Laws of Infrared Similitude

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## Laws of Infrared Similitude

Accurate physical scale models of complex systems are found to be of great use in fields as diverse as aerodynamics, fluid mechanics, radar, or nuclear physics. It now appears that the concept of scale modeling may offer distinct advantages to those interested in the study of thermodynamic processes that occur in large physical structures. In this investigation, it is shown from a study of the heat equation and its boundary conditions that physical scale modeling can be used to simulate realistic systems operating in realistic outdoor environments. The thermal properties of the construction materials used in the system under study are allowed to vary with position, thus allowing the structure to be divided into a number of compartments. Physical effects which involve heat exchange between the structure and the ocean, or with the atmosphere, are investigated. Both time-dependent and time-independent cases are examined. (Continued on reverse side)

### Abstract (Continue on reverse if necessary and identify by block number)

Accurate physical scale models of complex systems are found to be of great use in fields as diverse as aerodynamics, fluid mechanics, radar, or nuclear physics. It now appears that the concept of scale modeling may offer distinct advantages to those interested in the study of thermodynamic processes that occur in large physical structures. In this investigation, it is shown from a study of the heat equation and its boundary conditions that physical scale modeling can be used to simulate realistic systems operating in realistic outdoor environments. The thermal properties of the construction materials used in the system under study are allowed to vary with position, thus allowing the structure to be divided into a number of compartments. Physical effects which involve heat exchange between the structure and the ocean, or with the atmosphere, are investigated. Both time-dependent and time-independent cases are examined. (Continued on reverse side)
The scaling laws of thermodynamic similitude which govern the exchange of infrared radiation between any structure and its environment are derived. These scaling laws are then used to obtain a set of dimensionless variables which transform the heat equation and its boundary conditions into manifestly scale invariant form. In this form the heat equation applies to a scale model of arbitrary size. Consideration of the "temperature" function solution to the scale invariant heat equation is applied to the design of an actual physical scale model and to the simulation of a background environment in which the thermal properties of the model are to be measured. Methods for the calibration and validation of laboratory measurements using actual field measurements are described. The scaling laws that are derived apply to realistic physical structures interacting with an outdoor environment. In particular, they may be used to accurately describe such structures as aircraft in flight, ships at sea, tanks, or even buildings.
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ABSTRACT

Accurate physical scale models of complex systems are found to be of great use in fields as diverse as aerodynamics, fluid mechanics, radar, or nuclear physics. It now appears that the concept of scale modeling may offer distinct advantages to those interested in the study of thermodynamic processes that occur in large physical structures. In this investigation, it is shown from a study of the heat equation and its boundary conditions that physical scale modeling can be used to simulate realistic systems operating in realistic outdoor environments. The thermal properties of the construction materials used in the system under study are allowed to vary with position, thus allowing the structure to be divided into a number of compartments. Physical effects which involve heat exchange between the structure and the ocean, or with the atmosphere, are investigated. Both time-dependent and time-independent cases are examined. The scaling laws of thermodynamic similitude which govern the exchange of infrared radiation between any structure and its environment are derived. These scaling laws are then used to obtain a set of dimensionless variables which transform the heat equation and its boundary conditions into manifestly scale invariant form. In this form the heat equation applies to a scale model of arbitrary size. Consideration of the "temperature" function solution to the scale invariant heat equation is applied to the design of an actual physical scale model and to the simulation of a background environment in which the thermal properties of the model are to be measured. Methods for the calibration and validation of laboratory measurements using actual field measurements are described. The scaling laws that are derived apply to realistic physical structures interacting with an outdoor environment. In particular, they may be used to accurately describe such structures as aircraft in flight, ships at sea, tanks, or even buildings.

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INTRODUCTION

The purpose of this investigation is to obtain a better understanding of how thermal radiation is exchanged between a physical structure and its environment. At present, such studies are performed using experimental data and calculated results based upon computer codes. It is not generally recognized that valuable information can be obtained from laboratory measurements made on scale models. Much can be learned from the uses to which scale models have long been put in other fields of study; among others, in the areas of radar, acoustics, fluid mechanics, and aerodynamics. In the field of thermal radiation as well, it is thought that scale models can be a valuable investigation resource. The use of scale models has the potential for cost effectively enhancing the value of field measurements and computer code calculations. For example, scale-model studies may be used to verify those carried out in the field and provide important consistency checks with which computer codes must comply. This should lead to improvements in present-day computer codes based upon a more complete representation of the physical laws that give rise to the "temperature" function, which ultimately is responsible for the way a physical system radiates in the infrared.

