 'rudder-gust' corrections. This means that a certain rudder angle is set and, before the ship begins to move, brought back to the initial value again. Rudder-gusts have parameters T_1 (the duration) and A (the amplitude). Now, fuzzy sets are defined for D and the following observations of the helmsman are transformed to rules;
If ε is very small, but not zero, during a larger period of time a correction is made by computing a rudder-gust, with amplitude A and duration T_1, dependent on the mean value of ε. If ε is not very small, a faster correction is required, but still based upon the mean value of ε.
The mean value of ε is determined as follows:

$$\varepsilon_{av} = (1/T_2) \int_0^{T_2} \varepsilon \, dt$$

Some more auxiliary variables are determined which are not further discussed here. The idea is that based on these variables, which are derived from ε, control values for the duration T_1 and amplitude A and the application time T_2 of the gust are determined using fuzzy reasoning.

4.2.2 Target tracking

4.2.2.1 Kalman filter parameter tuning based on fuzzy logic

A typical engineering design problem is tuning of filter parameters based on measurements and filtered quantities. More specifically, in a Kalman filter several variables can be used to monitor the correct behaviour of the filter. For instance when the system model deviates from the 'truth model', the so-called residual will no longer be 'white' 'Gaussian' and may even have nonzero mean. In such a case, the modelled system noise variance must be increased to account for unmodelled system behaviour; a rule like the one below could for instance be used:

if (residual is large) *then* (modelled system noise variance must be high)

In the area of manoeuvre detection, different fuzzy rules can be thought of. A fuzzy rule that can be used is for instance based on the acceleration, obtained from the filter. When this passes a threshold, a different filter model could be switched to, or a combination of results from different filters can be combined. The rules could read:

if (acceleration is high) then (output from filter 1 is used)
if (acceleration is low) then (output from filter 2 is used)

4.2.2.2 Kalman filter based target tracking compared to fuzzy target tracking

This is an application that is discussed in ref. [10]. There the problem of pointing an antenna to a moving target is divided into elevation and azimuth tracking. Two tracker systems were designed, one based on fuzzy logic and one based on a Kalman filter approach. For each system an elevation- and azimuth channel was designed. The discussion is only done for one channel because both channels had equal structure. The input to a channel was the pointing error at time k: E(k), the derivative EDOT(k) and previous output of the tracker (at time k-1): A(k-1).

<u>Fuzzy controller:</u>
The fuzzy controller uses rules with three antecedents like:
If (E(k)=Medium Positive *and* EDOT(k)=Small Negative *and* A(k-1)=Zero) *then* A(k)=Small Positive

Correlation product encoding is used as an inference scheme and the fuzzy centroid method is used as a defuzzification scheme (see appendix).

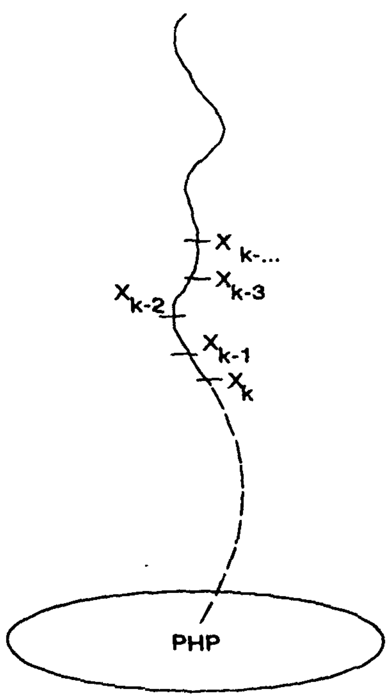
<u>Kalman filter:</u>
Simple (linear) system equations are used for the design of the Kalman filter.

Conclusions are that the simulations suggest that "The use of a fuzzy controller may provide a robust, computationally effective alternative to linear Kalman-filter, indeed to non-linear extended Kalman-filter, approaches to real time system control - even when we can accurately articulate an input-output math model."

