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Stochastic Resonance is a nonlinear stochastic phenomenon which can cause a transfer of energy from a random process (noise) to a periodic signal over a certain range of signal and system parameters. It has been observed in many diverse natural and physical systems, and may be one means by which biological sensor systems amplify weak sensory signals for detection. This paper is an evaluation of the use of the Stochastic Resonance phenomenon as a tool for signal processing in terms of its processing gain.

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An Evaluation of the Stochastic Resonance Phenomenon as a Potential Tool for Signal Processing

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Abstract

Stochastic Resonance is a nonlinear stochastic phenomenon which can cause a transfer of energy from a random process (noise) to a periodic signal over a certain range of signal and system parameters. It has been observed in many diverse natural and physical systems, and may be one means by which biological sensor systems amplify weak sensory signals for detection. This paper is an evaluation of the use of the Stochastic Resonance phenomenon as a tool for signal processing in terms of its processing gain.

1: Introduction

Stochastic Resonance is a nonlinear phenomenon which has been shown to occur in bi-stable systems or in two state devices. Over a certain range of signal and system parameters it can cause what amounts to a transfer of energy from a random process (noise) to a periodic process (signal). This has the potential to dramatically improve the signal to noise ratio at the output of the device. Use of a single nonlinear two state device will be shown to be lossy as compared to the use of the power spectrum for detecting a sinusoid in white noise. However, globally coupling many two state devices appears to have the potential to overcome these losses, and possibly exceed the gains achievable by linear systems due to an enhancement of the Stochastic Resonance effect over what might be expected in a single two state device.

The "classical" description of the phenomenon is that of a particle in a dual well potential which is excited by a strong stochastic process and a weak periodic process. Here, weak means that the force applied by the periodic excitation alone is not sufficient to overcome the barrier, so no state transitions occur in the absence of stochastic forcing. When noise of appropriate strength is added to the periodic forcing, state transitions which have a random component and a periodic component occur. The periodic component of the transitions (output) contains the fundamental frequency and sometimes odd harmonics of the periodic excitation. As one increases the strength of the random forcing, the component of the power spectrum of the transitions which is at the frequency of the periodic forcing increases to a peak value for a critical value of noise strength, and then decreases with further increases of noise strength until the stochastic forcing completely dominates the state transitions. It is this peaking that inspired the term Stochastic Resonance. There is also a peaking of the signal to noise ratio of the periodic component of the transitions, occurring near the amplitude peak.

The dual well model (Fig. 1) is the one most commonly found in the literature. An equivalent description which is better suited to the purposes of this paper is an input/output or transfer characteristic. Figure 2 shows two examples that should be familiar to the reader which support Stochastic Resonance; that of a two state magnetic device, and of a Schmitt Trigger. Here the Stochastic Resonance phenomenon may be viewed as the response of a two state device driven by periodic and a stochastic signal, wherein the periodic signal alternately increases the probability of a transition from one state to the other, with the noise facilitating the transitions.

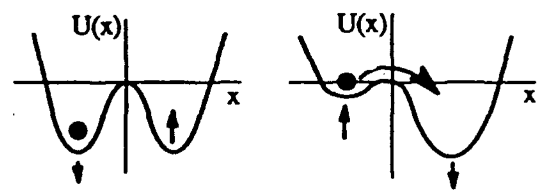

Figure 1. Particle in a Dual Well

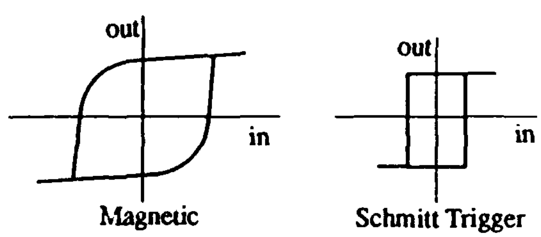

Figure 2. Transfer Characteristics

2: Background

Stochastic Resonance was first reported and analyzed in [1] as an interesting effect in nonlinear dynamics. It was soon after proposed as a possible explanation for the observed periodic occurrences of the earth's ice ages in [2] and [3]. For this case, the two states of the system are taken to be the earth's climate as it is at present and as it is during an ice age. The "noise" of the system is the variation in the absorption or reflection of incident solar energy caused by daily and seasonal weather patterns, cloud cover, ice cover, etc. The weak periodic force is the variation of incident solar energy caused by a small periodic eccentricity in the earth's orbit having a period of 100,000 years.