The feasibility of scale modeling is examined theoretically by analyzing the heat equation. This equation, together with the appropriate boundary conditions, offers a physically accurate description of all the convective and radiative processes that take place whenever a physical structure exchanges heat with its surroundings. For instance, one such physical structure may be a ship sailing through the ocean, an airplane flying through the air, or a tank moving through a smoke-charged wind. The equations are written to be applicable to thermodynamic systems that include compartments made from materials that may vary so that the thermal parameters that characterize these materials can also vary. The conclusions from this investigation of the heat equation cover the time-dependent as well as the time-independent cases. The results of analyzing the heat equation are the thermodynamic scaling laws appropriate for thermodynamic scaling. In the case of a ship, for example, the scaling laws enable experimental results from scale models to be compared with those from full-size ships at sea. The scaling of temperature, position, and time is related to scaling of heat sources, boundary conditions (including initial conditions), and material thermodynamic parameters. Thus, true scaling of a large ship involves much more than simply building a small geometrical replicate. Thermodynamic similitude requires that response times, heat loads, thermal properties of ship materials and of fluids in the ship's surroundings be taken into account as well.

Furthermore, the scaling laws suggest a set of dimensionless variables that transform the heat equation into a manifestly scale invariant form, which describes all geometrically and thermodynamically similar ships no matter what their sizes. Such a description is hence size-independent. The set of dimensionless variables define the minimum number of measurements that fix the "temperature function" and, therefore, define the most cost-effective procedure for the use of scale models. The set of dimensionless variables, once obtained, are in themselves implementation criteria.
serving to support a wide variety of applications which may be envisioned, such as the setting of thermal radiation goals for surface ships, the study of anti-ship seekers, engineering design choices related to ship size, and physical concepts that should lead to the next generation of thermal computer programs.

**THERMAL (INFRARED) SCALING LAWS: THE TIME-DEPENDENT CASE**

Scaling laws appropriate to a structure based on compartments made from materials with thermal properties that are allowed to vary from place to place are derived. The compartments are filled with fluids (air, water, oil, etc.). Within the compartments both convective and radiative heat transfer processes occur. All conductive processes that occur throughout the structure are described by the heat equation, Equation (1). Of all possible solutions to the heat equation, those applicable to a particular structure, in a particular environment, are selected by the prevailing boundary conditions. The heat transfer processes between compartments and their enclosing conductive walls are accounted for by the boundary conditions that are imposed. That is, the fluid-filled space of compartments are, from the heat equation describing the heat conduction processes in such a system, part of the internal thermodynamic surroundings. The fluids on the outside (the air of the atmosphere and the water of the ocean) compose the external thermodynamic surroundings. The total thermodynamic surroundings (the internal and external portions together) are separated from the conductive thermodynamic system by a surface termed the thermodynamic boundary. The heat exchange processes that occur connecting the thermodynamic system to any portion of its surroundings are described by means of appropriate boundary conditions. The heat equation contains, as parameters, the thermodynamic properties of materials from which the structure is made. To be as realistic as possible, it is necessary to consider these thermodynamic properties to be completely general functions of position.

The mathematical structure containing the thermodynamic description above is given by a form of the heat equation and boundary conditions that can be displayed as:

\[ \nabla \cdot [k(x) \nabla T(x,t)] + H(x,t) = \rho(x) c(x) \frac{\partial T(x,t)}{\partial t} \]  \hspace{1cm} (1)

\[ k(x) \nabla_n T(x,t) = h(x) \Delta T(x,t) \quad \text{for all } x \subset b \]  \hspace{1cm} (2)

\[ T(x,t) \big|_{t=0} = T_0(x) \quad \text{for all } x \]  \hspace{1cm} (3)
The first equation is the heat equation, the second is a boundary condition, and the third is an initial condition. The various symbols are defined as:

\begin{align*}
    b &= \text{boundary} \\
    \nabla &= \text{three-dimensional gradient operator} \\
    \nabla_n &= \text{operator yielding gradient magnitude in direction normal to boundary} \\
    k &= \text{thermal conductivity (power/distance/temperature)} \\
    T &= \text{temperature at position } x \\
    x &= \text{coordinates of a point in three-dimensional position space} \\
    t &= \text{coordinates of a point in time} \\
    H &= \text{rate of heat production per unit volume at position, } x, \text{ and time, } t, \text{ (power/volume)} \\
    \rho &= \text{mass density at position } x \text{ (mass/volume)} \\
    c &= \text{specific heat at position } x \text{ (heat/temperature/mass)} \\
    h &= \text{heat transfer coefficient at position, } x, \text{ belonging to boundary, } b \text{ (power/area/temperature differential)} \\
    \Delta T &= \text{temperature difference prevailing between ship and its surroundings at position, } x, \text{ and time, } t \\
    T_0 &= \text{initial temperature distribution that holds at every position, } x, \text{ throughout the ship}
\end{align*}

The solution of Equations (1) to (3) yields a temperature function

\[ T = T(x, t, H, k, \rho, c, h, \Delta T) \]  

which evolves from its initial value \( T_0 \). It is a function of position and time which depends upon the heat sources, the material thermodynamic properties, the heat transfer coefficient, and the temperature differences between system and surroundings.