5 FUZZY LOGIC IN FIRE CONTROL

5.1 The fire control problem

The fire control we are discussing here is the aiming problem that is encountered in air defense systems. So specific problems concerning intercept boundaries and guidance are not dealt with.
Projectiles, whether guided or unguided, are launched/fired in such a direction that the probability of intercept is optimal. The fire control problem for air defense systems can in general be described as follows:
Given a *sequence* of target position measurements (generally contaminated with noise): $x_1,...,x_k$ for times: $t_1,...,t_k$, determine the *aiming* direction of the gun γ_k for time t_k such that the probability of intercept is optimal.
We will now more specifically deal with unguided projectiles, or in other words with gun fire control.
In essence, the problem that has to be solved is the prediction problem. Based on target position measurements, until a given time t_k, a model for the target motion in the future (so for times $t>t_k$) must be assembled. This will in general be a statistical model. In the figure below, a (two-dimensional) example target trajectory is displayed. The drawn line is the measured target trajectory. The dotted line is the predicted target trajectory for times $t > t_k$. PHP is the Predicted Hitting Point, the ellipse in this case indicates the variances present in the PHP.



The dotted line is usually obtained from a so-called prediction filter. This filter estimates parameters of an assumed trajectory model, given position measurements. When for example the trajectory model is quadratic (so parabolic), the filter estimates, the position, velocity and acceleration. This trajectory model will be used for $t > t_k$, in other words it will be used for prediction. In an iteration process, PHP is determined given firing table data. This PHP will generally have a zero mean noise term superimposed upon a bias. This nonzero mean is of course only present when small time intervals are considered because the bias itself will fluctuate around zero. Depending on this mean, the variance can be set to obtain an optimal value for the hit probability, given the gun dispersion. In other words, the prediction filter can (at least in principle) be tuned for optimal hit probability.

5.2 The pure fuzzy approach

When designing a 'pure fuzzy' fire control system, one actually wonders how a human being performs the task of shooting down a manoeuvring target. The inputs he uses will probably be the range R to the target and the angle between the Line Of Sight (LOS) to the target and the velocity or stated differently, the LOS rate $d\theta/dt$. The output will be a lead angle φ.

He will probably use rules like:

if (R is large) *and* ($d\theta/dt$ is small)	*then*	φ is small
if (R is large) *and* ($d\theta/dt$ is large)	*then*	φ is large
if (R is small) *and* ($d\theta/dt$ is small)	*then*	φ is small
if (R is small) *and* ($d\theta/dt$ is large)	*then*	φ is very_large

With fuzzy sets: small, large, very_large for the range as well as the LOS rate. The rules are not exhaustive but are just given for the sake of example.

The problem that the fuzzy engineer encounters is to define the fuzzy sets. This may be rather easy for hunters shooting at birds, but human operators of air defense gun systems this is probably harder. Moreover, gunners will probably not give better performance than a computer system.

5.3 Fuzzy reasoning as a fire control system design aids

Fuzzy logic applications in fire control are mainly in the area of CC fuzzy systems. Although the Predicted Hitting Point with a corresponding uncertainty area may be fuzzy, the output of the fire control system that is fed to the servo will always be non-fuzzy; namely a gun aiming direction, so CF or FF are not applicable.

FC (combined with CC) could be a possibility. Think for instance of a gun system that is set by an operator as a function of the environment. Characteristics about the environment could be some *fuzzy statement* about expected threat type or some fuzzy statement about the weather.

These FC systems are possible in principle but are highly speculative. One would probably prefer a system that operates independently of operator settings.

CC systems are considered more likely. One could for instance think of systems that perform a selection or weighted average of filter outputs, based on the quality of these outputs.

Rules like the following would then apply:

if (Quality of filter is high) *then* (weigh the output high)
if (Quality is low) *then* (weigh the output low)

Other possible rules are:

if (the acceleration is high) *then* (time constant of the filter is low)
if (the acceleration is low) *then* (time constant of the filter is high)

Or:

if (the derivative of the acceleration is high) *then* (trajectory change detected)
if (the derivative of the acceleration is low) *then* (no trajectory change detected)

Or tactical rules like:

if (target range is large) *then* (do not use acceleration in the prediction but use linear prediction)

The choice of the values for the membership functions is still a problem that one is left with. Actually, all this would probably have been feasible without using fuzzy logic.

6 ADVANTAGES AND DISADVANTAGES OF THE USE OF FUZZY SYSTEMS

The use of fuzzy logic clearly enables a human being to interface easier with an automated system than in the conventional case. This is because human beings more or less have a natural tendency towards uncertainty. Advantages therefore may result in all cases where human beings are involved with systems, be it as a designer or as a user.

When a human being is seen as a user, a more natural system interface can be obtained in fuzzy systems. This is because the system can directly communicate with the user via natural language terms.

