DESIGN OF SPHERICAL AEROSOL PARTICLES TO MAXIMIZE SOUND ATTENUATION

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**Title and Subtitle**

Design of Spherical Aerosol Particles to Maximize Sound Attenuation

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**Summary**

Analytic expressions for the audible sound absorption and scatter cross sections of an aerosol consisting of spherical particles are identified. The acoustic cross sections per aerosol mass and volume are explored as functions of three independent variables (sound frequency, particle diameter, and particle density) to determine regions of strong attenuation of audible sound. Optimum diameters and densities that maximize cross sections per mass and volume at specific frequencies are found by reading contour plots contained in the figures of this report. Corresponding expected levels of acoustic attenuation are also contemplated. To a small extent, it is possible to scatter audible acoustic radiation with aerosol particles while aerosol absorption of sound can be significant. Comparing the levels of audible sound attenuation by atmospheric gases plus water vapor with the levels of attenuation achievable with an aerosol, we find that aerosol absorption of sound can greatly exceed that of the intervening air.
PREFACE

The work described in this report was authorized under Project No. 10162622A552, Smoke/Obscurants. This work was started in July 1993 and completed in October 1993.

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Introduction

This study investigates the design of aerosols to maximize their acoustic absorption cross section per aerosol mass or volume over the audible frequency range 20-15000 hertz. Air molecules, including water vapor will absorb audible frequencies with little scatter because of their molecular size. To a small extent it is possible to scatter audible acoustic radiation with aerosol particles while aerosol absorption of sound can be significant.

Theory of Attenuation of Sound by Aerosols

In addition to the inverse distance squared drop in sound intensity as acoustic radiation propagates away from the source, we observe attenuation due to absorption by air molecules as indicated by the figures 1 and 2. Attenuation as a function of relative humidity, temperature, and frequency of sound is indicated in units of m$^{-1}$ and dB/m. These units are related by m$^{-1} = 4.35$ dB/m. If an aerosol is present it will scatter and absorb sound with the scatter coefficient concentration product $\alpha_s C$ in units of reciprocal length given by the expression

$$\alpha_s C = \frac{7\pi(2\pi) \left(\frac{D}{2}\right)^6 n}{9\lambda^4}$$

where $\lambda$ is the wavelength of sound, $D$ is the aerosol particle diameter and $n = \frac{C_M}{\rho_s \pi^5 D^3}$ is the number concentration of particles where $C_M$ is the mass concentration and $\rho_s$ the particle density. Because the ratio $D^6/\lambda^4$ is so small even for the largest aerosol particles ($D \approx 2 \times 10^{-3}$ cm and the highest frequency of audible sound $\lambda \approx 2$ cm ) that at the upper concentration limit set by coagulation $\approx 10^8$cm$^{-3}$ the aerosol scatter remains negligible compared to absorption which will now be described.

Absorption of sound by an aerosol occurs as the result of two mechanisms - viscous losses and irreversible heat transfer losses. The viscous losses take place when the aerosol particles move with respect to the air surrounding the particle and the irreversible heat transfer losses take place when heat is transferred between an aerosol particle and the surrounding air. As sound propagates through an aerosol cloud the pressure waves propagate at about 340 m/s at standard temperature and pressure and accelerate particles to oscillate with the air that moves back and forth first in the direction of propagation and then opposite to that direction with the pressure gradient. As air pressure is increased, heat flows from the air into the aerosol particles and when air pressure decreases heat flows from the particles back into the air.
The absorption coefficient concentration product representing viscous losses may be written

$$\alpha^* C = \frac{3\pi \nu D n}{U_s} \left( 1 + \frac{1}{\beta} \right) \mu_\nu$$

where $\nu \approx .15 \text{ cm}^2/\text{s}$ is the viscosity of air, $U_s \approx 340 \text{ m/s}$ is the speed of sound, $D$ is the particle diameter, $n$ is the aerosol particle number concentration defined earlier,

$$\beta = \frac{2}{D} \left( \frac{2\nu}{\omega} \right)^{1/3}$$

where $\omega$ is the angular frequency and

$$\mu_\nu = \frac{\left[ \frac{2\rho_p}{3\rho_{air}} \right]^2}{\left[ \frac{2\rho_p}{3\rho_{air}} \right]^2 + 3 \left[ \frac{2\rho_p}{3\rho_{air}} \beta + \frac{9}{2} \beta^2 \left( 1 + \beta + \frac{\beta^2}{2} \right) \right]}$$

where $\rho_p$ is the aerosol particle density and $\rho_{air} \approx .0012 \text{ g/cm}^3$ is the density of air. If we express particle diameter in $\mu\text{m}$, particle density in $\text{g/cm}^3$ and frequency $f$ in hertz then the absorption coefficient $\alpha^*_v$ due to viscous losses in units of $\text{m}^2/\text{g}$ is

$$\alpha^*_v = \frac{5.3}{\rho_p} \left[ \frac{1}{D^2} + \frac{f^{1/2}}{4370D} \right]$$

$$1 + \frac{23.6}{D\rho_p f^{1/2}} + \frac{278}{D^2 \rho_p^2 f} \left[ \frac{1 + 4370}{Df^{1/2}} + \frac{9550000}{D^2 f} \right]$$

with contourplots of $\alpha^*_v$ appearing in Figures 3 and 4.