The material thermodynamic properties, \( k, \rho, \) and \( c \), are completely general functions of position, as is the heat transfer coefficient, \( h \). Note that the boundary condition, Equation (2), is Newton's law of heat transfer and properly describes natural convection and forced convection to fluid surroundings (air, water, oil, etc.) as well as radiation in accordance with Planck's frequency distribution or Stefan's law. In brief, the complex processes of heat transfer between a ship with realistic architecture and materials and its fluid surroundings, both internal and external, are accurately described by the heat equation and its companion boundary conditions.
The scaling laws are obtained by applying the heat equation and boundary conditions to both a ship, \( S \), and its physically scaled model, \( M \), and by stipulating that the equations hold in both instances. Geometric similarity of model and ship is obtained by enforcing the relation between analogous points \( x^M \) and \( x^S \) of model and ship

\[
K_x = \frac{x^M}{x^S}
\]  

(5)

where \( K_x \) is a scaling constant, namely that of position. Complete infrared similarity of model and ship is obtained by also requiring analogous relations that involve all pertinent quantities from the heat equation and boundary conditions. Thus, Equations (6) through (13) define the scaling constants of time, temperature, conductivity, mass density, specific heat, heat transfer coefficient, temperature difference, and initial temperature, respectively.

\[
K_t = \frac{t^M}{t^S}
\]  

(6)

\[
K_T = \frac{T^M}{T^S}
\]  

(7)

\[
K_k = \frac{k^M}{k^S}
\]  

(8)

\[
K_\rho = \frac{\rho^M}{\rho^S}
\]  

(9)

\[
K_c = \frac{c^M}{c^S}
\]  

(10)

\[
K_h = \frac{h^M}{h^S}
\]  

(11)
Equations (1) through (3) applied to a ship yield:

\[ \nabla^S \cdot (k^S \nabla^S T^S) + H^S = \rho^S c^S \frac{\partial T^S}{\partial t^S} \]  

(14)

\[ k^S \nabla^S T^S = h^S \Delta T^S \quad \text{for all} \ x^S \subset b^S \quad \text{(boundary of full-scale ship)} \]  

(15)

\[ T^S \big|_{t^S = 0} = T_o^S \quad \text{for all} \ x^S \]  

(16)

where all symbols retain their previous meanings. Superscript "S" indicates quantities pertaining to a ship, and for brevity of notation space and time dependence is not shown explicitly although it is implicitly understood. Similarly, for an analogous model,

\[ \nabla^M \cdot (k^M \nabla^M T^M) + H^M = \rho^M c^M \frac{\partial T^M}{\partial t^M} \]  

(17)

\[ k^M \nabla^M T^M = h^M \Delta T^M \quad \text{for all} \ x^M \subset b^M \quad \text{(boundary of scale model)} \]  

(18)

\[ T^M \big|_{t^M = 0} = T_o^M \quad \text{for all} \ x^M \]  

(19)

By invoking the scaling constants, Equations (6) through (13), the ship equations can be rewritten as:

\[ K_{\Delta T} = \frac{\Delta T^M}{\Delta T^S} \]  

(12)

\[ K_{T_o} = \frac{T_o^M}{T_o^S} \]  

(13)
\[
\frac{K_T}{K_x} K_T \nabla M \cdot (k M \nabla M T^M) + K_B H^M = \frac{K_T}{K_r} \frac{K_T}{\partial r^M} \quad (20)
\]

\[
\frac{K_T}{K_x} k M \nabla M T^M = K_h K_{\Delta h} h^M \Delta T^M \quad \text{for all } x^M \subset b^M \quad (21)
\]

\[
K^T T^M |_{x^M = a^M} = K_{T_o} T^M \quad \text{for all } x^M \quad (22)
\]

But by insisting that the ship equations, (20) to (22), and the model equations, (17) to (19), must both hold, requires that:

\[
K \frac{K_T}{\rho_c} \frac{K_T}{K_x^2} = 1 \quad (23)
\]

\[
K_h \frac{K_T}{K_{\rho c} K_T} = 1 \quad (24)
\]

\[
K_h \frac{K_T}{K_x} \frac{K_{\Delta T}}{K_T} = 1 \quad (25)
\]

\[
\frac{K_T}{K_{T_o}} = 1 \quad (26)
\]

The abbreviating definitions are obvious in context.

Equations (23) through (26) are the thermal scaling laws for the time-dependent case. They connect the temperatures to scaling of time and distance, conductivity, density, specific heat, heat sources, heat transfer coefficients, temperature differences between boundaries and surroundings, and initial temperature distributions. These laws apply to realistic naval ship architectures and materials in the context of heat transfer mechanisms including conduction, convection, and radiation. They also are the basis upon which an IR physical scale modeling measurements program can be designed.
DIMENSIONLESS VARIABLES ASSOCIATED WITH THERMAL (INFRARED) PHYSICAL SCALE MODELING

Essentially, physical scale models can be built to thermally behave like full-scale ships because the thermodynamic equations can be cast entirely in terms of dimensionless variables. By definition, these are variables with magnitudes completely independent of size, and they are exactly the same applied to a ship and to physical scaled models. An appropriate set of dimensionless variables and parameters is immediately suggested by the form of the scaling laws previously derived. From such dimensionless quantities, the entire set of thermodynamic equations can be cast into dimensionless form. Thus, a ship and its scaled model behave according to equations manifestly independent of size. This leads to the logical design of a physical scale modeling measurements laboratory because the physical variables that such a laboratory must control and measure include only the dimensionless quantities themselves. True physical scale modeling is therefore tantamount to a laboratory arrangement that fixes all dimensionless quantities to be the same values on a model and the ship it represents by virtue of displaying precisely the same IR dynamics as the ship itself.