In the design of systems that are less soft, fuzzy logic can be of assistance because of the fact that in the design of such systems often human knowledge can or must be used. One can think of expert knowledge from humans that already are able to perform tasks that must be automated, like for instance train control, mortgage analysis or target tracking. One can also think of fuzzy knowledge of expert system designers. Mostly, the tasks that can be performed with fuzzy logic can also be done in a nonfuzzy way. The key idea of using fuzzy logic however is that precision is expensive while not always necessary. People for instance are quite good at performing several decision tasks using only nonprecise data and generating nonprecise actions. One of the key reasons why fuzzy logic works well is the fact that many systems do not require very critical tuning. In other words, when parameters are set sub-optimal, the performance will not degrade very much.

Summarizing, the following benefits can be named:

- Fuzzy Logic describes systems in terms of a combination of numerics and linguistics (symbolics). This has advantages over pure mathematical (numerical) approaches or pure symbolic approaches because very often system knowledge is available in such a combination.
- Problems for which an exact mathematically precise description is lacking or is only available for very restricted conditions can often be tackled by fuzzy logic, provided a fuzzy model is present.
- Fuzzy logic sometimes uses only approximate data, so simple sensors can be used.
- The algorithms can be described with little data, so little memory is required.
- The algorithms are often quite understandable.
- Fuzzy algorithms are often robust, in the sense that they are not very sensitive to changing environments and erroneous or forgotten rules.

- The reasoning process is often simple, compared to computationally precise systems, so computing power is saved. This is a very interesting feature, especially in real time systems.
- Fuzzy methods usually have a shorter development time than conventional methods.

Although the above named advantages are very promising, one must be aware that fuzzy logic does not fit to every problem. The following remarks must be made:

- As has been shown in section 3, fuzzy logic amounts to function approximation in the case of Crisp-Input/Crisp-Output systems. This means that in many cases, using fuzzy logic is just a different way of performing interpolation. In the light of the fact that system knowledge is often available as a combination of numerics (quantitative) and linguistics (quantitative or qualitative) this approach may even be advantageous.
- In areas that have good mathematical descriptions and solutions, the use of fuzzy logic most often may be sensible when computing power (i.e. time and memory) restrictions are too severe for a complete mathematical implementation.
- I am convinced that results obtained in successful fuzzy applications that are given in literature can be reached with a conventional approach as well, possibly taking longer development time and possibly with the use of different interpolation methods.

  Careful analysis of comparison examples, 'proving' the superiority of fuzzy logic often shows that they compare the fuzzy approach with a very simple, non-optimized conventional approach.
- Proof of characteristics of fuzzy systems is difficult or impossible in most cases because of lacking mathematical descriptions; especially in the area of stability of control systems this is an important research item. On the other hand, when solving practical problems, this is often not a very severe restriction because when the system is tested the characteristics will also be found.

7 CONCLUSIONS

In this paper the fuzzy concept is introduced and several applications are briefly discussed. It is shown that fuzzy logic plays a role in problem areas which combine numerical (mathemathical) with symbolic (which in this case means linguistic) solutions. Therefore, when system knowledge is available in linguistic and/or numerical terms, fuzzy logic may be very helpful in the design of a solution.
Almost in every field in which 'fuzzy' knowledge plays a role, fuzzy logic may be used, although sometimes it is not the best solution to use fuzzy logic without regarding different methods. On the other hand, when computing power restrictions are severe, it seems wise to at least consider fuzzy logic.

In the previous sections we have seen some examples of fuzzy systems. Several categories were found, depending on the system interfaces. Crisp Input / Crisp Output is the most frequently encountered.
Two main application areas are generally discussed; the engineering design areas and the 'pure fuzzy' applications.
Engineering design applications deal with 'tuning' and designing. Applications of fuzzy logic in this area often result in 'hybrid' systems, i.e. systems with a fuzzy part as well as a pure numerical part. Frequently one encounters fuzzy rules for the selection of numerical algorithms. Although it is often claimed that systems can operate much smoother using fuzzy logic for switching or tuning, this is a misconception. Many (for instance) control systems exist that operate smoothly. This can be reached by careful (conventional) design.
In the pure fuzzy approach, the conventional methods are often thrown away and all the relations are defined purely fuzzy.

Several possibilities exist for 'inference' schemes; where this is frequently referred to as a 'richness' of the theory the question arises if this really helps the designer of a fuzzy system.

