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The Application of Nuclear Propulsion to Satellite Boosting

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The characteristics of rocket vehicles and the basic mechanics of boost trajectories for satellite orbits are presented with special emphasis on those of nuclear rockets. The advantages and disadvantages of nuclear rockets are compared to rocket systems of higher and lower impulse, and an operational philosophy for nuclear rockets is developed. The application of nuclear rockets for space vehicle take-off is discussed.

The energy imparted to the propellant of a nuclear rocket is derived from a nuclear reactor. A nuclear rocket engine necessarily has a higher specific weight than a chemical rocket engine, in which the energy is derived from the propellant itself. On the other hand, because propellants of lower molecular weight can be used, the performance available from a nuclear rocket is much higher. The specific impulse of a nuclear rocket ranges from 600 to 1200 sec, depending upon the propellant, chamber pressure, and reactor temperature (1). With further specification, an optimistic value of 900 sec will be assigned to the specific impulse of a nuclear-heated engine, and a value of 400 sec to chemically-fueled engines. These values represent approximately the same stage of advancement for each type.

If propellant usage is integrated along the flight trajectory of a rocket vehicle, the mass ratio thus calculated can be correlated with a fictitious velocity according to the relation

\[ V = I_{g0} \ln R \]

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which gives the velocity change of a rocket fired in field-free space. This mission velocity becomes then a measure of the work which the vehicle must do in accomplishing the mission. It can be seen that for each fraction of propellant used, the nuclear rocket will attain more than twice the velocity increment attained by a chemical engine. However, some understanding must be given to the use of a mission velocity in reaching a basis of comparison between rocket systems. The mission velocity increases slightly with specific impulse, so that systems of higher impulse appear to pay an energy penalty. For example, the trajectory computation for an ion rocket, which has very high impulses in the range of 8000 to 16,000 sec, indicates that the mission velocity for escaping from an Earth orbit and transferring to a Mars orbit about the Sun is more than 90,000 fps, while a quick calculation of this quantity, based on energy change, gives a value of 45,000 fps for a chemical rocket (2). A physical interpretation of this penalty has been suggested as resulting from imparting more gravitational energy to the propellant before it is exhausted through the engine (3). High-performance systems more than compensate for this penalty by reduction of the mass ratio, so that the total weight required for taking off with a given payload is significantly reduced. The difference between the nuclear rocket and the chemical rocket probably does not exceed 2000 fps for similar missions which do not escape from the earth.

The power which the reactor must generate is given by the relation

\[ P = \frac{1}{2} F g_0 \approx 20 \text{ kw/lb of thrust} \]

An engine developing a sizable amount of thrust must develop a large amount of power. The total amount of power which must be delivered to the payload for a mission is about the same, whether it is generated by nuclear fission or the burning of exotic, high-energy propellants. The capability of the power plant is one reason that nuclear power has become important and missions of greater and greater total energy are contemplated. The reactor can generate a tremendous amount of power (total energy) in a compact package. For applications such as boosting a small payload vehicle from the Earth's surface, a small nuclear rocket does not compare favorably with the chemical rocket, principally because of the higher engine weight. A reactor must contain at least a certain critical amount of fissionable material, ranging in weight from 500 to 4000 lb (4). However, as the thrust and power requirements increase into the range of practical reactor design, then the lower mass ratio required for the nuclear-powered mission begins to show definite advantages in take-off weight and thrust.

### Nuclear Versus Chemical - Rocket Power

A hypothetical example will serve to illustrate the difference. Consider a one-stage chemical rocket and a nuclear rocket which are each to carry a payload M through a velocity change of 20,000 fps. For the chemical rocket, the indicated mass ratio is 4.73, and for the nuclear, 1.42. Let the unfueled chemical rocket consist of 70 percent payload, and 30 percent tanks, engine, and structure. Furthermore, let the unfueled nuclear rocket consist of 40 percent payload and 60 percent engine, structure, and tank, although the tank weight is smaller in this case. The gross weights based on these estimates are then 3.52 M for the nuclear and 6.75 M for the chemical. This estimate is not intended to represent actual design data for a certain value of payload, but rather to indicate the decisive advantages of improved performance, even with a heavier engine. Some improvement in the weight of the chemical system might be derived from staging the missile.