To obtain appropriate dimensionless quantities, the heat equation and its boundary conditions are cast in dimensionless form. The dimensionless quantities required by the scaling of the IR problem are:

\[ \bar{x} = \frac{x}{L} \quad (27) \]

\[ \bar{t} = \frac{k \, t}{\rho c \, L^2} \quad (\text{Fourier number}) \quad (28) \]

\[ \bar{T} = \frac{T}{T_o} \quad (29) \]

\[ \bar{H} = \frac{HL^2}{kT_o} \quad (30) \]

\[ \bar{h} = \frac{hL}{k} \quad (\text{Nusselt number}) \quad (31) \]

where the bar symbol indicates a dimensionless quantity, and \( L \) represents a "typical" length; all other symbols are as previously defined. The five dimensionless variables
in Equations (27) to (31) are suggested immediately by the five scaling laws derived, viz., the condition of geometric similarity, Equation (5), and the four conditions of thermodynamic similarity which follow from the heat equation and its boundary conditions, Equations (23) to (26).

Thus, \( x \), which is consistent with Equation (5), is simply a reformulation of the condition of geometric similarity, and is obviously dimensionless. In \( \bar{t} \), a definition is immediately suggested by the cluster of subscripts in Equation (23). That \( \bar{t} \) is, in fact, dimensionless follows from a detailed inspection of units carried by the quantities contained in its definition. \( \bar{T} \) is suggested by the cluster of subscripts in Equation (26), and also is obviously dimensionless. \( \bar{H} \) is suggested by the subscript-cluster in Equation (24), writing \( t \) and \( T \) in terms of their dimensionless analogues. Thus,

\[
\frac{H}{\rho c} \frac{t}{\bar{T}} = \frac{H}{\rho c} \frac{\bar{t} \rho c L^2}{k} \frac{1}{\bar{T} T_o}
\]

which simplifies to

\[
\frac{H}{\rho c} \frac{t}{\bar{T}} = \frac{H L^2}{k T_o} \frac{\bar{t}}{\bar{T}}
\]

In Equation (33) the coefficient of dimensionless \( \bar{t}/\bar{T} \) defines \( \bar{H} \), whose own form is dimensionless as follows from a detailed inspection of units carried by the quantities entering its definition. Similarly, \( \bar{H} \) is suggested by the cluster of subscripts contained in Equation (25), from which:

\[
\frac{h}{k} \frac{L}{T} \frac{\Delta T}{\bar{T}} = \frac{h L}{k} \frac{T_o \Delta \bar{T}}{T_o \bar{T}}
\]

and on simplifying,

\[
\frac{h}{k} \frac{L}{T} \frac{\Delta T}{\bar{T}} = \frac{h L}{k} \frac{\Delta \bar{T}}{\bar{T}}
\]

Here \( \bar{H} \) as appears as the coefficient of dimensionless \( \Delta \bar{T}/\bar{T} \). Inspection indicates \( \bar{H} \) also is dimensionless, as required. Thus, the form of the IR scaling laws suggests the form of the dimensionless quantities, which are appropriate to the scaling of IR.
All of the dimensionless quantities in Equations (27) through (31), have well-defined physical interpretations. The Fourier number measures the ratio of heat conducted through a volume element to that "stored," causing an increase in temperature. The Nusselt number $\bar{H}$ measures the ratio of heat "convected" from a surface to that conducted through the surface. Less recognizable, $\bar{H}$ for an elemental volume is the ratio of heat from a source present to that "stored," causing an increase in temperature. More obviously, $\bar{x}$ is a measure of position in units of a representative length, and $\bar{T}$ of temperature in units of the initial temperature.

Using the dimensionless quantities suggested by the scaling laws naturally transforms the thermodynamic description of a ship interacting with its surroundings into one that is dimensionless, i.e., completely independent of size or scale. Using the relations

$$\nabla = \frac{1}{L} \vec{\nabla}$$

(36)

and

$$\frac{\partial}{\partial t} = \frac{k}{\rho c L^2} \frac{\partial}{\partial t}$$

(37)

which follow from Equations (27) and (28) and replacing $T$, $H$, and $h$ with their equivalents in terms of $\bar{T}$, $\bar{H}$, and $\bar{h}$ given by Equations (29) to (31), Equations (1) to (3) become:

$$\bar{\nabla}^2 \bar{T} + \frac{\bar{\nabla} k}{k} \bar{\nabla} \bar{T} + \bar{H} = \frac{\partial}{\partial t} \bar{T}$$

(38)

$$\bar{\nabla}_a \bar{T} = \bar{h} \Delta \bar{T} \quad \text{for all } \bar{x} \subset b$$

(39)

and

$$\bar{T}|_{\bar{r}=0} = 1 \quad \text{for all } \bar{x}$$

(40)