The main advantage at this moment seems to be the fact that fuzzy logic is promising for rapid prototyping. Clear solutions can be obtained within reasonable time, requiring less computing power and less precise sensing.
Lots of optimization problems don't have very critical solutions. In other words, tuning of system parameters will in general only slightly increase the system performance. This is the reason that a

first solution, such as a fuzzy logic solution, obtained via rapid prototyping generally is quite good and further tuning is not necessary. On the other hand, when more precise systems are required, tuning fuzzy systems may take a lot of time, perhaps even more than a careful conventional design.

8 RECOMMENDATIONS FOR FOLLOW-ON WORK

It is recommended to explore fuzzy systems theory in a real application. Doing this enables one to really experience the stated (dis)advantages rather than trying to understand them from literature examples.
It is desirable that example applications are chosen which already have conventional solutions to enable one to compare fuzzy to nonfuzzy methods.
It is also important to study how quick one can find a fuzzy solution, or in other words, to study the use of fuzzy logic for rapid prototyping.

Discussions about further study and example applications will continue after this study has been finished. As a first step towards these discussions the following three examples are given. These are in the areas of IFF (Identification Friend or Foe), plot extraction (fusion of observations made by radar or IR) and camera setting.
The first example refers to NIS (NATO Identification System). This consists of methods for combining data for identification purposes. In a NATO research group, studies have been performed to find a 'best' method. A method (namely Bayesian Inference) has already been chosen and is currently under study. Fuzzy Logic has been considered but was not chosen. A counterpart study for applying fuzzy logic therefore seems interesting.
The second example refers to the ARTIST program (Advanced Radar Techniques for Improved Surveillance and Tracking). In this program several methods are studied to find new radar techniques. One of the sub-studies (MUSICIAN) is in the area of plot extraction. One method that is studied here is the so-called Multiple Hypothesis Testing (MHT). The idea is to study a fuzzy plot extraction method, possibly by using fuzzy rules combined with MHT or by using a pure fuzzy approach.
The last example consists of an automated system for setting a camera/image-intensifier system. Camera setting is a process that is now done manually by an expert.

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H. Blok
(group leader)

N.M. de Reus
(author)

Appendix A: Binary, Multivalued and Fuzzy Logic

The generalisation of binary logic to multivalued logic and fuzzy logic is briefly discussed.

Binary logic

In binary logic, the operators AND ($\wedge$), OR ($\vee$) NEGATION ($\neg$) and IMPLICATION ($\rightarrow$) can be defined with truth tables as follows:

A	B	$\neg$A	A $\wedge$ B	A $\vee$ B	A $\rightarrow$ B
0	0	1	0	0	1
0	1	1	0	1	1
1	0	0	0	1	0
1	1	0	1	1	1

Let x and y be elements of the universe of discourse of the (crisp) sets A and B. And let $f_A(x)$ and $f_B(y)$ denote the indicator functions of A and B. I.e. for A such an indicator function is defined as:

$f_A(x) = 0 \quad \text{if } x \in A$

$f_A(x) = 1 \quad \text{if } x \notin A$

Then, the above table could be described by the following formulae:

$$f_{\neg A}(x) = 1 - f_A(x)$$

$$f_{A \wedge B}(x,y) = f_A(x)*f_B(y)$$

$$f^1{}_{A \vee B}(x,y) = \max(f_A(x),f_B(y))$$

A different OR operator that one can think of is: $f^2{}_{A \vee B}(x,y) = f_A(x) + f_B(y) - f_A(x)*f_B(y)$

The IMPLICATION operator can be defined in terms of the OR operator. The two possibilities that result from the two operators above are given below:

$$\begin{aligned} f^1{}_{A \rightarrow B}(x,y) &= f^1{}_{(\neg A) \vee B}(x,y) = \\ &= \max(f_{\neg A}(x),f_B(y)) \end{aligned}$$

$$f^2_{A\rightarrow B}(x,y) = f^2_{(\neg A)\vee B}(x,y) =$$
$$= f_{\neg A}(x) + f_B(y) - f_{\neg A}(x)*f_B(y)$$

Multivalued logic

When defining multivalued logic, the idea is to extend or generalize binary logic. That means that in the limit case, when the membership values are 0 or 1, the same results must be obtained compared to binary logic. In other words, the formulae that are derived for the binary logic above can also be used for multivalued logic with the only difference that now $f_A(x)$ and $f_B(y)$ are now elements from [0,1] instead of {0,1}.