The absorption coefficient concentration product due to irreversible heat transfer losses can be written

$$\alpha^*_C = \frac{2\pi \kappa D n}{U_s} \left( \gamma - 1 \right) \left[ 1 + \frac{1}{\beta} \right] \mu_\kappa$$
where $\kappa \approx 0.187 \text{ cm}^2/\text{s}$ is the thermal diffusivity of air, $\gamma \approx 1.4$ is the constant pressure over constant volume heat capacity ratio for air which is approximately that of a diatomic molecule and

$$
\mu_n = \frac{\left(\frac{2\rho_p}{3\rho_{air}}\right)^2}{\frac{2\rho_p}{3\rho_{air}} + 3 \left(\frac{2\rho_p}{3\rho_{air}}\right) \Theta + \frac{9}{2} \Theta^2 \left(1 + \Theta + \Theta^2\right)}
$$

where

$$
\Theta = \left(\frac{8\kappa/\omega}{D}\right)^{1/2}.
$$

The absorption coefficient in units of $m^2/g$ due to irreversible heat transfer losses becomes

$$
\alpha_n^e = \frac{1.76}{2\rho} \left[ \frac{1}{D^2} + \frac{f^{1/2}}{4870D} \right]
$$

$$
1 + \frac{26.3}{D\rho_p f^{1/2}} + \frac{347}{D\rho_p^2 f} \left[ 1 + \frac{4880}{Df^{1/2}} + \frac{11907200}{D^2 f} \right]
$$

where again particle diameter is expressed in microns, density in $g/cm^3$, and frequency in hertz. Contourplots of $\alpha_n^e$ are presented in Figures 5 and 6.

**Discussion**

Total aerosol absorption acoustic cross sections per volume (square meters per cubic centimeter) due to combined viscous and irreversible heat transfer losses are plotted in Figures 7 and 8 in units of square meters per cubic centimeter of aerosol at an aerosol particle density of $5\text{g/cc}$. In Figures 9 and 10 the cross sections per mass (square meters per gram) are similarly plotted. Optimum diameter regions producing maximum contour levels for the sound absorption cross sections per volume or per mass are clearly indicated. For example, the optimum diameter for a $\rho = 5\text{ g/cc}$ density particle attenuating 7000 Hz acoustic radiation would be about 1.5 $\mu$m and the absorption coefficient is about 2 $m^2/cc$. At lower frequencies the optimum diameters are larger and the absorption cross sections are smaller. For example, the optimum diameter for attenuating 700 Hz sound is about 4.5 $\mu$m and the absorption coefficient is about 0.2 $m^2/cc$. Figures 11 and 12 for unit density particles can be compared to Figures 9 and 10 for 5 $g/cc$ density particles. Although optimum diameters for the unit density particles are substantially greater than the optimum diameters for the 5 $g/cc$ particles, the absorption cross section per mass has not changed very much. Because we may roughly estimate the cross section per volume as the particle density multiplied by the cross section per mass, we can expect that the cross section per volume of the 5 $g/cc$ particles will be about 5 times greater than that of the unit density particles. Figures 13 and 14 show the sound absorption cross sections per volume and per mass as a function of particle diameter and density for 100 Hz sound. Similarly Figures 15
and 16 show it for 1000 Hz and Figures 17 and 18 for 10000 Hz frequencies. Optimum diameters are indicated in the plots of cross section per volume with increases in cross section per volume to be expected with increasing particle density assuming the necessary adjustments in diameter are made. On the other hand, there is a ridge of high cross section per mass in particle density-diameter space so that increasing particle density will not increase cross section per mass as long as optimum diameters are chosen. Optimum diameters can be seen to increase with decreasing density along the ridge. Again the contour levels for absorption coefficients are seen to be approximately proportional to the frequencies.

Comparing the levels of audible sound attenuation by atmospheric gases plus water vapor as indicated in the first two figures with the levels of attenuation achievable with an aerosol, we find that aerosol absorption of sound can greatly exceed that of the intervening air.

Conclusion

Aerosol acoustic absorption cross sections per aerosol volume and mass have been contourplotted by taking surfaces defined by constant values of one of the three variables - particle density, diameter and frequency of sound; while the remaining two variables define the abscissa and ordinate of the contourplot. Regions of high acoustic cross section per volume and mass have been shown and optimum ranges for the independent variables have been shown in the contourplots. We have found that the higher audible frequencies are absorbed by an aerosol much more strongly than are the lower audible frequencies roughly in proportion to the frequency. Optimum particle size was found to increase with decreasing acoustic frequencies and higher particle densities were found to increase aerosol absorption per mass but not per volume at the optimum diameter. Finally, aerosol absorption can be made much greater than atmospheric absorption of audible sound.
Figure 1. Values of the total attenuation coefficient versus percent relative humidity for air at STP for frequencies between 2000 and 12500 Hz.

Figure 2. Attenuation of sound in air vs. temperature for various values of relative humidity and 1000 Hz frequency, at atmospheric pressure.
Figure 3. Aerosol Sound Absorption Coefficient (m²m/g)

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