An experimental measurement of the behavior of a two state electronic device, the Schmitt Trigger, driven by a weak periodic signal in noise, was reported in [4]. The Stochastic Resonance phenomenon was observed, and the location of the resonance peak was shown to be in the vicinity of where the noise intensity had a Kramers rate [5] close to half of the frequency of the periodic signal. The Kramers rate (in the absence of signal) is given by:

$$r = \frac{1}{2\pi}\sqrt{|U''(0)|\ U''(c)}\ exp(-\frac{\Delta U}{\sigma^2}) \qquad (1)$$

where U"(0) and U"(c) are the curvatures of the potential at the barrier and in the wells, respectively, ΔU is the barrier height, and σ is the standard deviation of the noise. White noise is assumed here, and throughout this paper.

Stochastic Resonance has been observed in other physical systems such as ring lasers [6], bi-stable electron paramagnetic resonance systems [7], magnetoelastic ribbons [8], and simulated in bi-stable Superconducting Quantum Interference Devices (SQUIDs) [9]. References [10] and [11] contain a theoretical treatment of the Stochastic Resonance phenomenon based on the so-called adiabatic assumption, and [7] reported the observation of a phase shift occurring in the output of the periodic component as it passed through the output signal to noise ratio peak that is characteristic of Stochastic Resonance. This gives added validity to the use of the term resonance, as similar phase shifts are observed in systems with deterministic resonances. A treatment based on linear response theory is given in [12].

It has been speculated that Stochastic Resonance may be exploited by biological sensor systems as a means of increasing the signal to noise ratio of sensory inputs [13]. Though it has yet to be demonstrated in any known biological system, the elements are there for this to be possible. Sensory neurons are viewed as two state systems (firing or not firing), and noise induced switching has been observed in biological systems: [14], [15], and [16]. Stochastic Resonance can be induced in artificial neural networks [17], and if arrays of bi-stable elements are globally coupled, there are cooperative effects which enhance the phenomenon [18].

Good surveys on the subject of Stochastic Resonance with extensive reference lists can be found in [13], [19], and [20].

3: Stochastic Resonance as a Signal Processing Tool

The question to be addressed here is whether or not it is useful (or desirable) to incorporate a "stochastic resonator" in a signal processing system to aid signal detection. It is well known that the linear filter which maximizes output signal to noise ratio is the "matched filter" [21], [22], [23]. There is no such generalized theory for the class of nonlinear filters so it is possible (though not yet proven or disproved) that a nonlinear filter or operation could create a greater output signal to noise ratio.

The Schmitt Trigger is the "stochastic resonator" device chosen for the evaluation here. It is simple to implement in hardware or software, it reduces dynamics to just switching events, and is a constant output energy device. Thus it is easy to show that if the output signal to noise ratio increases with increasing input noise intensity, the increase in output signal power is at the expense of noise power. A computer simulation consisting of a sinusoid in "white" noise driving a Schmitt Trigger was used for the evaluation. Adjustable parameters were signal amplitude, noise variance, and threshold of the Schmitt Trigger. The power spectra of the Schmitt Trigger input and output were used to measure the input and output signal to noise ratios. Power spectral estimation via Fourier methods is commonly used for detecting sinusoids of unknown frequency and phase in noise, and is the closest approximation to the matched filter achievable for that case with Gaussian noise [22], [23], when the binwidth [24] is matched to the bandwidth of the unknown signal. For these reasons the power spectral estimate was chosen to be the baseline for comparison in this paper.

Figure 3 is a plot of typical input and output power spectra for the simulations. Figures 4 and 5 are averages of the signal to noise ratios at the input and output of the Schmitt Trigger measured from many power spectra for a variety of cases with different signal amplitudes and input noise power. Note that for a single Schmitt Trigger, output signal to noise ratio was always less than that of the input, but was within 2 dB for large noise variance. Thus the Stochastic Resonance process of a single device is lossy as compared to standard spectral estimation techniques.

4: Output Statistics, Non-Poissonian Behavior at High Noise Levels

A Schmitt Trigger driven by zero mean noise will have, under appropriate conditions, switching events that are