It must be stated that different approaches are possible when deriving formulae for multivalued logic. One approach is based on the fuzzy relation instead of the generalisation of binary logic, the meaning of the implication rule is used rather than a generalisation of binary logic. This is because even in binary logic it is philosophically difficult to understand or explain the implication when the antecedent is not true.

In general one can say that there are several possibilities and lots of theory is being developed for these possibilities. In ref. [13] a particular problem with the often used 'minimum operator' for approximating the meaning of the AND is discussed and proposals for enhancing this operator are made.

Fuzzy Logic

In fuzzy literature, many inference mechanisms are encountered, three of which are described here. I have called them *minmax*, *minsum* and *prodsum*.

Assume that a bank of FAM rules is available: $(A_1,B_1),\ldots,(A_n,B_n)$. Let each A_i and B_i be fuzzy sets with membership functions f_{Ai} and f_{Bi} respectively.

Consider a crisp input x^* given and the consequent fuzzy set B' from the bank to be determined.

First for $i=1,\ldots,n$ the consequent fuzzy sets B_i' are determined.

When using *prodsum*, the fuzzy sets B_i are simply multiplied by the corresponding membership value $f_{Ai}(x^*)$, so:

$f_{Bi'}(x)=f_{Ai}(x^*)\cdot f_{Bi}(x)$.

When using *minsum* and *minmax*, the consequent fuzzy set B_i' is determined for each element x from the universe of discourse of B_i by taking the minimum of $f_{Ai}(x^*)$ and $f_{Bi}(x)$, so:

$f_{Bi'}(x)=\min\{f_{Ai}(x^*),f_{Bi}(x)\}$.

After that the B_i' are combined to form B' so that defuzzification can be applied resulting in a crisp value.

The *prodsum* and *minsum* method both superimpose all the consequent membership functions.

So:

$$f_{B'}(x) = \sum_{i=1}^{n} f_{Bi'}(x)$$

When using *minmax*, the maximum of each membership value for each B_i' is determined for each x from the domain, so:

$$f_{B'}(x) = \max\{f_{B1'}(x),...,f_{Bn'}(x)\}$$

Defuzzification

Real life systems often require a crisp output value while in general the result of fuzzy reasoning is a fuzzy set. Generating a crisp value from a fuzzy set is called defuzzification. Two main techniques are encountered in the literature; the centroidal defuzzification and the maximum membership defuzzification scheme. In the centroid method, *an element* x_m *in the* universe of discourse of a fuzzy set B is determined comparable to the expectation in probability theory:

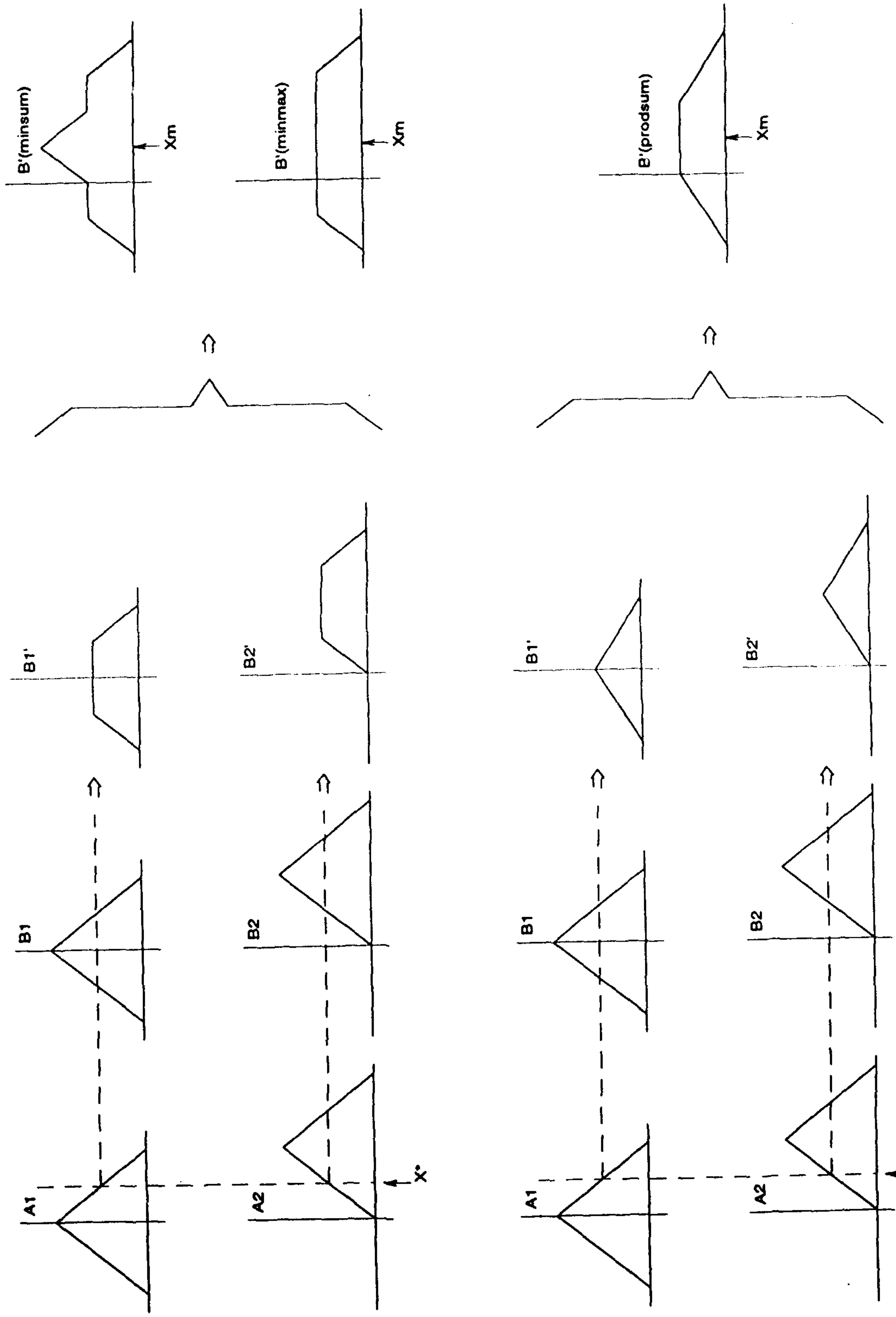
$$x_m = \sum_{i=1}^{n} x_i * f_B(x_i) \,/\, \sum_{i=1}^{n} f_B(x_i)$$

Where in this case B is a fuzzy set on a finite universe of discourse: $\{x_1,...,x_n)$. When the universe of discourse is continuous, the sums are replaced by integrals.

The maximum membership defuzzification scheme determines the x_m such that:

$$f_B(x_m) = \max_{i=1,..,n} f_B(x_i)$$

In the following figures, the above inference mechanisms; minmax, minsum and prodsum are clarified.


B'(minsum)
Xm
B'(minmax)
Xm
B'(prodsum)
Xm
B1'
B2'
B1'
B2'
B1
B2
B1
B2
A1
A2
X*
A1
A2
X*

Multi-antecedent rules

Rules with more than one antecedent require different handling from those with one antecedent. Consider as an example the rule with two antecedents: *if* ((x_1 is A_{11}) and (x_2 is A_{12})) *then* (y is B_1)

or using a different notation: (A_{11},A_{12};B_1)

Consider crisp values x_1^* and x_2^* given.

Then, two membership values result; $f_{A11}(x_1^*)$ and $f_{A12}(x_2^*)$ instead of one $f_{A1}(x^*)$, like in the single antecedent case. These are combined with the AND operator, so usually this means determining the minimum of $f_{A11}(x_1^*)$ and $f_{A12}(x_2^*)$. When the operator would be OR, the usual choice is the maximum. After that the procedure is as described for the single antecedent case.

BACK TO BASICS

Most of us have learned about relations, mappings and functions back in our earlier school years. Because of the importance of these ideas for the introduction of fuzzy logic as a function approximation technique, they are summarized here and defined formally.
The concept that not every function needs to be represented by a formula, although very simple and down to earth, is often a source of misunderstanding. This is the reason why this section elaborates upon it.

In the discussion below, sets are used. These are meant in the classical sense so they are not fuzzy sets.

Relations:

First the definition of Cartesian product is given, before the term relation is introduced.