The properties of satellite orbits about a planet can generally be derived from single-body, two-dimensional, classical mechanics (5). For exact determination of satellite lifetimes, an accurate model of the earth's gravitational field and atmosphere should be used, but for calculating the first-order characteristics of orbits, it is sufficiently accurate to consider the Earth as a point mass center of attraction of strength

\[ g_0 R_0 (\approx 24.2 \text{ m}^2/\text{sec}^2) \]

Then the total energy of a particle in a circular orbit is \( E_0 R_0 (1 - 1/2k) \), where k is the radius of the orbit in terms of \( R_0 \). For each radius, there corresponds a definite velocity, period, and energy requirement. A similar relationship can be worked out for elliptic orbits. If the total energy of the particle is less than the strength of the gravitational center, then it will describe a periodic orbit, either a circle or ellipse, about the attracting point. On the other hand, if the total energy is greater than the strength of the gravity field, the particle will be on an open, nonperiodic orbit and escape the earth along a parabolic or hyperbolic path.

Corresponding to an energy level, there is

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1 Numbers in parentheses designate References at end of paper.
an ideal velocity such that a particle in free space moving at this velocity will have the
escape energy in motion. For example, escape energy of \(24.2 \text{ mi}^2/\text{sec}^2\) corresponds to a veloc-
ity of 6.95 mi/sec, or about 37,000 fps. The escape, or parabolic, velocity at any altitude is \(\sqrt{2}\) times the circular velocity at that altitude. A computation of the mass ratio along the boost trajectory to the orbit can be con-
sidered then as an exact calculation of the energy losses involved in the flight. The optimum trajectory is that path for which the velocity increment due to loss is the least.

**Characteristics of a Representative Orbit**

The characteristics of some representative orbit should be mentioned. For an orbit of 100 miles altitude, the ideal energy velocity is 26,300 fps and the orbital velocity is 25,700 fps, with a period of about 88 min. At 500 miles, the ideal velocity has increased to 27,400 fps, but the circular velocity has de-
creased to 24,500 fps. The period is slightly over 100 min. As the orbit radius increases, the ideal velocity approaches the escape velo-
city, and the circular velocity continues to decrease. There is one radius whose period cor-
responds to that of the rotation of the Earth. This is the stationary orbit at 22,300 miles altitude. A vehicle in this orbit would have a velocity of 10,000 fps, but viewed from some equatorial point, would apparently oscillate about the zenith point in a narrow diurnal ellipse along the longitudinal meridian. If the orbit were equatorial, then the vehicle would appear to hang motionless in the sky.

The ideal velocity of the stationary orbit is 35,500 fps, or 92.4 per cent of the escape energy. The high orbits are literally on the threshold of space; the capability of reaching an outer orbit is tantamount to the capability of accomplishing a space mission.

**Optimum Firing and Thrust-Direction Program**

The ideal velocity is only a theoretical lower bound for the mission energy. Since the losses appearing in the formula for mass ratio result from both conservative and nonconservative forces, different paths or firing modes between two paths can require quite different mission velocities. In general, a thrust-direction program which acts against gravity, that is, radially, is less efficient in increasing the vehicle energy than a program which "fires" in an azimuthal direction, although in most cases, the radial thrust produces the shortest trip time between potential-energy levels. The problem of selecting the optimum firing and thrust-direction program has been studied ex-
tensively by machine computation and the methods of the calculus of variations. For the case of the airless, nonrotating Earth, Fried and Perkins have independently shown that an optimum tra-
jectory results from the tangent of the thrust angle with the local horizontal decreasing linearly in time; that is, the direction of thrust decreases from nearly straight up at take-off to horizontal at orbit injection ac-
cording to a linear decrease of a tangent \((6,7)\). In actual practice, the effects of the atmosphere and the rotation of the Earth cannot be neg-
lected, but a solution can still be obtained by machine computation.