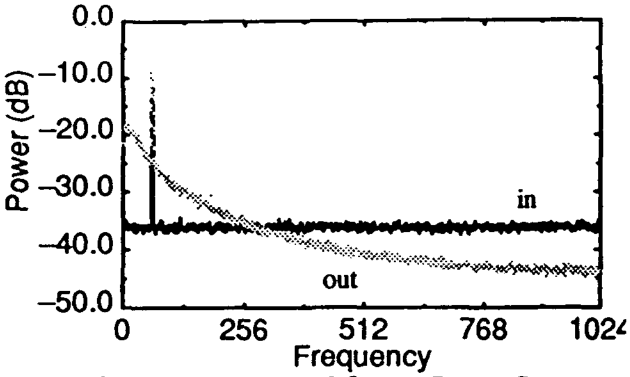

Figure 3. Input (black) and Output Power Spectra

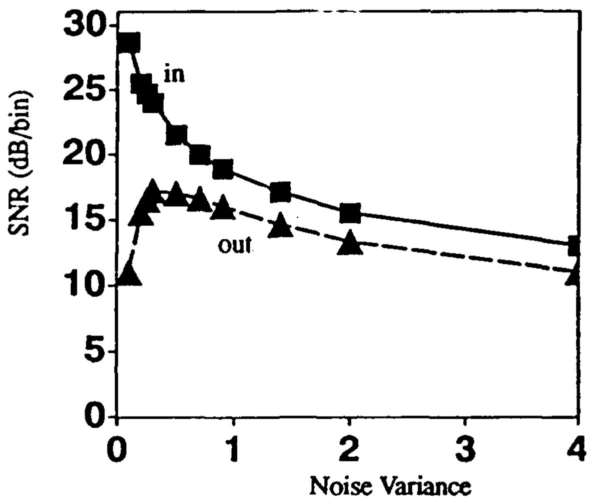

Figure 4. Input and Output Signal to Noise Ratio with Signal Amplitude .5, Threshold 1

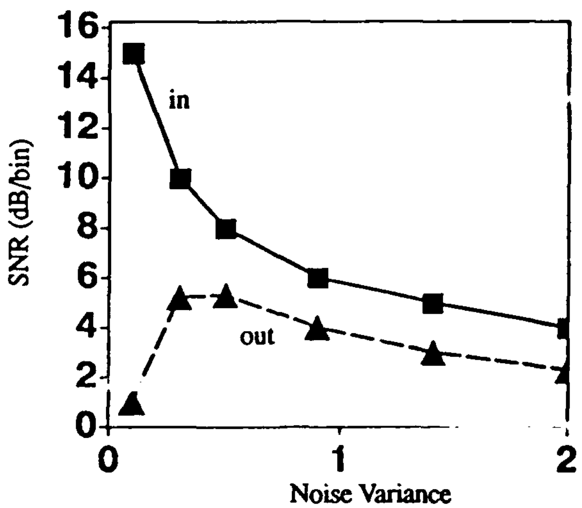

Figure 5. Input and Output Signal to Noise Ratio with Signal Amplitude .1, Threshold 1

Poisson distributed [25]. That is, the probability of k switching events over some time interval t will be:

$$P(k:t) = e^{-\lambda t}\left[\frac{(\lambda t)^k}{k!}\right] \tag{2}$$

where λ is the "rate" of the Poisson process. Then the autocorrelation function of the Schmitt Trigger output will be [25]:

$$R(\tau) = exp[-2\lambda\tau] \tag{3}$$

and its power spectrum will be:

$$S(\omega) = \frac{4\lambda}{4\lambda^2 + \omega^2} \tag{4}$$

Note that the rate, λ, can be estimated from the power spectral estimate of the Schmitt Trigger output since (from (4)):

$$\lambda = \frac{\omega_{3dB}}{2} = \pi f_{3dB} \tag{5}$$

It is noted here that the Poisson Distribution is in fact a limiting case of Bernoulli Trials:

$$P(\text{A occurs k times}) = \begin{bmatrix} n \\ k \end{bmatrix} p^k q^{n-k} \tag{6}$$

where n is the number of trials, p is the probability of occurrence of event A, and $q = 1 - p$. Following the Poisson Theorem, one can approximate (6) by (2) for a continuous time process as $n \rightarrow \infty$, $t \rightarrow 0$, and as $n/t \rightarrow \lambda$. For a discrete time process one can approximate (6) by:

$$P(\text{A occurs k times}) = e^{-np}\left[\frac{(np)^k}{k!}\right] \tag{7}$$

as $n \rightarrow \infty$, and $p \rightarrow 0$. Using (5), (7), and (2), one can check the validity of simulation results, or as will be shown, identify the point where there is a departure from Poisson behavior due to a violation of the assumptions of the Poisson Theorem.

If the noise driving the Schmitt Trigger is zero mean and Gaussian, the probability, p, of a switching event occurring at any instant in time is:

$$p = erfc(Th/\sigma) \tag{8}$$

where Th is the switching threshold of the Schmitt Trigger, and σ is the standard deviation of the noise, with erfc(x) defined as in [22].

Table 1 is a comparison of theoretical vs. experimental (simulated) determination of the Poisson rate, λ, for a Schmitt Trigger with switching thresholds at +/- 1, and the input noise variances shown. Note that the experimental rate is higher than the predicted value for input noise variance greater than 2, or for probability of a switching event greater than .2, due, no doubt, to a violation of Poisson Theorem assumptions.

