**Abstract**

Nonlinear wave propagation in gas flows, MHD, combustions, elasticity, multiphase flows etc. is studied. Shock waves which are either more or less compressive than the classical gas shocks are shown to be nonlinearly stable, but in the sense distinct from each other. This resolves the longstanding paradox on their physical admissibility by pointing out the sensitive role of dissipations. The phenomenon of non-equilibrium is systematically studied. The problem of zero relaxation limit is resolved for models in mechanics and phase transition.
NON-LINEAR WAVES
IN GAS DYNAMICS AND MECHANICS

FINAL REPORT

Tai-Ping Liu

April 29, 1994

U.S. ARMY RESEARCH OFFICE

DAAL03-91-G-0017

Stanford University

APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED.
Research Results.

During the past three years, under the partial support of Army Basic Research Grant DAAL-3-91-G-0017, the author has completed the following research on nonlinear wave propagation in gas dynamics and mechanics.

There are shocks in elasticity and magnetohydrodynamics, the intermediate shocks, which are more compressive than the gas shocks. The author has earlier shown that such a shock cannot be stable uniformly with respect to the strength of dissipations. In the following paper the nonlinear stability of intermediate shocks is shown for a rotational model derived from nonlinear elasticity and MHD.


This completely resolves the long-standing paradox of the admissibility of intermediate shocks.

Shocks occur in multiphase flow, on the other hand, are often less compressive than the gas shocks. These shocks are the saddle-saddle connections of the corresponding ODEs and their physical admissibility has also been in question. The following paper shows for the complex Burgers equation that such a shock is nonlinear stable.


The existence of undercompressive shocks depends sensitively on the relative strength of the dissipation parameters. We are currently studying more general physical models, including the weak detonations and deflagrations. The author’s new approach of point-wise estimates allows for the studying of nonlinear coupling of wave interactions and is the main tool for these studies.

The important physical phenomenon of relaxation, non-equilibrium, memory and delay can be modeled by quasilinear hyperbolic PDE’s. The author
has earlier proposed a simple model and studied the basic properties of stability, nonlinear waves and Chapman-Enskog-type expansion. Recently, the hard problem of zero relaxation limit is resolved in


Earlier results in this direction do not consider solutions with shocks. The weakly nonlinear limits and the stability criterion for more general systems are resolved in


In these two papers we study the analogues of the compressible and incompressible Navier-Stokes and Euler equations as the limits of the simple model as the relaxation time, or the mean free path, tends to zero. Phenomenon of initial and shock layers is included in our study.

In the following paper, a preliminary draft, we study the slowing propagation of stationary shock under the effect of dissipation and boundary.

5. (with S. H. Yu) “Propagation of stationary viscous shocks under the boundary effects.” (preprint)

This phenomenon is also observed in the numerical calculation of inviscid shocks with the presence of boundary.

Another critical phenomenon is found for the compressible Euler equations under the external damping due to porous media. The following two papers show that the hyperbolic PDE’s are time-asymptotically equivalent to the nonlinear diffusion equations as the momentum equations are simplified to the Darcy’s law.


Our results justified the Darcy's law for compressible flow.