Given n sets: M_i (i=1,...,n) the Cartesian product $M_1*...*M_n$ is defined as the set of all possible so-called n-tuples : $(x_1,...,x_n)$ with $x_i \in M_i$. The n-tuple can be seen as the generalisation of the ordered pair or 2-tuple.
So: $M_1*...*M_n = \{(x_1,...,x_n) | x_i \in M_i\}$

Every subset $R \subseteq M_1*...*M_n$ of $M_1*...*M_n$ is called a Relation of n variables.
Note that nothing is said about the sets M_i so the elements need not necessarily be numbers.

Of special importance are the so-called binary relations, these occur when there are two sets involved :
$R \subseteq M*N$. They can be seen as assignments (elements of M are assigned to elements of N according to a predefined procedure).

These assignments can be represented graphically. For example, the two sets M and N both have four elements: $M=\{x_1,x_2,x_3,x_4\}$ and $N=\{y_1,y_2,y_3,y_4\}$. In the picture below the stars represent the Cartesian product of M and N. The underlined stars are elements of the example relation:
$R=\{(x_1,y_1),(x_1,y_3),(x_2,y_2),(x_3,y_2),(x_3,y_3),(x_3,y_4)\}$

Graphical representation of an example binary relation

	x_1	x_2	x_3	x_4
y_1	*	*	*	*
y_2	*	*	*	*
y_3	*	*	*	*
y_4	*	*	*	*

Mappings:

Binary relations are generally not unambiguous, i.e. more elements can be assigned to one element as can be seen in the example mapping above.

A mapping is a binary relation $f \subseteq M*N$ for which holds that to every element x of M exactly one element y of N is assigned. Instead of writing $(x,y) \in f$ one also writes $x \rightarrow f(x)$ or $x \rightarrow y$ with $y=f(x)$. The sets M and N are called Domain and Range respectively. The element $y(=f(x))$ is called the image of x.

A graphical representation of an example mapping is given below.

Graphical representation of an example mapping

	x_1	x_2	x_3	x_4
y_1	*		*	
y_2		*		
y_3				
y_4				*

Functions:

In the above definitions nothing is said about the nature of the sets involved. When the sets are algebraic structures (number sets or vector spaces) then the mappings are called functions. In plain (but less precise) English: Functions are mappings for which hold that the domain as well as the range are sets of numbers, like for instance **R** or **N**.

Functions can often be given by formulae, for instance $f(x)=x^2$ but it should be clear from this paragraph that a formula description is not a necessary condition for a function to exist. Also enumerations (in the form of tables) or algorithms to construct the image for a given element from the domain can be equivalent to a function.

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N.M. DE REUS

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TNO PHYSICS AND ELECTRONICS LABORATORY, P.O. BOX 96864, 2509 JG THE HAGUE
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15. ABSTRACT (MAXIMUM 200 WORDS, 1044 POSITIONS)

RECENTLY, MANY APPLICATIONS OF FUZZY LOGIC ARE EMERGING. MORE AND MORE POPULAR ARTICLES ARE PUBLISHED, REVIEWING THE FIELD OF APPLICATIONS OR REVIEWING THE THEORY. THE PURPOSE OF THIS REPORT IS TO GIVE A COMMON SENSE INTRODUCTION TO THE POSSIBILITIES OF FUZZY LOGIC.

ONE OF THE MOST IMPORTANT AREAS WHERE FUZZY LOGIC IS USED IS THAT OF SYSTEMS WHERE THE INPUT AS WELL AS OUTPUT ARE NOT FUZZY BUT JUST NUMBERS. FOR SUCH SYSTEMS IT IS SHOWN THAT THE FUZZY APPROACH ACTUALLY IS A FUNCTION APPROXIMATION METHOD, OFTEN ALLOWING MORE INSIGHT THAN DIFFERENT METHODS. EVEN IN APPLICATION AREAS THAT MAY NOT SEEM FUZZY AT FIRST SIGHT, POSSIBILITIES APPEAR TO EXIST. THIS IS THE REASON FOR THIS SURVEYING LITERATURE STUDY ON FUZZY LOGIC APPLICATIONS IN GENERAL AND FIRE CONTROL APPLICATIONS IN PARTICULAR.

16. DESCRIPTORS	IDENTIFIERS
FUZZY SETS AND SYSTEMS	FUZZY LOGIC
FIRE CONTROL	FUZZY INFERENCE
TARGET TRACKING	FUNCTION REALISATION
SHIP STEERING	FUNCTION APPROXIMATION
PREDICTED TRAJECTORY	

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