Variance	Theoretical		Experimental	
	p = erfc(Th/σ)	np	λ	f_{3dB}
0.25	.023	47	45	14
0.5	.08	163	151	48
1.0	.159	327	332	105
2.0	.24	491	622	198
4.0	.39	634	854	272

Table 1. Theoretical vs. Experimental Determination of λ (note: Th = 1, n = 2048)

5: Dithering, Stochastic resonance, Coarse Quantization

Dithering is a technique used to ameliorate undesirable nonlinearities in a system. It consists of adding a small amount of (white) noise to a system. In digital signal processing, adding noise to the least significant bit of a quantized signal is often used to reduce the nonlinear affects of the quantization. It has been reported that dithering can allow the detection of signals smaller in amplitude than the least significant bit [26], or, for image processing, allow the detection of image features less intense than the lowest quantization level.

It was suggested in [26] that the ability to detect these under-resolved signals was due to a "rounding" of the corners of the quantization steps which would create a linearization of the stepped transfer characteristic from input to output of the quantization process. This could be the case if the noise were multiplicative, however this is not possible with a linear summation of noise. A much more plausible explanation is that there is a mechanism similar to Stochastic resonance which allows this detection of under-resolved signals. The diagram in Figure 6 illustrates this mechanism: an under-resolved sinusoid added to noise of appropriate intensity will alternately increase the probability of transition to a higher or lower quantization state at its high and low peaks, and otherwise tend to stay in the current quantization state.

Continuing this line of reasoning, a few simulations were run to compare the processing gain (loss) of coarse quantization to that of a Schmitt Trigger. Note that the key difference between the transfer characteristics of the two is that the Schmitt Trigger has hysteresis while the "staircase" characteristic of the quantizer is closer to being linear. The results of this cursory test indicated that the coarse quantizer outperformed the Schmitt Trigger, and matched the performance of a finely quantized simulation until a point just above the threshold of detection for the fine case. An exhaustive study or analysis of this was not done.

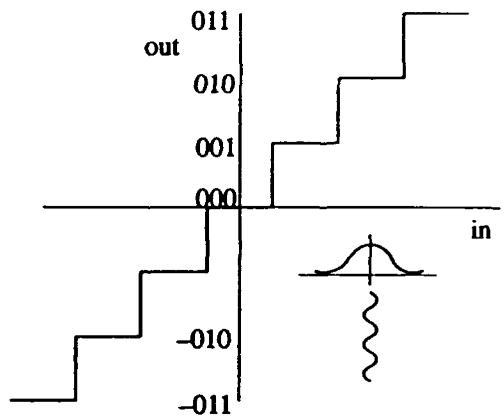

Figure 6. Quantization and Dithering

6: Coupled Bi-Stable Devices

A study of the influence of a "bath" of overdamped bistable oscillators coupled to a single bi-stable oscillator can be found in [18], with a more detailed analysis forthcoming in [27]. Equation (9) is the model for the oscillators, coupling, and periodic plus noisy forcing that was considered:

$$C_i \dot{u}_i = \sum_{j=1}^{N} J_{ij} \tanh u_j - \frac{u_i}{R_i} + F_i(t) + q \sin \omega t \qquad (9)$$

where the summation term defines the coupling, and the last two terms are the noisy and periodic forcing respectively. The variables u_i denote the state variables (analogous to the membrane potentials in a biological neuron application) of each element, and R_i and C_i represent the input capacitance and transmembrane resistance of each.

If one assumes the time constant of the element of interest to be much longer than that of the others, ie.:

$$R_1 C_1 >> R_i C_i \quad \text{for} \quad i > 1 \qquad (10)$$

and if the frequency of the periodic forcing, ω, is constrained to be less than the Kramers rate of the system when driven only by the noise, then one can adiabatically eliminate the "bath" variables (u_i, for $i > 1$) from Equation (9) to yield a reduced model of the following form [18], [28]:

$$\dot{u}_1 = -\alpha u_1 + \beta \tanh u_1 + \delta \sin \omega t + \sigma_e F(t) \qquad (11)$$

Simulations of this reduced model have been done, and enhancements of output signal to noise ratio (as compared to what is observed with a *single oscillator*) have been achieved. The behavior of the system is very sensitive to the choice of system parameters (coupling, time constants, etc.) and signal parameters (input noise, bath noise, etc.). Investigations into the applicability of this phenomenon to signal processing are continuing, including the generation of a simulation of the full dynamics represented by Equation (9).

7: Conclusion

Compared to standard power spectral estimation techniques, the use of a single "stochastic resonator" device will result in a processing loss of at least 2 dB in the region of signal to noise ratio where the "resonator" performs best. In addition, there does not appear to be any advantage to utilizing a single "resonator" device where signals are underresolved due to coarse quantization. There is, however, evidence which suggests that use of an array of globally coupled devices may be able to overcome the processing loss of a single device, and perhaps exceed the gains achievable with linear signal processing systems.

Acknowledgements

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