Equations (38) to (40) are the full information equivalents of the heat equation and boundary conditions, but are now in manifestly scale invariant form. Every term is dimensionless, thus corroborating the choice of dimensionless variables and parameters, Equations (27) to (31), suggested by the form of the IR scaling laws.
The scale invariant heat equation and boundary conditions imply the existence of a "temperature" function,

$$\bar{T} = \bar{T}(\bar{X}, \bar{T}, \bar{H}, \bar{h}, \Delta \bar{T})$$  \hspace{1cm} (41)

which evolves from its initial value of unity. It is a function of "position" and "time" which depends parametrically upon the "heat sources," the "heat transfer coefficient," and the "temperature differences" between systems and surroundings. This expression for the "temperature" is the analogue of the temperature solution given by Equation (4), but here given in manifestly scale invariant form. Moreover, \(\bar{x}\) displays an efficient reduction in the number parameters upon which it depends, when compared to the case of \(T\).

The temperature as given by Equation (4) depends upon eight parameters, viz., \(x, t, H, k, \rho, \sigma, c, h, \) and \(\Delta T\); whereas, the scale invariant "temperature" is seen to depend upon only five scale invariant parameters, viz., \(\bar{x}, \bar{T}, \bar{H}, \bar{h}, \) and \(\Delta \bar{T}\). The matter of reducing the number of parameters from eight to five is of importance to the feasibility of any measurements program which seeks to effectively study, experimentally, the solutions of the heat equation.

Reviewing the development thus far reveals that an IR physical scale modeling program for ships is a feasible undertaking. A ship's temperature function, together with the emissive/reflective properties of the outer skin, ultimately determines, through Planck radiation, the IR signal emitted by a ship. Thus, the problem of studying the IR emission of a ship is reducible to obtaining the temperature function of the ship. But this is given as a solution of the heat equation for the ship together with its companion boundary conditions. In order to reduce the problem of studying IR ship emissions to laboratory manageability, the heat equation must equally apply to a ship and its geometrically similar physical scale model. The conditions that a point-by-point comparison, in space and time, between a ship and model shall hold are embodied in the IR scaling laws. But once a model is built in accordance with the scaling laws another logical question arises; viz., what is the most efficient set of measurements for the study of the IR defining temperature function? The answer is obtained by casting the heat equation and its boundary conditions into dimensionless form. The solution is a temperature function parametrically dependent upon a minimal number of dimensionless quantities. In this way, the laboratory effort required to study the temperature function is optimized. Physical scale modeling in the IR becomes, therefore, a laboratory arrangement to equate the appropriate dimensionless quantities of a scale model to those of the actual ship under study. Thus, physical scale modeling is seen to be an activity which is entirely feasible in accordance with standard thermodynamic practice, and holds promise of being an efficient, cost-effective means of studying the IR contrast.
The following discussion examines particular cases in which certain dimensionless variables are associated with convection and radiation exchange at a boundary.

True scale modeling implies that the Nusselt number, \( \bar{h} \), defined in Equation (31), shall be fixed at the same value in both model and ship. The magnitude of \( \bar{h} \) controls the heat exchange mechanisms everywhere on the boundary, as may be seen from the boundary conditions, Equation (39). The temperature function, \( \bar{T} \), is parametrically dependent on \( \bar{h} \), as indicated in Equation (41).

The two principal mechanisms of heat exchange at a ship boundary are those of convection and radiation. Convection occurs between the ship and the sea, and between the ship and the air/atmosphere. Depending upon operating conditions and environmental conditions, free convection or forced convection may apply, with either laminar or turbulent flow conditions. Radiation exchange occurs between the ship and the air/atmosphere and between the ship and the sun. All of the heat exchange mechanisms can be occurring simultaneously at a given position, or separately at different segments of a ship. Thus, all of the mechanisms must be under the experimental control of the scale model laboratory. The Nusselt number \( \bar{h} \) can be written as a superposition of convection and radiation terms. Hence,

\[
\bar{h} = \bar{h}_c + \bar{h}_r
\]  

(42)

It can be established purely by dimensional analysis that the convective contribution to the Nusselt number has a functional dependence given by

\[
\bar{h}_c = \bar{h}_c (Re, Pr, Gr)
\]  

(43)

where the variables are respectively, the Reynolds number, the Prandtl number, and the Grashof number. Each of these is a dimensionless number and, therefore, \( \bar{h}_c \) embodies convective effects in a scale invariant form. The dimensionless quantities are defined as:

\[
Re = \frac{\rho L v}{\eta} \quad \text{(Reynolds number)}
\]  

(44)

\[
Pr = \frac{\eta c}{k} \quad \text{(Prandtl number)}
\]  

(45)
Grashof number \( Gr = \frac{\beta g \Delta T L^3 \rho^2}{\eta^2} \) (Grashof number) (46)

where:
- \( \rho \) = fluid mass density
- \( L \) = representative length
- \( v \) = fluid free-stream velocity
- \( \eta \) = fluid velocity
- \( c \) = fluid heat capacity
- \( k \) = thermal conductivity
- \( \beta \) = thermal expansion coefficient
- \( g \) = acceleration constant due to gravity
- \( \Delta T \) = temperature differential