For reaching the higher orbits, use should be made of the Hohmann ellipse. The injection point will be perigee of the ellipse, and the apo-
gee will lie at a higher altitude on the op-
posite side of the Earth. By firing again at apo-
gee, the velocity of the vehicle can be increased to the correct value so that it remains in a circular orbit at the higher altitude. It was proved by Hohmann that impulse firing at the apo-
gee and perigee of the ellipse results in a minimum expenditure of energy for transfer between circular orbit levels. It may be de-
sirable, however, for a vehicle to reach high-
altitude injection within the horizon of the launching site rather than on the opposite side of the earth, for reasons of control or guidance. In this case, the firing program would be similar to that for a low-altitude orbit, except that long coasting periods of climb would be neces-
sary. The mass-ratio expenditure would be at least nearly double that of the optimum Hohmann launch program \((8)\).

**Stationary Orbit Launch**

Some characteristics of the launch using the stationary orbit as an example will be ex-
amined. Using the formula for mass ratio,

\[ R = \left[1 - \frac{f}{I} t_b \right]^{-1} \]

where \(t_b\) is the burning time for the engine, \(I\) is the specific impulse, and \(f\) is the thrust to weight ratio at take-off (these parameters are taken as average and fixed for the trip). Remembering the relation between mission veloc-
ity and mass ratio, then a vehicle with an \(f = 1.4\) and \(I = 900\) sec, reaches a mission velocity of 40,000 fps in 480 sec of engine burning. Furthermore, the maximum radial (upward) veloc-
ity must decrease with altitude or the rocket will exceed escape velocity at that altitude.
A simple numerical integration of the motion of a falling particle will show that it takes 15,000 sec for a particle to fall straight down from the stationary orbit under the influence of gravity. Kepler's law for the period of revolution of planetary bodies gives around 40,000 sec for Hohmann transfer ellipses to the stationary orbit. Thus, the trip time to reach this orbit must lie between 15,000 and 20,000 sec, unless a vehicle is used which exceeds the free-fall velocity along the climb and uses thrust to cancel the excess rise at the desired orbital altitude. The difference between 480 and 15,000 sec is an indication of the magnitude of the cost period, or periods or greatly reduced thrust, which must be used to guide the vehicle correctly through the high-altitude boost.

A nuclear vehicle would not have a prohibitive mass ratio for missions of around escape energy. For 900 sec impulse, a mass ratio of 5 corresponds to 46,500 fps. If partial staging, that is, dropping of empty-tank structure, could be employed, the corresponding mission velocity can be extended. Chemical rockets for mission velocities of this magnitude must employ several stages. Thus, the nuclear rocket, with its higher trajectory losses and heavier engine weight, is competing with several stages of chemical rockets. The trajectory for the nuclear is slightly different from that for the chemical. One reason for this is the control problem at thrust cut-off (9). Another is that the nuclear rocket may want to climb through the atmosphere in the shortest length of time. Both of these factors increase in mission velocity slightly, but the capability of the engine derived from the higher impulse more than offsets these.

**Missions Beyond Escape**

For missions much beyond escape, the nuclear vehicle will probably have to be staged. This brackets the range of mission velocity in which the single-stage nuclear rocket can excel over all other types. This range corresponds to boosting large payloads into orbit or the initial stage of a space mission. Both of these tasks can be performed by a chemical system, but the thrust required per pound of payload will be much higher, as has been mentioned. For the space mission, a single-stage nuclear rocket may be used to accelerate the payload directly into the ballistic interplanetary transfer orbit. There will be a small guidance error resulting from the fact that the ballistic transfer injection occurs too close to the Earth (2,10). Furthermore, the single-stage nuclear vehicle may have enough propellant available after injection that it could retrofire and re-enter the atmosphere as a glide or parachuted vehicle coasting to some preselected landing region. Thus, the valuable power plant would be recoverable for processing and re-use.

While the use of any nuclear reactor creates a possible radiation hazard, there exists a radiation hazard in space itself, so that protection from both may not be much more of a problem than protection from either one (1,11).

**References**