Each of the dimensionless numbers defined in Equations (43) through (46) has a well-defined physical significance. The convective portion of the Nusselt number, \( \overline{h}_c \), is a measure of the ratio of the flux of power convected to that which would be conducted were the fluid at rest. The Reynolds number, \( Re \), which fixes the flow pattern around a solid body in a moving fluid, measures the ratio of the flux of fluid momentum to the flux of fluid viscous drag. The Prandtl number, \( Pr \), relates the temperature distribution in a fluid to its velocity distribution, and is a measure of the ratio of the kinematic viscosity, \( \eta/\rho \), to the thermal diffusivity, \( k/\rho c \). The Grashof number, \( Gr \), fixes the relative importance of buoyancy and viscous forces, and is a measure of the ratio of fluid buoyancy force per area to viscous drag per area. In the case of pure free convection, the Reynolds number is inconsequential, and the convective Nusselt number, Equation (43), simplifies to

\[ \overline{h}_c = \overline{h}_c (Pr, Gr) \] (47)

Similarly, for pure forced convection, the Grashof number drops out of consideration yielding

\[ \overline{h}_c = \overline{h}_c (Re, Pr) \] (48)

This last expression holds for either laminar or turbulent flow, although the specific functional dependence will change between the two types of flow. If the fluid to which this last pair of equations is applied is a gas, a further simplification occurs.
Over a wide range of temperature, the Prandtl number is practically the same for any gas and therefore may be neglected, obtaining from Equations (47) and (48):

\[ h_c = h_c (Gr) \]  \hspace{1cm} (49)

and

\[ h_c = h_c (Re) \]  \hspace{1cm} (50)

for gases only.

The radiation contribution to the Nusselt number has the functional dependence

\[ h_r = h_r (St) \]  \hspace{1cm} (51)

where the variable is the Stefan number that is dimensionless and, therefore, \( h_r \) embodies radiative effects in a scale invariant form. The Stefan number is defined as:

\[ St = \frac{\sigma e T^4 L}{k} \]  \hspace{1cm} (Stefan number)  \hspace{1cm} (52)

where:

\( \sigma \) = Stefan constant from the \( T^4 \) radiation law

\( e \) = emissivity

\( T \) = temperature

\( L \) = representative length

\( k \) = thermal conductivity

The physical significance of the Stefan number is that it measures the ratio of heat radiated to that conducted per unit area.

As stated, true physical scale modeling requires equating the total Nusselt number, Equation (31), in ship and model. Convective and radiative exchange at the boundaries each contribute to the overall Nusselt number. In the most general case, these contributions may be accounted for through control of four dimensionless quantities, viz., the Reynolds number, the Prandtl number, the Grashof number, and the Stefan number, of Equations (43) and (51). In the particular limiting cases
discussed, simplification occurs requiring control over fewer dimensionless quantities, as in Equations (47) to (50).

In addition to equating the overall Nusselt number, $\bar{N}$, applied to ship and model, the relative contributions of convective and radiative effects should also be represented properly. True scale modeling therefore entails separately equating $\bar{N}_c$ in ship and model as well as $\bar{N}_r$.

**THERMAL (INFRARED) SCALING LAWS: THE TIME-INDEPENDENT CASE**

To derive the scaling laws appropriate for a ship in a thermodynamic steady state, the formalism and definitions used in the time-dependent case are retained. The physical nature of the ship and its energy exchange processes with its surroundings are as previously discussed, except that changes in time are not considered; no transients are allowed. In particular, the ship is made of materials whose conductivity is allowed to vary from place to place, and has a naval architecture providing for a variety of compartments filled with fluids (air, water, oil, etc.).

The mathematical structure capturing the thermodynamic description in this steady state case is:

$$\nabla \cdot [k(x)\nabla T(x)] + H(x) = 0$$  \hspace{1cm} (53)

$$k(x)\nabla_n T(x) = h(x)\Delta T(x) \quad \text{for all } x \in b$$  \hspace{1cm} (54)

The solution of the time-independent heat equation and its companion boundary condition, Equations (53) and (54), respectively, yields a temperature function

$$T = T(x, H, k, h, \Delta T)$$  \hspace{1cm} (55)

which is a function of position only, and depends parametrically upon the heat sources, the thermodynamic conductivity, the heat transfer coefficient, and the temperature differences between system and surroundings. Note that $k$ and $h$ are general functions of position, and Newton's law again describes heat transfer mechanisms at the boundaries, e.g., convection and radiation.

As before, the time-independent scaling laws are extracted from the heat equation and its boundary conditions simply by applying them to both a ship and its physically scaled model. Hence, for a ship:
\[ \nabla^S \cdot (k^S \nabla^S T) + H^S = 0 \quad (56) \]

\[ k^S \nabla^S T^S = h^S \Delta T^S \quad \text{for all } x^S \subset b^S \quad (57) \]

and, for its analogous model:

\[ \nabla^M \cdot (k^M \nabla^M T^M) + H^M = 0 \quad (58) \]

\[ k^M \nabla^M T^M = h^M \Delta T^M \quad \text{for all } x^M \subset b^M \quad (59) \]

where all symbols in Equations (56) to (84) retain their previous meanings.

Invoking the scaling relations, Equations (5) through (13), the ship equations become:

\[ \frac{K_k K_T}{K_x^2} \nabla^M \cdot (k^M \nabla^M T^M) + K_H H^M = 0 \quad (60) \]

and

\[ \frac{K_k K_T}{K_x} k^M \nabla^M T^M = K_h K_{\Delta T} h^M \Delta T^M \quad \text{for all } x^M \subset b^M \quad (61) \]

For the ship equations, (60) and (61) and the model Equations (58) and (59) to both hold, it must be that

\[ \frac{K_H K_x^2}{K_k K_T} = 1 \quad (62) \]

\[ \frac{K_h K_x K_{\Delta T}}{K} = 1 \quad (63) \]

These are important scaling laws that must hold in the steady state case. They connect the temperatures to the scaling of distance, conductivity, heat sources, heat transfer coefficients, and temperature differences between boundaries and
surroundings. These laws are the basis for the design of physical scale modeling measurements in the steady state, and they supply restrictions to which computer models must conform.

The dimensionless variables appropriate to the steady state case are especially easy to obtain. They are already defined in Equations (27) through (31), with the exception of Equation (28) that depends explicitly on time. These equations now become:

\[ \bar{x} = \frac{x}{L} \] (64)

\[ \bar{T} = \frac{T}{T_o} \] (65)

\[ \bar{H} = \frac{HL^2}{kT_o} \] (66)

\[ \bar{h} = \frac{hL}{k} \] (67)

Here \( T_o \) is redefined to mean an average background temperature. The prime in the subscript symbol is a reminder of the new definition imposed. The physical interpretations previously given for the dimensionless quantities, Equations (64) to (67), are retained. As expected, these dimensionless quantities are adequate to transform the steady state thermodynamic description of a ship exchanging heat with its surroundings to one which is scale invariant.

Using

\[ \nabla = \frac{1}{L} \bar{\nabla} \] (68)

and replacing \( T, H, \) and \( h \) with their equivalents in terms of \( T, H, \) and \( \bar{H} \), Equations (65) to (67), Equations (53) and (64) become:

\[ \bar{\nabla}^2 \bar{T} + \frac{\bar{k}}{k} \bar{\nabla} \bar{T} + \bar{H} = 0 \] (69)
\[ \nabla_n \overline{T} = \overline{h} \Delta \overline{T} \text{ for all } \overline{x} \subset b \tag{70} \]

Of course, Equations (69) and (70) are the information equivalents of the steady state heat equations and boundary conditions; however, in manifestly scale invariant form, they imply the existence of a "temperature" function

\[ \overline{T} = \overline{T} (\overline{x}, \overline{H}, \overline{h}, \Delta \overline{T}) \tag{71} \]

which is a function of "position," and depends parametrically upon the "heat sources," the "heat transfer coefficient," and system/surrounding "temperature differences." This "temperature" is in manifestly invariant form and contains an optimal number of experimental parameters to be controlled in scale modeling measurements.

The dimensionless variables associated with convection and radiation exchange at the ship boundaries, Equations (42) through (52), carry over to the time-independent case without change, except that all boundary conditions conform to a steady-state by definition.

**PHYSICAL SCALE MODEL AND DESIGN OF A MEASUREMENTS LABORATORY**

Figures (1) and (2), respectively, display the design of a generic physical scale model and a measurements laboratory suitable for the study of IR contrast. The key design features include:

For the Physical Scale Model

1. Geometric similarity of architecture
2. Geometric similarity of material thermal properties \( k(x), \rho(x), c(x) \)
3. Geometric similarity of coatings absorptivity/emissivity \( a(x)/\varepsilon(x) \)
4. Electronically controlled thermal sensing-source-sink nodes per isothermal surface

For the Measurements Laboratory

1. Temperature controlled liquid basin and gaseous atmosphere
2. Wave making machine for scaled waves
3. Rotatable towing bridge, experimental platform, and accessories
Fig. 1. Generic physical scale model.
The design of the physical scale model and measurements laboratory is based upon the scaling laws and dimensionless quantities that transform the heat equation and its boundary conditions to scale invariant form. In the most general time dependent-case, the scaling laws are given by Equations (23) to (26) and the dimensionless quantities by Equations (27) to (31).

Consider first the design of the generic physical scale model; see Figure 1. Geometrical similarity of the scale model and the ship prototype is fundamental. This is enforced by requiring distances to be scaled, so that there is a one-to-one correspondence between geometric points of model and ship. The model is designed so that the dimensionless distance, Equation (27), is the same at analogous points of model and ship.

In summary, in laboratory size the scale model is to be a geometrical replicate, inside and out, of a ship prototype. Thus, it is not adequate to simply model the external surfaces of the ship. Instead, the interior compartmented architecture is replicated as well. Thermal similarity requires that the thermodynamic properties of the building materials, $k$, $\rho$, $c$, and $\epsilon$, shall have the same spatial dependence in model and ship. For example, the conductivity’s functional dependence upon the “position,” $\bar{x}$, shall be the same in model and ship. Similar remarks apply to the density, specific heat, emissivity, heat sources, coefficients of heat transfer, and the temperature differences with the surroundings. The last of these parameters are not solely physical properties of the model, but obviously depend on the environment with which it interacts; the interaction must have the same functional dependence on “position” in model and ship.

In addition to geometrical similarity and thermal similarity, the model is designed in a way that allows the necessary measurements to be made conveniently. A quantity of fundamental importance in IR is the temperature, and it should be measured in various independent ways. A calibrated IR radiometer (or FLIR) is installed in the measurements laboratory for this purpose. In addition, the scale model is lined with numerous experimental nodes that serve as thermal sensing/source/sink stations. In Figure 1, the model is divided into isothermal surfaces, and each number represents an S/S/S station. Each node carries out multiple functions under prearranged computer control. A thermocouple sensing device indicates the temperature. A heating coil serves as a heat source. A Peltier effect junction serves as a heat sink. If the geometrical area of the experimental nodes is sufficiently small compared to the total external area of the model, the perturbation to the IR signal will be negligible, as required. The surface of the model that forms the boundary to the surroundings has a sufficient number of nodes attached so that each approximately isothermal region will be thermally sampled. This also applies to the internal compartments; thus walls, ceilings, and floors are outfitted with S/S/S nodes, as shown in Figure 1. The design of the measurements laboratory (Figure 2) is such that effects due to a ship’s surroundings can be replicated.
Thus the movements of fluids (e.g., water and air) past the ship model surfaces can be controlled experimentally. This could be done by moving the model through stationary fluids or by moving the fluids past a stationary model. There are some experimental advantages in either of the measurements methods. But here a laboratory design choice in which arrangements are made to move the model through the laboratory past the relevant fluids is suggested.

A rotatable bridge for experimentation is suspended over the liquid basin. A towing car that runs on tracks the length of the bridge provides an experimentally chosen velocity to the scale model under tow. Wave making apparatus along two adjacent and orthogonal laboratory walls supply waves of chosen scale and direction. The velocity direction of the towed model is also under computer control since the orientation of the bridge is variable. Wind effects are replicated by suspension of a mechanical wind machine from the towing car. The wind velocity magnitude and direction are measurable. The fluids used in the model basin need not simply be water and air. By using fluids with differing Re, Pr, and Gr numbers, it is possible to vary the heat transfer coefficients $h_c (Re, Pr, Gr)$ to values convenient for a given experiment. For reasons of feasibility, an appropriate location for such a measurement must be carefully planned. The temperature of the fluids may be controlled. Solar heat load effects are replicated by means of a carbon arc light source with a frequency spectrum similar to that of the sun. The "solar source" is suspended from the towing car in a geometric configuration which allows for variable zenith and azimuth angles, measured from the position of the scale model. The intensity of the "solar source" may be varied so that the heat load at the model surface is an experimental variable. A computer workstation in a control room atop the bridge is used to "initialize" each experiment through automated readout and control of the S/S/S nodes, to collect the data of each run, and to analyze the data collected.

**SUMMARY, CONCLUSIONS, AND SUGGESTIONS FOR FURTHER RESEARCH**

The feasibility of studying the IR signal of surface ships by means of measurements upon physical scale models has been shown. This follows from analysis of the heat equation and its boundary conditions which leads to the laws of thermodynamic similitude, i.e., the scaling laws for the IR. The scaling laws have been derived with wide generality, applying to ships with position variable thermal properties, realistic compartmented architecture, general heat loads, convection and radiation heat exchange mechanisms with the surroundings, in transient or steady state conditions. The scaling laws are suggestive of a set of dimensionless variables which render the heat equation into scale invariant form whose descriptions therefore apply to geometrically similar ships and scale models of arbitrary size. True scale modeling is, therefore, seen as an activity based upon experimental arrangements so ordered that precisely the same dimensionless magnitudes apply to ship and model. The validity of scale modeling is ensured when "temperature" measurements, as a function of the dimensionless variables, fall on the same experimental curve for both ship and
scale model. The necessity of equating dimensionless variables in the ship prototype and its scale model replicate leads directly to the design features of both scale model and measurements laboratory. For the scale model, design features include geometric similarity and thermodynamic similarity in accordance with the derived scaling laws. Each important isothermal surface throughout the model is attached to an electronically automated sensing-source-sink thermal node. Each node serves the multiple purpose of detecting the temperature and in initializing procedures, either absorbing or supplying heat, as required. For the measurements laboratory, design features include a controlled-temperature liquid basin and gaseous atmosphere, scaled wave making capacity, solar and wind simulator, a FLIR, an experimental platform, and a model towing capacity. The platform would include a computer workstation to control the automatic collection of measured data and to carry out subsequent data analysis.
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